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Chiral fermions and gauge-fixing in five-dimensional theories

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Abstract

We study in detail the issue of gauge-fixing in theories with one *universal* extra dimension, i.e. theories where both bosons and fermions display Kaluza-Klein (KK) excitations. The extra dimension is compactified using the standard orbifold construction for a massless chiral fermion. We carry out the gauge-fixing procedure at the level of the five-dimensional theory and determine the tree-level propagators and interaction vertices needed for performing perturbative calculations with the effective four-dimensional theory resulting after the compactification. The gauge-independence of the tree-level S-matrix involving massive KK modes is verified using specific examples. In order to obtain massive fermionic zero modes one has to enlarge the theory by introducing a set of mirror fermions, a construction which is carried out in detail. Finally, the gauge-independence of the tree-level S-matrix involving the resulting new mass-eigenstates is proved by resorting to generalized current conservation equations.

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I. INTRODUCTION

Field theories with large extra dimensions have been the focal point of extensive research in recent years. Extra dimensions may or may not be accessible to all known fields, depending on the specifics of the underlying, more fundamental theory. In particular, a plethora of variants have been put forth, where the usual Standard Model fields feel a smaller number of extra dimensions compared to gravity, a fact which is usually enforced by means of standard string-inspired constructions [1–10]. The studies of such theories presented so far have mainly focused on scenarios where the fermions live in the 4-d (no KK modes) and the gauge-bosons in 5-d or higher, displaying KK modes [11]. The phenomenology of such models is rich and has been widely explored in the recent literature (see for instance [12–19]). On the other hand, scenarios where all Standard Model fields live in higher dimensions are less explored [18]. Following Appelquist, Chang, and Dobrescu [20], we will refer to the former type of extra dimension(s) as non-universal, and to the latter type as universal. Models with universal extra dimensions differ from the non-universal ones both theoretically and phenomenologically. From the theoretical point of view, accommodating chiral fermions in five dimensions presents well-known subtleties, and necessitates non-trivial constructions. From the phenomenological point of view, the most characteristic feature of such theories is the conservation of the KK number at each elementary interaction vertex [18,20]. As a result, and contrary to what happens in the non-universal case, the coupling of any excited (massive) KK mode to two zero modes is prohibited. One immediate phenomenological consequence is that the lightest massive gauge-bosons corresponding to the first excited states cannot be resonantly produced using normal (zero-mode) particles as initial sates; instead they must be pair-produced, a fact which could place them beyond the reach of the next generation of colliders. In addition, as has been recently shown, in the framework of theories with universal extra dimension(s) one is able to relax relatively tight phenomenological bounds on the size of the compactification scale obtained in theories with non-universal extra dimensions. For example, the process $Z \to b\bar{b}$ is reported [20] to furnish less stringent a bound in the universal case than in the non-universal one, first considered in [21]; in particular the former yields a 300 GeV bound, to be compared with the 1TeV bound obtained in the latter. Finally, the conservation of the KK number leads to the appearance of heavy stable (charged and neutral) particles, which may pose cosmological complications (e.g. nucleosynthesis) [20].

Given the interest in such theories, one would like to establish a well-defined calculational framework in order to accomplish a detailed quantitative study of their various phenomenological implications. One open issue in this effort, which becomes relevant when attempting to extend formal considerations to actual calculations, is the question of how to carry out correctly the gauge-fixing procedure in the context of such theories. There are at least three different cases where the gauge fixing can be of phenomenological importance. Calculation of a simple S-matrix element of massive fermions (non-conserved currents) involving a gauge boson as an intermediate particle are directly affected by the gauge fixing, already at tree level. Moreover, one of the main features of theories with extra dimensions is the power-law running of the couplings, which in turn offers the phenomenologically interesting possibility of an early unification [22, 23]. In general, when computing the non-Abelian contributions to β functions non-trivial gauge cancellations take place, which guarantee that the resulting expressions are gauge-independent. Therefore, if one wants to compute the entire set of quantum corrections due to both fermionic and bosonic loops, one needs to use the correct propagators and vertices, in order not to distort the aforementioned cancellations. The gauge-fixing procedure used here as well as most of the conclusions regarding the form of the tree-level propagators carry over to a non-Abelian context, augmented by minor modifications when elementary scalars with tree-level vacuum expectation values are included. In addition, when computing the effective potential traditionally one chooses a Landau type of gauge, in order to eliminate a certain class of diagrams [24]. Therefore, the problem of defining the Landau gauge in models with extra dimensions is a relevant one.

In this paper we study in detail the following points in the context of an Abelian gauge theory with one extra universal dimension:

We carry out the gauge-fixing both *before* and *after* the compactification of the single extra universal dimension. We find that if the (covariant) gauge-fixing is carried out before the compactification the Landau gauge cannot be defined and that the unitary gauge cannot be reached continuously from this type of gauges. On the other hand, if the gauge fixing is carried out after the compactification one arrives at a Standard Model-like result for the various propagators; in particular both the unitary and the Landau gauges can be reached continuously. Notice in addition that (a) both constructions give rise to the same propagators in the Feynman gauge, and (b) the unitary gauge may be obtained directly before compactification by means of a non-covariant, axial-gauge type of gauge fixing [22].

Throughout this paper we have not employed any formal argument which guarantees that one arrives at the same answer for physical observables regardless of whether one carries out the gauge fixing before or after the compactification, especially in the presence of interactions. Given the importance of this issue we have instead resorted to the study of explicit examples in the context of an interacting theory, in order to establish whether at least some of the typical features, known to be true for standard theories, persist. In order to accomplish that, we have carried out in detail the construction necessary for defining fermions in 5 dimensions. We have adopted the orbifold compactification, which is necessary in order to eliminate from the spectrum the massless zero mode corresponding to the fifth component of the gauge field, which is phenomenologically bothersome. If one adopts the orbifold construction a chiral structure is introduced to a theory which is vector-like at the level of the 5-d Lagrangian. Thus, after compactification the fermionic zero modes are chiral and massless. On the other hand, the fermionic KK modes come in chiral pairs and can be combined to form Dirac fermions with masses which are integer multiples of the compactification scale. The coupling of the KK modes to the bosons (vector and scalar) display the most general Lorentz structure in four dimensions. In this paper we do not consider problems related with chiral anomalies and assume that they cancel by adding the fermions with the right quantum numbers or the relevant Chern-Simons terms [25]. Notice that any realistic model should reduce, at low energies, to the standard model in which chiral anomalies are canceled by a very particular choice of quantum numbers.

Using the boson propagators derived earlier we study tree-level S-matrix elements involving the massive KK fermions as external particles. Due to the KK conservation reflected in the elementary vertices, together with the particular mass spectrum mentioned above, we have shown that one arrives at the same answer for the S-matrix elements considered, regardless of when the gauge-fixing was carried out.

If one wants to obtain normal QED at low energies it is necessary to have massive Dirac fermions as zero fermionic modes. However, the exercise of endowing the zero modes with mass is rather subtle. If one was to assume that the Abelian theory we consider will be eventually embedded into a larger group, like the $SU(2) \otimes U(1)$, one would expect to give masses to the zero modes by means of the standard Yukawa coupling of the fermions to a scalar field, which develops a non-zero expectation value through the usual Higgs mechanism. This standard construction however appears to be more involved in the presence of an orbifold-type of compactification, giving rise to a subtle interplay of various field-theoretical mechanisms [26]. In addition, from the theoretical point of view, one would like to be able to deal with QED as if it were a self-contained theory without having to introduce elementary scalars in the spectrum. Therefore, in the simple Abelian case we will adopt the construction whereby the task of giving masses to the fermionic zero modes is accomplished through the introduction of a set of mirror fermions, with opposite chirality properties. The proper definition of mass-eigenstates after the zero modes have acquired masses requires a re-diagonalization of the Lagrangian, a fact which in turn complicates the proof of the gauge-cancellations when computing simple tree-level processes. The demonstration of these cancellations, once accomplished, furnishes an additional non-trivial check for the robustness of both the gauge-fixing procedure and the orbifold construction. We have indeed shown that these cancellations take place by virtue of general expressions reflecting current (non)conservation.

