Economic and Statistical Measurement of Physical Capital: From Theory to Practice*

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May 14, 2018.

*The authors acknowledge the financial support from the Spanish Ministerio de Economía y Competitividad, Projects ECO2015-65049-C2-1-P and ECO2016-76818-C3-3-P as well as the support of the Belgian research programmes ARC on Sustainability. The authors also acknowledge the financial support from the Generalitat Valenciana PROMETEO/2016/097.

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Abstract:

The standard measurements of capital and depreciation are statistical measures based on assumptions about the average service life of capital goods, which are accumulated according to the perpetual inventory method. The purpose of this paper is to obtain a true economic measure of capital stock according to the prescriptions of the neoclassical theory. In this way, we develop an alternative method based on the equations that solve the dynamic optimization problem of the firm, yielding an economic estimation based on indicators of profitability, such as the distributed profits and the Tobin's q ratio. Thus, this method enables us to endogenously calculate the variables' rate of depreciation and capital stock. We apply this procedure to the Spanish economy, getting the series of an economic measure of depreciation and the economic value of capital stock. Our results show an economic depreciation rate that fluctuates around the statistical rate. Moreover, we get two time profiles for the economic and statistical capital stocks that differ significantly from each other.

Keywords: Capital, Depreciation, Maintenance, Profitability, Tobin's q.

JEL classification: C61, D21, E22.

1 Introduction

Capital theory was subject to important convulsions during the years of the Cambridge controversies between Neokeynesian and Neoclassical schools. Standing out from the different dimensions of such a clash of views are, the theory of economic growth, the theory of value, and the theory of distribution between the factors of production. It is well known that the different elements of confrontation are not independent of each other. However, our interest here is more on the consequences of the debate concerning the measure of capital, which involves the other debates about the role of concepts like production function, factor substitution, prices on shares, and marginal products of factors at equilibrium. After some contributions from both sides, including the proposal of measuring capital in physical units as the labor hours, the neoclassical view ended up hegemonic in the literature on capital, placing the concept of aggregate capital measured in terms of value at the very center of the subsequent modelization.

According to Harcourt (1972) " *Aggregate production functions which invoke the concept of aggregate capital have been used not only in the pure theory of value, distribution, and growth, but also in the early post-war econometric studies of productivity growth over time, and of the possibilities of capital-labour substitution in economies and individual industries.*" Associated with this dominant methodology, malleability, which allows abstracting from specificity and heterogeneity of capital goods, came to occupy a central position in the neoclassical scene. In addition, a kind of consensus was established among mainstream researchers who accepted the measure of aggregate capital at equilibrium under the assumptions of a perfectly competitive economy, perfect foresight, lack of uncertainty, and static expectations.

In this context, as detailed in Bitros (2010), Jorgenson (1963, 1974) placed the so-called proportionality theorem at the forefront of capital theory research. This theorem is now well known because of the hypothesis concerning depreciation in the process of getting a measure of capital. According to Jorgenson, the rate of depreciation of physical capital is a constant proportion of the capital stock. In any case, this assumption of the theorem became a central feature of the algorithm designed to produce measures of capital stock through the perpetual inventory method.

Why is this methodological proposal so important? The neoclassical theory of capital is based on the hypothesis of an aggregate production function, which establishes a relationship between output, labor, and capital. Of the three elements, capital is the most problematic to measure, and this is because, among other reasons, the theory claims a measure in terms of value for capital. With respect to production and labor, there is a fairly broad consensus as to how they should be quantified. A consequence of this general agreement is the existence of standardized records that are widely accepted by researchers and society as a whole. In contrast, things are different when talking about the physical capital input. There are significant discrepancies both at the theoretical level and in practical application.

First, capital is a stock that provides services.[1] This stock has its own law of motion, with additions and deductions. On the one hand, capital formation is, more or less, a clear-cut concept that is well known and easily identifiable as gross investment. On the other, the meaning of capital deductions is not so clear, or at least it entails a disparity of opinions about what is and what is not. People often distinguish between consumption of fixed capital, decline in productive capacity, efficiency losses, obsolescence, replacement, retirements, scrapping, and so on. Instead, we will refer to all of these as depreciation, but we shall still consider a fundamental differentiation depending on its immediate cause. In this way, we could have

[1] See Baily (1981) and Oulton and Wallis (2016).

deterioration and obsolescence. Deterioration, which may be both physical (output decay) and economic (input decay), is an inherent characteristic of capital goods associated with the aging, use, and maintenance of equipment. Obsolescence, which may be both technological and structural, comes from outside the productive assets suffering from obsolescence, and appears to be associated with either technical progress (mainly embodied but also disembodied), energy prices, patterns of international trade, regulatory programs linked to environmental policy, or changes in the output composition that affect the relative prices.

Second, there is the main issue of valuation. Despite the difficulty involved from an empirical point of view, the estimation of price and quantity indices for the capital goods that stand for gross investment is a feasible task, as transactions are market observable transactions. Consequently, the measure of capital in terms of value, as the neoclassical theory claims, depends fundamentally on the possibility of obtaining a measure of depreciation in terms of value. This is a more difficult task because the economic transactions that have to do with capital depreciation activities are basically unobservable transactions. Therefore, what we really need is the shadow price that makes up the net present value condition, a condition stating that the market value of an asset is equal to the discounted stream of future benefits that it is expected to yield. We agree with Bitros when he asks for a model with strong microeconomic foundations that treats depreciation as an endogenous variable, because the capital stock cannot be correctly measured in the absence of a theory that explains how investment and depreciation are determined.

Faced with this requirement, Jorgenson's hypothesis of proportionality offers a weak theoretical answer, in that only depreciation caused by output decay receives due attention, leading researchers to make flawed economic predictions in the short run when economies experience important demand fluctuations as well as in periods of accelerated technological and structural change. However, from the point of view of empirical applications, and according to the OECD (2009) report, national and international organizations, as well as private and government agencies, continue to provide capital stock series on the basis of constant depreciation rates that are inversely proportional to the useful economic life of assets.[2]

The measure of capital stock obtained following the precepts of the OECD is a statistical measure of capital, not an economic measure in terms of value. This is because depreciation is estimated by adopting arbitrary assumptions about the functional form of three interconnected statistical functions: retirement, age-price, and age-efficiency functions, the parameters of which are taken from empirical studies. Consequently, because the crucial role of utilization, maintenance, and embodied technical progress is ignored, depreciation is considered a technical necessity rather than the result of economic decisions.

