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Joint life care annuities to help retired couples to finance the cost of long-term care

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ABSTRACT

This paper examines the possibility of including cash-for-care benefits in life care annuities (LCAs) to help retired couples to cope with the cost of long-term care (LTC). The paper develops an actuarial model to assess how much it would cost to add an extra stream of payments to annuities for couples should either or both require LTC. We also analyse how willing couples would be to choose this type of LCA. Using Australian LTC transition probability data for a realistic calibration and assuming independence of the risks involved, we numerically illustrate the model and the theoretical findings implied. The paper highlights the importance of reporting the joint life expectancy and the survivor life expectancy broken down by health state, given that this information makes the computation of the actuarial factors transparent and provides highly useful information to help the couple understand the need to be protected against the cost of LTC services. Our proposal could be welfare-improving, especially for "dual-earner" couples, given that under the assumption that coordinated retirement behaviour exists within the couple, the availability of LCAs for couples may complete the market over and above the use of products for individuals alone.

1. Introduction

Unlike health and disability programs which refer to working age population, long-term care (LTC) is generally consumed later in life (Costa-Font and Raut, 2022). Uncertainty about the need for LTC services is a major challenge in retirement planning. The financing of LTC is a complicated issue that raises a number of considerations including economic efficiency and incentives, equity including intergenerational equity, the balance of risk between public and private funding, and the sustainability of public expenditures (Karagiannidou and Wittenberg, 2022). In a well-designed aged care financing system (Sherris, 2021) there is a role for innovation in private market products, for example life care annuities (LCAs) and reverse mortgages, combined with LTC insurance. Eling and Ghavibazoo (2019) document the exponential growth of academic interest in LTC insurance by showing the number of papers and citations from the Web of Science. They conclude that new methods and products are needed in order to finance increased LTC expenditure. These include LCAs (Murtaugh et al., 2001; Spillman et al., 2003; Brown and Warshawsky, 2013; Chen et al., 2021; Ramsay and Oguledo, 2022), reverse mortgages (Ahlstrom et al., 2004; Ji et al., 2012; Alai et al., 2014; Andrews and Oberoi, 2014; Mayhew et al., 2021), variable LCAs with guaranteed lifetime withdrawal benefits (Hsieh et al., 2018), personal care savings bonds (Mayhew and Smith, 2014), the bundling of reverse mortgages with private LTC insurance (Shao et al., 2019), life care tontines (Chen et al., 2019; Hieber and Lucas, 2022) and the embedding of LTC insurance costs in pre-existing public pay-as-you-go defined benefit (DB) or notional defined contribution (NDC) pension schemes (Pla-Porcel et al., 2016 and 2017; Vidal-Meliá et al., 2020), to name just a few proposals.

There are many reasons for wanting LTC insurance, such as to protect resources or leave an inheritance, to not burden others with nursing home bills, to be able to choose the type of care and where it is received, to enjoy peace of mind and/or to be independent of outside support (NAIC, 2019). From the perspective of retired couples, protecting a spouse/domestic partner is very important given that there is substantial evidence that (spousal) caregiving can have negative effects on the caregiver's health (Klimaviciute, 2020).

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Fuino et al. (2022) find that factors relating to the awareness and understanding of LTC are extremely relevant when purchasing LTC insurance. Self-perceived health, behaviour and trust relationships between customers and insurers are important. Socioeconomic factors only play a secondary role in the decision-making process. However, most people would prefer to receive LTC in their known physical and social environment if their care needs are moderate, but residential care if they are more extensive (Lehnert et al., 2019).

This paper deals with LCAs, which are lifetime annuities in which the LTC benefit is defined in terms of an uplift with respect to the basic amount. In return for the payment of a premium (either in a lump sum or collected over time), the LCA provides a stream of fixed-income payments for the lifetime of the annuitant. It also provides an extra stream of payments if the annuitant requires LTC. In other words, an LCA bundles an annuity and LTC insurance into a single product.

The idea of LCAs has not yet been addressed from the perspective of a "dual-earner" married or cohabiting couple, which refers to a cohabiting couple where both partners work in the labour market (Boye, 2014).[2] This is not surprising, given that the couple's perspective is often ignored when it comes to products and strategies for the decumulation of wealth during retirement (Bär and Gatzert, 2022). An LCA for couples (LCAc) or joint LCA implies that the amount paid depends on the health state of both the annuitant and the co-annuitant, i.e. benefits are dependent on the joint mortality and morbidity of two lives, typically a dual-earner married or cohabiting couple.

Based on the main types of life annuity for couples (Brown and Poterba, 2000; Vidal-Meliá and Lejárraga-García, 2006; Pitacco, 2017; Dickson et al., 2019), the idea of using joint LCAs comes naturally to actuarial thinking as a way of helping couples cope with the cost of needing LTC. We propose two types of joint LCA which would be used by the state or a private insurer, both of which enable the mortality and disability risks of each member of the couple to be transferred: a joint LCA with a last survivor payout rule (LCAc1), or an LCA with a reversionary payout rule (LCAc2). With both these types of annuity, the payments cease with the second death. As we will see later in this paper, the difference between an LCAc1 and an LCAc2 is not only actuarial. They also have very different economic interpretations. Only the LCAc1 makes sense for a "dual earner" couple.

We are convinced that our paper is a timely and welcome addition to the literature given that the availability of LCAs for couples, assuming that coordinated retirement behaviour exists within the couple, may complete the market over and above the use of products for individuals alone and, what it is more important, could help dual-earner couples to cope with the cost of LTC. There is clear evidence (Cetin, 2021; Michaud et al., 2020; Kridahl and Kolk, 2018; Warren, 2015; Hospido and Zamarro, 2014; Bingley and Lanot, 2007 to name just a few) that retirement decisions within couples are interdependent, with a significant proportion of spouses retiring at approximately the same time irrespective of their age difference (retirement coordination).

There are various reasons why it would be preferable to buy an LCA for couples instead of an individual LCA for each partner. Policyholders would opt for the joint LCA in a single contract because it is cheaper insofar as it avoids too much diversification in the contractual documents by presenting the coverage in the same policy. The expenses loading and safety loadings would be also lower. As will be seen in Section 4, under conservative assumptions about the loading expenses, the lump-sum premium for purchasing a joint LCA would be up to about 3.36% lower than the price of the same annuities purchased separately under conventional insurance underwriting.

The paper is organized as follows. After this introduction, Section 2 briefly reviews the main literature on the issue of bringing together the retirement and LTC contingencies from a double perspective: within a social insurance framework and in the area of private market insurance. It also details the research gap and our contribution to the field. Section 3 presents the methodology for valuing a synthetic LCA for couples that relies on the illness-death multistate framework, in which transitions are modelled from the initial (healthy) state of both members of the couple to the absorbing death state. The impact of introducing the LTC contingency on the annuity for couples is assessed by comparing the initial benefits in both cases – it appears that the difference arises due to the annuity factors used to compute the benefits. We also analyse the annuitants' willingness to choose this type of LCA for couples. Section 4 shows the results obtained for a numerical example representative of the model developed in the previous section. The paper ends with some concluding comments and future research needs. It also includes supplementary material with five appendices. The first of these develops the main multiyear transition probabilities, the second shows the formula for the average time the couple will spend in the various (non-absorbing) states until the death of both members (extinction of the couple), the third provides detailed information on the LCA with the reversionary payout rule (LCAc2), the fourth presents additional tables and comments in connection with the numerical illustration, and the fifth establishes the link between two single LCAs and an LCA for couples,

2. Background

The idea of combining retirement and LTC contingencies within a social insurance framework is not new. The link between old-age income and LTC has long been acknowledged in the literature, as can be seen in Table 1 (below).

Early proposals by Christopherson (1992), Scanlon (1992), Nuttall et al. (1994) and Chen (1994, 2001 and 2003) suggested a social insurance programme that would provide a cash payment to pensioners needing LTC services. Generally speaking, these proposals were underdeveloped from an actuarial point of view.

A second group of works by Pitacco (2002), Pla-Porcel et al. (2016, 2017), Tanaka (2016), Vidal-Meliá et al. (2019, 2020) go much deeper into the actuarial aspects and it also discusses the policy implications of various issues that should be taken into account before any decision is made to put the model into practice. The paper by Sherris (2021) argues that actuarial methods of quantifying LTC risk are fundamental to quantifying the costs of aged care for private LTC insurance and for government-funded aged care.

With reference to the link with Life annuities and LTC from a private insurance perspective (see Table 2 for details), various researchers (e.g. Getzen, 1988; Murtaugh et al., 2001; Spillman et al., 2003, Brown and Warshawsky, 2013, to name just a few) proposed financial defined contribution (FDC) schemes in which LCAs are designed to deal with the major problems that arise in the currently separate markets for life annuities and LTC insurance. Yakoboski (2002) argued that for most retirement planners, financial advisors and pension benefit actuaries, retirement and LTC planning often go hand-in-hand. More recently, Sherris and Wei (2021) and Wu et al. (2022a) also conclude that there are many benefits to be gained from combining LTC insurance with a life annuity.

Despite the good characteristics that LCAs present as regards helping pensioners to cope with the cost of LTC, the market for LCAs is small (Lambregts and Schut, 2020). These authors find that

[2] A *single-earner couple* is defined as a married or cohabiting couple where only one member has income from employment or self-employment.

Table 1
Overview of the sources reviewed: retirement and LTC contingencies within a social insurance framework.

Author(s)	Country	(Brief) Summary points
Christopherson (1992)	USA	He launched the idea of an annuity that, increasing with the severity of the disability, could serve as a new design for social security benefits.
Scanlon (1992)	USA	He suggested a programme that would provide a cash payment or indemnity benefit to pensioners with specified levels of disability.
Nuttall et al. (1994)	UK	They proposed that when pensioners retired, they should be allowed to defer a certain fraction of their pension in return for the right to an increase in benefit in the event that they or their spouse should require LTC.
Chen (1994, 2001, 2003)	USA	She suggested that a social insurance programme could be created to provide basic LTC coverage by diverting a small portion of a retiree's social security cash benefits for LTC. Further details were added in 2001 and 2003.
Pitacco (2002)	Italy	He put forward the idea of establishing an LTC scheme embedded in the retirement pension system, specifically the introduction of enhanced pension annuities (EPA) funded by contributions.
Pla-Porcel et al. (2016)	Japan-Spain	They explored the idea of embedding a public LTC insurance scheme in a notional defined contribution (NDC) pensions framework by using LCAs.
Tanaka (2016)	Japan	The author proposed redesign of the social security LTC pension in Japan that involves introducing a pension benefit increase mechanism that would depend on the level of care needed by beneficiaries.
Costa-Font et al. (2016)	Europe	They argued that LTC finance should be considered part of an overall retirement strategy rather than just an extension of health insurance, even if it were possible to separate the goals of consumption smoothing (retirement) from insurance (LTC).
Pla-Porcel et al. (2017)	USA-Australia	They assessed the cost of converting retirement benefit into an LCA with graded benefits using a pre-existing public PAYG pension scheme.
Vidal-Meliá et al. (2019)	Australia	The authors demonstrated that converting retirement benefit into an LCA with graded benefits within a pre-existing public PAYG pension scheme using gender-neutral annuity factors involves a big increase in ex-ante gender redistribution compared to a system without LTC coverage.
Vidal-Meliá et al. (2020)	USA	The authors developed the necessary technicalities to fully integrate an LTC benefit, graded according to the annuitant's degree of disability, into a generic NDC retirement framework with a minimum pension benefit for both contingencies.
Sherris (2021)	Australia	The author argued that in a well-designed sustainable financing system, there should be a role for private long-term care insurance and the use of home equity, taking into account differing individual health and wealth levels at and during retirement.
Source: Own		

uptake is lowered through substitution by social security, adverse selection, nonstandard preferences and limited rationality due to low financial literacy and unawareness of risk. Wu et al. (2022b) conclude that LCAs are seen as complementary to informal care and is attractive to seniors who plan to rely on family members for extensive care. Tennyson et al. (2022) examine household decisions on LTC insurance purchases through a bargaining lens. LTC insurance purchase is a discrete decision and spouses' opinions on the subject may differ greatly.