The paper is organized as follows: In section II we present the gauge fixing, which is carried out both before and after the orbifold compactification. In section III we add interactions, complete the orbifold construction for fermions, and derive the elementary interaction vertices. In section IV we demonstrate with some simple examples the gauge-invariance of the S-matrix, and that the two gauge-fixing procedures are equivalent, at least at this basic level. In section V we enlarge the spectrum in order to give masses to the fermionic zero modes in a way compatible with the orbifold symmetry. We also check that the gauge symmetry of the S-matrix is preserved in the extended theory. Finally, in section VI we present our conclusions.

II. GAUGE-FIXING AND TREE-LEVEL PROPAGATORS

In this section we will carry out the gauge-fixing in detail and derive the expressions for the various propagators. First we will start from a five-dimensional Lagrangian which is already gauge-fixed before compactification, and we will derive the propagators obtained after the compactification of the fifth dimension on an orbifold, following two different but equivalent procedures. Then we will derive the propagators starting from a five-dimensional Lagrangian which is not gauged fixed before the compactification, and will carry out the gauge fixing on the four-dimensional Lagrangian obtained after the compactification.

A. The gauge-fixing before the compactification

We will use a notation in which $M, N, \cdots = 0, 1, 2, 3, 4$ denote five-dimensional indices and $\mu, \nu, \cdots = 0, 1, 2, 3$ are four-dimensional indices. The metric g^{MN} is the standard five-dimensional Lorentz metric with spatial signature for the extra dimension. We will denote the extra spatial coordinate as y and will write $x^M = (x^\mu, y)$ (so $x^4 = -x_4 = y$). On the other hand, for the gauge fields we will write $A_M(x,y) = (A_\mu(x,y), A_5(x,y))$, where x denotes collectively the four-dimensional coordinates.

We start out with the free part $\mathcal{L}_0$ of the QED_5 Lagrangian and add a covariant gauge fixing term $\mathcal{L}_{gf}$. With the notation introduced above we write the five-dimensional action as

$$S^{(5)} = \int d^4x dy (\mathcal{L}_0 + \mathcal{L}_{gf}) \ , \tag{2.1}$$

with

$$\begin{aligned}\mathcal{L}_0 + \mathcal{L}_{gf} &= -\frac{1}{4}(\partial_M A^N - \partial_N A^M)^2 - \frac{1}{2a}(\partial_M A^M)^2 \\ &= \frac{1}{2} A^M \left[\Box_5 g_{MN} - \left(1 - \frac{1}{a}\right)\partial_M \partial_N \right] A^N \, ,\end{aligned} \tag{2.2}$$

where in the second line we have carried out partial integrations, and $\Box_5 = \Box_4 - \partial_y^2$ with $\Box_4 = \partial_\mu \partial^\mu$. Next assume that y is compactified on a circle of radius R with the points y and $-y$ identified, i.e. on an orbifold $S^1/\mathbb{Z}_2$. In general, fields even under the $\mathbb{Z}_2$ symmetry will have zero modes which will be present in the low-energy theory, whereas fields odd under $\mathbb{Z}_2$ will only have KK modes, and their zero modes will disappear. For the case at hand, we have that A_μ transforms like ∂_μ (even under $\mathbb{Z}_2$), whereas A_5 transforms like ∂_y (odd under $\mathbb{Z}_2$). Therefore, the Fourier expansion of the fields has the form

$$\begin{aligned}A_\mu(x,y) &= \frac{1}{\sqrt{L}}\left[A_\mu^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty} A_\mu^{(n)}(x)\cos(m_n y)\right] \\ A_5(x,y) &= \sqrt{\frac{2}{L}}\sum_{n=1}^{\infty} A_5^{(n)}(x)\sin(m_n y) \ ,\end{aligned} \tag{2.3}$$

with

$$m_n = \frac{n}{R} \qquad \text{and} \qquad L = \pi R \, . \tag{2.4}$$

The above normalization of the Fourier expansion results in the canonical form for the kinetic terms. Following the standard KK construction, we arrive at

$$S^{(4)} = \int d^4x \sum_{n=0}^{\infty} \left(\mathcal{L}_V^{(n)} + \mathcal{L}_S^{(n)} + \mathcal{L}_M^{(n)} \right) \ , \tag{2.5}$$

with

$$\begin{aligned}\mathcal{L}_V^{(n)} &= \frac{1}{2} A_\mu^{(n)} \left[(\Box_4 + m_n^2) g^{\mu\nu} - \left(1 - \frac{1}{a}\right)\partial^\mu \partial^\nu \right] A_\nu^{(n)} \\ \mathcal{L}_S^{(n)} &= -\frac{1}{2} A_5^{(n)} \left[\Box_4 + \frac{1}{a} m_n^2 \right] A_5^{(n)} \\ \mathcal{L}_M^{(n)} &= -\frac{1}{2}\left(1 - \frac{1}{a}\right) m_n \left[A_\mu^{(n)} \partial^\mu A_5^{(n)} - A_5^{(n)} \partial^\mu A_\mu^{(n)} \right] \, ,\end{aligned} \tag{2.6}$$

where the subscripts stand for vector (V), scalar (S), and mixed (M). Clearly, $\mathcal{L}_M^{(0)} = 0$, and $\mathcal{L}_S^{(0)} = 0$ because $A_5^{(0)} = 0$. Notice moreover that the mass term for the $A_5^{(n)}$ is multiplied by the inverse of the gauge-fixing parameter, and that the mixing term vanishes in the Feynman gauge, $a = 1$.

In order to determine the propagators one must re-diagonalize the Lagrangian. The quadratic part given above may be cast in the form

$$\left(A_\mu^{(n)} A_5^{(n)} \right) \left(\begin{array}{cc} (\Box_4 + m_n^2) g^{\mu\nu} - (1-1/a)\partial^\mu \partial^\nu & -(1-1/a) m_n \partial^\mu \\ (1-1/a) m_n \partial^\nu & -\Box_4 - m_n^2/a \end{array} \right) \left(\begin{array}{c} A_\nu^{(n)} \\ A_5^{(n)} \end{array} \right) . \tag{2.7}$$

In the momentum space the 2×2 matrix given above, to be denoted by D_n, assumes the form

$$D_n = \left(\begin{array}{cc} (m_n^2 - q^2) g^{\mu\nu} + (1-1/a) q^\mu q^\nu & i(1-1/a) m_n q^\mu \\ -i(1-1/a) m_n q^\nu & q^2 - m_n^2/a \end{array} \right) . \tag{2.8}$$

Next we must find the inverse of the matrix; using the condition $D_n D_n^{-1} = 1$, and using the parametrization

$$D_n^{-1} = \left(\begin{array}{cc} \Delta_{\mu\nu}^{(n)}(q) & \Delta_\mu^{(n)}(q) \\ -\Delta_\nu^{(n)}(q) & \Delta^{(n)}(q) \end{array} \right) \tag{2.9}$$

we find

$$\begin{aligned} i\Delta_{\mu\nu}^{(n)}(q) &= \frac{-i}{q^2 - m_n^2} \left[g_{\mu\nu} - \frac{(1-a) q_\mu q_\nu}{q^2 - m_n^2} \right] \\ i\Delta_\mu^{(n)}(q) &= \frac{m_n (1-a)}{(q^2 - m_n^2)^2} q_\mu \\ i\Delta^{(n)}(q) &= i \left[\frac{1}{q^2 - m_n^2} + \frac{m_n^2 (1-a)}{(q^2 - m_n^2)^2} \right] , \end{aligned} \tag{2.10}$$

shown in Fig. 1a – Fig. 1c . Notice that there is no value for a such that one would recover a unitary gauge type of propagator. In particular, the choice ($a = 0$) does not decouple the scalar sector and does not lead to the unitary gauge type of spectrum. In addition, as is evident from the first expression in Eq. (2.10) the Landau gauge, i.e. a gauge where $\Delta_{\mu\nu}^{(n)}(q)$ is proportional to the usual transverse structure $(g_{\mu\nu} - q_\mu q_\nu / q^2)$ cannot be defined. Instead, as we will see in the last sub-section, both the unitary gauge as well as the Landau gauge may be reached if the gauge fixing is performed at the level of the four-dimensional Lagrangian obtained after carrying out the compactification of the extra dimension.

B. The Dyson summation

Next we will show that one arrives at exactly the expressions for the propagators given in Eq. (2.10) if one treats the mixing term as an interaction, and carries out the Dyson summation.