The purpose of this paper is to address the challenge posed by the neoclassical theory of capital and to achieve the goal of obtaining a true economic measure of capital stock at the aggregate level. To do this, we intend to act in three areas simultaneously. First, we develop a dynamic optimization model that endogenizes the depreciation rate by adding it to the set of controls, together with the variables gross investment and employment. This optimal control problem may be read as a first step toward a more general theory explaining simultaneously the behavior of investment and depreciation. The key element here is the maintenance and repair expenditures, which, combined with the adjustment costs, will contribute to show that decisions about investment and depreciation, the major decisions involved in the dynamics of capital stock, are interrelated and are not independent of each other. Despite the similitude

[2]This is the type of capital measures provided by the main databases in Europe (EU-KLEMS and AMECO), in the US (BEA and BLS), as well as in Spain (IVIE-BBVA and BD.MORES).

with those models found in the theoretical literature on endogenous depreciation, our model has the advantage of going beyond the routine study of the comparative dynamics focused on control variables. Here, we explore the possibility of obtaining empirical measurements of the endogenous variables with the theoretical support of the model and the equations that characterize it.

Second, from the solution of the model, we get two equations organized in an algorithmic form that are ready to be implemented. We apply this procedure to the Spanish data, getting the series of an economic measure of depreciation and the economic value of capital stock. This method yields an estimation based on profitability indicators such as firms' distributed profits and Tobin's q ratio. Although these ideas were initially raised in Escribá-Pérez and Ruiz-Tamarit (1995a,b), we take an important step forward and develop the old ideas in a renewed and integrated theoretical-empirical framework. According to Triplett (1996), the market-based measure of economic depreciation and the corresponding economic value of capital stock are suitable for both the economic accounts of income and wealth and production analysis and productivity measurement. Consequently, in this paper, we get a measure of the capital stock that may represent simultaneously the net stock of capital in terms of wealth, and the productive capital stock, from which it is immediate to deduce a proportional flow of capital services.

With these results in hand, we revisit the measures of depreciation and capital obtained under the rules of the proportionality theorem. Taking the Spanish economy as reference, we compare our series with those supplied by agencies that follow the recommendations of the OECD. In other words, we compare the economic measures of depreciation and capital stock with the more standard measures, which are statistical in nature, based on the assumption of constant useful lives, and accumulated according to the perpetual inventory method. The scenario of the Spanish economy during the period 1964-2011 differs substantially in their short-run evolutions, but it is possible to establish a close correspondence between the long-run profiles. In particular, we observe that the economic depreciation rate fluctuates around the statistical depreciation rate, which may be read as if the long-run statistical measurement of depreciation were a good approximation of the long-run economic depreciation.

This paper is organized as follows. Section 2 provides a theoretical model of optimizing behavior for the representative competitive firm. In this context, we derive an algorithmic procedure that allows us to calculate, in economic terms, the depreciation rate and the capital stock. Section 3 briefly describes the standard procedure employed to obtain the statistical measures of capital stock and the rate of depreciation. In Section 4, we apply our model to the Spanish data and show the *traditional* statistical and the *new* economic measures of capital and depreciation rate. Section 5 summarizes and concludes.

2 The economic measurement of capital

Let us consider the supply side of an economy consisting of a relatively large fixed number of identical firms. In this section, we provide the theoretical model that shows the optimizing behavior of the individual price-taking firm in a competitive environment. However, given the representative agent assumption, the variables of the model may also represent aggregate levels and, therefore, we can read it as if the decisions were made by the economy as a whole. The optimization problem that solves our agent is an intertemporal maximization problem that generalizes the standard problem in which the employment $L(t)$ and the investment $I^G(t)$ are controlled in order to maximize the present discounted value of cash-flow. The generalization consists in adding to the set of controls the depreciation rate of capital $\delta^*(t)$.[3] Output is produced according to a neoclassical production function $Y(t) = A^*(t) F(L(t), K^*(t))$, where $A^*(t)$ is the exogenous technological level and $F(.)$ is an homogeneous of degree one function. This optimal control problem has a single state variable so it includes a dynamic constraint that expresses the accumulation process of capital stock $K^*(t)$. In general we can write

$$\max_{\{K^*, L, I^G, \delta^*\}} V(t_0) =$$

$$\int_{t_0}^{+\infty} \left(G\left(p(t), A^*(t), K^*(t), L(t), I^G(t), \delta^*(t)\right) - W(t) L(t) - p^k(t) I^G(t)\right) e^{-\int_{t_0}^{t} R(s)ds} dt$$

$$s.t. \qquad \overset{\bullet}{K}{}^*(t) = I^G(t) - \delta^*(t) K^*(t), \tag{1}$$

$$K^*(t_0) = K_0^* > 0.$$

For the sake of simplicity we normalize to unity the price of output, $p(t) = 1$. The price of labor $W(t)$, the market price of capital goods $p^k(t)$ and the nominal interest rate $R(t)$ are given for the individual firm. Function $G(.)$ represents the value of net production after discounting investment-related adjustment costs $C\left(I^G, K^*\right)$ and the maintenance expenditures that contribute to determine the rate of depreciation $M(\delta^* K^*, K^*)$. These two functions are assumed homogenous of degree one in its corresponding determinants.[4] Under the above assumptions we can write $G\left(A^*, K^*, L, I^G, \delta^*\right) = A^* F(L, K^*) - \phi\left(\frac{I^G}{K^*}\right) K^* - m(\delta^*) K^*$, and we can also characterize the function by means of the sign of the first and second derivatives with respect to the controls and the state variable. That is, $G_L = A^* F_L > 0$, $G_{LL} = A^* F_{LL} < 0$, $G_{I^G} = -C_{I^G} = -\phi' < 0$, $G_{I^G I^G} = -C_{I^G I^G} = -\frac{\phi''}{K^*} < 0$, $G_{\delta^*} = -m' K^* > 0$, $G_{\delta^* \delta^*} = -m'' K^* < 0$. Moreover, given that $\phi(.)$ is strictly convex we get $C_{K^*} = \phi - \phi' \frac{I^G}{K^*} < 0$ and $C_{K^* K^*} = \phi'' \frac{\left(I^G\right)^2}{(K^*)^3} > 0$, consequently $G_{K^* K^*} = A^* F_{K^* K^*} - C_{K^* K^*} < 0$. Finally, we assume that the net marginal productivity of capital is positive, $G_{K^*} = A^* F_{K^*} - C_{K^*} - m > 0$.

When we add the dynamic constraint to the objective functional by introducing one more variable (the multiplier μ) as expression of the shadow price of capital, we get the following Hamiltonian function written in current value

$$H^c = G\left(A^*(t), K^*(t), L(t), I^G(t), \delta^*(t)\right) - W(t) L(t) - p^k(t) I^G(t) + \mu(t)\left(I^G(t) - \delta^*(t) K^*(t)\right). \tag{2}$$

In addition, partly for technical reasons and partly for economic reasons, we assume that the discount rate for the cash-flow is given exogenously and is perceived as a constant R.[5] We

[3] We consider Hayashi's (1982) model as the standard problem, which perfectly summarizes all the previous literature corresponding to the neoclassical model with investment-related adjustment costs. The incorporation of the depreciation rate to the endogenous variables of the model is possible thanks to its link with maintenance and repair expenditures. See Escribá-Pérez and Ruiz-Tamarit (1996), Boucekkine and Ruiz-Tamarit (2003), and Kalyvitis (2006).

