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An effective field theory approach to the QCD corrections to the large-m_t $Zb\bar{b}$ vertex

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Abstract

Using effective field theory techniques we discuss the QCD corrections to the large-m_t contributions to the process $Z \to b\bar{b}$. In particular we obtain the α_s correction to the non-universal $\log m_t$ contribution to the $Zb\bar{b}$ vertex.

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1 Introduction

Electroweak (EW) radiative corrections are presently achieving an extremely high degree of sophistication and complexity. Indeed, after the pioneering one-loop calculations of Marciano and Sirlin [1] and the two-loop ones of van der Bij and Veltman [2], and because of the high-precision experiments recently performed at LEP and the SLC [3], there is a clear need for increasingly higher-order calculations; even if only for assessing the size of the theoretical error when comparing to the experiment. Currently two-loop EW corrections (pure or mixed with QCD) are being analyzed rather systematically [4] and , sometimes, even up to three loops are being accomplished [5]. Needless to say these calculations are extremely complicated and usually heavily rely on the use of the computer. In this paper we would like to point out that in some situations thinking in terms of effective field theories (EFTs) [6, 7] can help much in this development.

Built as a systematic approximation scheme for problems with widely separated scales [8], EFTs organize the calculation in a transparent way dealing with one scale at a time and clearly separating the physics of the ultraviolet from the physics of the infrared. They are based on the observation that, instead of obtaining the full answer and then take the appropriate interesting limits, a more efficient strategy consists in taking the limit first, whereby considerably reducing the amount of complexity one has to deal with, right from the start. For this kind of problems EFTs are never more complicated than the actual loopwise perturbative calculation and in some specific cases they may even be more advantageous, even able to render an extremely complicated calculation something very simple.

By EFT we specifically mean the systematic construction of the effective Lagrangian that results when a heavy particle is integrated out. The procedure goes as follows [6, 7]. Let us imagine we are interested in studying the physics at an energy scale E_0. Starting at a scale $\mu >> E_0$ one uses the powerful machinery of the renormalization group equations (RGE's) to scale the initial Lagrangian from the scale μ down to the energy E_0 one is interested in. If in doing so one encounters a certain particle with mass m, one must integrate this particle out and find the corresponding matching conditions so that the physics below and above the scale $\mu = m$ (that is to say the physics described by the Lagrangian with and without the heavy particle in question) is the same. This is technically achieved by equating the one-particle irreducible Green functions (with respect to the other light fields) in both theories to a certain order in inverse powers of the heavy mass m[1]. This usually requires the introduction of local counterterms [9] in the effective Lagrangian for $\mu < m$. Once this is done, one keeps using the RGE's until the energy E_0 is reached. If another particle's threshold is crossed, the above matching has to be performed again. All this procedure is most efficiently carried out by using the $\overline{\rm MS}$ renormalization scheme where the RGE's are mass independent and can be gotten directly from the $1/\epsilon$ poles of dimensional regularization.

In this work we would like to apply this technique by concentrating on the QCD corrections to the large-m_t EW contributions *specific* to the $Zb\bar{b}$ vertex. Other corrections common to the other fermions originate from vacuum polarization and have been already studied in [10–12]; therefore we shall not consider them. The decay width $Z \to b\bar{b}$ can be written as [13, 14]

$$\Gamma(Z \to b\bar{b}) = N_c \, \frac{M_Z^3 \sqrt{2} G_F}{48\pi} \; \rho \; R_{QCD} \, R_{QED} \; [A^2 + V^2] \; , \tag{1.1}$$

with

$$A = 1 + \frac{1}{2}\Delta\rho^{vertex} \qquad ; \qquad V = 1 + \frac{1}{2}\Delta\rho^{vertex} - \frac{4}{3}\kappa s_0^2 \; , \tag{1.2}$$

[1]One could also match S-matrix elements.

$$\kappa \approx 1 - \frac{c^2}{c^2 - s^2}\,\Delta\rho \;+ \frac{g^2}{(4\pi)^2}\;\frac{1}{6\,(c^2-s^2)}\log\frac{M_W^2}{m_t^2}\left(1+\frac{\alpha_s(\mu)}{\pi}\right)\;, \tag{1.3}$$

$$\rho = 1+\Delta\rho \qquad ; \qquad \Delta\rho \approx \frac{3}{(4\pi)^2} m_t^2(m_t) G_F\sqrt{2}\left(1-\frac{\alpha_s(\mu)}{\pi}\;\frac{2}{9}\,(\pi^2-9)\right)\;, \tag{1.4}$$

and

$$s_0^2 = \frac{1}{2}\left(1-\sqrt{1-\frac{4\pi\alpha(M_Z)}{\sqrt{2}G_F M_Z^2}}\right)\;, \tag{1.5}$$

where

$$R_{QCD} \approx 1+\frac{\alpha_s(\mu)}{\pi} \quad ; \quad R_{QED}\approx 1+\frac{\alpha(\mu)}{12\pi}\;, \tag{1.6}$$

$$\Delta\rho^{vertex} \approx -\frac{4m_t^2(m_t)G_F\sqrt{2}}{(4\pi)^2}\left(1-\frac{\alpha_s(\mu)}{\pi}\;\frac{\pi^2-8}{3}\right)+ \\ +\frac{g^2}{(4\pi)^2}\;\log\left(\frac{M_W^2}{m_t^2}\right)\left(\frac{8}{3}+\frac{1}{6c^2}\right)\left(1+\,\mathcal{C}\,\frac{\alpha_s(\mu)}{\pi}\right)\;. \tag{1.7}$$

It is more natural, when using the EFT language, to employ the running $\overline{\mathrm{MS}}$ $m_t(\mu = m_t)$ rather than $m_t(pole)$ and this is what we have done in the previous expressions.

What is the natural scale for μ in these contributions? As explained in refs. [11, 12, 15] the $\alpha_s(\mu)$ appearing in $\Delta\rho$ of eq. (1.4) is to be interpreted as $\alpha_s(\mu \approx m_t)$ because it originates at the matching between the full theory with top and the effective field theory without top at the scale $\mu = m_t$. As discussed in ref. [16], it turns out this even encompasses most of the $\mathcal{O}(\alpha_s^2)$ contributions. On the contrary, as explained in ref. [12], the logarithmic term of eq. (1.3) comes from the running of the effective Lagrangian from the scale $\mu = m_t$ down to the scale $\mu = M_W \simeq M_Z$ and consequently does not probe a single μ scale but rather integrates over the whole range. The result of this integration leads to the substitution [2]

$$\log\frac{M_W^2}{m_t^2}\;\left(1+\frac{\alpha_s(\mu)}{\pi}\right)\rightarrow \log\frac{M_W^2}{m_t^2}\;+\;\log\left(\frac{\alpha_s(M_W)}{\alpha_s(m_t)}\right)^{-4/\beta_0} \tag{1.8}$$

in eq. (1.3); here $\beta_0 \equiv 11 - 2n_f/3 = 23/3$ is the β function of the QCD coupling constant $\alpha_s(\mu)$ for $n_f = 5$ flavors:

$$\frac{d\alpha_s}{dt} = -\,\frac{\beta_0}{(4\pi)}\,\alpha_s^2(t) \quad ; \quad t\equiv \log\mu^2\;\;. \tag{1.9}$$

Equation (1.8) actually resums all the QCD leading logarithms, i.e. all terms of the form $\alpha_s^n \log^n$.

In this paper we shall describe an effective field theory calculation of the physical process $Z \to b\bar{b}$. As a result we shall obtain the value of the coefficient $\mathcal{C}$ in eq. (1.7). This coefficient has also been recently obtained in ref. [17] and our result agrees with theirs. Moreover, our construction of the EFT will also yield the value for the natural scale μ that appears in the different terms of eqs. (1.6),(1.7).