2.1. Research gap and our contribution

Although the treatment of dual-earner married/cohabiting couples is an important issue in retirement and LTC planning and despite the increasing presence of dual-earner couples, virtually all of the previous research on annuities has focused on individuals rather than couples as decision-making units. As far as we know, only two papers – Rickayzen (2007) and more recently Shao et al. (2017) – describe an insurance product for couples with LTC coverage, although without technical details. The present paper helps to fill a gap in the literature because, as far as we are aware, the costs and main implications of transforming retirement and survivor benefits into an LCA for couples have not yet been explored in detail.

The product most similar to a joint LCA is that described by Rickayzen (2007), the so-called disability-linked last-survivor annuity. As the name implies, this is a last survivor annuity, but the income level depends on the health state of both of the individuals involved. Shao et al. (2017), on the other hand, investigate the premiums for shared LTC insurance issued to couples. The n-year shared LTC insurance pools the couple's LTC insurance into one policy and increases the maximum benefit period for the pooled policy to 2n.

Two differentiated joint LCAs (LCAc1 and LCAc2) are proposed which in any given case, assuming that coordinated retirement behaviour exists within the couple, may complete the market over and above the use of products for singles alone.

1) Joint life care annuity with a last survivor payout rule (LCAc1). This is a contract whereby the insurer/the state undertakes to pay a periodic amount while both members of the couple are alive and in an able state, and a fraction of this amount when one of them has died and for as long as the other is alive. If at least one of the partners requires LTC, an uplift with respect to the basic amount is added. If both members are alive and both require LTC, the amount of the uplift will be greater.

2) Life care annuity with a reversionary payout rule (LCAc2). With this type of LCA, a periodic payment is made to the primary annuitant, which they receive until they die. An extra stream of payments is also provided if the primary annuitant requires LTC. On the death of the primary annuitant, their spouse/partner, assuming they have survived until this date, will start to receive an amount calculated as a fraction of what the deceased annuitant was receiving, whether or not they require LTC. The periodic payment is not modified if the co-annuitant dies or becomes dependent before the death of the primary annuitant.

The difference between an LCAc1 and an LCAc2 is not only actuarial. They also have very different economic interpretations. With an LCAc1, both members have entitlement to receive a peri-

Table 2
Overview of the sources reviewed: life annuities and LTC insurance.

Author(s)	Country	(Brief) Summary points
Getzen (1988)	USA	He proposed the first type of LCA, the so-called "longlife insurance", a mechanism to provide for comprehensive financial support throughout retirement regardless of the state of health.
Murtaugh et al. (2001)	USA	They showed that by combining an immediate income annuity with LTC coverage at retirement age they could reduce the cost of both and make them available to more people by reducing adverse selection in the income annuity and minimizing the need for medical underwriting in the LTC coverage.
Yakobosky (2002)	USA	He argued that the link with retirement an LTC is a logical extension of the rationale underlying all pension arrangements, which is to provide an adequate income to meet retirement needs regardless of the retiree's state of health.
Spillman et al. (2003)	USA	They proposed that when pensioners retired, they should be allowed to defer a certain fraction of their pension in return for the right to an increase in benefit in the event that they or their spouse should require LTC.
Brown and Warshawsky (2013)	USA	They carried out a new empirical examination of LCA. Their main findings were that different risk groups at age 65 had similar projected LTC expenses, but that the level-periodic premium structure of most LTC insurance policies created incentives for individuals to be separated into different risk pools according to observable characteristics, thereby justifying the underwriting observed in the market.
Lambregts and Schut (2020)	Developed countries	They have shown that similar factors hinder the uptake of both LTC insurance and annuities. They find that uptake is lowered through substitution by social security, adverse selection, nonstandard preferences and limited rationality due to low financial literacy and unawareness of risk. They conclude that LCAs are unlikely to solve other aspects that limit uptake.
Sherris and Wei (2021)	USA	They showed that there are significant benefits to be gained from combining LTC with a life annuity in terms of lower insurance premiums and less uncertainty surrounding the costs of aged care arising from systematic trends.
Wu et al. (2022a)	USA	They found that LCAs are more attractive than life annuities and would enhance annuity demand by 12 percentage points. LCAs allow individuals to reduce precautionary savings and consume more throughout retirement, with individuals willing to pay a loading of up to 21% for access to the product
Wu et al. (2022b)	Australia	They measured potential purchasers' interest in LTC income insurance. Their results show that LTC income insurance is seen as complementary to informal care and is attractive to seniors who plan to rely on family members for extensive care. Women who expect to rely exclusively on extensive care from family members are willing to buy more cover than men.
Tennyson et al. (2022)	USA	The cost of buying LTC insurance is typically borne by both spouses, but the benefits go disproportionately to women, who are more likely to need LTC for themselves and benefit from the asset protection and other support LTC insurance offers in the event their husband needs care.
Source: Own		

odic payment and the benefit is treated jointly. This is important, given that most studies of intra-household resource allocation focus on consumption and income and have found that differences arise depending on whether it is the primary annuitant or both that has access to and/or control of the resources. The more equal the access and control, the more balanced the income use and the household financial decisions (Grabka et al., 2015). This is the most suitable LCA for a dual-earner married/cohabiting couple.

With an LCAc2, only one member of the couple has entitlement to income from the annuity and the other is a mere dependent. This product is a type of reversionary annuity that includes LTC coverage and is the most suitable LCA for a single-earner married/cohabiting couple.

To explore the main implications of transforming retirement and survivor benefits into an LCA for couples, we develop a methodology for valuing a synthetic joint LCA in which the health dynamics rely on the illness-death multistate framework. One important aspect of the model is that it enables us to express the actuarial factor for both types of LCAc by using the joint and survivor life expectancy of active couples disaggregated into healthy and unhealthy life years. Such measures are potentially useful to those designing or evaluating policies affecting older couples and to those couples making decisions about retirement, savings and long-term care.

The importance of reporting the joint and survivor life expectancy of active couples disaggregated into healthy and unhealthy life years is twofold. First, it makes the computation of the actuarial factors transparent, and these explicit demographic data are very difficult to find in the specialized literature on LCAs. And second, these data provide very useful information to help the couple understand the need to be protected against the cost of LTC services.

Finally, we also analyse annuitants' willingness to choose this type of joint LCA by extending to the case of LCAs for couples the methodology used by Chen et al. (2021) to determine the policyholder's willingness to pay for the care option. On the assumption that the couple base their decisions on the (discounted) expected lifetime utility, we also show a great number of parameters that influence the couple's willingness to buy (choose) an LCAc, the most important of these being risk aversion, transition probabilities, the design of the uplifts, the indexation rule of benefits in payment, the rate of time preference, the technical interest rate, and the value of the payment weighting factors. In short, the couple will be willing to add LTC coverage if the welfare gain is higher than the incremental cost of purchasing the LTC insurance.

3. The model for valuing a notional LCA for couples

We use array calculus to value two types of LCA for couples. Specifically, we use a discrete-time Markov multistate model for a couple in which no more than one transition is assumed for each annuitant each year. Although the Markovian property does not take into account past transitions or the time spent in previous states, these models are more computationally efficient than the semi-Markov processes (Yu et al., 2022).

We assume that the survival probabilities are stochastically independent, i.e. the death of one member of the couple does not affect the survival probability of the other. In most countries insurance practice conventionally assumes independence between couples' lives for computational convenience despite the fact that a certain dependence between them has been described and highlighted in the literature for some time now.[3] The importance of the effect of dependent mortality on annuity valuation is not very clear in the literature. Frees et al. (1996) find that annuity values are reduced by approximately 5% when dependent mortality models are used compared to the standard models that assume independence. More recently, Spreeuw and Owadally (2013) estimate that, for a typical joint life and survivor annuity (without LTC coverage), assuming independence when short-term dependence is prevalent results in over-pricing of about 2.3%, and that assuming dependence of the wrong type (i.e. long-term rather than short-term) leads to a premium that is too low by about 1.8%. Standard industry practice assumes independence of mortality rates among the insured, adding loading factors to compensate for possible underestimations of the joint last survivor mortality due to common life style, broken-heart factor or common disaster[4] (Youn and Shemyakin, 2001).

The age of the primary annuitant at the start of the contract is denoted by x and that of the co-annuitant by y; ω and ω' are the maximum respective lifespans for – in this specific case – the annuitant (male) and the co-annuitant (female), obviously other couple combinations are possible.

The maximum number of years between the time that the LCAc is purchased and the maximum length of life can be expressed as $T = \max\{\omega - x, \omega' - y\}$, and since it is a discrete model of uniform annual periods, the time reference is given by $k \in \{0, 1, 2, \cdots, T\}$. Initially, we assume that at the time of inception, $k = 0$, x and y are alive and healthy (able).

In this section our model considers only one level of dependence (severe), d, and the space state is completed with active (able), a, and death, f. As we will see in Section 4, the numerical illustration is performed taking into account four states of dependence. The couple's health dynamics rely on the illness-death multistate framework, in which transitions are modelled from the initial healthy state of both members of the couple to the absorbing death state. Time trends and systematic uncertainty in health transitions are not considered in this model.[5]

As in Hubener et al. (2014), we ignore all changes in the family state – such as a divorce or a new partnership after one of the partners has died – other than those driven by mortality and morbidity. For the sake of simplicity, we also ignore expenses and fees in this section, but in Section 4 we revisit the issue when comparing single LCAs with LCAs for couples.

The set of direct possible transitions is $\mathcal{T} = \{i \in \mathcal{S}, j \in \mathcal{S} : i \to j\}$. Table 3 shows the yearly transition probabilities that form the discrete one-year transition matrix $\Pi^{i\to j}_{x+k:y+k} = \mathbb{P}[\mathcal{S}(k+1) = j \mid \mathcal{S}(k) = i]$.