[4] Actually, the maintenance cost function should be written as $M(D^*, K^*)$, where $D^* = \delta^* K^*$ represents the depreciation in level. Therefore, our homogeneity assumption involves the variables D^* and K^* instead of δ^* and K^*.

[5] It is well-known after Obstfeld (1990) and Marin-Solano and Navas (2009) that under a non-constant discount rate, either exogenous or endogenous, the Pontryagin's maximum principle cannot be applied directly

can then apply in a simple way the Principle of Maximum, from which we get the necessary conditions for the control variables

$$H^c_L(.) = 0 = A^*(t)\, F_L(L(t), K^*(t)) - W(t), \tag{3}$$

$$H^c_{I^G}(.) = 0 = -\phi'\left(\frac{I^G(t)}{K^*(t)}\right) - p^k(t) + \mu(t), \tag{4}$$

$$H^c_{\delta^*}(.) = 0 = -m'(\delta^*(t))\, K^*(t) - \mu(t)\, K^*(t). \tag{5}$$

As well as the following Euler equation

$$\overset{\bullet}{\mu}(t) = R\mu(t) - H^c_{K^*}\left(A^*(t), K^*(t), L(t), I^G(t), \delta^*(t)\right), \tag{6}$$

where $H^c_{K^*}(.) = A^* F_{K^*} - \left(\phi - \phi' \frac{I^G}{K^*}\right) - m - \mu\delta^*$, the dynamic constraint

$$\overset{\bullet}{K^*}(t) = I^G(t) - \delta^*(t)\, K^*(t), \tag{7}$$

and the transversality condition

$$\lim_{t\to+\infty} \mu(t)\, K^*(t)\, e^{-R(t-t_0)} = 0. \tag{8}$$

The first order conditions (3)-(5) implicitly define a system of three control functions. After total differentiation, and according to the implicit function theorem, we get the results that allow us, with the help of the sign of the partial effects, to identify some of the main results from the neoclassical theory of factor demand[6]

$$L = L\left(\overset{+}{K^*}(t); \overset{+}{A^*}(t), \overset{-}{W}(t)\right), \tag{9}$$

$$I^G = I^G\left(\overset{+}{K^*}(t), \overset{+}{\mu}(t); \overset{-}{p^k}(t)\right), \tag{10}$$

$$\delta^* = \delta^*\left(\overset{-}{\mu}(t)\right). \tag{11}$$

Beyond these generic results and according to the main goal of our paper, we find that the differential equation (6) may be integrated forward solving for $\mu(t)$, under the non-explosivity condition $\lim_{t_F\to+\infty} \mu(t_F) \exp\left\{-\int_t^{t_F} (R + \delta^*(\tau))\, d\tau\right\} = 0$. The result we get may be put in terms of the model-based definition of marginal q,

$$q^M(t) = \frac{\mu(t)}{p^k(t)} = \tag{12}$$

because the corresponding intertemporally dependent preferences can create a time-consistency problem. Examples of how to proceed in the case of a variable discount rate may be found in Palivos et al. (1997), Ayong and Schubert (2007), and Boucekkine et al. (2018).

[6] These are not yet the usually pursued explicit labor demand function and the corresponding investment and depreciation functions. For that, we first would need to substitute (9)-(11) into (6) and (7). Then, we should have to solve the resulting dynamic system for K^* and μ depending on the exogenous variables and parameters. Finally, with the solution trajectories just found we could come back to the control functions and get the expected true factor demand functions depending only on the exogenous variables and parameters. However, this far-reaching goal is beyond the scope of the present paper.

$$\frac{\int_t^{+\infty}\left(A^*(s)\,F_{K^*}(L(s),K^*(s))-\left(\phi\left(\frac{I^G(s)}{K^*(s)}\right)-\frac{I^G(s)}{K^*(s)}\phi'\left(\frac{I^G(s)}{K^*(s)}\right)\right)-m(\delta^*(s))\right)e^{-\int_t^s(R+\delta^*(v))dv}ds}{p^k(t)}.$$

That is, the present value of the future stream of the net marginal productivity of capital, discounted by the sum of the constant discount rate plus the variable depreciation rate, and all that divided by the current market price of one unit of capital. In other words, the quotient between the shadow price of one unit of capital (the Hamiltonian multiplier μ) and its replacement cost (the market price of capital goods p^k). This is the variable which directly explains the flows of investment and depreciation that determine the dynamics of the capital stock.

Moreover, the property of homogeneity assumed on the production and cost functions as well as the first order conditions arising from the dynamic optimization problem, allow us to set the following linear ordinary differential equation in $X=\mu K^*$: $\overset{\bullet}{X}=RX-A^*F(L,K^*)+\phi\left(\frac{I^G}{K^*}\right)K^*+m(\delta^*)K^*+WL+p^kI^G$. It may be integrated forward solving for the product $\mu(t)\,K^*(t)$, under the transversality condition (8). The result we get may be put in terms of the model-based definition of average q, the quotient between the market value of (all) the firm(s) and the economic value of the capital stock measured in nominal terms at its replacement cost,[7]

$$q^A(t)=\frac{\mu(t)\,K^*(t)}{p^k(t)\,K^*(t)}= \tag{13}$$

$$=\frac{\int_t^{+\infty}\left(G\left(A^*(s),K^*(s),L(s),I^G(s),\delta^*(s)\right)-W(s)\,L(s)-p^k(s)\,I^G(s)\right)e^{-R(s-t)}ds}{p^k(t)\,K^*(t)}.$$

Given (12) and (13) it is apparent the equality between marginal and average q.[8] That is, the two theoretical q ratios are equivalent and, according to the literature, they can be empirically approximated by the observable Tobin's q_t, which is the ratio of the stock market value of the firm to the current-cost book value of capital assets. Available measures of Tobin's q ratio are based on financial market valuation of the corporate assets corresponding to the fundamentals of the firm, as well as on data obtained from balance sheets. Consequently we can use them as exogenous proxies for our model-based q ratios.[9]

From now on, we will simplify notation calling the market value of the firm along the optimal equilibrium path as V_t^*, and we shall rewrite variables in discrete terms to make the relevant expressions computationally operative. Then, on the side of the value of the firm we have

$$q_t=\frac{V_t^*}{p_t^kK_t^*}. \tag{14}$$

On the other side, the stock of capital is determined at each moment according to the first-order difference equation

$$K_t^*=I_t^G+(1-\delta_t^*)\,K_{t-1}^*. \tag{15}$$

[7]Although we are expressing economic or market values, we can use, as in the literature, different names to refer to them. The valuation at replacement cost, in nominal terms or at current prices are equivalent, and means that the capital goods are valued at the prices of the current period. This is opposed to valuation at historical prices, which means that the assets are valued at the prices at which they were originally purchased.

[8]See Hayashi (1982), Blanchard et al. (1993) or Kalyvitis (2006).