To that end let us invert naively the quadratic parts of $A_5^{(n)}$ and $A_5^{(n)}$ and obtain the tree-level propagators $d_{\mu\nu}^{(n)}(q)$ and $d^{(n)}(q)$, respectively, given by [24] (Fig. 2a and Fig. 2b)

$$
\begin{aligned}
id_{\mu\nu}^{(n)}(q) &= \frac{-i}{q^2 - m_n^2}\left[g_{\mu\nu} - \frac{(1-a)q_\mu q_\nu}{q^2 - am_n^2}\right] \\
id^{(n)}(q) &= \frac{i}{q^2 - m_n^2/a} \ .
\end{aligned}
\tag{2.11}
$$

The mixing interaction is given by the vertex (the momentum flow convention is shown in Fig. 2c)

$$
\mathcal{V}_n^\mu = \left(1 - \frac{1}{a}\right) m_n q^\mu \ . \tag{2.12}
$$

The basic quantity appearing in the Dyson sum is (no sum over n)

$$
\begin{aligned}
K_n &= d^{(n)} \mathcal{V}_n^\mu \mathcal{V}_n^\nu d_{\mu\nu}^{(n)} \\
&= -a\left(1 - \frac{1}{a}\right)^2 q^2 m_n^2 \left[\left(q^2 - \frac{m_n^2}{a}\right)\left(q^2 - am_n^2\right)\right]^{-1} \ .
\end{aligned}
\tag{2.13}
$$

Then $\Delta_{\mu\nu}^{(n)}(q)$ is given by (Fig. 2d)

$$
\begin{aligned}
i\Delta_{\mu\nu}^{(n)} &= id_{\mu\nu}^{(n)} + id^{(n)}\left(\mathcal{V}_n^\rho d_{\mu\rho}^{(n)}\right)\left(\mathcal{V}_n^\sigma d_{\nu\sigma}^{(n)}\right)\left[1 + \sum_{\ell=1}^{\infty}(K_n)^\ell\right] \\
&= id_{\mu\nu}^{(n)} + \left(\frac{i(1-a)^2 m_n^2}{(q^2 - m_n^2/a)(q^2 - am_n^2)^2}\right)\left[\frac{1}{1 - K_n}\right] q_\mu q_\nu \\
&= id_{\mu\nu}^{(n)} + \frac{i(1-a)^2 m_n^2}{(q^2 - m_n^2)^2 (q^2 - am_n^2)} q_\mu q_\nu \ .
\end{aligned}
\tag{2.14}
$$

Similarly, for $\Delta^{(n)}(q)$ (Fig. 2e) we have

$$
i\Delta^{(n)} = id^{(n)} + id^{(n)} K_n \left[1 + \sum_{\ell=1}^{\infty}(K_n)^\ell\right] = \frac{id^{(n)}}{1 - K_n} = id^{(n)} \frac{(q^2 - am_n^2)(q^2 - m_n^2/a)}{(q^2 - m_n^2)^2} \ . \tag{2.15}
$$

It is elementary to verify, by using the expressions for $d_{\mu\nu}^{(n)}$ and $d^{(n)}$ given in Eq. (2.11), that the right-hand sides of Eq. (2.14) and Eq. (2.15) indeed reduce to those reported to the expressions for $\Delta_{\mu\nu}^{(n)}$ and $\Delta^{(n)}$, respectively, given in Eq. (2.10). The mixing term $\Delta_\mu^{(n)}$ can be obtained in a similar fashion.

C. The Gauge Fixing after the compactification

In this subsection we will carry out the KK construction first, and then we will do the gauge-fixing directly in the 4-d theory. The result of compactifying the y before adding a gauge fixing term may be worked out straightforwardly, or equivalently gleaned off directly from Eq. (2.6) by setting $a \to \infty$. We repeat that this limit does not amount to the unitary gauge; in particular, due to the mixing term which does not vanish, we do not arrive at the Proca Lagrangian for the n-th massive gauge boson. Instead we have

$$\begin{aligned}
\widehat{\mathcal{L}}_{V}^{(n)} &= \frac{1}{2} A_{\mu}^{(n)} \left[(\Box_4 + m_n^2) g^{\mu\nu} - \partial^{\mu}\partial^{\nu} \right] A_{\nu}^{(n)} \\
\widehat{\mathcal{L}}_{S}^{(n)} &= -\frac{1}{2} A_{5}^{(n)} \Box_4 A_{5}^{(n)} \\
\widehat{\mathcal{L}}_{M}^{(n)} &= m_n A_{5}^{(n)} \partial^{\mu} A_{\mu}^{(n)} \ .
\end{aligned} \tag{2.16}$$

Then we add a gauge-fixing term which corresponds to the generalization of the usual R_ξ gauge-fixing, used in the electroweak sector of the Standard Model,

$$\widehat{\mathcal{L}}_{R}^{(n)} = -\frac{1}{2\widehat{a}_n} \left(\partial^{\mu} A_{\mu}^{(n)} + \widehat{a}_n m_n A_{5}^{(n)} \right)^2 \ . \tag{2.17}$$

Notice that we introduce an infinity of arbitrary gauge fixing parameters $\widehat{a}_n$; thus one can carry out the gauge-fixing independently for every single gauge boson, exactly as in the Standard Model case, when one may choose three completely independent gauge-fixing parameters ξ_γ, ξ_Z, and ξ_W for the photon, the Z-boson, and the W-boson, respectively. By construction this gauge fixing removes the tree-level mixing and gives rise to the following two propagators

$$\begin{aligned}
\widehat{\Delta}_{\mu\nu}^{(n)}(q) &= \frac{-i}{q^2 - m_n^2} \left[g_{\mu\nu} - \frac{(1-\widehat{a}_n) q_\mu q_\nu}{(q^2 - \widehat{a}_n m_n^2)} \right] \\
\widehat{\Delta}^{(n)}(q) &= \frac{i}{q^2 - \widehat{a}_n m_n^2} \ .
\end{aligned} \tag{2.18}$$

Clearly, in the Feynman gauge ($\widehat{a}_n = 1$) we recover the same propagators as in the Feynman gauge of the previous subsection, $a = 1$. But in addition, now the Landau gauge ($\widehat{a}_n = 0$) gives indeed a transverse propagator for the massive gauge boson, and a massless scalar propagator, the analogue of a massless would-be Goldstone boson. In addition, in the limit $\widehat{a}_n \to \infty$ one recovers the standard unitary propagator, i.e.

$$U_{\mu\nu}^{(n)}(q) = \frac{-i}{q^2 - m_n^2} \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{m_n^2} \right] \ . \tag{2.19}$$

Notice that, as in the standard model

$$U_{\mu\nu}^{(n)}(q) = \widehat{\Delta}_{\mu\nu}^{(n)}(q, \widehat{a}_n = 0) + \widehat{\Delta}^{(n)}(q, \widehat{a}_n = 0) \frac{q_\mu q_\nu}{m_n^2} \ . \tag{2.20}$$

Finally, it is known [22] that one may reach directly the unitary gauge by resorting to the following field redefinition

$$A_\mu^{(n)} \to A_\mu^{(n)} + \partial_\mu \theta^{(n)}$$
$$A_5^{(n)} \to A_5^{(n)} - m_n \theta^{(n)} \ , \tag{2.21}$$

which is, at the same time, a non-linear gauge transformation with $\theta^{(n)}$ the gauge transformation parameter and leaves the Lagrangian invariant. The choice $\theta^{(n)} = m_n^{-1} A_5^{(n)}$ eliminates the scalar component completely, leading to the unitary propagator of Eq. (2.19) for all massive vector bosons.

Given the important differences in the form of the propagators derived following the two gauge-fixing procedures presented above, it is important to verify whether one arrives at the same answer for physical observables calculated within either scheme. Even though no formal proof to that effect will be offered here, the next sections are devoted to the study of the above question in the context of specific examples. To accomplish that within a well-defined framework we will introduce an Abelian interaction at the level of the five-dimensional Lagrangian, allowing photons to interact with fermions.