We think that our discussion in terms of effective Lagrangians is a good guide for dealing with questions of this sort and also for computing things like "the QCD corrections to the $\log m_t$ term" for large m_t, i.e. the coefficient $\mathcal{C}$. As we will see, one could even resum, if necessary, the leading logarithms. However one should keep in mind that in the real world the top mass is not that large with respect to the Z and, therefore, one should expect sizeable corrections to the simple case $m_t \to \infty$.

[2] For small values of α_s of course the next two expressions coincide.

2 Case without QCD corrections

Integrating the top quark out affects the coupling to the W and Z gauge bosons of every lighter fermion through vacuum polarization. Moreover it also affects specifically the coupling of the bottom quark to the Z boson; an effect that is not felt by any other fermion. To set the stage for the QCD corrections of the next section we shall now review, in an effective field theory language, how this comes about. The standard strategy is the following:

1. Matching the effective theory to the full theory at m_t.
2. Running the effective Lagrangian from m_t down to M_Z.
3. Calculating matrix elements with the effective Lagrangian at the scale M_Z.

The integration of the top quark is done in several steps. Firstly, at tree level, there is the contribution given by the diagram of fig. 1. This contribution gives rise to an effective operator that is suppressed by two inverse powers of the top mass. We shall consistently neglect this type of contributions since they can never give rise to the terms we are interested in, i.e. eq. (1.7). This is the only contribution in the unitary gauge, which is the one we shall employ in this work. In any other gauge other effective operators arise because the would-be Nambu–Goldstone bosons couple proportionally to the top mass and may compensate the m_t^2 factor in the denominator (see for example the diagram of fig. 2). This is a welcome simplification, most notably when QCD corrections will be considered in the next section.

Since at this order the matching condition turns out to be trivial, the effective Lagrangian below the top quark mass looks exactly the same as the full standard model Lagrangian except that the top is absent; e.g. there is no tbW vertex in this Lagrangian :

$$\mathcal{L} = \bar{b}\, i\not{D} b - \frac{1}{2} c_L^b(\mu)\, \bar{b}\, \not{Z} P_L\, b + \frac{1}{3} c_V^b(\mu)\, \bar{b}\, \not{Z} b +$$

$$+ \bar{e}\, i\not{D} e - \frac{1}{2} c_L(\mu)\, \bar{e}\, \not{Z} P_L\, e + c_V(\mu) \bar{e}\, \not{Z} e + \frac{c_+(\mu)}{\sqrt{2}} \left(\bar{e}\, \not{W} P_L\, \nu + h.c. \right) \tag{2.1}$$

where P_L is the lefthanded projector and $\not{D}$ stands only for the QED covariant derivative since $\alpha_s = 0$ in this section. The $c(\mu)$'s of the electron are actually common to all the fermions but the bottom quark. For instance, the $Z\nu\bar{\nu}$ would be $+c_L(\mu)/2$ since the neutrino has no vector coupling $c_V(\mu)$. We also take all the fermions but the top as massless and assume that $M_H \simeq M_{Z,W}$ to avoid unnecessary complications in the form of terms $\log(M_H/M_Z)$. Notice that we have decomposed the $Zf\bar{f}$ vertex in terms of a lefthanded and vector couplings instead of the more conventional left and righthanded, or vector and axial counterparts. The matching conditions at this order yield Standard Model tree-level values for the effective couplings:

$$c_L^b(m_t) = \frac{g}{c} \quad ; \quad c_V^b(m_t) = \frac{g}{c}\, s^2\, ; \tag{2.2}$$

$$c_L(m_t) = \frac{g}{c} \quad ; \quad c_V(m_t) = \frac{g}{c}\, s^2 \quad ; \quad c_+(m_t) = g\ . \tag{2.3}$$

We can select the non-universal part of the $Zb\bar{b}$ vertex by comparing the $c_L^b(\mu)$ coupling on shell with the analogous coupling for the electron $c_L(\mu)$ at the scale $\mu \sim M_Z \sim M_W \equiv M$. One defines[3]

$$1 + \frac{1}{2}\Delta\rho^{vertex} = \frac{c_L^b(M)}{c_L(M)}\ . \tag{2.4}$$

[3] This ratio is called $1 + \epsilon_b$ in ref. [18]

In order to make contact with the physics at the scale $\mu = M$, one has to scale the Lagrangian (2.1) down to this particular μ. Since we are only interested in one-loop electroweak contributions, the RGEs that govern this scaling will be computed using the lowest order in the electroweak couplings, i.e. the tree level. In this process of scaling, $c_L^b(\mu)$ and $c_L(\mu)$ run differently. The calculation can be done by setting the external particles on shell. This is actually a subtle point. If one were to compute off-shell Green functions one would discover that extra structures other than those appearing in the operators of eq. (2.1) are generated in the running. This has nothing to do with the effective field theory construction but is a reflection of the known fact that the Standard Model is not manifestly renormalizable in the unitary gauge at the level of off-shell Green functions. Only when S-matrix elements are taken can one renormalize it [19]. Of course this is all we need since in the physical process $Z \to b\bar{b}$ all particles are on shell, so that the physical amplitude is governed by the coefficient $c_L^b(M)$.

For $c_L^b(\mu)$ the running is given by the $1/\epsilon$ poles of the diagrams of fig. 3. One can see, however, that the diagrams of figs. 3a,b do not contribute to the running of $c_L^b(\mu)$ since they are actually finite (this is akin to the Ward–Takahashi identity relating the vertex to the wave function renormalization in QED) and the only contribution comes from fig. 3c. Therefore

$$\frac{dc_L^b(t)}{dt} = \text{fig. 3c} \quad . \tag{2.5}$$

However things look differently for $c_L(\mu)$ since *there is* an $e\nu W$ vertex. The running of $c_L(\mu)$ is given by the diagrams of fig. 4. As before, the diagrams 4d,e do not contribute, and an explicit calculation in the unitary gauge for on-mass-shell matrix elements shows that the diagrams of fig. 4b,c are each finite. Therefore the only contribution is that of figs. 4a and 4f, and one finds

$$\frac{dc_L(t)}{dt} = -\frac{g}{c}\,\frac{g^2}{(4\pi)^2}\left(\frac{4}{3} + \frac{1}{12c^2}\right) + \text{fig. 4f} \quad , \tag{2.6}$$

where the relation $M_Z^2 c^2 = M_W^2$ has been used. One can now trivially integrate eqs. (2.5)-(2.6) with respect to t between $\log m_t^2$ and $\log M^2$ to find

$$c_L^b(M) \simeq c_L^b(m_t) + (\text{fig. 3c}) \log \frac{\mathrm{M}^2}{\mathrm{m_t^2}} \tag{2.7}$$

$$c_L(M) \simeq c_L(m_t) - \frac{g}{c}\,\frac{g^2}{(4\pi)^2}\left(\frac{4}{3} + \frac{1}{12c^2}\right) \log \frac{M^2}{m_t^2} + (\text{fig. 4f}) \log \frac{\mathrm{M}^2}{\mathrm{m_t^2}} \quad . \tag{2.8}$$

This yields

$$\frac{c_L^b(M)}{c_L(M)} \simeq \frac{c_L^b(m_t)}{c_L(m_t)}\left(1 + \frac{g^2}{(4\pi)^2}\left(\frac{4}{3} + \frac{1}{12c^2}\right) \log \frac{M^2}{m_t^2}\right) \quad , \tag{2.9}$$

since the contribution of figs. 3c and 4f cancel each other out in this ratio. The tree-level matching conditions of eqs. (2.2), (2.3) lead to $c_L^b(m_t)/c_L(m_t) = 1$ and, therefore, in this approximation, one finds the right logarithmic piece of eq. (1.7) but the power-like one that goes like m_t^2 is still missing. To obtain it one has to go to the next order, i.e. do a one-loop matching. In order to do this one has to consider the integration of the top quark to one loop. This not only modifies the boundary conditions of eq. (2.2),(2.3) but also produces new nontrivial operators besides those of eq. (2.1). The relevant diagrams are depicted in fig. 5. Then the part of the Lagrangian involving the bottom quark interactions reads[4]

$$\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_6 \quad , \tag{2.10}$$

[4]Because of the diagram of fig. 5a there is a wave function renormalization factor. $\mathcal{L}_4$ only follows after making the field redefinition that renders the kinetic term in a standard form.