[9]According to Siegel (2008) most of the empirical measures of Tobin's q use financial data and adjust for inflation to compute the replacement cost of the assets and liabilities. However, some authors such as Madsen (2016) distrust these measures due to the fear that they might be (re-)introducing the value of capital at acquisition cost into the calculations. Consequently, they prefer to use other methods to approach the Tobin's q, such as the deviation of the log of real share prices from its time trend, which minimizes the risk of inaccuracy and circularity when calculating the stock of capital.

Given the capital stock of the previous period, by adding the flow of gross investment I_t^G and subtracting the depreciation flow $\delta_t^* K_{t-1}^*$, we obtain the stock of capital of the current period. Although the gross investment is a control variable in the model, from the point of view of our computations it will be considered as a forcing variable. Since the transactions related to the acquisition of capital goods are observable transactions in the market, we shall consider these records as the result of optimal decisions and will introduce them in our calculations. In this way, we can try to obtain a simpler algorithm to generate the values of depreciation and capital stock that are not directly observable in the market.

Furthermore, under the assumption that financial markets are competitive, we can specify the market value of the firm V_t^* as the discounted present value of the infinite flow of distributed profits, B_t^*. The fact that both variables are in nominal terms imply that discount is made with the nominal interest rate, R_t,

$$V_t^* = \sum_{s=t}^{\infty} \frac{B_s^*}{\prod\limits_{\tau=t+1}^{s} (1 + R_\tau)}. \tag{16}$$

In this intertemporal context, it is important to know the way in which agents form their expectations about the future values of the variables. We will assume that economic agents have static expectations, so that in each period they consider that the current value will be repeatedly observed in the future. If we apply such an assumption to the terms of equation (16) we will find that, $\forall s, \tau \in [t, \infty[$, $B_s^* = B_t^* \left(1 + \pi_s^k\right)^{s-t}$ being $\pi_s^k = \pi_t^k$ the inflation rate associated with the price index of capital goods p^k, and $\prod\limits_{\tau=t+1}^{s} (1 + R_\tau) = (1 + R_\tau)^{s-t}$ with $R_\tau = R_t$. We define the real interest rate $r_t = R_t - \pi_t^k > 0$ and approximate the term $\frac{1+\pi_t^k}{1+R_t} = 1 + \pi_t^k - R_t$, taking the product $r_t R_t$ as negligible. Then, we can write

$$V_t^* = B_t^* \sum_{s=t}^{\infty} \left(\frac{1 + \pi_t^k}{1 + R_t}\right)^{s-t} = B_t^* \sum_{s=t}^{\infty} (1 - r_t)^{s-t} = \frac{B_t^*}{r_t}. \tag{17}$$

Substituting this result in (14) we get

$$q_t = \frac{B_t^*}{r_t p_t^k K_t^*}. \tag{18}$$

Equations (18) and (15) give us a clear idea of how the process of capital accumulation is defined in economic terms but they also tell us how the markets evaluate this process. In these equations we are considering the economic or market value of each of the variables. On the quantity-variables side: distributed profits, flows of gross investment and depreciation, and the capital stock; and on the side of the price-variables: interest rate, price of investment goods and the Tobin's q ratio.

Concerning the revenues of productive factors generated by firms and distributed through the market mechanisms, it must be remarked that the sum of the economic value of net distributed profits, B_t^*, and the flow of economic depreciation in nominal terms is equal to the distributed gross profits, $B_t^G = B_t^* + \delta_t^* p_t^k K_{t-1}^*$. Substituting in (18) we get

$$q_t r_t p_t^k K_t^* = B_t^G - \delta_t^* p_t^k K_{t-1}^*. \tag{19}$$

Therefore, if we know the value of all the price variables as well as the value of the economic-accounting flows of gross investment and gross distributed profits, we can use sequentially the equations (15) and (19) to obtain the values of the depreciation rate δ_t^* and the capital stock K_t^*. This dynamic system of two first-order difference equations allows us to express the values of the two previous endogenous variables as a function of the variables q_t, r_t, p_t^k, B_t^G, and I_t^G, given the predetermined value of K_{t-1}^*. In other words, from a known initial value K_0^* we can solve forward to obtain the complete series for the capital stock and the depreciation rate. Next, we show in closed-form their explicit solutions:

$$\delta_t^* = \frac{\frac{B_t^G}{q_t r_t p_t^k} - K_{t-1}^* - I_t^G}{\left(\frac{1}{q_t r_t} - 1\right) K_{t-1}^*}, \tag{20}$$

$$K_t^* = \frac{K_{t-1}^* + I_t^G - \left(B_t^G / p_t^k\right)}{1 - q_t r_t}. \tag{21}$$

Hence, the comparative statics of these two variables is symmetric with each other, for all independent variables and prices. Ceteris paribus, we can see that an increase in investment expenditures will increase the capital stock but reduces the depreciation rate. Moreover, the higher the level of distributed profits the higher the depreciation rate and the lower the capital stock. Finally, Tobin's q ratio and the real interest rate are inversely (positively) correlated with the depreciation rate (capital stock).

3 The statistical measurement of capital

In the previous section we have discussed an algorithmic strategy for calculating the depreciation rate and the capital stock based on the optimal decisions of economic agents. That procedure is built on the basis that the depreciation of capital is endogenous, and, consequently, we can expect that it represents a variable percentage of capital. However, from Jorgenson's (1963) pioneering work, a very different interpretative model, known as the proportionality theorem, has been widely used. This theorem states that the depreciation and replacement of capital goods is done at a constant rate, proportional to the corresponding capital stock, which fits perfectly to an exogenously predetermined (average) service life. The hypothesis of proportionality as a dominant paradigm concealed in the background all the previous discussion that for years had focused on the study of the endogenous determination of the optimal service life for different vintages and types of capital assets, which suddenly eclipsed. As a result, a good approximation to the capital stock was considered to be the following weighted sum of current and lagged investment values,

$$K_t = \int_{-\infty}^{t} I_v^G e^{-\delta(t-v)} dv = \sum_{-\infty}^{v=t} I_v^G \left(\frac{1}{1+\delta}\right)^{t-v}. \tag{22}$$

This method, which adds at any moment the remaining undepreciated quantity of every past investment, is known as the perpetual inventory method (PIM)[10] and gives us, when differentiating (22) with respect to the time index t, the most well-known equation of accumulation

$$K_t - K_{t-1} = I_t^G - \delta K_{t-1}. \tag{23}$$

[10] In the more basic view of the inventory method, where capital goods experience withdrawals without decay or obsolescence, the measurement of capital stock involves accumulating all past capital formation, and deducting the value of assets that have reached the end of their service lives.