III. THE CHIRAL FERMIONS

The orbifold compactification adopted in the previous section allows one to remove from the spectrum the unwanted massless scalar corresponding to the zero mode of the fifth component $A_5^{(0)}$ of the photon. As explained in [26] this introduces a chiral structure which appears at the level of the effective four-dimensional theory, even though the five-dimensional Lagrangian one starts with is vector-like. In this section we will carry out this construction in detail, not only for the kinetic term, but also for the interaction term. The resulting interactions involve four different types of vertices, whose Lorentz structure corresponds to vector, axial, scalar and pseudo-scalar. We start with the Lagrangian

$$\begin{aligned}\mathcal{L}_\psi(x) &= \int_0^L dy \bar{\psi}(x,y)\gamma^M \left(i\partial_M + e_5 A_M\right)\psi(x,y) \\ &= \int_0^L dy \bar{\psi}(x,y)\left(i\not{\partial} - \gamma_5 \partial_y + e_5 \not{A}(x,y) + ie_5\gamma_5 A_5(x,y)\right)\psi(x,y) \ ,\end{aligned} \tag{3.1}$$

with $\gamma^{M=0,1,2,3} = \gamma^\mu$ and $\gamma^4 = i\gamma_5$, where γ_5 is defined as usual as $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. We also used the standard notation $\not{\partial} = \gamma^\mu \partial_\mu$ and $\not{A} = \gamma^\mu A_\mu$. Finally, e_5 denotes the five-dimensional gauge coupling.

It is elementary to verify that, in the absence of a mass term, $\mathcal{L}_\psi$ is invariant under the transformation

$$\psi(x,y) \to e^{i\alpha}\gamma_5 \psi(x, C-y) \ , \quad A_\mu(x,y) \to A_\mu(x, C-y) \ , \quad A_5(x,y) \to -A_5(x, C-y) \tag{3.2}$$

where α is arbitrary and $C = L$ in order to map the interval $y \in [0, L = \pi R]$ into itself. If we require that

$$\psi \to \psi' \to \psi'' = \psi \ , \tag{3.3}$$

we have that $e^{i\alpha} = \pm 1$. Next we should impose boundary conditions, i.e. periodicity properties outside the interval $[0, L]$. One can impose

$$\psi(x, y) = \psi'(x, L + y) = \pm\gamma_5 \psi(x, -y) \tag{3.4}$$

for every y (inside or outside the interval $[0, L]$). Next one requires that the above relation be satisfied also for the transformed fields, i.e.

$$\pm\,\gamma_5 \psi(x, L - y) = \pm\gamma_5(\pm\gamma_5 \psi(x, L + y) = \psi(x, L + y)\ . \tag{3.5}$$

Since this last equation is satisfied for every y, it is also satisfied for $y = L + y$, from which follows that

$$\psi(x, y) = \psi(x, y + 2L)\ . \tag{3.6}$$

Then we carry out the Fourier expansion for the ψ field in the interval $[-L, L]$ (or equivalently expand the even and odd components in Fourier-cosine and Fourier-sine series, respectively, in the interval $[0, L]$), i.e.

$$\psi(x, y) = \frac{1}{\sqrt{L}}\left[\psi_R^{(0)} + \sqrt{2}\sum_{n=1}^{\infty}\left(\psi_R^{(n)}(x)\cos(m_n y) + \psi_L^{(n)}(x)\sin(m_n y)\right)\right]\ . \tag{3.7}$$

Imposing the boundary condition of Eq. (3.4) with the + sign, we see that

$$\gamma_5 \psi_R^{(n)}(x) = \psi_R^{(n)}(x),\ \ \gamma_5 \psi_L^{(n)}(x) = -\psi_L^{(n)}(x)\ , \tag{3.8}$$

i.e. $\psi_R^{(n)}(x)$ and $\psi_L^{(n)}(x)$ can be identified with the chiral right-handed and left-handed components, respectively.

Next we carry out the KK construction and we see that all the modes with $n > 0$ pick up a mass, whereas the zero mode ($n = 0$) remains massless, i.e.

$$\mathcal{L}_{\psi,Q} = \bar{\psi}_R^{(0)}(i\gamma_\mu\partial^\mu)\psi_R^{(0)} + \sum_{n=1}^{\infty}\bar{\psi}^{(n)}(i\gamma_\mu\partial^\mu + m_n)\psi^{(n)}\ , \tag{3.9}$$

where $\psi^{(n)} = \psi_R^{(n)} + \psi_L^{(n)}$

Using the Fourier decompositions given above and the auxiliary formulas (notice that since the five-dimensional Lagrangian is even under a reflection in the fifth component we can change $\int_0^L dy \to 1/2\int_{-L}^L dy$)

$$\begin{aligned}
&\frac{2}{L}\int_{-L}^{L} dy\cos(m_i y)\cos(m_j y)\cos(m_k y) = \delta_{i,j+k} + \delta_{k,i+j} + \delta_{j,i+k}\\
&\frac{2}{L}\int_{-L}^{L} dy\sin(m_i y)\sin(m_j y)\cos(m_k y) = \delta_{i,j+k} - \delta_{k,i+j} + \delta_{j,i+k}\ ,
\end{aligned} \tag{3.10}$$

where $i, j, k \geq 1$, we obtain the following expressions for the interacting part $\mathcal{L}_{\psi,I}$ of the Lagrangian

$$\mathcal{L}_{\psi,I} = \mathcal{L}_{\psi,I}^{(0)} + \mathcal{L}_{\psi,I}^{(0K)} + \mathcal{L}_{\psi,I}^{(K)}\ , \tag{3.11}$$

with

$$\mathcal{L}^{(0)}_{\psi,I}(x) = e\bar{\psi}^{(0)}_R \not{A}^{(0)}\psi^{(0)}_R$$
$$\mathcal{L}^{(0K)}_{\psi,I}(x) = e\sum_{n=1}\bar{\psi}^{(n)}\not{A}^{(0)}\psi^{(n)} + e\sum_{n=1}\left[\left(\bar{\psi}^{(0)}_R\not{A}^{(n)}\psi^{(n)}_R + \text{h.c.}\right) + \left(-i\bar{\psi}^{(0)}_R A^{(n)}_5\psi^{(n)}_L + \text{h.c.}\right)\right]$$
$$\mathcal{L}^{(K)}_{\psi,I}(x) = \frac{e}{\sqrt{2}}\sum_{m,n}\left(\bar{\psi}^{(n+m)}\left(\not{A}^{(m)} - iA^{(m)}_5\right)\psi^{(n)} + \text{h.c.}\right)$$
$$+ \frac{e}{\sqrt{2}}\sum_{m,n}\bar{\psi}^{(m)}\left(\not{A}^{(n+m)} + iA^{(n+m)}_5\right)\gamma_5\psi^{(n)}\,, \tag{3.12}$$

where we have rewritten the five-dimensional coupling, e_5 in terms of the four-dimensional coupling e as $e \to e_5\sqrt{L}$.

As we can see from the above Lagrangian, there are four types of interaction vertices, shown in Fig. 3, with different Lorentz structures: Γ^μ_V is vector, Γ^μ_A is axial, Γ_S is scalar, and Γ_P pseudo-scalar. Notice the conservation of the KK number at each elementary vertex. One direct field-theoretical consequence of this conservation, in addition to those already mentioned in the introduction, is that there is no higher order mixing between the different bosonic KK modes. Therefore the gauge boson mass-eigenstates derived in the previous section do not get modified by quantum corrections.

IV. THE GAUGE INVARIANCE OF THE TREE-LEVEL S-MATRIX

In the last two sections we have derived the tree-level expressions for the vector bosons and scalar fields appearing in the Lagrangian, as well as their interaction vertices with the fermions. As a basic application and a useful self-consistency check we will next demonstrate explicitly the gauge invariance (independence of the gauge-fixing parameter a) of the tree-level S-matrix . In addition, we will show that one arrives at the same expressions for the S-matrix regardless of whether one uses the expressions for the propagators obtained before or after the compactification (viz. Eq. (2.10) and Eq. (2.19)) We will show how the cancellations proceed using special examples and carrying out the explicit calculation; a more formal proof will be presented in section V. We will study two different types of scattering processes, one that involves neutral fermions as external particles, and one that involves charged ones.