$$\mathcal{L}_4 = \bar{b}\, i\not{D} b - \frac{1}{2} c_L^b(\mu)\, \bar{b}\, \not{Z} P_L\, b + \frac{1}{3}\, c_V^b(\mu)\, \bar{b}\, \not{Z} b \tag{2.11}$$

and

$$\mathcal{L}_6 = \frac{1}{\Lambda_F^2} \sum_i c_i(\mu) \mathcal{O}_i \ , \tag{2.12}$$

where $\Lambda_F = 4\pi v$, $v = (\sqrt{2} G_F)^{-1/2} = 246$ GeV and the $\mathcal{O}_i$'s are a set of dimension six operators involving the (lefthanded) bottom quark and three derivatives; or the (lefthanded) bottom quark, the Z and two derivatives[5]. They arise from the longitudinal part of the W propagators. This is why the scale Λ_F appears: it is the combination of the ordinary $1/m_t^2$ suppression of any six dimensional operator in an effective field theory and the fact that the would-be Nambu–Goldstone bosons couple proportionally to the top mass. There are also other operators generated at this stage, like for instance a Wess-Zumino term (which ensures in the theory without top the cancellation of anomalies that occurs in the theory with top) or a four bottom-quark operator (which comes from box diagrams with the top flowing in the loop). However they can only affect the $Zb\bar{b}$ vertex with contributions that are two-loop electroweak, i.e. subleading with respect to eq. (1.7). Still higher dimensional operators may exist but they are truly suppressed by inverse powers of the top mass.

It is easy to convince oneself that, for $\alpha_s = 0$, one can forget about $\mathcal{L}_6$. Firstly, we are only interested in considering matrix elements of $\mathcal{L}_6$ at tree level since $\mathcal{L}_6$ itself has been generated at one electroweak loop. With a massless bottom, dimensional analysis leaves M_Z as the only scale to compensate the scale Λ_F in eq. (2.12). This is an effect of $\mathcal{O}(g^2)$ in the $Zb\bar{b}$ vertex, i.e. without the $\log m_t$ enhancement and therefore is subleading with respect to those of eq. (1.7).

However, this is not the whole story. The longitudinal part of the W boson propagator also produces a contribution to the matching condition of the $c_L^b(\mu)$ coupling of eq. (2.11) and modifies the first of the boundary conditions in eq. (2.2). An explicit calculation yields the following result

$$\frac{c_L^b(m_t)}{c_L(m_t)} = 1 - 2\, \frac{m_t^2}{(4\pi v)^2} \ , \tag{2.13}$$

since $c_L(m_t) = g/c$ remains unchanged. Inserting this into eq. (2.9) one obtains the desired final result: eq. (2.9) one

$$\frac{c_L^b(M)}{c_L(M)} \simeq 1 - 2\, \frac{m_t^2}{(4\pi v)^2} + \frac{g^2}{(4\pi)^2} \left(\frac{4}{3} + \frac{1}{12c^2} \right) \log \frac{M^2}{m_t^2} \ , \tag{2.14}$$

i.e. eq. (1.7) with $\alpha_s = 0$. For the effective field theory aficionado eq. (2.14) is somewhat unconventional in that it mixes matching (i.e. the m_t^2 term) with running (i.e. the logarithm) both at one loop. The reason is of course the non-decoupling of the top quark which enhances the one-loop matching contribution with the sizeable m_t^2 term. The rest of the one-loop matching *is* of $\mathcal{O}(\alpha)$ and therefore subleading.

Clearly the effect of integrating the top quark out affects only the lefthanded projection of the bottom-quark field, $c_L^b(\mu)$, but leaves untouched the coefficient $c_V^b(\mu)$. This coefficient not only equals $c_V(\mu)$ at $\mu = m_t$ (eqs. (2.2-2.3)) but also runs with μ as $c_V(\mu)$ does. In other words, $c_V^b(\mu) = c_V(\mu)$. Of course this fact will be unaffected by QCD corrections.

3 QCD corrections

As discussed in the previous section, the unitary gauge has the advantage that, at the tree level, the matching corrections that appear when one integrates out the top quark are suppressed by

[5] See next section for more discussion.

two inverse powers of the top quark mass (fig. 1). Since ultimately this fact is due to dimensional analysis, it cannot change once QCD is switched on and one-loop α_s corrections to the diagram of fig. 1 are also considered in the matching conditions. Therefore, with our choice of the unitary gauge, dimensional analysis still dictates that these corrections are also suppressed by two inverse powers of the top mass when $\alpha_s \neq 0$. Since in the matching conditions one has to calculate not only the divergent parts but also the finite pieces, the fact that the unitary gauge does away with these particular matching conditions altogether is a major simplification and justifies our choice of this gauge.

Not all the matching corrections disappear, however. The diagrams of fig. 6 do give rise to new dimension six operators. Since no gluon loop appears in these diagrams, these operators are explicitly QCD gauge invariant and, in fact, together with the diagrams of fig. 5 they are none other than the Lagrangian $\mathcal{L}$ of eqs. (2.10)-(2.12) with the only change that, now, every derivative has been appropriately promoted to a QCD covariant derivative. It is important to notice that both sets of diagrams (fig. 5 and fig. 6) are necessary in order to resolve the ambiguities one encounters when trying to implement this promotion to covariant derivatives.

In principle one should now calculate how all these operators mix back into the $Zb\bar{b}$ operator of eq. (2.11) and make the coefficients $c^b_{L,V}(\mu)$ evolve with μ as one runs from m_t down to M_Z. However, one notices that since we are only interested in on-shell Z gauge bosons we can use the free-field equations of motion for the Z to trade derivatives by Z masses. Moreover, although the b-quark field is not on-shell and will be closed in loops, one is also free to use the *full* field equations of motion for the b quark as a means of defining and simplifying the operator basis [20].

In this way, a clever use of the equations of motion helps us get rid of most of the operator structures that are generated in the matching and leaves us with only three (in principle) relevant operators. These are[6,7]

$$\begin{aligned} \mathcal{O}_1 &= \bar{b}_L \gamma^\nu \frac{\lambda^A}{2} b_L \; g_s \; D^\mu G^A_{\mu\nu} \\ \mathcal{O}_2 &= \frac{g}{c} \bar{b}_L \sigma^{\mu\nu} \; \not{Z} \frac{\lambda^A}{2} b_L g_s G^A_{\mu\nu} \qquad (3.1) \\ \mathcal{O}_3 &= \frac{g}{c} \bar{b}_L \gamma^\mu \frac{\lambda^A}{2} b_L Z^\nu g_s G^A_{\mu\nu} \quad . \qquad (3.2) \end{aligned}$$

Of course these operators only affect the running of $c^b_L(\mu)$ and not of $c^b_V(\mu)$.

For instance an operator like $\mathcal{O}_4 = i\bar{b}_L \not{D}^3 b_L$ will give rise, upon integration by parts and use of the equations of motion for both b fields, to another operator with two Z's which can only mix back into the $Zb\bar{b}$ operators of eq. (2.11) by closing one Z in a loop, i.e. at the two-electroweak-loop level since $\mathcal{O}_4$ originated already at one electroweak loop, and therefore we can neglect it. One can check the fact that the $\mathcal{O}_4$ can be neglected by computing its contribution to the diagrams of figs. 7a and 7b and seeing that they cancel each other. Throughout this work we shall always use the Feynman gauge propagator for the gluon.