This way of interpreting the phenomenon of depreciation was challenged from the outset because it was based on assumptions that were too restrictive. The main problem with Jorgenson's theorem is that it focuses on age and ignores the role of economic variables, such as the intensity of use, maintenance and repair, obsolescence caused by embodied technical progress, uncertainty, and the business cycle, in determining the depreciation of capital assets. The problem was widely pointed out by Feldstein and Foot (1971), Eisner (1972), Feldstein and Rothschild (1974), Bitros and Kelejian (1974), Malcomson (1975), Nickell (1975), and Cowing and Smith (1977). These studies showed that depreciation varied considerably under the influence of conventional economic forces. Then, to be consequent with this fact, the dynamic process generating the capital stock should be represented as follows:

$$K_t = K_{t-1} + I_t^G - D_t. \tag{24}$$

Total depreciation D_t may change considerably, due not only to the variability generated by the own dynamics of the capital stock but also to the influence of the economic factors mentioned above. At this point, we probably should abandon the strict assumption of a unique and constant rate of depreciation that applies to the aggregate capital stock and implicitly define a variable rate of depreciation as

$$\delta_t = \frac{D_t}{K_{t-1}}. \tag{25}$$

This implicit depreciation rate reflects the variability of both the denominator and the numerator. But it may also be the case that it shows some variability simply because the composition of the capital stock is changing. The latter is what happens when we replace the assumption of a single constant rate with multiple constant rates, each one associated with a different type of capital asset.[11]

After a long period of discussion, the concrete application of the previous issues to the actual calculation of depreciation and capital stock has been carried out within the framework of methodological proposals from international organizations. The purpose of these proposals is twofold: to harmonize the uses and criteria of the different national statistical agencies and achieve the highest degree of homogeneity among the indicators calculated for the different countries.[12]

Following the recommendations of the OECD, most governmental and private statistical offices now accept the geometric depreciation pattern as the most suitable approximation to the loss in the value of assets as they age. Very often, the method employed to estimate the depreciation rate is the well-known double-declining balance method, summarized in the expression $\delta_i = 2/\bar{T}_i$, where $\bar{T}_i$ is the average service life for assets of type i. According to this method, the measurement of depreciation is directly associated with the fixed service life of the different assets. Beyond the complex statistical forms of survival-retirement distributions and age-efficiency functions chosen ad hoc, this is the simplest statistical method available to obtain a measure of depreciation, which is absolutely compatible with the postulates of the proportionality theorem.

[11] In this case, the endogeneity of the depreciation rate is not a substantial characteristic, because it is not a decision variable, but induced, because it depends on variables that are determined endogenously.

[12] A good description of the methodology used to estimate capital stock by the statistical agencies BEA and BLS may be found in Katz (2015).

Consequently, after aggregation, we can arrive expressing the depreciation as in (25) and the dynamics of the capital stock with the following equation that generalizes the permanent inventory method:

$$K_t = I_t^G + (1 - \delta_t)\, K_{t-1}. \tag{26}$$

At this point, we have equations (25) and (26) which allow us to perform a statistical measurement of depreciation and capital stock. Previously, equations (20) and (21) allowed us to reach an economic measurement of depreciation and capital stock. Now, a link, more conceptual than technical, can be established between the two accumulation processes. Although we have remarked on the short-run differences between the two measures of depreciation and capital stock, we have something new to state about the long-run relationship. The assumption regarding depreciation as a fixed, or almost fixed, proportion of capital stock is invalid in the short run, when the economy experiences important fluctuations due to the effects of continuous demand and supply shocks. However, after a while in which the economy has had time to absorb and adjust to the different monetary and real shocks, the statistical measurement of depreciation may be a good approximation of the economic depreciation. That is, in the long run the average service life of assets and the perpetual inventory method can offer a satisfactory measure of depreciation and the capital stock.

4 Capital and depreciation in Spain

In this section, we obtain the measures of economic capital stock and depreciation rate for the Spanish economy and compare them with the corresponding statistical measures. Therefore, we posit two different and apparently independent dynamic processes of capital accumulation. However, it is possible to establish some connections between them. Despite the two processes differing in the measure of depreciation, and the statistical depreciation flow implicitly defining a rate δ_t that does not have to coincide with the economic depreciation rate δ_t^*, both processes coincide in their measure of gross investment, I_t^G.

On the other hand, we have to decide among the several paths that are obtained integrating the dynamic process of capital accumulation and particularize specification of a boundary condition. We chose, as it is usual in these cases, the initial condition. Thus, in this context, and to establish a reasonable comparison between the two capital trajectories, the statistical and the economic series, we assume that, in the initial period, both measures of the capital stock coincide, $K_0^* = K_0$.

In this way, given the algorithmic procedure for calculating the depreciation rate δ_t^* and the capital stock K_t^* based on the optimal decisions of economic agents, as shown in section 2, we next apply this procedure to the non-financial business sector data of the Spanish economy during the period 1964-2011.[13]

As observables, we use gross investment I_t^G, price of capital goods p_t^k, gross distributed profits B_t^G, Tobin's ratio q_t for the period 1964-2011, and the predetermined value of the initial capital stock K_0. These series of the variables considered in the system of equations (20) and (21) are available in Table A.1 of the Appendix. Similarly, in that appendix, the statistical sources of the used variables as well as the process followed for their elaboration, are detailed.

Once the series of economic capital stock and depreciation rate are obtained, the next step is to analyze their evolution over the period compared with the corresponding series by using

[13] The non-financial business sector is defined as total activities in the economy excluding the financial intermediation sector, real estate and non-market services.

statistical measures of depreciation and capital, as described in section 3. Table 1 contains the detailed figures for the economic and statistical depreciation rates and stock of capital over the period 1964-2011. Figures 1 and 2 plot the evolution of the different measures of capital stocks and depreciation rates. In Figure 1, the economic depreciation rate compared to the statistical depreciation rate is presented, while, in Figure 2, the two time profiles for the economic and statistical capital stocks for the non-financial business sector for 1964-2011 are shown.

As can be seen in Table 1, we have established subperiods in which the values of the economic depreciation rate are lower than are those of the statistical depreciation rate (shaded in the table) and subperiods in which the reverse is true. The former subperiods correspond to expansions in the Spanish economy, and the latter subperiods correspond to recessions.

During the expansionary period of 1964-1974, the Spanish economy grew as a result of the Stabilization Plan to deal with the liberalization of the economy. In Figure 3, an enormous growth of capital stocks, $\overset{\wedge}{K^*} > \hat{K}$, can be observed as a consequence of the process of industrialization, the adoption of more intensive techniques in capital, and strong importing of capital assets. The economic depreciation rate over this period is much less, with an average value of 3.88%, than is the statistical rate, with a value of 6.25%, as a consequence of equipment maintenance expenditure. Furthermore, during these years, Tobin's q is always higher than unity, as can be observed in Table A.1 in the Appendix.

The next period is a period of crisis that started in the mid-1970s and lasted almost a decade, through to the mid-1980s. The economic capital stock stagnates significantly with $\overset{\wedge}{K^*} < \hat{K}$, as a result of the massive depreciation of manufacturing equipment. That, in turn, is a direct consequence of the high vulnerability stemming from the energy crisis and major industrial restructuring process –traditional and/or heavy industry subject to competition from the newly industrialized countries– that occurred during this period. The economic deterioration, but primarily the structural obsolescence, could be at the heart of the significant rise in depreciation, which cannot be explained by the simple physical deterioration of the equipment from age and normal use, as seen by the statistical depreciation rate. During this period, the economic depreciation rate is greater than is the statistical rate, with average values 10.59% and 6.86%, respectively. The disparity of values that can be seen by comparing our new estimates with the more traditional estimates shows the inappropriateness of the perpetual inventory method for calculating the capital stock in periods of major economic upheaval.