At any particular point in time the couple will be in one of nine possible different health states: $\mathcal{S} = \{aa, ad, da, dd, af, fa, df, fd, ff\}$. The first component of each letter pair (a = able, d = dependent, f = deceased) refers to the state of the primary annuitant (x), and the second to the state of the co-annuitant (y). Thus the first state (aa, in which both members are able) can lead immediately to any of the other states, and the final state (ff, in which both members are deceased) can come about from any of the other states. However, any state in which one of the members is deceased (af, fa, df, fd), for example, will obviously have fewer options for future changes. The main probabilities are:

$p^{i}_{x+k:y+k}$: the probability that a couple with the primary annuitant aged $x+k$, $k \in \{0, 1, \cdots, \omega - x\}$ and the co-annuitant aged $y+k$, $k \in \{0, 1, \cdots, \omega' - y\}$ will continue one year more in the same state.

$q^{i\to j}_{x+k:y+k}$: the probability that a couple currently in state i, with the primary annuitant aged $x+k$ and the co-annuitant aged $y+k$, will be in status j one year later. This notation is only valid if both members are alive in state i.

If one of the members is dead in state i, the notation is similar to that for individuals.

$q^{i\to j}_{x+k}$: the probability that the primary annuitant (in state i) aged $x+k$ will reach age $x+k+1$ in status j.

$q^{i\to j}_{y+k}$: the probability that the co-annuitant (in state i) aged $y+k$ will reach age $y+k+1$ in status j.

It is obvious that $p^{ff}_{x+k:y+k} = 1$, given that the death of both members of the couple is an absorbing state.

As Table 3 shows, the model explicitly takes the "recovery assumption" into account. This is certainly controversial in most European countries because the possibility of recovery is generally disregarded given the prevailing chronic character of LTC disability (see for example; Pitacco, 2014 and 2016; and Albarrán-Lozano et al., 2017). Li et al. (2017), using US Health and Retirement Study data between 1998 and 2012, documented that recovery rates are highly sensitive to the stochastic frailty factor. With Swiss data, Fuino and Wagner (2018) have reported very low probabilities of recovery transitions of less than 0.05%. More recently and using US data, Sherris and Wei (2021) find that recovery rates from disability decrease with age but that there is no significant difference between males and females. Finally, the paper by Yu et al. (2022), which compares mortality and functional disability experiences in China and the US, finds a significant time trend in recovery rates.

Multi-year transition probabilities can be derived from one-year transition probabilities (Appendix 1, supplementary material) by following the procedure described in Pitacco (2014), i.e. using recurrent Chapman-Kolmogorov equations.

After developing the key probabilities and adapting the methodology used by Pla-Porcel et al. (2016, 2017) to the case of two lives, we can build the model for determining the actuarial factor for LCAc1 ($AF^{LCAc1}_{x:y}$).

The benefits corresponding to each of the couple's possible states are:

[3] To name just a few, see the papers by Frees et al. (1996), Denuit and Cornet (1999), Ji et al. (2011), Spreeuw and Owadally (2013), and Ji and Zhou (2019).

[4] Dependence within a couple is often influenced by any of three factors: (1) catastrophic events that affect both lives (instantaneous dependence), (2) the impact of spousal death (short-term dependence or the broken-heart factor), and (3) the long-term association due to a common lifestyle. According to Dufresne et al. (2018), the gender of the older member of the couple also has an influence on the level of dependence between lifetimes.

[5] Li et al. (2017) show that the effect of improvement trends results in an increase in expected future lifetimes as well as an increase in future healthy life expectancy, with the proportion of lifetime spent in functional disability on average remaining similar. For the case of the USA and China, Yu et al. (2022) show that incorporating time trends is important, since failing to do so will underestimate both total life expectancy and healthy life expectancy and overestimate the proportion of the elderly in functional disability (the prevalence rate) in both countries. Incorporating systematic uncertainty allows the authors to quantify the confidence we have in the estimated proportion spent in functional disability.

Table 3
Discrete one-year transition matrix.

Starting state, i	Ending state, j								
	aa	ad	da	dd	af	fa	df	fd	ff
$\boldsymbol{aa}$	$p^{aa}_{x+k:y+k}$	$q^{aa\to ad}_{x+k:y+k}$	$q^{aa\to da}_{x+k:y+k}$	$q^{aa\to dd}_{x+k:y+k}$	$q^{aa\to af}_{x+k:y+k}$	$q^{aa\to fa}_{x+k:y+k}$	$q^{aa\to df}_{x+k:y+k}$	$q^{aa\to fd}_{x+k:y+k}$	$q^{aa\to ff}_{x+k:y+k}$
$\boldsymbol{ad}$	$q^{ad\to aa}_{x+k:y+k}$	$p^{ad}_{x+k:y+k}$	$q^{ad\to da}_{x+k:y+k}$	$q^{ad\to dd}_{x+k:y+k}$	$q^{ad\to af}_{x+k:y+k}$	$q^{ad\to fa}_{x+k:y+k}$	$q^{ad\to df}_{x+k:y+k}$	$q^{ad\to fd}_{x+k:y+k}$	$q^{ad\to ff}_{x+k:y+k}$
$\boldsymbol{da}$	$q^{da\to aa}_{x+k:y+k}$	$q^{da\to ad}_{x+k:y+k}$	$p^{da}_{x+k:y+k}$	$q^{da\to dd}_{x+k:y+k}$	$q^{da\to af}_{x+k:y+k}$	$q^{da\to af}_{x+k:y+k}$	$q^{da\to df}_{x+k:y+k}$	$q^{da\to fd}_{x+k:y+k}$	$q^{da\to ff}_{x+k:y+k}$
$\boldsymbol{dd}$	$q^{dd\to aa}_{x+k:y+k}$	$q^{dd\to ad}_{x+k:y+k}$	$q^{dd\to da}_{x+k:y+k}$	$p^{dd}_{x+k:x+k}$	$q^{dd\to af}_{x+k:y+k}$	$q^{dd\to fa}_{x+k:y+k}$	$q^{dd\to df}_{x+k:y+k}$	$q^{dd\to fd}_{x+k:y+k}$	$q^{dd\to ff}_{x+k:y+k}$
$\boldsymbol{af}$	0	0	0	0	$p^{af}_{x+k:y+k}$	0	$q^{a\to d}_{x+k}$	0	$q^{a\to f}_{x+k}$
$\boldsymbol{fa}$	0	0	0	0	0	$p^{fa}_{x+k:y+k}$	0	$q^{a\to d}_{y+k}$	$q^{a\to f}_{y+k}$
$\boldsymbol{df}$	0	0	0	0	$q^{d\to a}_{x+k}$	0	$p^{df}_{x+k:y+k}$	0	$q^{d\to f}_{x+k}$
$\boldsymbol{fd}$	0	0	0	0	0	$q^{d\to a}_{y+k}$	0	$p^{fd}_{x+k:y+k}$	$q^{d\to f}_{y+k}$
$\boldsymbol{ff}$	0	0	0	0	0	0	0	0	1

Source: Own

$$b^1(k) = \begin{cases} b & \text{if } j \in \{aa\};\ \text{both alive and able} \\ b\cdot(1+\xi) & \text{if } j \in \{ad\};\ \text{able and dependent} \\ b\cdot(1+\xi) & \text{if } j \in \{da\};\ \text{dependent and able} \\ b\cdot(1+\xi') & \text{if } j \in \{dd\};\ \text{both dependent} \\ b' = b\cdot\nu & \text{if } j\ \in \{af\}; \\ & \text{widower (able) and deceased} \\ b'' = b\cdot\eta & \text{if } j \in \{fa\};\ \text{deceased and widow (able)} \\ b'\cdot(1+\xi) & \text{if } j \in \{df\}; \\ & \text{widower (dependent) and deceased} \\ b''\cdot(1+\xi) & \text{if } j \in \{fd\}; \\ & \text{deceased and widow (dependent)} \\ 0 & \text{if } j \in \{ff\};\ \text{both deceased} \end{cases} \tag{1}$$

Benefit payouts from LTC insurance are typically triggered when the individual (i.e. the insured person) becomes functionally disabled or chronically ill and requires long-term support in various facets of living. Entitlement to LTC benefits depends on a mandatory medical assessment of the individual's ability to perform the basic activities of daily living (ADLs) and the instrumental activities of daily living (IADLs) (Fuino et al., 2020).

Generally speaking as regards LCAs, the LTC benefit is defined in terms of an uplift (ξ for one dependent member or ξ' if both members require LTC) with respect to the basic payment, b. This initial benefit b is paid from retirement onwards as long as both members of the couple remain in their initial state, but is replaced by the LTC annuity benefit – $b\cdot(1+\xi),\ j \in \{ad, da\}$ or $b\cdot(1+\xi'),\ j\in\{dd\}$ – in the case of a successful LTC claim while both members are alive.

Should one member of the couple die while in the initial state, the benefit is reduced to $b',\ j\in\{af\}$ or $b'',\ j\in\{fa\}$. The surviving member will usually receive a certain percentage ($\nu,\ \eta$) of the original payout, and this is called the survivor benefit ratio or reduction factor.

When the recovery assumption is admitted (see Section 4), the amount of the annuity will have to be reduced if the couple makes a transition to a better health state.

The process for obtaining the actuarial factor involves the valuation at time t of the couple's expected lifetime benefits to be paid out at time $t+k$, where $k\in\{0,1,\cdots,T\}$. This takes into account the financial factor ($F^k = [\frac{1+\alpha}{1+G}]^k$), which depends on α (indexation of benefits/pension in payment) and G (technical interest rate or wage bill growth depending on the type of pension system), the matrix row of annual benefits (B^1) and the transposed vector of multiyear transition probabilities (${}_kM^t_{x:y}$):

$$\begin{aligned} AF^{LCAc1}_{x:y} &= \sum_{k=0}^{T} B^1\cdot {}_kM^t_{x:y}\cdot F^k \\ &= \sum_{k=0}^{T} b\cdot {}_kp^{aa}_{x:y}\cdot F^k + \sum_{k=1}^{T} b\cdot(1+\xi)\cdot\left[{}_kp^{ad}_{x:y} + {}_kp^{da}_{x:y}\right]\cdot F^k \\ &\quad + \sum_{k=1}^{T} b\cdot\left(1+\xi'\right)\cdot {}_kp^{dd}_{x:y}\cdot F^k + \sum_{k=1}^{T} b'\cdot {}_kp^{af}_{x:y}\cdot F^k \\ &\quad + \sum_{k=1}^{T} b''\cdot {}_kp^{fa}_{x:y}\cdot F^k \\ &\quad + \sum_{k=1}^{T} b'\cdot(1+\xi)\cdot {}_kp^{df}_{x:y}\cdot F^k \\ &\quad + \sum_{k=1}^{T} b''\cdot(1+\xi)\cdot {}_kp^{fd}_{x:y}\cdot F^k \\ &= b\cdot {}^{\alpha}\ddot{a}^{aa}_{x:y} + b'\cdot {}^{\alpha}\ddot{a}^{\,a}_{y|x} + b''\cdot {}^{\alpha}\ddot{a}^{\,a}_{x|y} \\ &\quad + b\cdot\left({}^{\alpha}_{\xi}A^{ad}_{x:y} + {}^{\alpha}_{\xi}A^{da}_{x:y} + {}^{\alpha}_{\xi'}A^{dd}_{x:y}\right) \\ &\quad + b'\cdot {}^{\alpha}_{\xi}A^{d}_{y|x} + b''\cdot {}^{\alpha}_{\xi}A^{d}_{x|y} \\ &= b\cdot\overbrace{\left(\underbrace{{}^{\alpha}\ddot{a}^{aa}_{x:y}}_{\substack{\text{both}\\ \text{able}}} + {}^{\alpha}_{\xi}A^{ad}_{x:y} + {}^{\alpha}_{\xi}A^{da}_{x:y} + \underbrace{{}^{\alpha}_{\xi'}A^{dd}_{x:y}}_{\substack{\text{both}\\ \text{dependent}}}\right)}^{\text{both alive}} \\ &\quad + b'\overbrace{\left(\underbrace{{}^{\alpha}\ddot{a}^{\,a}_{y|x}}_{\text{able}} + {}^{\alpha}_{\xi}A^{d}_{y|x}\right)}^{\text{widower}} + b''\cdot\overbrace{\left(\underbrace{{}^{\alpha}\ddot{a}^{\,a}_{x|y}}_{\text{able}} + {}^{\alpha}_{\xi}A^{d}_{x|y}\right)}^{\text{widow}} \end{aligned} \tag{2}$$

where:

$b\cdot {}^{\alpha}\ddot{a}^{aa}_{x:y} = b\cdot\sum_{k=0}^{T} {}_kp^{aa}_{x:y}\cdot F^k$: the present value of the basic payment of a lifetime pension payable in advance while both members remain in the able state, indexed at rate α with a technical interest rate equal to G (Unless otherwise stated, all actuarial values are computed using these rates). We assume that all payments are made regularly in advance, i.e. at the beginning of the respective time intervals (years).

$b\cdot {}^{\alpha}_{\xi}A^{ad}_{x:y} = \sum_{k=1}^{T} b\cdot(1+\xi)\cdot {}_kp^{ad}_{x:y}\cdot F^k$: the present actuarial value of an LCAc1 that supplements the initial benefit (b) by percentage ξ while the couple remain in state $j\in\{ad\}$ (the primary annuitant able, the co-annuitant dependent).

$b \cdot {}^{\alpha}_{\xi} A^{da}_{x:y} = \sum_{k=1}^{T} b \cdot (1+\xi) \cdot {}_k p^{da}_{x:y} \cdot F^k$: the present actuarial value of the LCAc1 that supplements the initial benefit (b) by percentage ξ while the couple remain in state $j \in \{da\}$ (the primary annuitant dependent, the co-annuitant able).

$b \cdot {}^{\alpha}_{\xi'} A^{dd}_{x:y} = \sum_{k=1}^{T} b \cdot (1+\xi') \cdot {}_k p^{dd}_{x:y} \cdot F^k$: the present actuarial value of the LCAc1 that supplements the initial benefit (b) by percentage ξ' while both members remain dependent, $j \in \{dd\}$.

$b'^{\alpha} \ddot{a}\ \underbrace{\overset{a}{y}}_{\text{dead}} | \underbrace{x}_{\text{alive}} = \sum_{k=1}^{T} b' \cdot {}_k p^{af}_{x:y} \cdot F^k$: the present value of the reduced basic payment, b', of a lifetime pension payable in advance while the primary annuitant is able and the co-annuitant deceased, i.e. the couple is in state $j \in \{af\}$.

$b'^{\alpha}_{\xi} A_{y|x}^{\ d} = \sum_{k=1}^{T} b' \cdot (1+\xi) \cdot {}_k p^{df}_{x:y} \cdot F^k$: the present actuarial value of the LCAc1 that supplements the reduced basic payment, b', by percentage ξ while the primary annuitant is dependent and the co-annuitant deceased, i.e. the couple is in state $j \in \{df\}$.

$b''^{\alpha} \ddot{a}_{x|y}^{\ a} = \sum_{k=1}^{T} b'' \cdot {}_k p^{fa}_{x:y} \cdot F^k$: the present value of the reduced basic payment, b'', of a lifetime pension payable in advance while the co-annuitant is able and the primary annuitant deceased, i.e. the couple is in state $j \in \{fa\}$.

$b''^{\alpha}_{\xi} A_{x|y}^{\ d} = \sum_{k=1}^{T} b'' \cdot (1+\xi) \cdot {}_k p^{fd}_{x:y} \cdot F^k$: the present actuarial value of the LCAc1 that supplements the reduced basic payment, b'', by percentage ξ while the co-annuitant is dependent and the primary annuitant deceased, i.e. the couple is in state $j \in \{fd\}$.

3.1. The cost of adding an extra stream of payments to annuities for couples if one or more of the annuitants requires LTC

To assess the cost of including cash-for-care benefits if any of the annuitants were to require LTC services, we use the so-called "coverage ratio". This shows in present value the number of equivalent monetary units needed to determine the initial LCAc benefit for each monetary unit of the initial basic annuity for couples (without LTC coverage). To calculate this we need to compare the initial benefits awarded in both cases: the basic annuity for couples and the extended annuity for couples with LTC coverage. For a joint LCA with a last survivor payout rule (LCAc1), the value of the coverage ratio (CR_t^{LCAc1}) can be expressed as:

$$
\begin{aligned}
CR_t^{LCAc1} &= \frac{AF_{x:y}^{LCAc1}}{AF_{x:y}^{JAc1}} \\
&= \frac{AF_{x:y}^{LCAc1}}{b \cdot {}^{\alpha}\ddot{a}_{x:y} + b' \cdot {}^{\alpha}\ddot{a}_{y|x} + b'' \cdot {}^{\alpha}\ddot{a}_{x|y}} \\
&= 1 + \xi \cdot (dxLTCWc1_t + dyLTCWc1_t) \\
&\quad + \xi' \cdot (dxyLTCWc1_t)
\end{aligned} \tag{3}
$$

where $dxLTCWc1_t$, $dyLTCWc1_t$ and $dxyLTCWc1_t$ represent the actuarial cost of covering LTC for the couple in each possible state of dependence as a proportion of the total cost of a classical joint annuity without LTC coverage, i.e. the higher the resulting values of $dxLTCWc1_t$, $dyLTCWc1_t$ and $dxyLTCWc1_t$, the lower the amount of the basic benefit of the joint LCA with a last survivor payout rule (LCAc1). These values do not take uplifts into account, only biometric assumptions.

Given that $\xi > 0$, $\xi' > 0$, $b' > 0$ and $b'' > 0$, it is easy to see that $AF_{x:y}^{LCAc1} > AF_{x:y}^{JAc1}$ and obviously $CR_t^{LCAc1} > 1$.

An analysis of the coverage ratio shows the key parameters that determine the amount of the initial benefit (both members of the couple are able) when LTC is included:

1. -The higher the value assigned to any ξ, ξ', the lower the amount of the initial benefit for the LCAc. It is easy to see that for all ξ, $\xi' = 0$, $CR_t = 1$, given that ($AF_{x:y}^{LCAc1} = AF_{x:y}^{JAc1}$), i.e. the amount of the periodic payment is not increased when either member of the couple becomes dependent.

2. -The higher the probability of the couple becoming dependent for a given age, ${}_k p^{j}_{x:y}$, the lower the amount of the initial pension of the LCAc benefit.

3. -The higher the survival probability of the couple in the state of dependence, ${}_k p^{dd}_{x:y}$, the lower the amount of the initial pension of the LCAc benefit.

3.2. The actuarial factor when the operating rule for indexing benefits in payment is $\alpha = G$

If the operating rule for indexing benefits in payment is $\alpha = G$, then the financial factor is $F = 1$. This is especially interesting because the actuarial factor for both types of LCAc can be expressed using the joint and survivor life expectancy of active couples disaggregated into healthy and unhealthy life years (Appendix 2, supplementary material). Such measures are potentially useful to those designing or evaluating policies affecting older couples and to those couples making decisions about retirement, savings and LTC (Compton and Pollak, 2021).

If we substitute $F = 1$ in formula (2) we get:

$$
\begin{aligned}
& AF_{x:y}^{LCAc1[\alpha=G]} \\
&= b \cdot \left(\left(1 + e^{aa}_{x:y}\right) + (1+\xi) \cdot \left(e^{ad}_{x:y} + e^{da}_{x:y}\right) + \left(1+\xi'\right) \cdot e^{dd}_{x:y} \right) \\
&\quad + b' \cdot \left(\left(1 + e_{y|x}^{\ a}\right) + (1+\xi) \cdot e_{y|x}^{\ d} \right) \\
&\quad + b'' \cdot \left(\left(1 + e_{x|y}^{\ a}\right) + (1+\xi) \cdot e_{x|y}^{\ d} \right)
\end{aligned} \tag{4}
$$

where:

$e^{aa}_{x:y}$: the expected years that both spouses will be alive and in an able state.

$e^{ad}_{x:y}$: the number of years that the primary annuitant is likely to spend free of activity limitation while the co-annuitant requires LTC services.

$e^{da}_{x:y}$: the number of years that the co-annuitant is likely to spend free of activity limitation while the primary annuitant requires LTC services.

$e^{dd}_{x:y}$: the expected years that both spouses will be alive and requiring LTC.

$e_{y|x}^{\ a}$: the expected years that the primary annuitant is likely to spend free of activity limitation after the death of the co-annuitant.

$e_{y|x}^{\ d}$: the expected years that the primary annuitant is likely to need LTC services after the death of the co-annuitant.

$e_{x|y}^{\ a}$: the expected years that the co-annuitant is likely to spend free of activity limitation after the death of the primary annuitant.

$e_{x|y}^{\ d}$: the expected years that the co-annuitant is likely to need LTC services after the death of the primary annuitant.

For the case of an LCAc2, the equivalent formulas are developed in Appendix 3 (supplementary material).

3.3. Analysing people's willingness to purchase an LCA for couples

Examining decision outcomes in the context of couples is pertinent for understanding the complexity of LTC planning (Barnett and Stum, 2012). For example, it is widely recognized that financial decisions are typically made in the context of families/couples and not simply by one individual. Financial LTC planning is no exception. This section will analyse how willing people might be to acquire joint LCA, specifically an LCAc1 as opposed to a joint annuity with survivor payout rule (JAc1), the closest financial-insurance product without LTC coverage.

To a large extent we follow the methodology proposed by Chen et al. (2021, 2022). Generally speaking, when deciding whether or not to buy an LCAc1, if the couple can compare different situations they buy a JAc1. The couple is assumed to have standard preferences and full rationality due to high financial literacy and is risk adverse, i.e. they base their decisions on the (discounted) expected lifetime utility. We also assume that the couple's subjective perceptions concerning transition probabilities coincide with the information available to the insurer/sponsor on the subject.