An explicit straightforward evaluation of the diagrams of fig. 6a yields for the coefficient $c_1(\mu)$ accompanying the operator $\mathcal{O}_1$ the value

$$c_1(m_t) = -\frac{7}{18} \quad . \qquad (3.3)$$

In principle we should also compute $c_{2,3}(m_t)$ for the corresponding operators $\mathcal{O}_{2,3}$. However, for our purposes this is totally unnecessary. A moment's thought reveals that $\mathcal{O}_{2,3}$ can

[6]One could still use the equations of motion for the gluon field but we found more convenient not to do so.

[7]There are two Hermitian linear combinations of $\mathcal{O}_2$ that must be considered.

never mix back into the $Zb\bar{b}$ operators of eq. (2.11), where the three particles are on shell, because closing the gluon line in a loop will produce –through a $\not{p}$ or a p^2– a bottom mass which we have taken to be zero.

In order to make contact with the physics at the scale $\mu \simeq M$ one has to find how $c_L^b(\mu)$ scales with μ. What is the RGE governing the running of the coefficient $c_L^b(\mu)$ now that α_s corrections are included? For $\alpha_s = 0$ we know that the answer is given by eq. (2.5). Not much is changed when $\alpha_s \neq 0$. In particular, the cancellation of the diagrams of figs. 3a and 3b still takes place, even with QCD corrections. First of all there is the diagram of fig. 3c (now of course including $\alpha_s$corrections in the Z vacuum polarization). Notice that there can be no gluon attaching the vacuum polarization to the external bottom-quark line to order α_s. Secondly there is the contribution of the coefficients $c_i(\mu)$ of $\mathcal{L}_6$ to the RGE for $c_L^b(\mu)$. However, as we discussed above, only $c_1(\mu)$ needs to be considered; the relevant diagrams are those of fig. 7b. Notice also in this regard that there is no wave function renormalization due to $\mathcal{O}_1$ because of the masslessness of the bottom quark. Finally notice that $c_L^b(\mu)$ does not renormalize itself through α_s corrections because the operator $\bar{b}_L \not{Z} b_L$ behaves like a conserved current under QCD.

Consequently, gathering all the pieces, one obtains that

$$\frac{dc_L^b(t)}{dt} = \text{fig. 3c} \; + \; \frac{\mathrm{g}}{\mathrm{c}} \, \frac{\mathrm{g}^2}{(4\pi)^2} \, \gamma_1 \; \mathrm{c}_1(\mathrm{t}) \frac{\alpha_\mathrm{s}(\mathrm{t})}{\pi} \quad . \tag{3.4}$$

We have explicitly checked that, as we argued before, the contributions of diagrams of figs. 8a and 8b cancel out when the vertices obtained from figs. 5b and 6b are inserted thus confirming that the only running, besides the vacuum polarization diagram of fig. 3c, comes from the penguin operator $\mathcal{O}_1$, so that eq. (3.4) indeed follows. We obtain the following value for the coefficient γ_1:

$$\gamma_1 = -\frac{1}{9c^2}\left(1 - \frac{2}{3}s^2\right) \quad . \tag{3.5}$$

Since $\mathcal{O}_1$ only involves the lefthanded bottom quark it is clear why the coefficient γ_1 turns out to be proportional to the lefthanded bottom coupling to the Z, i.e. the combination $1 - \frac{2}{3}s^2$.

Now we would like to integrate eq. (3.4). In first approximation, one may take $\alpha_s(t)$ and $c_1(t)$ as constants independent of t, i.e. $\alpha_s(\mu) \simeq \alpha_s(m_t) \simeq \alpha_s(M) \equiv \alpha_s$ and $c_1(\mu) \simeq c_1(m_t) \simeq c_1(M) \equiv c_1 = -7/18$. The integration over t between $\log m_t^2$ and $\log M^2$ gives

$$c_L^b(M) \simeq c_L^b(m_t) + \text{fig. 3c} \; \log\frac{\mathrm{M}^2}{\mathrm{m_t^2}} + \frac{\mathrm{g}}{\mathrm{c}}\frac{\mathrm{g}^2}{(4\pi)^2} \; \gamma_1 \; \mathrm{c}_1 \; \frac{\alpha_\mathrm{s}}{\pi} \log\frac{\mathrm{M}^2}{\mathrm{m_t^2}} \quad . \tag{3.6}$$

It is in principle possible to improve on this approximation by considering the μ-dependence of $\alpha_s(\mu)$ and $c_1(\mu)$ in eq. (3.4). The μ-dependence of $\alpha_s(\mu)$ is given by the usual one-loop β function. However the μ-dependence of $c_1(\mu)$ is more complicated to obtain because it requires performing a complete operator mixing analysis of the penguin operator along the lines of, for instance, the work carried out in the studies of $b \to s\gamma$ or K^0-$\bar{K}^0$ mixing [21] from where most of the results could be taken over to our case. However, the fact that $\gamma_1 c_1(m_t) = \frac{7}{162c^2}\,(1 - \frac{2}{3}\,s^2) \approx 0.05$ turns out to be so small renders this improvement moot and we shall content ourselves with eq. (3.6) as it is. As we shall see later on, there are other sources of QCD corrections that are numerically more important.

Of course leptons do not have strong interactions and for them the answer for the running of $c_L(\mu)$ is still eq. (2.8) (notice however that fig. 4f will now have some α_s-dependence inside the vacuum polarization, just as fig. 3c does). Consequently one obtains from eqs. (3.6),(2.8):

$$\frac{c_L^b(M)}{c_L(M)} \approx \frac{c_L^b(m_t)}{c_L(m_t)}\left[1 + \frac{g^2}{(4\pi)^2}\left(\frac{4}{3} + \frac{1}{12c^2}\right)\log\frac{M^2}{m_t^2} + \frac{g^2}{(4\pi)^2}\;\gamma_1\;c_1\frac{\alpha_s}{\pi}\log\frac{M^2}{m_t^2}\right] \quad , \tag{3.7}$$

since, also in this case, the contributions from figs. 3c and 4f cancel each other out in this ratio. This fixes the coefficient $\mathcal{C}$ in eq. (1.7) to be

$$\mathcal{C} = 2\,\gamma_1\,c_1\,\left(\frac{8}{3}+\frac{1}{6c^2}\right)^{-1} \approx 0.03\ . \tag{3.8}$$

We would like to mention at this point that all of the above calculations (included those in the previous section without QCD) have been repeated using the Landau gauge for the W propagator with exactly the same results. In particular we would like to point out that in this gauge the coefficient $c_1(m_t)$ comes entirely from fig. 6a with the W replaced by a Nambu–Goldstone boson. This agreement has as a consequence the vanishing of the contribution to $c_L^b(M)$ of the QCD corrections to the matching conditions of operators with external Nambu–Goldstone bosons like for instance the one in fig. 2, which are present in this gauge. Of course in the unitary gauge they trivially disappear.

The ratio $c_L^b(m_t)/c_L(m_t)$ is still given by eq. (2.13) in this approximation. One could now consider α_s corrections to eq. (2.13). This is a hard two-loop calculation of the matching conditions in the presence of QCD when the top is integrated out. Here the effective field theory language does not help much and the calculation has to be done. Fortunately the result is already available in the literature [22]. Translated into our context it amounts to

$$\frac{c_L^b(m_t)}{c_L(m_t)} = 1 - 2\,\frac{m_t^2(m_t)}{(4\pi v)^2}\left[1-\frac{\alpha_s(m_t)}{\pi}\,\left(\frac{\pi^2-8}{3}\right)\right]\ . \tag{3.9}$$

Again, what the EFT does tell us is that the μ scale of $\alpha_s(\mu)$ in this equation has to be m_t since it originates at the matching condition when the top is integrated out.[8] Therefore we get to eq. (1.7) with $\alpha_s(\mu = m_t)$ in the m_t^2-dependent term.