In the new expansionary period from the mid-80s to the start of the 2000s, the two time profiles, for the economic and statistical capital stock, are similar and indicate strong growth for both series, greater in the case of economic capital. Associated with this general recovery, the Tobin's q continues to be greater than one (except between 1991 and 1993) reaching its highest value in the year 2000. In this year, the turning point along the temporal line of evolution of capital stock, which shoots up to levels similar to those of the pre-crisis period, is shown. Only during the recession of 1992-1993 is a slowdown in the trend over this long a period observed. The economic depreciation rate is quite volatile and smaller than is the statistical rate in this period.

As of 2002, there is a slowdown in the rate of growth of economic capital stock, $\overset{\wedge}{K^*} < \hat{K}$, breaking away from the general trend and opening a large gap between K^* and K. The economic depreciation rate rises with an average value of 10.36% above the statistical rate, with a value of 7.52%. This could be the result of the information and communication technology (ICT) investment boom, which modifies production methods and increases the depreciation caused by economic deterioration and obsolescence. Moreover, the ICT investment causes non-residential investment to fall sharply, which can be seen in the official statistics in European countries in the early 2000s. The Tobin's q gradually decreases to values lower than 1. The last year in the sample marks the start of the recovery.

During the period 1964-2011, the performance of the Spanish non-financial business sector differs substantially in its short-run evolutions, but it is possible to establish a close correspondence between the long-run profiles. In fact, as can be observed in Figures 1 and 2, the economic depreciation rate fluctuates around the statistical depreciation rate and the economic capital stock fluctuates around the statistic capital stock. So, it can be concluded that the long-run statistical depreciation rate is a good approximation of the long-run economic depreciation. In the same way, when the economies have had time to adjust to the shocks, the statistical capital stock accumulated according to the perpetual inventory method is a satisfactory measure of the capital stock in the long run.

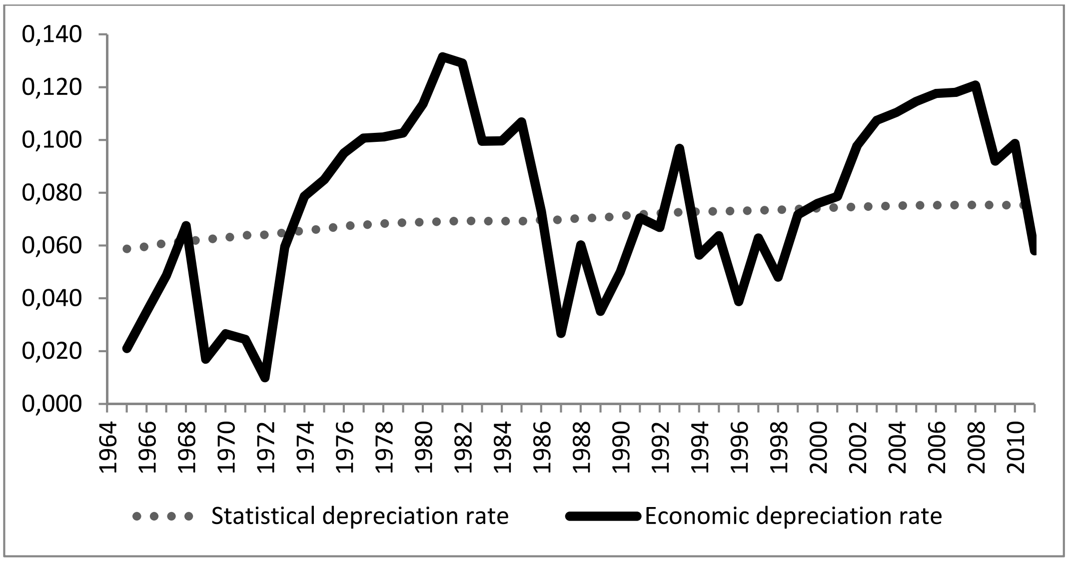

Figure 1. Economic and statistical depreciation rate: Spanish non-financial business sector, 1965-2011. Source: Own elaboration and official statistics (see Appendix).

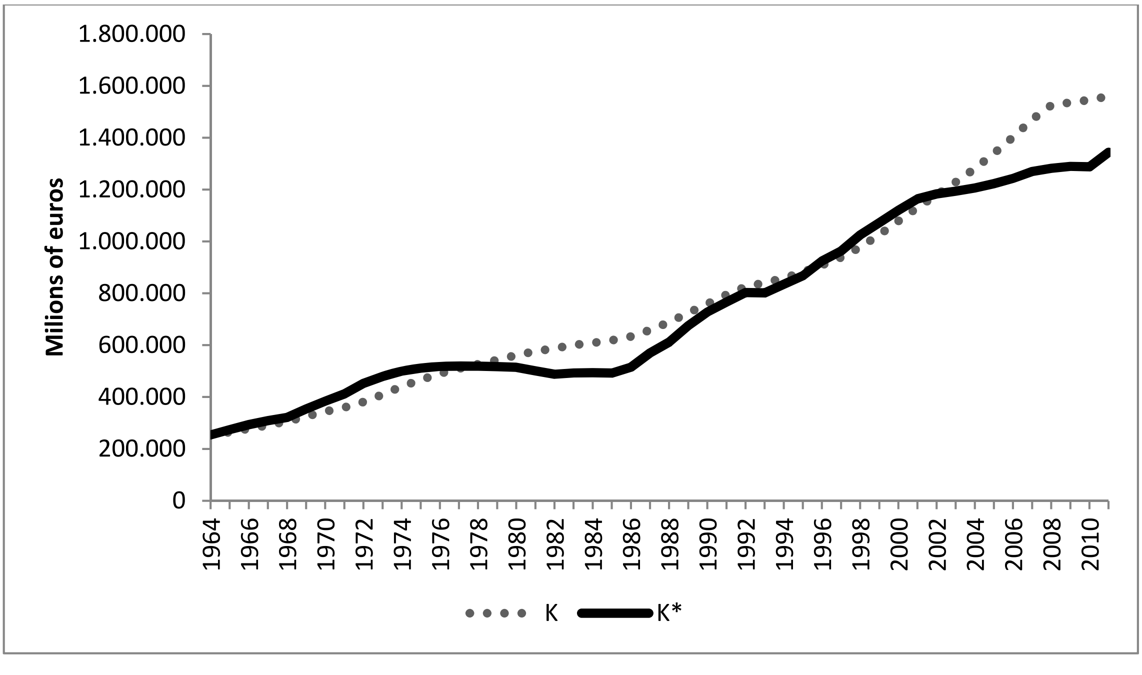

Figure 2. Economic and statistical capital stock: Spanish non-financial business sector, 1964-2011. Source: Own elaboration and official statistics (see Appendix).