Adding the LTC contingency means that (1) the premium (the contribution) has to be greater for an LCAc1 ($AF^{LCAc1}_{x:y}$) than for a JAc1 ($AF^{JAc1}_{x:y}$), and (2) the (discounted) expected lifetime utility for the couple also has to be greater in the case of purchasing an LCAc1 than a JAc1.

(1) The actuarial pricing technique for the couple means that in the end the risk loading fee, $F_{x:y}$, for adding the LTC coverage is:

$$\begin{aligned} F_{x:y} &= AF^{LCAc1}_{x:y} - AF^{JAc1}_{x:y} \\ &= \xi \cdot \left(b \cdot {}^{\alpha}A^{da}_{x:y} + b \cdot {}^{\alpha}A^{ad}_{x:y} + b' \cdot {}^{\alpha}A_{y|x}{}^{d} + b'' \cdot {}^{\alpha}A_{x|y}{}^{d}\right) \\ &\quad + \xi' \cdot b \cdot {}^{\alpha}A^{dd}_{x:y} \end{aligned} \tag{5}$$

otherwise a reduction would be needed in the initial amount of benefit corresponding to the initial healthy state of both members of the couple, i.e. $b < 1$. In this case, $AF^{LCAc1}_{x:y} = AF^{JAc1}_{x:y}$.

(2) The couple is willing to pay an extra-premium ($\hat{F}_{x:y}$) to add LTC coverage to the JAc1, since such an addition leads to greater utility:

$$\begin{aligned} \hat{F}_{x:y} &= \hat{AF}^{LTCc1}_{x:y} - AF^{JAc1}_{x:y} \\ &= Q \cdot AF^{JAc1}_{x:y} - AF^{JAc1}_{x:y} = (Q-1) \cdot AF^{JAc1}_{x:y} \end{aligned} \tag{6}$$

where the $\hat{AF}^{LTCc1}_{x:y}$ value is the couple's subjective assessment of a given LCAc1, i.e. the value that would make the policyholder (the couple) utility-indifferent to choosing between an LCAc1 and a JAc1 at an actuarially fair price. To link (6) with the couple's utility we introduce quantity Q, referred to as a utility indifference number, which denotes the number of JAc1 contracts that would make the couple indifferent – in terms of the expected discounted lifetime utility it would represent to them – to choosing between buying (accepting) an LCAc1 and these Q JAc1 contracts:

$$\mathbb{E}\left[U^{LCAc1}_{x:y}\right] \equiv \mathbb{E}\left[Q \cdot U^{JAc1}_{x:y}\right] \tag{7}$$

where:

$$\begin{aligned} \mathbb{E}\left[U^{LCAc1}_{x:y}\right] = &\, U^{aa}_{x:y}[b] \cdot {}^{\alpha}\ddot{a}^{aa}_{x:y(\rho)} \\ &+ U^{ad}_{x:y}\left[(1+\xi) \cdot b\right] \cdot {}^{\alpha}A^{ad}_{x:y(\rho)} \\ &+ U^{da}_{x:y}\left[(1+\xi) \cdot b\right] \cdot {}^{\alpha}A^{da}_{x:y(\rho)} \\ &+ U^{dd}_{x:y}\left[(1+\xi') \cdot b\right] \cdot {}^{\alpha}A^{dd}_{x:y(\rho)} \\ &+ U_{y|x}{}^{a}\left[b'\right] \cdot {}^{\alpha}\ddot{a}_{y|x(\rho)}{}^{a} \\ &+ U_{y|x}{}^{d}\left[(1+\xi) \cdot b'\right] \cdot {}^{\alpha}A_{y|x(\rho)}{}^{d} \\ &+ U_{x|y}{}^{a}\left[b''\right] \cdot {}^{\alpha}\ddot{a}_{x|y(\rho)}{}^{a} \\ &+ U_{x|y}{}^{d}\left[(1+\xi) \cdot b''\right] \cdot {}^{\alpha}A_{x|y(\rho)}{}^{d} \end{aligned} \tag{8}$$

and:

$$\begin{aligned} \mathrm{E}\left[Q \cdot U^{JAc1}_{x:y}\right] = &\, U^{aa}_{x:y}[Q \cdot b] \cdot {}^{\alpha}\ddot{a}^{aa}_{x:y(\rho)} \\ &+ U^{ad}_{x:y}[Q \cdot b] \cdot {}^{\alpha}A^{ad}_{x:y(\rho)} \\ &+ U^{da}_{x:y}[Q \cdot b] \cdot {}^{\alpha}A^{da}_{x:y(\rho)} \\ &+ U^{dd}_{x:y}[Q \cdot b] \cdot {}^{\alpha}A^{dd}_{x:y(\rho)} \\ &+ U_{y|x}{}^{a}\left[Q \cdot b'\right] \cdot {}^{\alpha}\ddot{a}_{y|x(\rho)}{}^{a} \\ &+ U_{y|x}{}^{d}\left[Q \cdot b'\right] \cdot {}^{\alpha}A_{y|x(\rho)}{}^{d} \\ &+ U_{x|y}{}^{a}\left[Q \cdot b''\right] \cdot {}^{\alpha}\ddot{a}_{x|y(\rho)}{}^{a} \\ &+ U_{x|y}{}^{d}\left[Q \cdot b''\right] \cdot {}^{\alpha}A_{x|y(\rho)}{}^{d} \end{aligned} \tag{9}$$

where $b = 1$. The alternative would be to receive a lower periodic benefit in initial health state $b < 1$, but maintaining the same value of expected utility as with the JAc1.[6]

In (8) and (9), ρ stands for the subjective pure rate of time preference, which captures the couple's time preferences.

The utility functions, on the other hand, depend on the couple's health state and have mathematical properties that are strictly monotone increasing and strictly concave with respect to the periodic benefit. In addition, given that the utility provided by a given benefit when any member makes a transition to a state of dependence is reduced, they fulfil the following relationships:

$$\begin{aligned} &U^{aa}_{x:y}[b] \geq U^{ad}_{x:y}[b] = U^{da}_{x:y}[b] \geq U^{dd}_{x:y}[b] \\ &\quad U_{y|x}{}^{a}\left[b'\right] \geq U_{y|x}{}^{d}\left[b'\right] \\ &\quad U_{x|y}{}^{a}\left[b''\right] \geq U_{x|y}{}^{d}\left[b''\right] \end{aligned} \tag{10}$$

and furthermore, for the case of non-negative utility functions, the respective marginal utility functions[7] fulfil also:

$$\begin{aligned} &\frac{\partial U^{aa}_{x:y}[b]}{\partial b} \geq \frac{\partial U^{ad}_{x:y}[b]}{\partial b} = \frac{\partial U^{da}_{x:y}[b]}{\partial b} \geq \frac{\partial U^{dd}_{x:y}[b]}{\partial b} \\ &\quad \frac{\partial U_{y|x}{}^{a}[b']}{\partial b'} \geq \frac{\partial U_{y|x}{}^{d}[b']}{\partial b} \\ &\quad \frac{\partial U_{x|y}{}^{a}[b'']}{\partial b''} \geq \frac{\partial U_{x|y}{}^{d}[b'']}{\partial b''} \end{aligned} \tag{11}$$

which means that the couple profits proportionally more with an extra income unit as long as both members are able as opposed to when either both or one of them require LTC services. The relations included in (11) are reversed for the case of non-positive utility functions.

It is assumed that all the benefit the couple receives from the LCA, regardless of their health state, is consumed in the same period, i.e. it is assumed that there are no savings or dissavings.

We can find the Q^* that fulfils (7) and, according to its conceptual interpretation, provides the nexus between the couple's assessment of LCAc1 and the actuarially fair price of JAc1:

$$\hat{AF}^{LTCc1}_{x:y} = Q^* \cdot AF^{JAC1}_{x:y} \tag{12}$$

where, under the conditions outlined, it holds that[8]

$$Q^* \in \left(1,\ \left(1+\xi'\right)\right) \tag{13}$$

[6] In this case, quantity q denotes the number of LCAc1 contracts that would make the couple indifferent – in terms of the expected utility it would represent for them – to choosing between the JAc1 contract and these $q \cdot LCAc1$ contracts. The value of q is determined by $\mathbb{E}[q \cdot U^{LCAc1}_{x:y}] \equiv \mathbb{E}[U^{JAc1}_{x:y}]$.

[7] For further details, interested readers can consult the paper by Chen et al. (2021).

[8] The proof is similar to that shown in *Appendix A. Proof of Proposition I* in Chen et al. (2021).

By substituting (12) into (6), we get

$$\hat{F}_{x:y} = Q^* \cdot AF_{x:y}^{JAc1} - AF_{x:y}^{JAc1} = \left(Q^* - 1\right) \cdot AF_{x:y}^{JAc1} \tag{14}$$

$\hat{F}_{x:y}$ in (14) can be interpreted as being the maximum lump-sum premium the couple would be willing to pay to add LTC coverage to the JAc1 contract, i.e. an upper bound for the possible application of a safety loading by the insurer.

The decision rule to choose whether an LCAc1 is attractive when the risk loading is taken into account is as follows.

An extra premium, $F_{x:y}$, that is higher than $\hat{F}_{x:y}$ – i.e. the minimum premium (the additional contributions) required by the insurer (state) exceeds the maximum premium the couple is willing to pay – would prevent the couple from purchasing an LCAc1. In theory at least, the couple would choose the LCAc1 as long as $F_{x:y} \leq \hat{F}_{x:y}$, i.e. if the state or provider required an extra premium or additional contribution, $F_{x:y}$, that was smaller than or equal to what the couple was willing to pay (accept) for the LTC coverage: $\hat{F}_{x:y}$. Given that $F_{x:y} \leq \hat{F}_{x:y}$ involves a positive difference between $\hat{F}_{x:y}$ and $F_{x:y}$:

$$\begin{aligned} \hat{F}_{x:y} - F_{x:y} &= \left(Q^* - 1\right) \cdot AF_{x:y}^{JA} - \left(AF_{x:y}^{LCAc1} - AF_{x:y}^{JA}\right) \\ &= Q^* \cdot AF_{x:y}^{JA} - AF_{x:y}^{LCAc1} \geq 0 \end{aligned} \tag{15}$$

It is easy to see that an equivalent decision rule to (15) is:

$$Q^* \geq \frac{AF_{x:y}^{LCAc1}}{AF_{x:y}^{JA}} \equiv CR_t^{LCAc1} \rightarrow \frac{Q^*}{CR_t^{LCAc1}} \geq 1 \tag{16}$$

In short, the couple will be willing to add LTC coverage if the welfare gain is higher than the incremental cost of purchasing the LTC insurance.

As is usual in the literature, we assume that the primary annuitant and co-annuitant have constant relative risk aversion (CRRA) utility functions and that both members are in the healthy state (aa):

$$U_{x:y}^{aa}[b] = \frac{1}{1-\gamma} \cdot \left(\frac{b}{\phi}\right)^{1-\gamma} \tag{17}$$

where $\gamma \in [0, \infty)$, $\gamma \neq 1$. This parameter is the Arrow-Pratt coefficient, representing both risk aversion and the inverse of the elasticity of intertemporal consumption substitution. If $\gamma = 0$, the couple is indifferent to risk, and the greater the value of γ, the greater the risk aversion, i.e. the risk tolerance is lower. We also assume that the risk aversion parameter γ is the same for the husband and the wife.