However this is not yet all. Up to now all the physics has been described with RGEs (i.e. running) and their initial conditions (i.e. matching) which is only ultraviolet physics, and no reference to infrared physics has been made. For instance, where are the infrared divergences that appear when a gluon is radiated off a bottom-quark leg? As we shall now see, this physics is in the matrix element for $Z \to b\bar{b}$. After all, we have only obtained the effective Lagrangian (2.10),(2.12) at the scale $\mu = M$; we still have to compute the physical matrix element with it, and here is where all the infrared physics takes place.

Indeed, when computing the matrix element for $Z \to b\bar{b}$ with the effective Lagrangian (2.10),(2.12) expressed in terms of $c_{L,V}^b(\mu)$ at $\mu = M$, one has the contribution of the diagrams of figs. 9a, 9b, where the $\otimes$ stands for the effective vertices proportional to $c_{L,V}^b(M)$. These diagrams give rise to infrared divergences. These divergences disappear in the standard way once bremsstrahlung diagrams like those of fig. 9c are (incoherently) added [23, 24]. Note that similar diagrams with effective $g-b-b$ couplings are subleading and never give rise to corrections of the form of eq. (1.7).

As is well known [14], the net result of all this (a similar calculation can be performed for the QED corrections) is the appearance of the factors R_{QCD} and R_{QED} of eqs. (1.1),(1.6), where b-quark mass effects can also be included [24, 25] if needed.

The EFT technology adds to this the choice of scale for μ, namely $\mu = M$, in these factors[9]:

$$R_{QCD} \simeq 1 + \frac{\alpha_s(M)}{\pi}\quad ;\quad R_{QED} \simeq 1 + \frac{\alpha(M)}{12\pi}\ , \tag{3.10}$$

[8] Another advantage is that matching conditions are free from infrared divergences [6], which is a nice simplification. For some more discussion on infrared divergences, see below.

[9] This has been previously suggested by D. Bardin (private communication).

and naturally leads to the factorized expression (1.1)-(1.7) (see the Appendix), with the value of $\mathcal{C}$ given by eq. (3.8). As stated in the introduction, our result agrees with that of ref. [17]. Since the "intrinsic" α_s contribution of $\Delta\rho^{vertex}$ is, due to the smallness of the coefficient $\mathcal{C}$, much less important than that of R_{QCD} one sees that the QCD corrections to the non-universal $\log m_t$ piece of the $Zb\bar{b}$ vertex are, to a very good approximation, of the form one-loop QCD ($m_t << M_Z$) times one-loop electroweak ($m_t >> M_Z$) [26].

4 Discussion and conclusions

In this paper we have presented an effective field theory study of the process $Z \to b\bar{b}$ in the limit of large m_t including all the QCD corrections to the leading and the subleading contributions. In particular we have explicitly shown that the QCD corrections to the term $\log(m_t/M_Z)$ can be easily obtained in the EFT framework by computing only a couple of one-loop diagrams in the limit of small external momenta. We have also shown that the EFT framework answers quite naturally the question of the renormalization points to be used for the coupling constants in the different terms.

In addition, it is important to remark that in the EFT language all the physics above M is absorbed (in particular, all m_t effects) in the coefficients of the effective operators so that infrared physics is relegated to the calculation of the physical process one is interested in. With our effective Lagrangian one could in principle compute any physical quantity, and not only the Z width, like for example jet production (i.e. where cuts are needed) , forward–backward asymmetries, etc. This is to be compared with more standard methods for computing radiative corrections to the Z width in which this width is extracted from the imaginary part of the Z self-energy to avoid problems with infrared divergences. For instance, a more standard calculation of the coefficient $\mathcal{C}$ [17] requires starting with the Z self-energy at three loops (with all the complications of the renormalization program at that order) to then compute its imaginary part. Extracting from this spatial asymmetries or jet rates, for instance, will be much harder because it is not easy to limit to one's needs the entire phase space. The EFT calculation clearly separates ultraviolet from infrared physics and as a consequence it is more flexible. And it is also simpler since, after all, we never had to compute anything more complicated than a one-loop diagram.

Of course, our results become more accurate as the top mass becomes larger. In practice it is unlikely that the top quark be much heavier than, say, 200 GeV so due caution is recommended in the phenomenological use of eq. (1.1). In the lack of a (very hard !) full $\mathcal{O}(g^2\alpha_s)$ calculation, this is the best one can offer. Furthermore, we think it is interesting that at least there exists a limit (i.e. $m_t >> M_Z$) where the various contributions are under full theoretical control.

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APPENDIX

A The width for $Z \to b\bar{b}$

In ref. [12] the effective couplings $g_3(\mu), g_+(\mu)$ and $g'(\mu)$ were employed. The connection with $c_L(\mu), c_V(\mu)$ and $c_+(\mu)$ is

$$c_L(\mu) = \frac{g_3(\mu)}{c(\mu)} \; ; \; c_V(\mu) = \frac{g_3(\mu) s^2(\mu)}{c(\mu)} \; ; \; c_+(\mu) = g_+(\mu) \;\; ,$$

where $s^2(\mu) = \sin^2\theta_W(\mu)$ and $\tan\theta_W(\mu) = g'(\mu)/g_3(\mu)$.

According to our discussion, the width $Z \to b\bar{b}$ can be computed from our Lagrangian (2.10) as

$$\Gamma \propto M_Z \; R_{QCD} \; R_{QED} \; \left[V^2 + A^2 \right] \;\; ,$$

where $R_{QCD,QED}$ are given by eq (1.6) with $\mu = M$, and

$$V = \frac{1}{3}\, c_V^b(M) - \frac{1}{4}\, c_L^b(M) \;\; ,$$

$$A = \frac{1}{4}\, c_L^b(M) \;\; .$$

Therefore

$$V^2 + A^2 = \frac{1}{16} \left[\left(c_L^b(M) \right)^2 + \left(c_L^b(M) - \frac{4}{3} c_V^b(M) \right)^2 \right] \;\; .$$

Since $c_V^b(\mu) = c_V(\mu)$ and $c_V(\mu) = c_L(\mu) s^2(\mu)$ one obtains that

$$V^2 + A^2 = \frac{1}{16} \; (c_L(M))^2 \; \left[\left(\frac{c_L^b(M)}{c_L(M)} \right)^2 + \left(\frac{c_L^b(M)}{c_L(M)} - \frac{4}{3} s^2(M) \right)^2 \right] \;\; ,$$

and, using (3.7)-(3.9), one sees that

$$\frac{c_L^b(M)}{c_L(M)} = 1 + \frac{\Delta\rho^{vertex}}{2} \;\; .$$

From ref. [12] we know that

$$M_W^2 = \frac{c_+^2(M)}{4} \; v_+^2(M) = \frac{c_+^2(M)}{4} \; \left(\sqrt{2} G_F \right)^{-1} \;\; ,$$

and

$$M_Z^2 = \frac{c_L^2(M)}{4} \; \left(v_3^2(M) + 4 M_Z^2 \delta Z_{3Y}(M) \right) \;\; ,$$

so that one can relate $c_L(M)$ to M_Z and G_F. One finally obtains eq. (1.1) after noticing that

$$s^2(M) = \kappa s_0^2 \;\; ,$$

with

$$\kappa = 1 - \frac{c^2}{c^2 - s^2} \; \Delta\rho + \frac{g^2}{c^2 - s^2} \; \delta Z_{3Y}(M) \;\; ,$$

and

$$\delta Z_{3Y}(M) = \frac{1}{6(4\pi)^2} \left[\log \frac{M^2}{m_t^2} + \log \left(\frac{\alpha_s(M)}{\alpha_s(m_t)} \right)^{-4/\beta_0} \right] \;\; .$$

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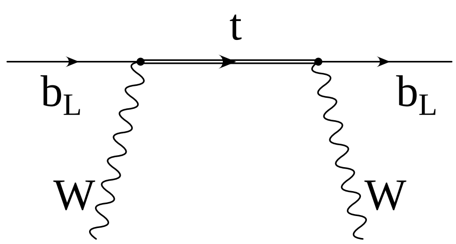


Figure 1: Diagram contributing to the matching in the unitary gauge. It is suppressed by $1/m_t^2$.