Let us first do the neutral case: The tree-level scattering amplitude $\mathcal{S}_\mathcal{N}$ for the process $\bar{\psi}^{(n)}\psi^{(n)} \to \bar{\psi}^{(n)}\psi^{(n)}$ involving the n-th KK fermion of mass m_n is given by (Fig. 4) (we consider only s-channel amplitudes; the argument for t-channel amplitudes is identical)

$$\mathcal{S}_\mathcal{N} = \Gamma^{\mu\,(2n)}_A\Delta^{(n)}_{\mu\nu}(q)\Gamma^{\nu\,(2n)}_A + \Gamma^{\mu\,(2n)}_A\Delta^{(2n)}_\mu(q)\Gamma^{(2n)}_P + \Gamma^{(2n)}_P\Delta^{(2n)}_\nu(q)\Gamma^{\nu\,(2n)}_A + \Gamma^{(2n)}_P\Delta^{(2n)}(q)\Gamma^{(2n)}_P\,, \tag{4.1}$$

where the elementary vertices Γ_A, etc are given in Fig. 3. Using that

$$q_\mu\Gamma^{\mu\,(2n)}_A = -2im_n\Gamma^{(2n)}_P \tag{4.2}$$

we see that the condition for the cancellation of the terms with the double poles, proportional to $(1-a)$, is simply

$$4m_n^2 - 4m_n m_{2n} + m_{2n}^2 = 0\,, \tag{4.3}$$

which is automatically satisfied by virtue of Eq. (2.4). Notice that the omission of the mixing terms would result in a residual dependence of the S matrix on a. After the double poles have canceled, the remaining contribution is effectively given by fixing $a = 1$. It is elementary to verify that the result obtained in that case is identical to the one computed by using the unitary-type of gauge given in Eq. (2.19), derived when the gauge fixing is carried out after the compactification; indeed, the two answers coincide provided that $(2m_n)^2/m_{2n}^2 = 1$, which is the same condition given in Eq. (4.3), and holds for every n .

Turning to the charged case, it is straightforward to see that again due to the special mass relations the dependence on a cancels, and the final answer coincides with the Feynman and unitary gauges. For example, the amplitude $\mathcal{S}_{\mathcal{C}}$ for the process $\bar{\psi}^{(\ell)}\psi^{(i+j)} \to \bar{\psi}^{(i)}\psi^{(\ell+j)}$ is given by

$$\mathcal{S}_{\mathcal{C}} = \Gamma_V^{\mu\,(j)}\Delta_{\mu\nu}^{(j)}(q)\Gamma_V^{\nu\,(j)} + \Gamma_A^{\mu\,(j)}\Delta_{\mu}^{(j)}(q)\Gamma_S^{(j)} + \Gamma_S^{(j)}\Delta_{\nu}^{(j)}(q)\Gamma_V^{\nu\,(j)} + \Gamma_S^{(j)}\Delta^{(j)}(q)\Gamma_S^{(j)} \ . \tag{4.4}$$

Using that

$$q_\mu \Gamma_V^{\mu\,(j)} = i(m_{i+j} - m_i)\Gamma_S^{(j)} \ , \tag{4.5}$$

we find that the condition for the gauge cancellation reads

$$(m_{i+j} - m_i)^2 - m_j(m_{i+j} - m_i) - m_j(m_{\ell+j} - m_\ell) + m_j^2 = 0 \ , \tag{4.6}$$

which is again automatically satisfied, since $m_{i+j} - m_i = m_j$.

V. GIVING MASS TO THE ZERO MODES

In order to recover conventional QED with massive electrons from the 5-d construction presented in the previous section, we must give mass to the fermionic zero modes. In this section we will carry out the construction which gives masses to the zero modes. The presence of a new mass-term leads to the need of redefining the mass-eigenstates of the theory, which in turn alters the form of the interaction term.

The construction we will adopt for giving mass to the fermionic zero modes proceeds as follows (for an alternative construction see [27]). We introduce an additional set of fermions, to be denoted by χ, which have the opposite chirality properties compared to the ψ. Specifically, add to the Lagrangian density $\mathcal{L}_\psi$ of Eq. (3.1) the Lagrangian density $\mathcal{L}_\chi$ of a new fermionic field χ, obtained from $\mathcal{L}_\psi$ by simply changing $\psi \to \chi$, and impose the following transformation properties on the ψ and χ fields

$$\begin{aligned} \psi(x,y) \to \psi' &= \gamma_5\psi(x, L-y) \\ \chi(x,y) \to \chi' &= -\gamma_5\psi(x, L-y) \ . \end{aligned} \tag{5.1}$$

Then, a mass-term is allowed, which does not violate the chiral symmetry,

$$\mathcal{L}_{\text{mass}} = -\int_0^L dy m_0 \left(\bar{\psi}(x,y)\chi(x,y) + \bar{\chi}(x,y)\psi(x,y)\right) \tag{5.2}$$

giving rise to the final Lagrangian

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{mass} \ . \tag{5.3}$$

The fact that χ has the opposite chiral transformation property than ψ results in the following Fourier expansion

$$\chi(x,y) = \frac{1}{\sqrt{L}}\left[\chi_L^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty}\left(\chi_L^{(n)}(x)\cos(m_n y) + \chi_R^{(n)}(x)\sin(m_n y)\right)\right] , \tag{5.4}$$

and after the standard KK construction we find for the quadratic part of the Lagrangian

$$\mathcal{L}_Q(x) = \bar{f}^{(0)}\left(i\partial\!\!\!/ - m_0\right) f^{(0)} + \sum_{n=1} \bar{\xi}^{(n)} i\partial\!\!\!/ \xi^{(n)} - \bar{\xi}^{(n)} M_n \xi^{(n)} , \tag{5.5}$$

where

$$f^{(0)} = \psi_R^{(0)} + \chi_L^{(0)}, \qquad \xi^{(n)} = \begin{pmatrix} \psi_L^{(n)} + \psi_R^{(n)} \\ \chi_L^{(n)} + \chi_R^{(n)} \end{pmatrix}, \qquad M_n = \begin{pmatrix} -m_n & m_0 \\ m_0 & m_n \end{pmatrix} . \tag{5.6}$$

The interaction terms can be separated as before into three pieces, one containing only zero modes, one containing a mixture of zero modes and KK modes, and one containing only KK modes, i.e.

$$\mathcal{L}_I = \mathcal{L}_I^{(0)} + \mathcal{L}_I^{(0K)} + \mathcal{L}_I^{(K)} , \tag{5.7}$$

with

$$\begin{aligned}
\mathcal{L}_I^{(0)}(x) &= e\bar{f}^{(0)} A\!\!\!/^{(0)} f^{(0)} \\
\mathcal{L}_I^{(0K)}(x) &= e\sum_{n=1} \bar{\xi}^{(n)} A\!\!\!/^{(0)} \xi^{(n)} \\
&\quad + e\sum_{n=1}\left[\left(\bar{f}^{(0)} A\!\!\!/^{(n)}\left(\psi_R^{(n)} + \chi_L^{(n)}\right) + \text{h.c.}\right) + \left(-i\bar{f}^{(0)} A_5^{(n)}\left(\psi_L^{(n)} - \chi_R^{(n)}\right) + \text{h.c.}\right)\right] \\
\mathcal{L}_I^{(K)}(x) &= \frac{e}{\sqrt{2}}\sum_{m,n}\left(\bar{\xi}^{(n+m)}\left(A\!\!\!/^{(m)} - i\sigma_3 A_5^{(m)}\right)\xi^{(n)} + \text{h.c.}\right) \\
&\quad + \frac{e}{\sqrt{2}}\sum_{m,n} \bar{\xi}^{(m)}\left(A\!\!\!/^{(n+m)}\sigma_3 + iA_5^{(n+m)}\right)\gamma_5 \xi^{(n)} ,
\end{aligned} \tag{5.8}$$

where σ_3 is the Pauli matrix and acts on the fields $\xi^{(n)}$. Next one should rewrite the interaction Lagrangian in terms of the mass eigenstates; this may be done in different ways. The mass matrix M_n is real and symmetric, so it can be diagonalized by an orthogonal transformation. Since the corresponding eigenvalues are $\pm\lambda_n \equiv \pm\sqrt{m_n^2 + m_0^2}$ we have

$$U_n^\dagger M_n U_n = -\lambda_n \sigma_3, \qquad U_n = \begin{pmatrix} c_n & s_n \\ -s_n & c_n \end{pmatrix}, \qquad c_n = \sqrt{\frac{\lambda_n + m_n}{2\lambda_n}}, \qquad s_n = \sqrt{\frac{\lambda_n - m_n}{2\lambda_n}} , \tag{5.9}$$

which obviously satisfy the relations

$$c_n^2 + s_n^2 = 1, \qquad c_n^2 - s_n^2 = \frac{m_n}{\lambda_n}, \qquad 2c_n s_n = \frac{m_0}{\lambda_n}\,. \tag{5.10}$$