Table 1. Economic and statistical depreciation rates and capital stock (millions of euros of 2008) in Spanish non-financial business sector (1964-2011).

Year	Depreciation rate Economic δ^*	Depreciation rate Statistical δ	Capital Stock Economic K^*	Capital Stock Statistical K	Year	Depreciation rate Economic δ^*	Depreciation rate Statistical δ	Capital Stock Economic K^*	Capital Stock Statistical K
1964			254,181	254,181	**1988**	0.0602	0.0704	612,356	688,898
1965	0.0211	0.0588	274,421	264,832	**1989**	0.0351	0.0706	675,318	724,751
1966	0.0349	0.0596	294,076	278,275	**1990**	0.0499	0.0713	728,277	759,779
1967	0.0487	0.0610	309,549	291,098	**1991**	0.0705	0.0718	766,555	794,893
1968	0.0676	0.0616	321,346	305,883	**1992**	0.0669	0.0723	803,245	825,419
1969	0.0170	0.0623	354,306	325,236	**1993**	0.0968	0.0727	801,618	841,582
1970	0.0266	0.0630	384,836	344,693	**1994**	0.0564	0.0729	834,929	858,780
1971	0.0245	0.0639	412,411	359,668	**1995**	0.0637	0.0730	868,298	882,688
1972	0.0100	0.0641	453,383	381,692	**1996**	0.0388	0.0732	925,454	908,963
1973	0.0595	0.0649	479,626	410,142	**1997**	0.0629	0.0734	963,780	938,831
1974	0.0787	0.0657	499,718	441,038	**1998**	0.0481	0.0736	1,025,848	978,137
1975	0.0850	0.0664	511,839	466,365	**1999**	0.0717	0.0739	1,072,631	1,026,272
1976	0.0950	0.0675	518,704	490,417	**2000**	0.0760	0.0741	1,120,565	1,079,612
1977	0.1008	0.0678	519,251	509,968	**2001**	0.0786	0.0744	1,164,844	1,131,660
1978	0.1011	0.0684	519,031	527,405	**2002**	0.0979	0.0747	1,183,795	1,180,095
1979	0.1027	0.0687	517,559	542,993	**2003**	0.1075	0.0749	1,194,287	1,229,456
1980	0.1136	0.0689	514,626	561,482	**2004**	0.1105	0.0751	1,206,816	1,281,636
1981	0.1315	0.0691	500,956	576,681	**2005**	0.1146	0.0752	1,223,034	1,339,742
1982	0.1292	0.0694	488,089	588,516	**2006**	0.1176	0.0753	1,243,424	1,403,097
1983	0.0995	0.0693	493,167	601,376	**2007**	0.1181	0.0754	1,269,952	1,470,684
1984	0.0997	0.0693	493,811	609,509	**2008**	0.1208	0.0754	1,282,322	1,525,592
1985	0.1068	0.0692	492,606	618,855	**2009**	0.0920	0.0753	1,290,060	1,536,363
1986	0.0729	0.0697	515,129	634,158	**2010**	0.0987	0.0753	1,288,119	1,546,047
1987	0.0267	0.0698	570,127	658,682	**2011**	0.0581	0.0753	1,343,990	1,560,230

Source: Own elaboration and official statistics (see Appendix).

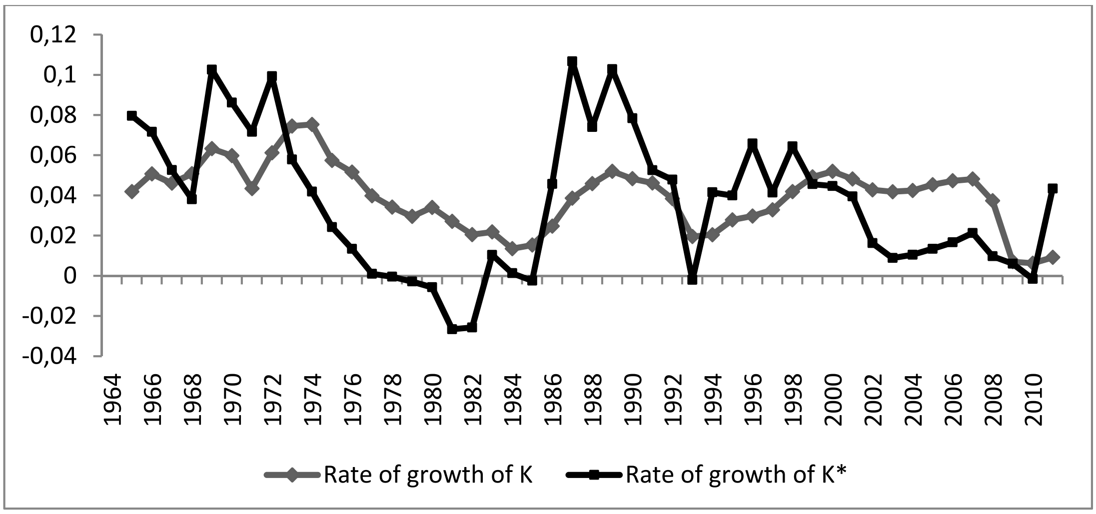

Figure 3. Rate of growth of the economic and statistical capital stock. 1965-2011. Source: Own elaboration and official statistics (see Appendix).

5 Conclusions

Most of the empirical studies in macroeconomics and the economic growth literature depend on the measure of the physical capital stock. The variables that explain the main phenomena and problems in these areas of economics always appear to be related, whether directly or indirectly, to capital stock and depreciation. The standard measurements of capital and depreciation are statistical measures based on assumptions about the average service life of capital goods, which are accumulated according to the perpetual inventory method.

In this paper, we propose an alternative method based on the equations that solve the dynamic optimization problem of the neoclassical firm. In our model, an adjustment cost function coexists with a maintenance cost function; these, are linearly homogeneous. This method enables us to endogenously calculate the variables' rate of depreciation and capital stock, yielding an economic estimation based on indicators of profitability, such as the distributed profits and the Tobin's q ratio.

The above represents a change of paradigm in measuring capital and depreciation, from statistical terms to economic values, which we supply and apply to the Spanish economy. The results show an economic depreciation rate (endogenous) that fluctuates around the statistical rate (exogenous), and two time profiles for the economic and statistical capital stocks that are markedly different to each other. These short-run differences are the result of either greater or lesser destructions of capital in different periods that are not recorded in the official statistics. Consequently, it is not unusual for misleading economic interpretations to exist in the short run, based on the statistical depreciation that corresponds to a geometric pattern and the capital stock that arises from the perpetual inventory methodology. Instead, in the long run these two measures are good approximations of our endogenous measures in terms of economic value. This is because, in the long-term, economies have had enough time to absorb and adjust to shocks, and the rule of double declining balance rate, built upon the average service life of assets, correctly captures the true depreciation.