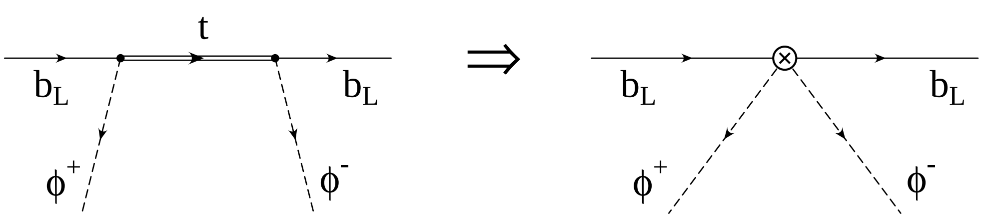


Figure 2: Diagram contributing to the matching in non-unitary gauges. It is not suppressed by $1/m_t^2$.

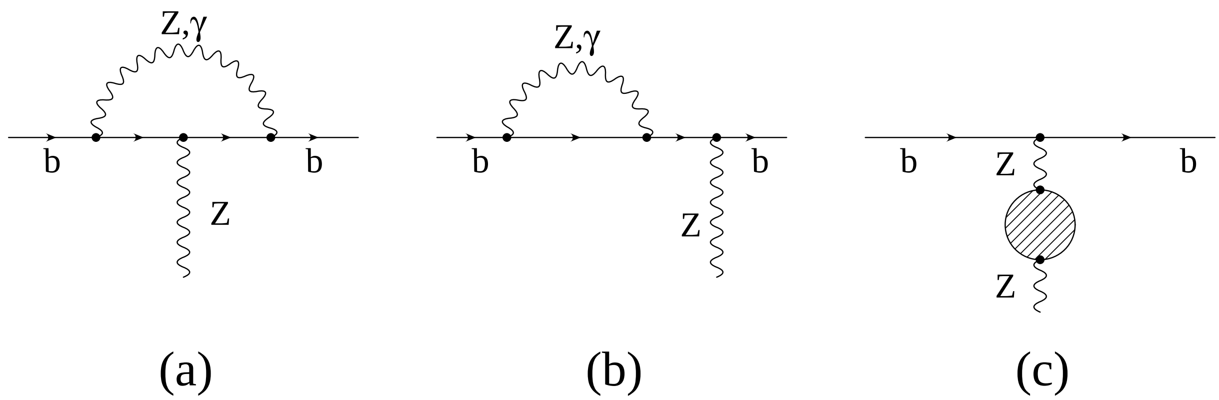


Figure 3: Running of the vertex Z-b-b in the effective theory.

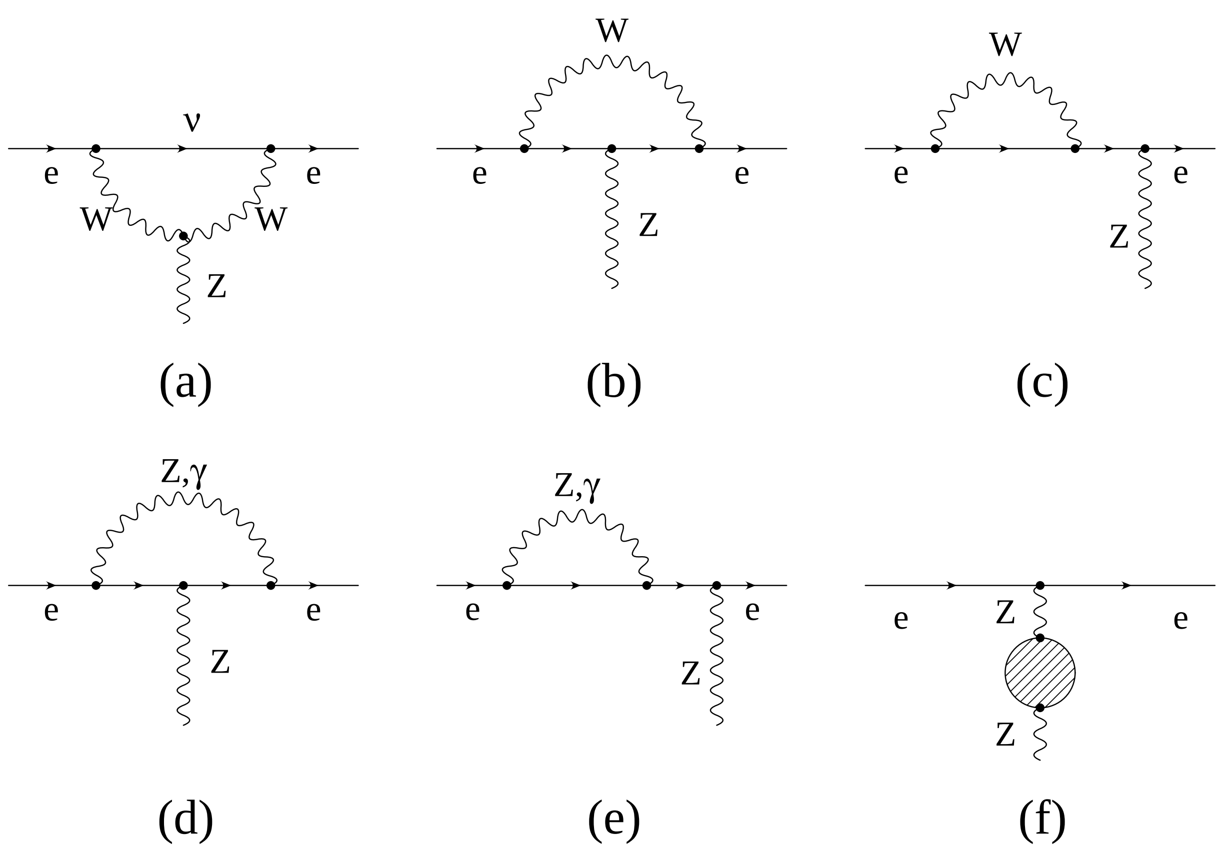


Figure 4: Running of the vertex Z-e-e in the effective theory.