Note that if $\gamma \in [0, 1)$, the utility function is positive, and if $\gamma > 1$, the utility function is negative. For the assumption that $\gamma = 1$ we have the Bernoulli logarithmic utility function.

The total consumption is normalized by the scaling factor (ϕ), which depends on family size n.

Given that the CRRA function fulfils expressions included in (10) and (11), we need to introduce what are known as "payment weighting factors", $(\theta; \varphi)$, to reduce the value of the utility functions according to the couple's state of dependence:

$$\begin{aligned} U_{x:y}^{da}\left[(1+\xi) \cdot b\right] &= U_{x:y}^{ad}\left[(1+\xi) \cdot b\right] = \frac{\theta}{1-\gamma} \cdot \left(\frac{(1+\xi) \cdot b}{\phi}\right)^{1-\gamma} \\ U_{x:y}^{da}\left[(1+\xi) \cdot b\right] &= U_{x:y}^{ad}\left[(1+\xi) \cdot b\right] = \frac{\theta \cdot}{1-\gamma} \cdot \left(\frac{(1+\xi) \cdot b}{\phi}\right)^{1-\gamma} \\ U_{x:y}^{dd}\left[(1+\xi') \cdot b\right] &= \frac{\varphi}{1-\gamma} \cdot \left(\frac{(1+\xi') \cdot b}{\phi}\right)^{1-\gamma} \end{aligned} \tag{18}$$

where the relationships between payment weighting factors are $1 > \theta \geq \varphi > 0$ if $\gamma \in (0, 1]$, in which case the utility function is positive, and $1 < \theta \leq \varphi$ if $\gamma > 1$, in which case the utility function is negative (Chen et al., 2021).[9]

If we consider the states in which only one member of the couple is alive, $\{af, fa, df, fd\}$, with $\phi = 1$, the payment weighting factors being $k_x \in (0, 1)$ and $k_y \in (0, 1)$ for the primary annuitant and co-annuitant respectively, and under the assumption that the enhanced benefits should compensate for the loss of utility in the states of dependence, it is reasonable to assume that:

$$\begin{aligned} & U_{x:y}^{ad}\left[b \cdot (1+\xi)\right] = U_{x:y}^{da}\left[b \cdot (1+\xi)\right] \geq U_{x:y}^{aa}[b] \\ & U_{x:y}^{dd}\left[b \cdot \left(1+\xi'\right)\right] \geq U_{x:y}^{ad}\left[b \cdot (1+\xi)\right] \\ & U_{y|x}{}^{d}\left[b' \cdot (1+\xi)\right] \geq U_{y|x}{}^{a}\left[b'\right] \\ & U_{x|y}{}^{d}\left[b'' \cdot (1+\xi)\right] \geq U_{x|y}{}^{a}\left[b''\right] \end{aligned} \tag{19}$$

Based on the power utility functions described above, we can find the Q^* that fulfils (7):

$$\mathbb{E}\left[U_{x:y}^{LCAc1}\right] = Q^{1-\gamma} \cdot \mathbb{E}\left[U_{x:y}^{JAc1}\right] \tag{20}$$

$$Q^* = \left(\frac{\mathbb{E}[U_{x:y}^{LCAc1}]}{\mathbb{E}[U_{x:y}^{JAc1}]}\right)^{\frac{1}{1-\gamma}} = \left(\frac{\text{Numerator}}{\text{Denominator}}\right)^{\frac{1}{1-\gamma}} \tag{21}$$

where, in the case of utility functions (17) to (19), we get

$$\begin{aligned} \text{Numerator} = & \\ & \left(\frac{b}{\phi}\right)^{1-\gamma} \cdot \Big({}^{\alpha}\ddot{a}_{x:y(\rho)}^{aa} + \theta \cdot \left((1+\xi)\right)^{1-\gamma} \cdot \left({}^{\alpha}A_{x:y(\rho)}^{ad} + {}^{\alpha}A_{x:y(\rho)}^{da}\right) \\ & \quad + \varphi \cdot \left((1+\xi')\right)^{1-\gamma} \cdot {}^{\alpha}A_{x:y(\rho)}^{dd}\Big) \\ & \quad + {}^{\alpha}\ddot{a}_{y|x(\rho)}{}^{a} + k_x \cdot \left((1+\xi) \cdot b'\right)^{1-\gamma} \cdot {}^{\alpha}A_{y|x(\rho)}{}^{d} \\ & \quad + {}^{\alpha}\ddot{a}_{x|y(\rho)}{}^{a} + k_y \cdot \left((1+\xi) \cdot b''\right)^{1-\gamma} \cdot {}^{\alpha}A_{x|y(\rho)}{}^{d} \end{aligned} \tag{22}$$

and

$$\begin{aligned} \text{Denominator} = & \\ & \left(\frac{b}{\phi}\right)^{1-\gamma} \cdot \left({}^{\alpha}\ddot{a}_{x:y(\rho)}^{aa} + \theta \cdot \left({}_{\rho}^{\alpha}A_{x:y(\rho)}^{ad} + {}_{\rho}^{\alpha}A_{x:y(\rho)}^{da}\right) + \varphi \cdot {}_{\rho}^{\alpha}A_{x:y(\rho)}^{dd}\right) \\ & \quad + {}^{\alpha}\ddot{a}_{y|x(\rho)}{}^{a} + k_x \cdot \left(b'\right)^{1-\gamma} \cdot {}^{\alpha}A_{y|x(\rho)}{}^{d} + {}^{\alpha}\ddot{a}_{x|y(\rho)}{}^{a} \\ & \quad + k_y \cdot \left(b''\right)^{1-\gamma} \cdot {}^{\alpha}A_{x|y(\rho)}{}^{d} \end{aligned} \tag{23}$$

Given the decision rule established in (16), the couple's willingness to buy (choose) an LCAc1 depends on several parameters, the most important being risk aversion, transition probabilities, the design of the uplifts, the indexation rule for benefits in payment, the rate of time preference, the technical interest rate and the value of the payment weighting factors.

4. Numerical illustration

This section shows the results obtained for a numerical example representative of the model developed in the previous section for valuing a notional LCA for couples and also analyses their willingness to purchase it. We present this to give readers a clearer

[9] Alternative expressions to those included in (18) can be found in Chen et al. (2022), where the two payment weighting factors fulfil the relationship $0 < \varphi \leq \theta < 1$, or Wu et al. (2022a), with a different consideration for the payment weighting factors.

idea of how we assess the cost of adding an extra stream of payments to annuities for couples should either or both of them come to require LTC, subject to a predefined structure of uplifts. An additional purpose of this section is to convey an idea of how variable the results can be depending on the type of LCA for couples used, the rule of indexation applied, the uplift structure chosen and the initial health state of the couple. We also illustrate that, due to the presence of loading expenses, a combination of individual LCAs would be a more expensive way of replicating a joint LCA. Furthermore, we explore the couple's willingness to purchase (accept) an LCAc1. However, the exercise should be viewed simply as an illustration for policymakers because the structure of the uplifts and decreases, the rate used for discounting pension wealth and the way we use all the other parameters are fairly arbitrary. We also want to make it clear in this section that the results can vary greatly depending on the set of assumptions used.

4.1. Biometric data, couple life expectancy and basic assumptions

4.1.1. Biometric data and the joint and survivor life expectancy of active couples disaggregated into healthy and unhealthy life years

We use data for Australia (Hariyanto et al., 2014a and 2014b). Apart from the data actually used, several alternative biometric data sets were also considered before performing this exercise, namely those for the USA (Robinson, 1996), Portugal (Esquível et al., 2021), Spain (Albarrán-Lozano et al., 2021) and China (Cui et al., 2022). We decided to rule out them for various reasons: for the USA, the data upon which it is based are now more than 40 years old; for Portugal, the authors do not provide yearly transition probabilities by gender and the transition probabilities are presented for different non-uniform age intervals; for Spain, the transition probabilities for the healthy population are missing; and for China, the authors estimate one-year transition matrices on the basis of three-year transition probability matrices, assuming that the transition probability is constant over three years.

Given that the data[10] from Hariyanto et al. (2014a and 2014b) do not provide all the probabilities by age that we need to perform our numerical example, a cubic smoothing spline[11] with R software was fitted to the data in order to obtain them.

In practice, people are categorized into several levels of dependence according to their inability to perform a given number of ADLs and IADLs. The data we use can be considered quite realistic, since Hariyanto et al. (2014a, 2014b) provide five health states for each sex: one healthy and four dependent on care (with core activity limitation in their terminology). In state a, the individual is able or active with no activity limitation.

In state d_1, the individual has mild limitations. In state d_2, the individual has moderate limitations. They need no help but have difficulty with a core activity task. In state d_3, the individual has severe limitations. In state d_4, the individual has profound limitations. They are unable to do, or always need help with, a core activity task. The data source gives no information about transitions between care received at home and care received in institutions.

The calculation of the life expectancy in years at age 67 and the percentage of that life expectancy likely to be spent in each of the above states for males and females respectively, show clear differences in mortality by sex and health state and contain no surprises (Tables 4.1 and 4.2 in appendix 4, supplementary material).

For the age range as a whole, the time expected to be spent in any state of dependence is higher for females than for males, and this is true irrespective of the initial health state. Such a pattern is indicative of the so-called "male-female health-survival paradox", a phenomenon seen in developed countries in which women experience greater longevity but higher rates of disability and poor health than men (Nusselder et al., 2019; Nielsen et al., 2021; Sherris and Wei, 2021).

The couples we form are defined by age, gender and health state. Here we follow traditional gendered patterns in which the man is the older spouse. For a set of couples where we list a variety of different initial ages for the annuitant (male) and the co-annuitant (female), Table 4 sets out the average time in years and percentages that the couple is likely to spend in each of the states before the death of the second life.

Table 4 also shows for these particular couples the expected years both spouses will be alive (joint life expectancy) and the expected years the surviving spouse will be a widow(er) (survivor life expectancy), broken down by health state. We calculate joint and survivor life expectancies from individual age-specific mortality rates but cannot take into account the correlation in mortality rates between husbands and wives. Evidence suggests that the age-specific mortality rates of married individuals are lower than those of unmarried individuals both because those that marry tend to be healthier and because marriage has protective effects (Compton and Pollak, 2021). Our calculations ignore these effects and assume that the age-specific mortality rates for individuals are independent of their marital status. This assumption could over- or undervalue our estimation of ($e_{\overline{x:y}}^{1}$), but this (small) error decreases when the age difference between spouses increases (Dufresne et al., 2018). Taking marital status and the protective effects of marriage into account when calculating joint and survivor life expectancies would require detailed longitudinal data on couples.