Then we can use the matrix U_n to diagonalize the quadratic term by defining a new set of fields $\xi^{(n)}$ and $f^{(n)}$ as follows

$$\xi^{(n)} = U_n f^{(n)}, \qquad f^{(n)} = \begin{pmatrix} f_1^{(n)} \\ f_2^{(n)} \end{pmatrix}. \tag{5.11}$$

In terms of these fields the quadratic term is rewritten as

$$\mathcal{L}_{Q}(x) = \bar{f}^{(0)}\left(i\partial\!\!\!/ - m_0\right) f^{(0)} + \sum_{n=1} \bar{f}^{(n)} i\partial\!\!\!/ f^{(n)} + \lambda_n \bar{f}^{(n)}\sigma_3 f^{(n)}\,. \tag{5.12}$$

Notice that the fields $f_2^{(n)}$ have mass λ_n while the fields $f_1^{(n)}$ have mass $-\lambda_n$. This sign can be reversed by further redefining $f_1^{(n)} \to \gamma_5 f_1^{(n)}$; in that case we would obtain two completely degenerate Dirac fields. For the time being however we will continue working with the basis in which the two fermions have masses with opposite sign. The field redefinition of Eq. (5.11) immediately leads to the following interaction Lagrangians

$$\begin{aligned}
\mathcal{L}_I^{(0)}(x) &= e\bar{f}^{(0)} A\!\!\!/^{(0)} f^{(0)} \\
\mathcal{L}_I^{(0K)}(x) &= e\sum_{n=1} \bar{f}^{(n)} A\!\!\!/^{(0)} f^{(n)} \\
&+ e\sum_{n=1}\left(\bar{f} A\!\!\!/^{(n)}\left(c_n f_{1R}^{(n)} + s_n f_{2R}^{(n)} - s_n f_{1L}^{(n)} + c_n f_{2L}^{(n)}\right) + \text{h.c.}\right) \\
&+ e\sum_{n=1}\left(-i\bar{f} A_5^{(n)}\left(c_n f_{1L}^{(n)} + s_n f_{2L}^{(n)} + s_n f_{1R}^{(n)} - c_n f_{2R}^{(n)}\right) + \text{h.c.}\right) \\
\mathcal{L}_I^{(K)}(x) &= \frac{e}{\sqrt{2}}\sum_{m,n}\left(\bar{f}^{(n+m)} U_{n+m}^{\dagger}\left(A\!\!\!/^{(m)} - i\sigma_3 A_5^{(m)}\right) U_n f^{(n)} + \text{h.c.}\right) \\
&+ \frac{e}{\sqrt{2}}\sum_{m,n} \bar{f}^{(m)} U_m^{\dagger}\left(A\!\!\!/^{(n+m)}\sigma_3 + iA_5^{(n+m)}\right)\gamma_5 U_n f^{(n)}\,.
\end{aligned} \tag{5.13}$$

Next we turn to the issue of the gauge-invariance of the tree-level S-matrix, and the current (non)conservation relations which enforce it. In particular, we will derive the Ward identities relating the various terms in the interaction Lagrangian of Eq. (5.13); they will constitute the generalizations of the elementary tree-level Ward identities employed in section IV, i.e. of Eq. (4.2) and Eq. (4.5).

From the kinetic terms of Eq. (5.12) we obtain the following Dirac equations:

$$\partial\!\!\!/ f^{(0)} = -im_0 f^{(0)}, \qquad \bar{f}^{(0)}\overleftarrow{\partial\!\!\!/} = im_0\bar{f}^{(0)}, \qquad \partial\!\!\!/ f^{(n)} = i\lambda_n\sigma_3 f^{(n)}, \qquad \bar{f}^{(n)}\overleftarrow{\partial\!\!\!/} = -i\lambda_n\bar{f}^{(n)}\sigma_3\,. \tag{5.14}$$

By employing these Dirac equations one can compute the following divergence

$$
\begin{aligned}
\partial_\mu \left(\bar{f}^{(m)} U_m^\dagger \sigma_3 U_n \gamma^\mu \gamma_5 f^{(n)} \right) &= \bar{f}^{(m)} \overleftarrow{\not\partial} U_m^\dagger \sigma_3 U_n \gamma_5 f^{(n)} - \bar{f}^{(m)} U_m^\dagger \sigma_3 U_n \gamma_5 \not\partial f^{(n)} \\
&= -i \bar{f}^{(m)} \lambda_m \sigma_3 U_m^\dagger \sigma_3 U_n \gamma_5 f^{(n)} - i \bar{f}^{(m)} U_m^\dagger \sigma_3 U_n \sigma_3 \lambda_n \gamma_5 f^{(n)} \\
&= i \bar{f}^{(m)} \left(U_m^\dagger M_m \sigma_3 U_n + U_m^\dagger \sigma_3 M_n U_n \right) \gamma_5 f^{(n)} \\
&= i \bar{f}^{(m)} \left(U_m^\dagger \left(M_m \sigma_3 + \sigma_3 M_n \right) U_n \right) \gamma_5 f^{(n)} \\
&= -i (m_m + m_n) \bar{f}^{(m)} \left(U_m^\dagger U_n \right) \gamma_5 f^{(n)} \; . \qquad (5.15)
\end{aligned}
$$

In deriving Eq. (5.15) we have used the relations

$$
M_n U_n = -U_n \sigma_3 \lambda_n, \qquad U_m^\dagger M_m = -\lambda_m \sigma_3 U_m^\dagger \; , \qquad (5.16)
$$

which follow directly from the fact that $U_n^\dagger M_n U_n = -\lambda_n \sigma_3$.

The identity of Eq. (5.15) guarantees the correct gauge cancellations for S-matrix elements involving these interaction terms. Notice that for the special case $m = n$ Eq. (5.15) reduces to Eq. (4.2)

Similarly, for the other two interacting terms in Eq. (5.13) we have

$$
\begin{aligned}
\partial_\mu \left(\bar{f}^{(m+n)} U_{m+n}^\dagger U_n \gamma^\mu f^{(n)} \right) &= \bar{f}^{(m+n)} \overleftarrow{\not\partial} U_{m+n}^\dagger U_n f^{(n)} + \bar{f}^{(m+n)} U_{m+n}^\dagger U_n \not\partial f^{(n)} \\
&= -i \bar{f}^{(m+n)} \lambda_{m+n} \sigma_3 U_{m+n}^\dagger U_n f^{(n)} + i \bar{f}^{(m+n)} U_{m+n}^\dagger U_n \sigma_3 \lambda_n f^{(n)} \\
&= i \bar{f}^{(m+n)} \left(U_{m+n}^\dagger M_{m+n} U_n - U_{m+n}^\dagger M_n U_n \right) f^{(n)} \\
&= i \bar{f}^{(m+n)} U_{m+n}^\dagger \left(M_{m+n} - M_n \right) U_n f^{(n)} \\
&= -i m_m \bar{f}^{(m+n)} U_{m+n}^\dagger \sigma_3 U_n f^{(n)} \; , \qquad (5.17)
\end{aligned}
$$

which again guarantees gauge invariance of diagrams including these couplings Again, Eq. (5.17) is the generalization of Eq. (4.5) Similar arguments apply to the couplings involving one zero mode fermion.