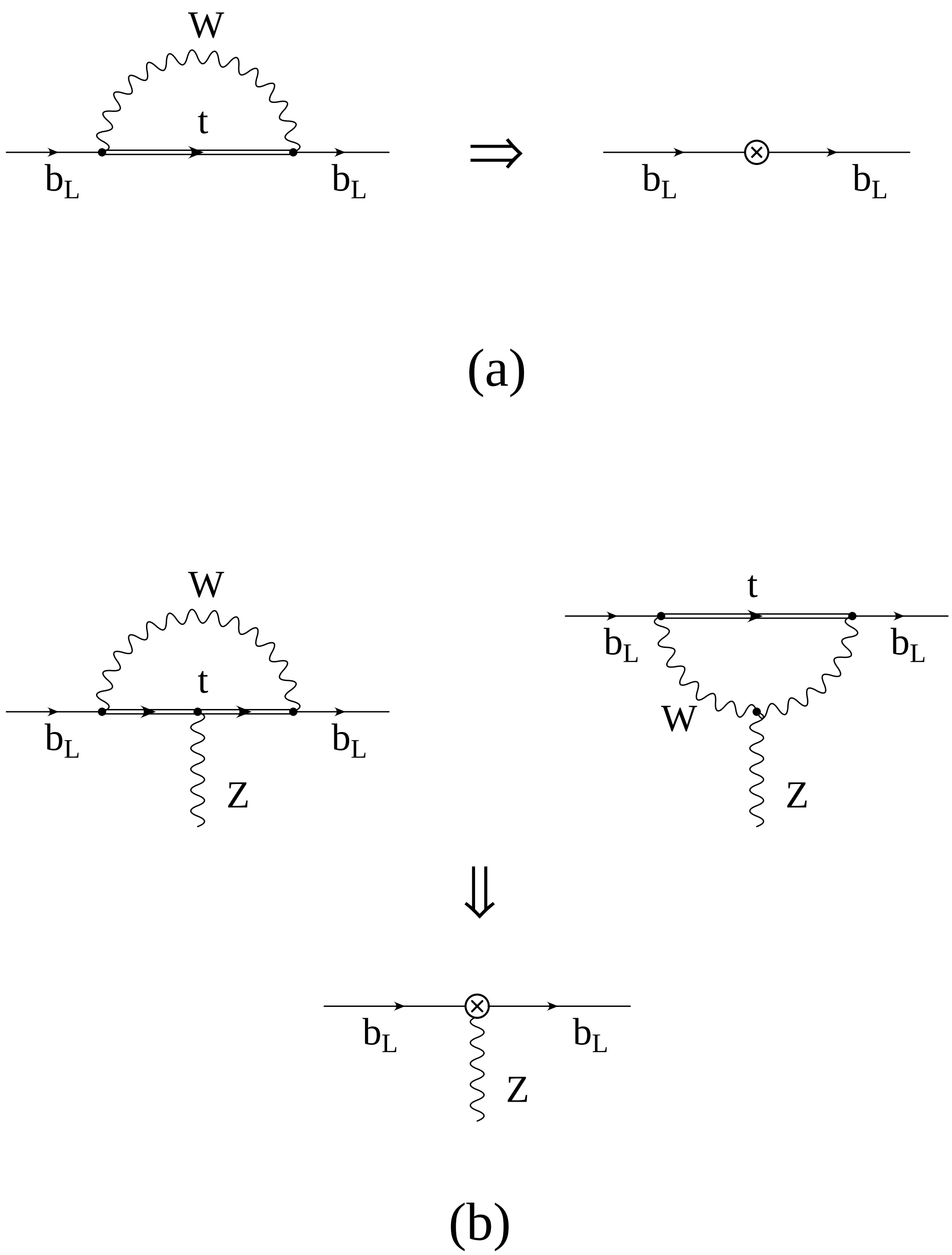
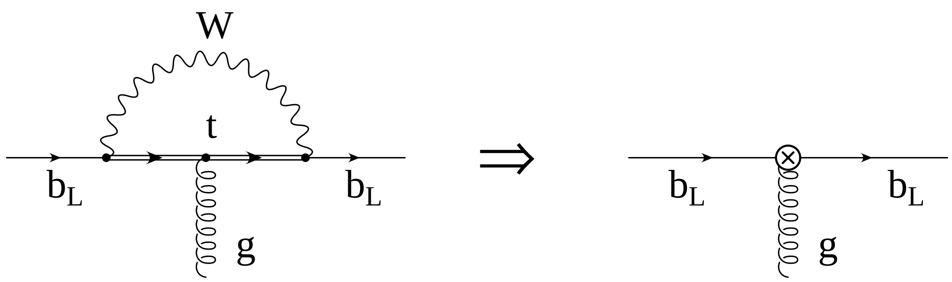
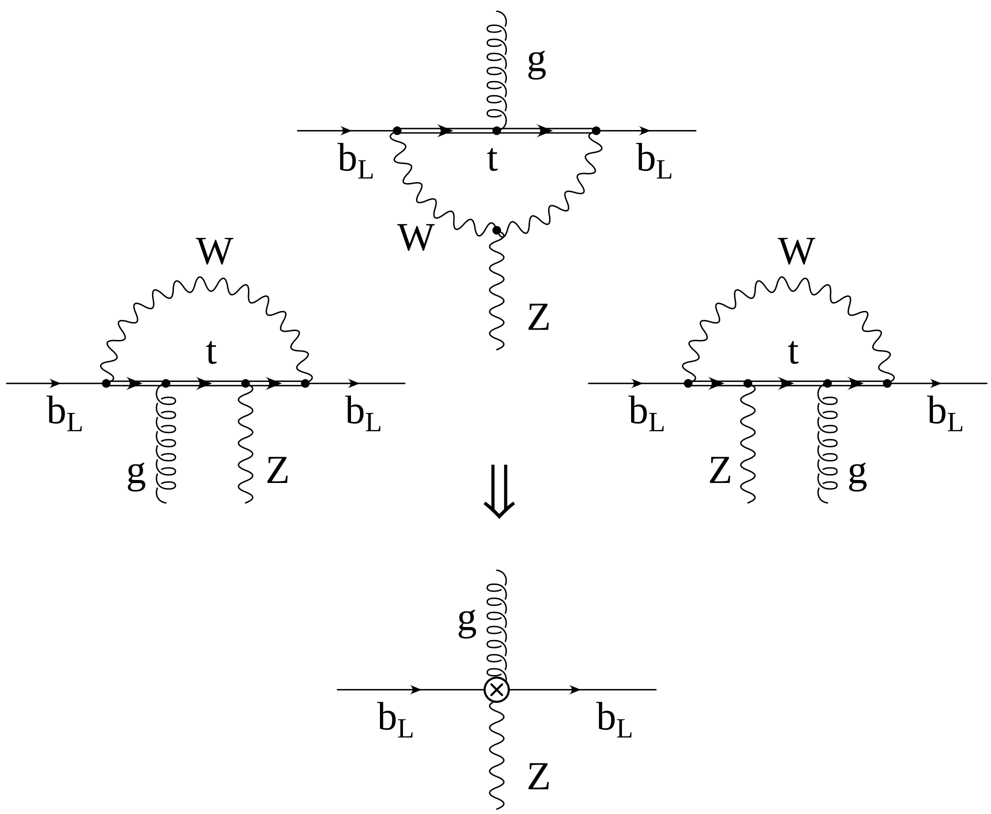


Figure 5: One-loop matching: QCD switched off.



(a)



(b)

Figure 6: One-loop matching: QCD switched on.