For our benchmark couple, the male is 3 years older than the female on average and it is assumed that the annuitant (male) and the co-annuitant (female) are 67 and 64 years old respectively and both are healthy (able) when they become beneficiaries ($x = 67,\ \ y = 64$). It can be seen that the expected length of time before the death of the second life is 24.17 years, this being the sum of the joint life expectancy (the average time in years before the death of the first life) and survivor life expectancy (the difference between the average time in years until the extinction of the couple and the average time in years before the death of the first life), which are 13.79 and 10.38 years respectively. During this period only 6.35 years are expected to be spent with both lives in the healthy state, i.e. 26.27% of the expected time until the extinction of the couple. For this initially healthy couple, 14.34 years will be spent in states with some type of activity limitation (for at least one of them), i.e. 59.32% of their remaining life expectancy (24.17 years) is expected to be spent in some state of dependence. The expected length of time that both spouses will be alive and requiring LTC is 2.43 years, 10.05% of the average time in years before the death of the second life.

As would be expected, the younger the couple, the lower the percentage of time they would likely spend in any situation of dependence.

Given that the data used provide four states of dependence, the information we showed in Table 4 can be given in greater detail (Tables 4.3, 4.4. and 4.5 in Appendix 4, supplementary material).

The importance of reporting the joint and survivor life expectancies of active couples disaggregated into healthy and unhealthy life years is twofold. First, it makes the computation of the actuarial factors transparent, and these explicit demographic data are very difficult to find in the specialized literature on LCAs. And second, these data provide very useful information to help the couple understand the need to be protected against the cost of LTC services. As we saw in Table 4, the data tell us that couples will spend an average of between 13.6 and 15.6 years with at least one member being care-dependent. In other words, between 57.3% and

[10] To be more specific, in the paper by Hariyanto et al. (2014b) Tables 7.1 and 7.2 present the graduated disability transition probabilities for males and females at a selection of ages.

[11] https://stat.ethz.ch/R-manual/R-devel/library/stats/html/smooth.spline.html.

Table 4
The average time in years ($e^{1}_{\overline{x:y}}$) and percentages that a couple is likely to spend in each of the states before the death of the second member.

Couple $(x,\ y)$	$e^{1}_{\overline{x:y}}$	Joint life expectancy					Survivor life expectancy				
		Total	$e^{aa}_{x:y}$	$e^{ad}_{x:y}$	$e^{da}_{x:y}$	$e^{dd}_{x:y}$	Total	$e^{a}_{y\|x}$	$e^{d}_{y\|x}$	$e^{a}_{x\|y}$	$e^{d}_{x\|y}$
(70,65)	22.84	12.19	5.43	1.66	2.88	2.22	10.65	0.54	1.30	3.08	5.72
%	100	53.37	23.77	7.27	12.61	9.72	46.63	2.36	5.69	13.49	25.04
(67,67)	22.42	13.08	5.86	2.35	2.41	2.46	9.34	0.98	2.06	1.97	4.32
%	100	58.34	26.14	10.48	10.75	10.97	41.66	4.37	9.19	8.79	19.27
(67,64)	24.17	13.79	6.35	2.11	2.90	2.43	10.38	0.73	1.60	2.75	5.30
%	100	57.05	26.27	8.73	12.00	10.05	42.95	3.02	6.62	11.38	21.93
(67,62)	25.48	14.21	6.65	1.96	3.23	2.37	11.27	0.59	1.33	3.37	5.99
%	100	55.77	26.10	7.69	12.68	9.30	44.23	2.32	5.22	13.23	23.51
(65,60)	27.31	15.67	7.54	2.17	3.49	2.47	11.64	0.61	1.35	3.55	6.15
%	100	57.38	27.61	7.95	12.78	9.04	42.62	2.23	4.94	13.00	22.52

Source: Own based on data from Hariyanto et al. (2014a, 2014b) NB: The totals will not necessarily equal the sums of the rounded components

Table 5
The increases and decreases to be applied depending on the annuitant's transitions between the various states.

Starting state, i	Ending state, j				
	a	d_1	d_2	d_3	d_4
$\boldsymbol{a}$	0.000	0.250	0.500	0.750	1.000
$\boldsymbol{d_1}$	−0.200	0.000	0.200	0.400	0.600
$\boldsymbol{d_2}$		−0.167	0.000	0.167	0.333
$\boldsymbol{d_3}$			−0.143	0.000	0.143
$\boldsymbol{d_4}$				−0.125	0.000

Source: Own

60.7% of the couple's life expectancy before the death of the second life ($e^{1}_{\overline{x:y}}$) will be spent in states of dependence.

In the data from Hariyanto et al. (2014a, 2014b), it is assumed that an individual in any state of core activity limitation can only improve by one category over a one-year period, if and only if they survive the year and do not then deteriorate to a more severe state of dependence.

4.1.2. Basic assumptions

Table 5 shows the increases and decreases to be applied depending on the annuitant's transitions between the various states. The diagonal of the matrix embedded in Table 5 shows a zero value because it denotes where the individual's health state remains unchanged. To give an example of a possible transition, when an active (healthy) person becomes dependent at level 4 (a, d_4), the amount of the annuity doubles, given that $\xi_{a4} = 1$.

The darker grey cells show where this transition is not possible. For example, a level-4 dependent cannot become able within a one-year period, i.e. (d_4, a) is not a possible transition.

Once the basic (arbitrary) uplifts have been chosen – meaning those transitions from the able state to any other that give coherence to the uplift structure – the rule for computing any one of the others (the transitions from any one dependent state to another in Table 5) can be expressed as

$$\xi_{ij} = \frac{(1+\xi_{aj})}{(1+\xi_{ai})} - 1; \quad i \in \{1,\ 2, \cdots, 4\},\ j \in \{a, 1, \cdots, 4\}; \qquad (24)$$
$$\xi_{ii} = 0$$

Since this concerns a couple, the structure of uplifts and decreases needs to be combined for two annuitants. It is assumed that both partners receive the same amount of benefit. (Table 4.6 in Appendix 4, supplementary material, which is divided into four parts, shows the increases and decreases applicable to the couple in the case of an LCAc1).

Like in the case of an individual but based on the benefits calculated for a healthy couple (first rows in Table 4.6, parts I and II), all the other increases and decreases to be applied are determined by carrying out a comparison with the benefits assigned to each state $(B_i;\ B_j)$:

$$\xi^{*}_{ij} = \frac{(B_j)}{(B_i)} - 1; \quad j \in \{ad_1,\ ad_2, \cdots, fd_4\},\ i \in \{aa,\ ad_1, \cdots, fd_4\};$$
$$B_{ii} = 0 \qquad (25)$$

In principle, given the initial assumption about the couple's health state (able), only the first row (Table 4.6, parts I and II, in Appendix 4, supplementary material) is used for computing the conversion factor (and the initial retirement benefit). As we will see later, a different row would need to be used if the assumption about the couple's health were to change (sub-section 4.2.2.)

The actuarial valuation is gender-based given that mortality and morbidity are provided for each sex. The financial factor used to discount pension wealth is $(F^k = [0.9804]^k)$, which also means that the initial benefit remains constant over time in real terms (pensions in payment are indexed to prices).

As mentioned earlier, the total consumption is normalized by the scaling factor (ϕ), which depends on family size n. For the numerical example, we apply the "square root equivalent scale" (OECD, 2008), which in our case is $\phi = \sqrt{2}$ when both members are alive and $\phi = 1$ if only one member is alive. Generally, for couples $\phi \in (1,\ 2)$, most authors use values of between 1.06 and 1.7. This specification assumes that consumption is equally shared between the partners, $b/2$, and that there is jointness in consumption (Hubener et al., 2014). This is equivalent to the approach taken by Brown and Poterba (2000) under the assumption of equally weighted, identical subutility functions.

Finally, the values of the payment weighting factors depend on the structure of uplifts and the risk aversion coefficient chosen. For the baseline scenario the values to use are the following:

States	$\mathbf{ad_1}$	$\mathbf{ad_2}$	$\mathbf{ad_3}$	$\mathbf{ad_4}$	$\mathbf{d_1a}$	$\mathbf{d_2a}$	$\mathbf{d_3a}$	$\mathbf{d_4a}$
$\boldsymbol{\theta}$	1.1	1.2	1.3	1.4	1.1	1.2	1.3	1.4
States s	$\mathbf{d_1d_1}$	$\mathbf{d_1d_2}$	$\mathbf{d_1d_3}$	$\mathbf{d_1d_4}$	$\mathbf{d_2d_1}$	$\mathbf{d_2d_2}$	$\mathbf{d_2d_3}$	$\mathbf{d_2d_4}$
$\boldsymbol{\varphi}$	1.2	1.3	1.4	1.5	1.3	1.4	1.5	1.6
States	$\mathbf{d_3d_1}$	$\mathbf{d_3d_2}$	$\mathbf{d_3d_3}$	$\mathbf{d_3d_4}$	$\mathbf{d_4d_1}$	$\mathbf{d_4d_2}$	$\mathbf{d_4d_3}$	$\mathbf{d_4d_4}$
$\boldsymbol{\varphi}$	1.4	1.5	1.6	1.7	1.5	1.6	1.7	1.8

Table 6 summarizes the main assumptions and parameter values and characterizes the baseline case.

Table 6
The baseline case: main assumptions and parameter values.

Biometric data		
Hariyanto et al. (2014a, 2014b). These data provide five health states for each gender: one healthy and four dependent.		
Main assumptions		
1.	Joint life care annuity with a last survivor payout rule (LCAc1).	
2.	Both members of the couple are able (active) at the onset.	
3.	Traditional cohabiting or married couple (male-female).	
4.	Gender-based actuarial valuation.	
5.	Coordinated retirement behaviour within the couple.	
6.	35 non-absorbing couple states (Table 5.6, in appendix 4, supplementary material).	
7.	Independence of risks, i.e. any possible correlation in mortality and morbidity rates between husband and wife is ignored.	
8.	All changes in the couple state other than those driven by mortality and morbidity are ignored.	
Parameter values		
Notation	Description	Values/Source
$(\boldsymbol{x},\ \boldsymbol{y})$	Benchmark couple	Ages (67,64); we also consider the following couples: (70,67); (67,67); (67,62) and (65,60).
$\boldsymbol{\omega},\ \boldsymbol{\omega}'$	Maximum age of the annuitants	109 years
$\boldsymbol{T}$	Maximum length of life (benchmark couple)	$\max\{\omega - x, \omega' - y\} = 45$ years
$\boldsymbol{\xi^*_{ij}}$	Uplift structure	Arbitrary; Table 4.6 compiled by combining the individual structure of increases and decreases (Table 5).
$\boldsymbol{F_b}$	Indexation rule	The initial benefit remains constant over time in real terms, i.e. pensions in payment are indexed to prices; $\alpha = 0$; $G = 0.02 \Rightarrow F_b = 0.9804$).
$\boldsymbol{\rho}$	The subjective pure rate of time preference	$G = 0.02 = \rho$
$\boldsymbol{\gamma}$	Risk aversion coefficient	2
$\boldsymbol{b}/\boldsymbol{2}$	Consumption	Consumption is equally shared between the partners.
$\boldsymbol{\phi}$	Scaling factor of consumption	$\sqrt{2}$ when both members are alive, and 1 if only one of the members is alive.
$\boldsymbol{\theta};\ \boldsymbol{\varphi}$	Payment weighting factors	The values depend on the structure of uplifts and the risk aversion coefficient chosen.
Source: Own		

4.2. Results

4.2.1. The actuarial valuation of an LCA for couples (LCAc1)

Table 7 below presents some selected values for the actuarial valuation of LCAc1 for the set of couples listed in the previous sub-section.