The diagonalization presented above leads to a rather compact Lagrangian (except perhaps for the terms containing zero modes). On the other hand, the fact that $f_1^{(n)}$ and $f_2^{(n)}$ have masses with different signs may appear unappealing; even though this may be remedied by means of the the field-redefinition mentioned earlier it would spoil the simple structure of the interaction Lagrangian . In addition, it would be difficult to exploit the degeneracy (once the wrong sign in the mass is removed by a chiral transformation) of the pair of Dirac fermions. We now present an alternative diagonalization which could be useful in some cases. We write quadratic terms for the KK modes in the following form

$$
\mathcal{L}_Q = \sum_{n=1} \bar{\xi}_R^{(n)} i \not\partial \xi_R^{(n)} + \bar{\xi}_L^{(n)} i \not\partial \xi_L^{(n)} - \bar{\xi}_R^{(n)} M_n \xi_L^{(n)} - \bar{\xi}_L^{(n)} M_n \xi_R^{(n)} \; . \qquad (5.18)
$$

Now since we have that

$$
M_n^2 = \lambda_n^2 \qquad (5.19)
$$

we find that the matrix M_n/λ_n is unitary, symmetric, self-adjoint and real, so we can define

$$
g_R^{(n)} = \xi_R^{(n)}, \qquad g_L^{(n)} = \frac{M_n}{\lambda_n} \xi_L^{(n)} \; , \qquad (5.20)
$$

and find

$$\mathcal{L}_Q = \sum_{n=1}\left[\bar{g}_R^{(n)} i\partial\!\!\!/ g_R^{(n)} + \bar{g}_L^{(n)} i\partial\!\!\!/ g_L^{(n)} - \lambda_n\left(\bar{g}_R^{(n)} g_L^{(n)} + \bar{g}_L^{(n)} g_R^{(n)}\right)\right]$$
$$= \sum_{n=1}\left[\bar{g}^{(n)} i\partial\!\!\!/ g^{(n)} - \lambda_n \bar{g}^{(n)} g^{(n)}\right] . \qquad (5.21)$$

In addition this Lagrangian is invariant under any rotation of the g fields (g_L and g_R must be rotated in the same way). Although this formulation is rather compact and leads naturally to positive masses it is not obvious how to simplify further the interaction Lagrangian, since it treats differently left from right components. The two basis are related by the following transformation

$$g_R^{(n)} = U_n f_R^{(n)}, \qquad g_L^{(n)} = \frac{M_n}{\lambda_n} U_n f_L^{(n)} = -U_n \sigma_3 f_L^{(n)} . \qquad (5.22)$$

VI. CONCLUSIONS

We have constructed a generalization of QED in five dimensions with the extra dimension compactified on a $S^1/\mathbb{Z}_2$ orbifold. This type of compactification leads naturally to a low energy theory with just one massless Weyl fermion and one Abelian gauge boson. We have derived the Feynman rules of four-dimensional theory obtained after compactification which contain an infinity of Kaluza-Klein modes. In particular we have focused on the issue of the gauge fixing, and have derived the tree-level propagators when the gauge fixing has been carried out before or after the compactification of the extra dimension. It turns out that in the former case the derivation of tree-level boson propagators is rather involved, at least in the context of the covariant (in 5-d) linear gauge fixing enforced by the addition of a term $\frac{1}{2a}(\partial_M A^M)^2$ to the original Lagrangian. The resulting expressions for the propagators have rather particular features; for example, one cannot arrive at a unitary-gauge type of spectrum for any choice of the gauge-fixing parameter a, nor can one choose a is such a way as to give rise to transverse, Landau gauge type of propagators. The subtleties associated with the gauge-fixing in this case can be traced back to the fact that the standard KK construction introduces at the level of the 4-d effective Lagrangian a residual tree-level mixing between the vector bosons (photons) and their (scalar) fifth components; the latter must be properly taken into account when deriving the various tree-level propagators. On the other hand, if the gauge fixing is carried out after the compactification one arrives at a Standard Model-like result for the various propagators; in particular both the unitary and the Landau gauges can be reached continuously. The Landau gauge defined after compactification can be useful when computing the effective potential in theories with extra dimensions. Notice however that in that case graphs with additional massless scalars need be considered, which in general are non-vanishing. Notice in addition that (a) both constructions give rise to the same propagators in the Feynman gauge, and (b) the unitary gauge may be obtained directly before compactification by means of a non-covariant, axial-gauge type of gauge fixing. In addition, we have demonstrated how the gauge cancellations proceed in tree-level S matrix elements, and that the two gauge-fixing procedures (before and after the compactification) give rise to the same answer.

Four-dimensional QED is vector-like and involves massive Dirac fermions, however the orbifold compactification we have described above starting with only one fermion leads, at low energies, to only one massless chiral fermion. To obtain QED as a low-energy theory we have included an additional five-dimensional fermion in the spectrum which has opposite transformation properties with respect to the orbifold symmetry. Then, one can write a mass term which preserves the orbifold symmetry. The low-energy limit of this theory is a theory with just one Abelian gauge boson and one massive Dirac fermion. The spectrum of fermionic KK modes is doubled and displays non-trivial mixing among the different components at each KK level. This requires a simple diagonalization which, however, complicates the structure of the interactions. In this theory we have verified, by means of compact identities involving the divergences of currents, that the gauge cancellations demonstrated before go through after the final diagonalization.

The above considerations provide a well-defined minimal calculational framework which allows for the further detailed study of theories with one universal extra dimension.

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FIGURE CAPTIONS

Fig.1: The three basic tree-level propagators arising after covariant 5-d gauge-fixing.

Fig.2: The tree-level propagators arising by ignoring the mixing term, and the Dyson series obtained by treating the mixing term as an interaction.

Fig.3: The four types of interaction vertices for non-zero modes.

Fig.4: The tree-level S-matrix for the process $\bar{\psi}^{(n)}\psi^{(n)} \to \bar{\psi}^{(n)}\psi^{(n)}$.

Fig.5: The tree-level S-matrix for the process $\bar{\psi}^{(\ell)}\psi^{(i+j)} \to \bar{\psi}^{(i)}\psi^{(\ell+j)}$.

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$$i\Delta^{(n)}_{\mu\nu}(q) = -i\left[\frac{g_{\mu\nu}}{q^2-m_n^2} - \frac{(1-a)q_\mu q_\nu}{(q^2-m_n^2)^2}\right]$$

(a)

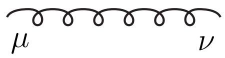
$$i\Delta^{(n)}(q) = i\left[\frac{1}{q^2-m_n^2} + \frac{m_n^2(1-a)}{(q^2-m_n^2)^2}\right]$$

(b)

$$i\Delta^{(n)}_{\nu}(q) = \frac{m_n(1-a)}{(q^2-m_n^2)^2}q_\nu$$

(c)

Fig. 1

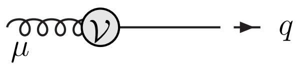


$$id^{(n)}_{\mu\nu}(q) = \frac{-i}{q^2-m_n^2}\left[g_{\mu\nu} - \frac{(1-a)}{q^2-am_n^2}q_\mu q_\nu\right]$$

(a)

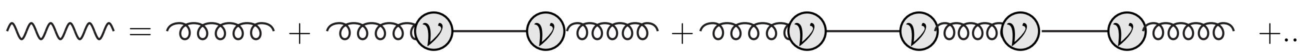
$$id^{(n)}(q) = \frac{i}{q^2-m_n^2/a}$$

(b)

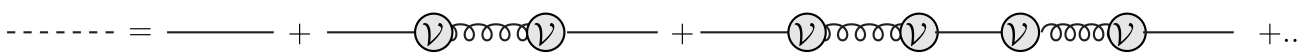


$$\mathcal{V}^\mu_n = m_n(1-\frac{1}{a})q^\mu$$

(c)

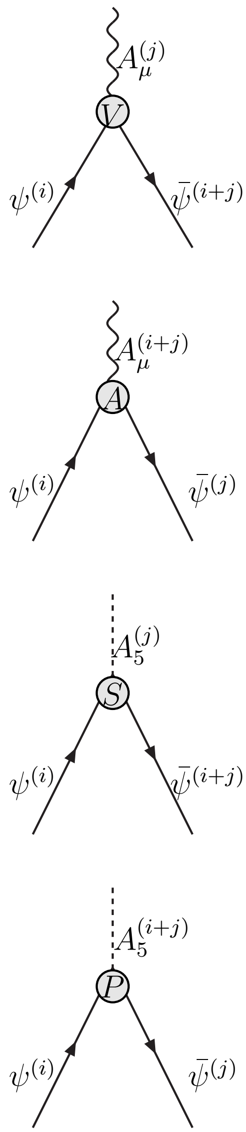


(d)



(e)

Fig. 2



$$\Gamma_V^{\mu\,(j)} : \left(\tfrac{ie}{\sqrt{2}}\right) \bar{\psi}^{(i+j)} \gamma^\mu \psi^{(i)}$$

$$\Gamma_A^{\mu\,(i+j)} : \left(\tfrac{ie}{\sqrt{2}}\right) \bar{\psi}^{(j)} \gamma^\mu \gamma_5 \psi^{(i)}$$

$$\Gamma_S^{(j)} : \left(\tfrac{e}{\sqrt{2}}\right) \bar{\psi}^{(i+j)} \psi^{(i)}$$

$$\Gamma_P^{(i+j)} : -\left(\tfrac{e}{\sqrt{2}}\right) \bar{\psi}^{(j)} \gamma_5 \psi^{(i)}$$