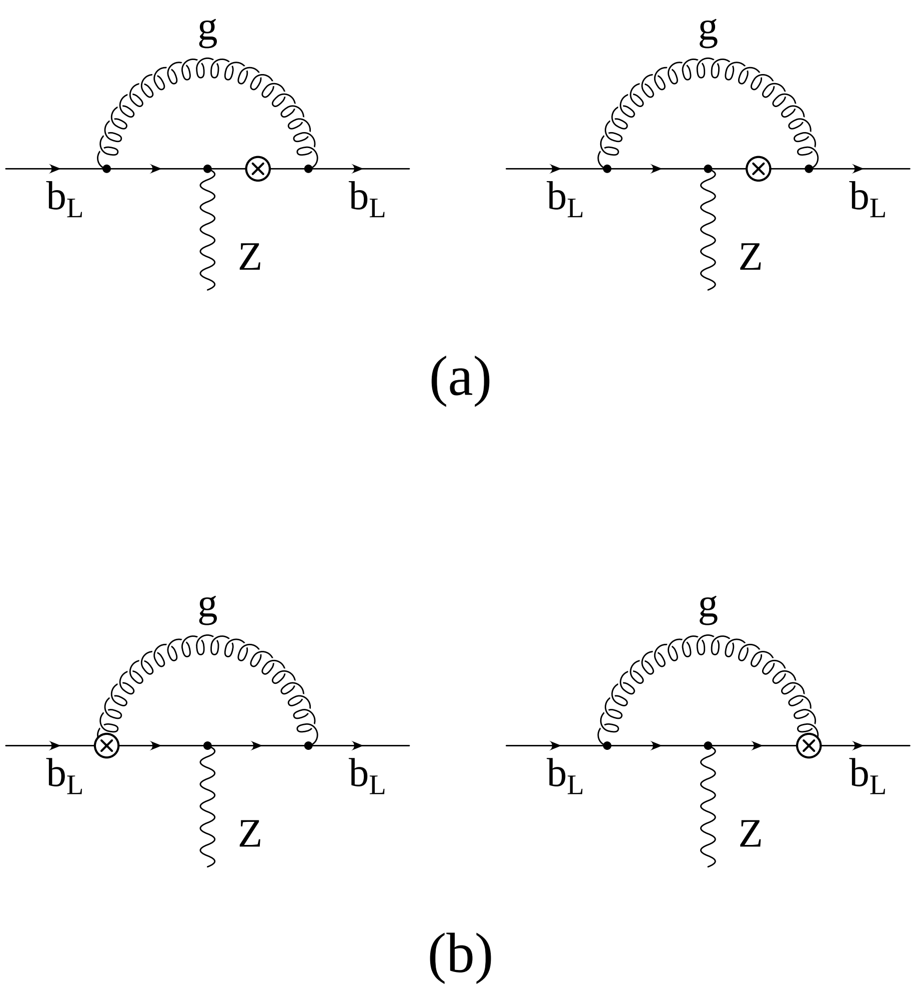
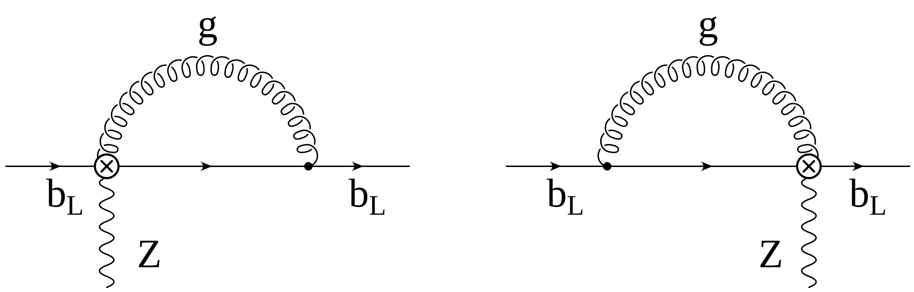
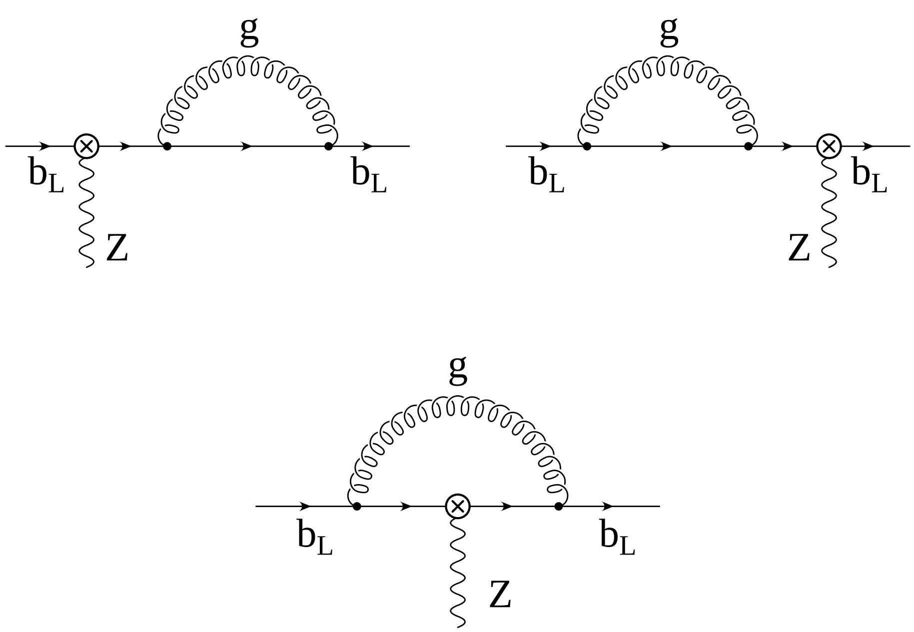


Figure 7: QCD running: Insertion of b-b and b-b-g operators.



(a)



(b)

Figure 8: QCD running: Insertion of b-b-Z and b-b-Z-g operators.

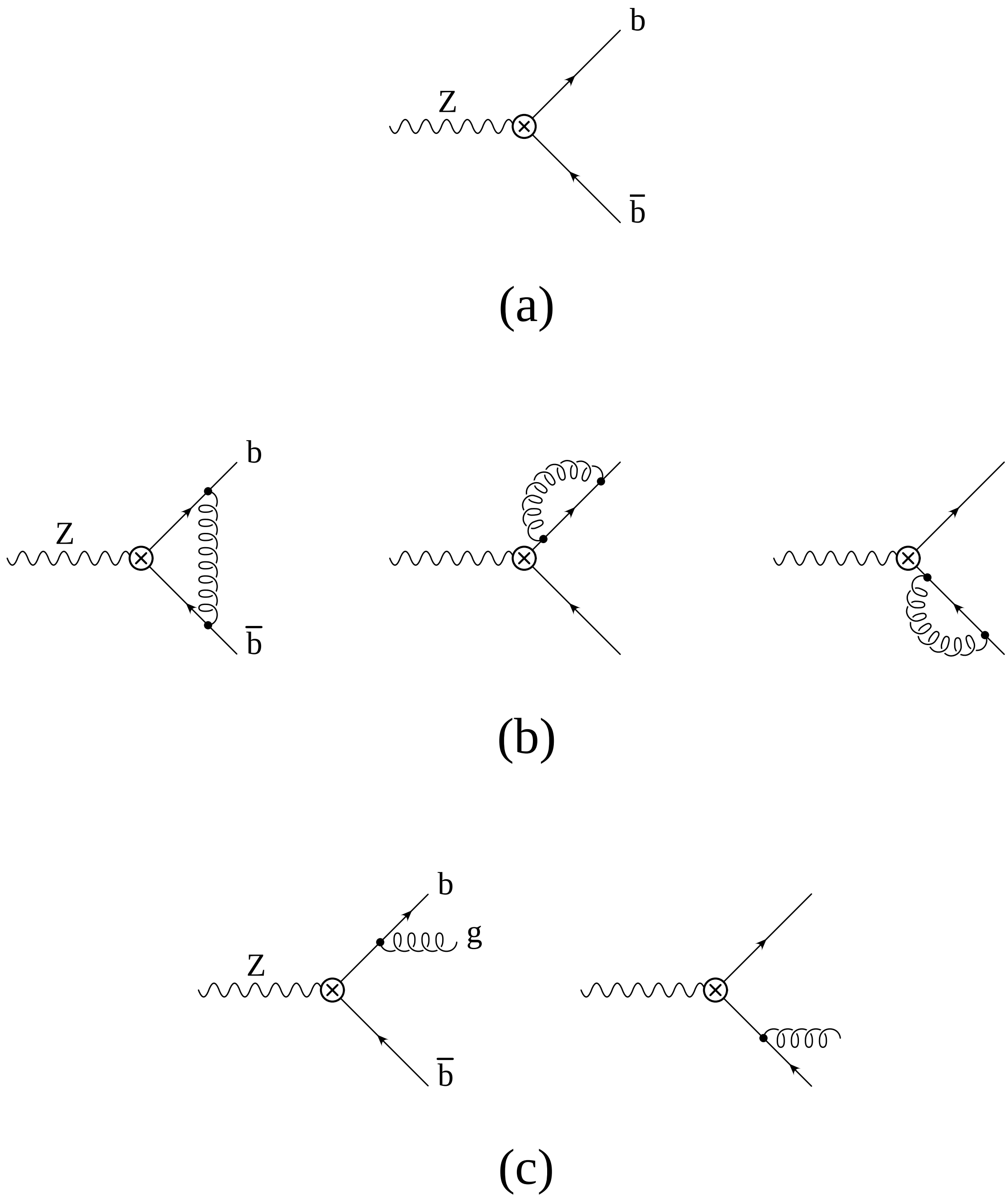


Figure 9: Diagrams contributing to the matrix element of $Z \to b\bar{b}$ in the effective theory.