Our agenda for future research includes a review of the main macroeconomic indicators that describe the growth of the Spanish economy over the past 50 years. Our economic measure of the capital stock can be used as an indicator of the productive capacity and as a measure of capital input in studies of multifactor productivity. It can also be related to the added value to calculate capital-output ratios. The new data on capital and total factor productivity deserve a new exercise of growth accounting to compare with the results arising when the statistical measures of capital and depreciation are used. Another issue we will address in the near future is the role of information and communication technologies in explaining the recent evolution of labor productivity and total factor productivity. An important point in this matter is to know if our economic measures of capital and depreciation can help us understand the paradox of productivity.

6 Appendix: Data

The database of reference for most of the series used in the paper is the BD. MORES database.[14] The variables and their statistical sources, as well as the process followed for their elaboration when this has been necessary are detailed bellow

- Y_t: The Gross Value Added of non-financial business sector in constant euros in 2008. BD. MORES b.2008
- L_t: The number of employees of non-financial business sector. BD. MORES b.2008
- I_t^G: The Gross Fixed Capital Formation of non-financial business sector in constant euros in 2008. BD. MORES b.2008
- p_t^k: The non-financial business capital goods price. BD. MORES b.2008.
- K_0: The capital stock of non-financial business sector in constant euros in 2008 in the initial period. BD.MORES b.2008.
- R_t: The long term nominal interest rate. From 1964 to 1977 the nominal series by Escribá and Ruiz (1995b) and from this year until 2011 the nominal series by AMECO database are used.
- q_t: Tobin's q ratio. We assemble this series using different sources. First, we take the values of Espitia (1987) for the period from 1964 to 1982. For the years 1983 to 1987 we use the values of Alonso and Bentolila (1992), which are based on information from *Central de Balances* of the Bank of Spain. Ramírez, Rosell and Salas (2003) shows Tobin's q values averaged for different subperiods, but the authors have kindly provided us with the annualized values for the period from 1988 to 2000. Finally, to lengthen the series until 2011, we use the implicit growth rates of Tobin's q values supplied by IMF's (2015) Report on Prospects for Europe and the Global Economy.
- B_t^G: Gross distributed profits in current euros. These include series of dividends, interest payments as well as depreciation of fixed capital. These series have been developed as described below:

  Dividends: Series of dividends has been built from the CNE base 2010 by INE. Within the non-financial accounts of institutional sectors annual series *property´s income* (D.4) is used and within this only *distributed incomes of societies* (D.42) and *reinvested profits* (D.43) are used. These series for the time period 1999 to 2014 are available. The CNE base 95 provides data on about the *property's income* being without disaggregated and this has not been used. However, the CNE base 86 provide series from 1985 to 1997 of *dividends and other income distributed by societies* (R.44) for corporations and quasi-nonfinancial corporations. Based on this information we have built a homogenous series for the period 1985-2011 linking both series.

[14] The Spanish regional database BD.MORES b.2008 (see De Bustos et al., 2008) is compiled by the Budget General Directorate of the Spanish Ministry of Finance and Civil Service and is available on the following web page:
http://www.sepg.pap.minhafp.gob.es/sitios/sepg
/es-ES/Presupuestos/Documentacion/paginas/basesdatosestudiosregionales.aspx

From 1964 to 1985 the dividends series produced by Escribá and Ruiz (1995b) has been used. The initial values of the series (1964-1973) are respected and using the value of the dividends in 1985 according to CNE b.86 and using the growth tendency described by Escribá and Ruiz (1995b) to connect the values of 1973 and 1985.

Interest payments: The series of interest payments has been built in a similar way to the series of dividends: from 1985 to 2014 using the information provided by the CNE base 2010 and base 86 about *interest* (R.41). From 1964 to 1985 interest series by Escribá and Ruiz (1995b) are used.

Depreciation: Series of depreciation of non-financial business capital built using database BD. MORES b.2008.

Table A.1. Spanish non-financial business sector 1964-2011.

Year	I_t^G	p_t^k	R_t	q_t	B_t^G	Year	I_t^G	p_t^k	R_t	q_t	B_t^G
1964	21402	0.054	0.08	1.531		**1988**	76578	0.47	0.1175	1.34	41539
1965	25593	0.055	0.083	1.414	1671	**1989**	84457	0.495	0.137	1.26	46024
1966	29230	0.057	0.086	1.289	1618	**1990**	86677	0.525	0.1468	1.02	51350
1967	29797	0.06	0.089	1.159	1642	**1991**	89646	0.545	0.1138	0.91	56650
1968	32714	0.064	0.091	1.19	1934	**1992**	87974	0.557	0.1172	0.73	59482
1969	38415	0.067	0.096	1.467	2076	**1993**	76143	0.586	0.1021	0.58	59276
1970	39950	0.071	0.11	1.341	2512	**1994**	78521	0.605	0.0999	1.03	62357
1971	37006	0.075	0.113	1.416	3189	**1995**	86591	0.641	0.1127	1.00	63723
1972	45084	0.079	0.105	1.548	3189	**1996**	90848	0.655	0.0874	1.11	66338
1973	53212	0.086	0.107	1.411	3390	**1997**	96545	0.674	0.064	1.33	69404
1974	57844	0.102	0.127	1.079	4793	**1998**	108383	0.684	0.0483	1.64	70187
1975	54606	0.117	0.134	0.965	5894	**1999**	120379	0.705	0.0473	1.63	72345
1976	55509	0.135	0.124	0.705	7302	**2000**	129424	0.753	0.0553	1.70	85213
1977	52820	0.164	0.12	0.533	9202	**2001**	132406	0.776	0.0512	1.53	97003
1978	52298	0.19	0.1203	0.461	10566	**2002**	132945	0.808	0.0496	1.30	102529
1979	51811	0.213	0.1331	0.425	11914	**2003**	137730	0.84	0.0412	1.04	108548
1980	55887	0.242	0.1596	0.412	15438	**2004**	144514	0.877	0.041	1.09	119093
1981	54008	0.277	0.1581	0.44	19570	**2005**	154523	0.921	0.0339	1.15	134133
1982	51838	0.313	0.1599	0.678	23353	**2006**	164249	0.96	0.0378	1.21	151692
1983	53667	0.348	0.1691	0.93	26051	**2007**	173334	0.985	0.0431	1.21	170375
1984	49797	0.378	0.1652	0.84	30966	**2008**	165792	1	0.0437	1.00	190020
1985	51552	0.407	0.1337	0.98	32668	**2009**	125714	0.975	0.0398	0.70	172218
1986	58441	0.428	0.1136	1.32	33417	**2010**	125410	0.979	0.0425	0.79	162731
1987	68774	0.447	0.1281	1.43	36663	**2011**	130674	0.941	0.0544	0.87	173228

Note: The variable B_t^G is expressed in millions of current euros and I_t^G in millions of euros of 2008

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