Fig. 3

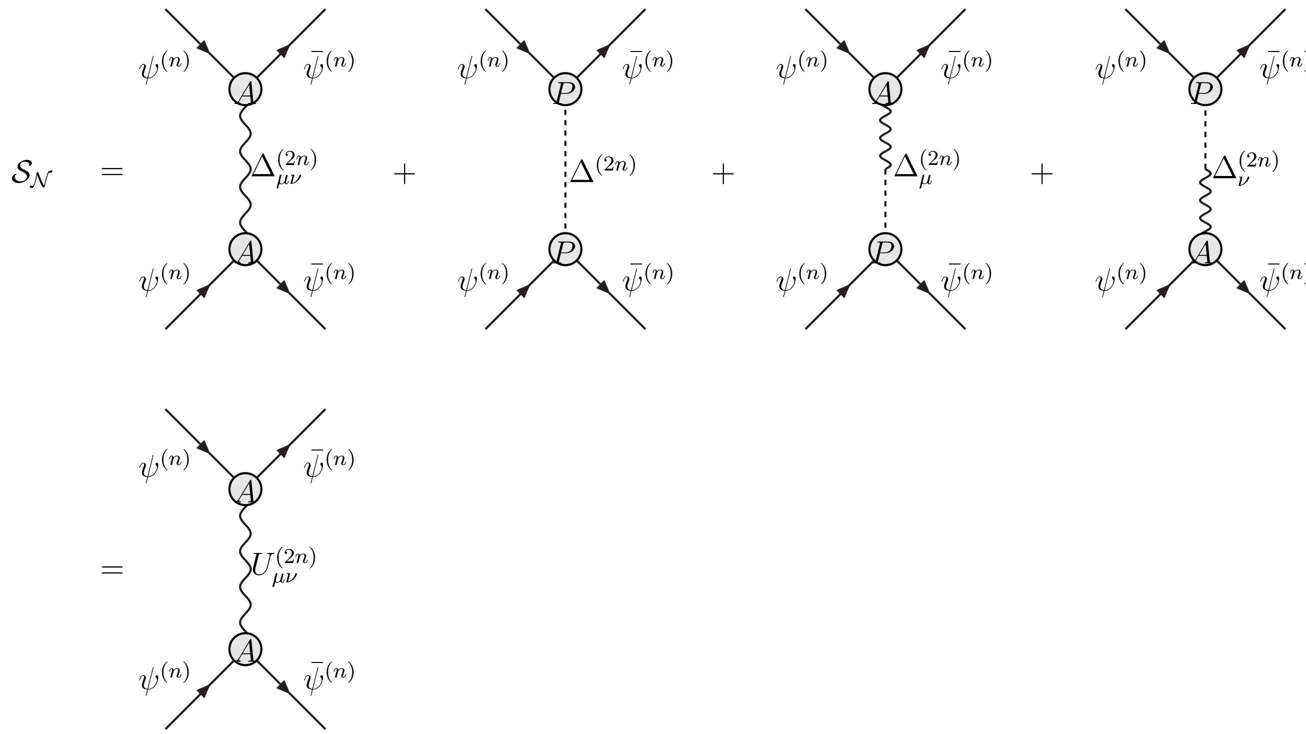

$\mathcal{S}_{\mathcal{N}}$
=
+
=
$\psi^{(n)}$
$\bar{\psi}^{(n)}$
A
P
$\Delta^{(2n)}_{\mu\nu}$
$\Delta^{(2n)}$
$\Delta^{(2n)}_{\mu}$
$\Delta^{(2n)}_{\nu}$
$U^{(2n)}_{\mu\nu}$

Fig. 4

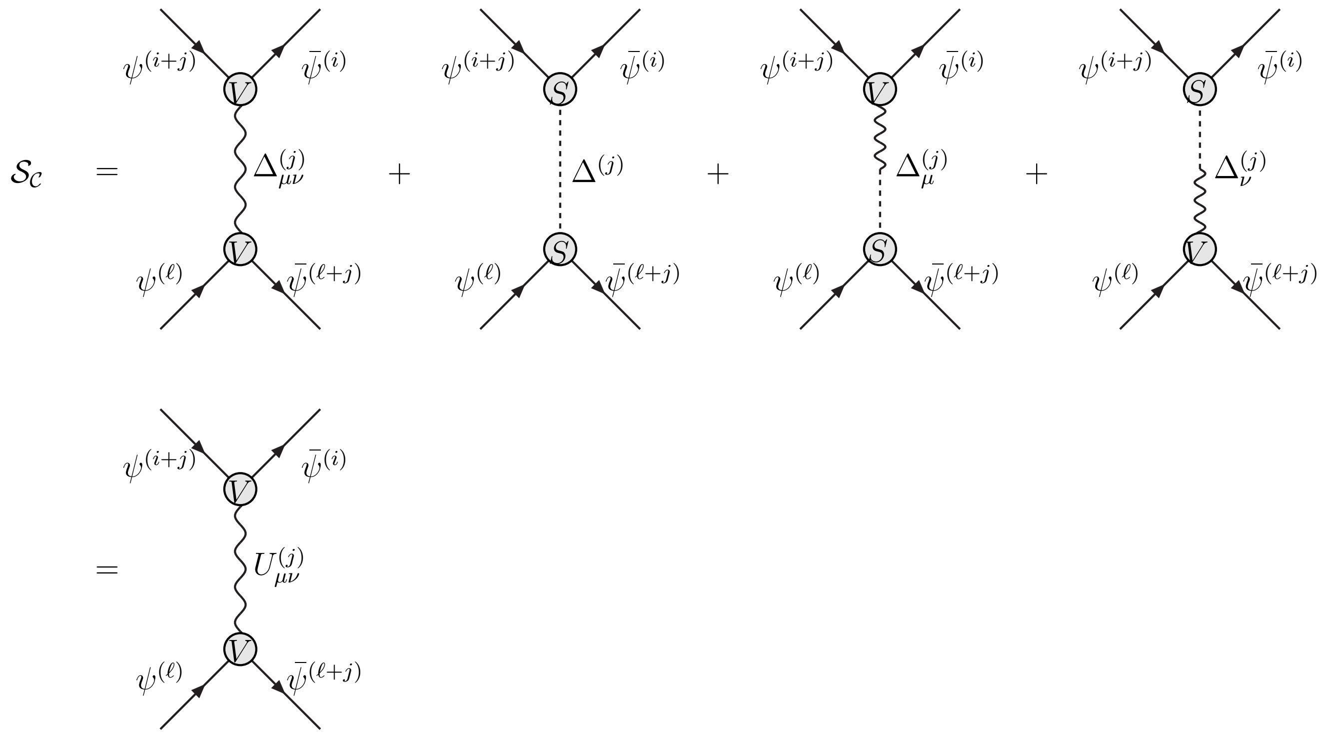

$\mathcal{S}_{\mathcal{C}}$
=
$\psi^{(i+j)}$
$\bar{\psi}^{(i)}$
V
$\Delta^{(j)}_{\mu\nu}$
V
$\psi^{(\ell)}$
$\bar{\psi}^{(\ell+j)}$
+
$\psi^{(i+j)}$
$\bar{\psi}^{(i)}$
S
$\Delta^{(j)}$
S
$\psi^{(\ell)}$
$\bar{\psi}^{(\ell+j)}$
+
$\psi^{(i+j)}$
$\bar{\psi}^{(i)}$
V
$\Delta^{(j)}_{\mu}$
S
$\psi^{(\ell)}$
$\bar{\psi}^{(\ell+j)}$
+
$\psi^{(i+j)}$
$\bar{\psi}^{(i)}$
S
$\Delta^{(j)}_{\nu}$
V
$\psi^{(\ell)}$
$\bar{\psi}^{(\ell+j)}$
=
$\psi^{(i+j)}$
$\bar{\psi}^{(i)}$
V
$U^{(j)}_{\mu\nu}$
V
$\psi^{(\ell)}$
$\bar{\psi}^{(\ell+j)}$

Fig. 5