Rows 1 and 2 for each couple show the values for the actuarial factors for an LCA for couples (LCAc1) and for a joint annuity with survivor payout rule (JAc1). The difference between the two is the higher value corresponding to the dependence states in the case of LTC coverage, i.e. being in any of these states means a higher cost for the insurer/sponsor. The higher the uplifts established, the greater the difference between the two actuarial factors.

Row 3 for each couple represents the actuarial cost of covering their LTC in each possible state of dependence as a proportion of the total costs of a classical joint annuity without LTC coverage, i.e. the higher the resulting values of $dxLTCWc1_t$, $dyLTCWc1_t$ and $dxyLTCWc1_t$, the lower the amount of the basic benefit for the LCAc1. As mentioned earlier, these values do not take uplifts into account, only biometric assumptions.

Table 7 also shows the value for the coverage ratio (CR_t^{LCAc1}), which indicates in present value the number of equivalent monetary units needed to determine the initial LCAc1 benefit for each monetary unit of the initial JAc1 basic annuity for couples. This figure provides very important information when the new contingency is included. The lump-sum premium – i.e. the savings accumulated at the time of retirement – for the LCAc1 would be need to between 23.85% and 27.47% higher than for the JAc1 depending on the ages of both members of the couple.

The last column of Table 7, benefit reduction ($BR_t\%$), tells us about the effect that the introduction of the LTC coverage – in this numerical example with graded benefit – would have on the initial benefit of the joint annuity with survivor payout rule (JAc1). In order to maintain the same premium (accumulated savings or contributions) computed for the JAc1, the initial benefit would have to be between 21.55% and 19.26% lower than the estimated benefit without LTC. In fact, we would be using an enhanced pension annuity, which is a special type of LCA in which the uplifts are financed by a reduction – with respect to the initial benefit – in the benefit paid while both annuitants are healthy.

If our benchmark couple, for example, were to ask about the possibility of adding LTC coverage, the information shown in the two last columns of Table 7 could be used to test the attractiveness of the proposed LCA through the use of questions such as *Would you (the couple) be willing to reduce your retirement pension by 20.62% in order to be entitled to LTC benefits* or *Would you be willing to increase your contribution rate (the premium) by25.98% in order to have LTC coverage during your retirement period?*

Table 4.8. (Appendix 4, supplementary material) replicates the values shown in Table 7 but for an LCAc2. Results of the actuarial factors are slightly higher for LCAc2 than for LCAc1. This seems logical since, although the couple in both cases receives the same level of initial benefit, the resulting benefits for most of the transitions from the initial state to other health states are usually higher for LCAc2 than for LCAc1

From a practical social insurance perspective, the actuarial factors applied in order to add the couple's LTC would need to be updated frequently (at least every three years) because the expected longevity of healthy and dependent persons and the transition probabilities between health states change over time and the rate of return applied to discount pension wealth is not known in advance. Obtaining timely, accurate estimations of mortality, transition and incidence rates is highly complex. The introduction of a time trend in insurance policy pricing and systematic uncertainty in LTC product design is strongly recommended for insurers/social security administrations. (Shao et al., 2017; Sherris and Wei, 2021). In developed countries, government authorities should make an effort to construct nationally representative estimates of functional status transition rates.

Table 7
Actuarial valuation of LCAc1: some selected values (basic assumptions).

Couple $(x,\ y)$	Actuarial factors	Total	R	LTC	x	y	x, y	CR_t^{LCAc1}	$BR_t\%$
	1.- $AF^{LCAc1}_{70:65}$	35.18	11.74	2.44	7.62	10.31	5.51		
(70,65)	2.-$AF^{JAc1}_{70:65}$	27.60	11.74	15.86	5.75	6.67	3.43	1.2747	21.55
	3.- $d70,65LTCWc1_t\%$			57.46	20.83	24.17	12.43		
	1.- $AF^{LCAc1}_{67:67}$	35.66	11.96	23.70	7.35	10.29	6.06	1.2719	21.38
(67,67)	2.- $AF^{JAc1}_{67:67}$	28.04	11.96	16.07	5.44	6.88	3.76		
	3.- $d6767LTCWc1_t\%$			57.31	19.40	24.54	13.41		
	1.- $AF^{LCAc1}_{67:64}$	37.35	13.13	24.22	7.80	10.58	5.84	1.2598	20.62
(67,64)	2.- $AF^{JAc1}_{67:64}$	29.65	13.13	16.51	5.87	7.00	3.65		
	3.- $d6764LTCWc1_t\%$			55.68	19.80	23.61	12.31		
	1.- $AF^{LCAc1}_{67:62}$	38.48	13.94	24.54	8.14	10.79	5.62	1.2520	20.13
(67,62)	2.- $AF^{JAc1}_{67:62}$	30.73	13.94	16.80	6.18	7.09	3.52		
	3.- $d6762LTCWc1_t\%$			54.67	20.11	23.07	11.45		
	1.- $AF^{LCAc1}_{65:60}$	40.73	15.46	25.27	8.49	11.09	5.69	1.2385	19.26
(65,60)	2.- $AF^{JAc1}_{65:60}$	32.89	15.46	17.43	6.48	7.36	3.59		
	3.- $d6560LTCWc1_t\%$			52.99	19.70	22.38	10.92		

Source: Own

NB: The totals will not necessarily equal the sums of the rounded components

Table 8
The average time in years ($e^1_{\overline{x:y}}$) and percentage that the couple is likely to spend in each of the states before the death of the second life.

Couple (67, 64)	$e^1_{\overline{x:y}}$	Joint life expectancy					Survivor life expectancy				
		Total	$e^{aa}_{x:y}$	$e^{ad}_{x:y}$	$e^{da}_{x:y}$	$e^{dd}_{x:y}$	Total	$e^{a}_{y\|x}$	$e^{d}_{y\|x}$	$e^{a}_{x\|y}$	$e^{d}_{x\|y}$
$(\boldsymbol{aa})$	24.17	13.79	6.35	2.11	2.90	2.43	10.38	0.73	1.60	2.75	5.30
%	100	57.03	26.27	8.73	11.99	10.04	42.97	3.03	6.63	11.36	21.94
$(\boldsymbol{ad_4})$	20.64	10.1	0.08	6.55	0.13	3.34	10.54	2.57	3.47	0.26	4.25
%	100	48.87	0.37	31.72	0.61	16.17	51.13	12.46	16.80	1.27	20.61
$(\boldsymbol{d_4a})$	22.76	8.27	0.08	0.10	5.99	2.10	14.49	0.07	0.85	5.93	7.64
%	100	36.33	0.35	0.43	26.30	9.25	63.67	0.30	3.75	26.05	33.57
$(\boldsymbol{d_2d_2})$	23.48	13.01	0.49	1.64	1.68	9.20	10.47	0.50	1.92	1.64	6.41
%	100	55.4	2.07	7.00	7.16	39.17	44.6	2.15	8.19	6.96	27.29
$(\boldsymbol{d_4d_4})$	17.39	6.4	0.01	0.10	0.08	6.21	10.99	0.14	2.65	0.38	7.83
%	100	36.77	0.03	0.57	0.46	35.71	63.23	0.81	15.25	2.17	45.00

Source: Own based on data from Hariyanto et al. (2014a, 2014b)

NB: The totals will not necessarily equal the sums of the rounded components

4.2.2. The impact of changing the health state of either or both of the annuitants on the actuarial valuation of a joint LCA)

So what might happen if the initial health state of either or both of the annuitants is not able when they become entitled to receive benefits?

The average time in years ($e^1_{\overline{x:y}}$) and percentage that the couple is likely to spend in each of the states before the death of the second life depends on the couple's initial health state can be seen in Table 8.

For the assumption that both members of the couple are in the most severe state of dependence, life expectancy until the death of the second life is only 17.39 years and the amount of time the couple is likely to spend in any dependence state reaches 96.42% (16.77). The worse the couple's initial health state, therefore, the shorter the life expectancy until the death of the second life and the greater the percentage of time likely to be spent in states of dependence.

To compute the actuarial factor of JAc1, two assumptions are examined. First, the insurer/sponsor does not take the couple's health state into account to determine the retirement benefit, i.e. the actuarial factor is independent of the couple's real health state (2.- $AF^{JAc1aa}_{67:64}$). And second, the actuarial factor (3.- $AF^{JAc1x\boldsymbol{y}}_{67:64}$) is based on the mortality assumption driven by the couple's health state, i.e. the insurer/sponsor knows their health state and uses the remaining life expectancy until the death of the second life to compute the annuity rate. These are commonly referred to as special-rate life annuities, impaired annuities or enhanced annuities. Here we apply this type of annuity to couples. As we saw in Table 8, given that their life expectancy is shorter than for an able couple, their benefit is higher.

As Table 9 (above) shows, the couple's initial health state has a great impact on the value for the coverage ratio (CR_t^{LCAc1}), especially when the insurer/sponsor takes the couple's health state into account to compute the actuarial factor for the JAc1. Unsurprisingly, the worse the couple's initial health state, the higher the value for the coverage ratio. The information conveyed by this figure is important when the new contingency is included or when the initial health state is modified. The lump-sum premium, i.e. the accumulated saving at the time of retirement for the LCAc1, would need to be between 44.66% and 81.82% higher than for the "en-

Table 9
Actuarial valuation of LCAc1 when the initial health state of either or both of the annuitants is not able when they become entitled to receive benefits. Benchmark couple (67,64).

Couple state	Actuarial factors	Total	R	LTC	x	y	x, y	CR_t^{LCAc1}
$(\boldsymbol{aa})$	1. $-\ AF_{67:64}^{LCAc1\boldsymbol{aa}}$	37.35	13.13	24.22	7.80	10.58	5.84	1.2598
	2. $-\ AF_{67:64}^{JAc1aa}$	29.65	13.13	16.51	5.87	7.00	3.65	
	3. $-\ AF_{67:64}^{JAc1aa}$	29.65	13.13	16.51	5.87	7.00	3.65	
$(\boldsymbol{ad_4})$	1. $-\ AF_{67:64}^{LCAc1a\boldsymbol{d_4}}$	36.62	2.38	34.24	4.23	20.99	9.02	1.2351
	3. $-\ AF_{67:64}^{JAc1a\boldsymbol{d_4}}$	24.30	2.38	21.92	2.69	13.82	5.40	1.5074
$(\boldsymbol{d_4a})$	1. $-\ AF_{67:64}^{LCAc1\boldsymbol{d_4}a}$	35.05	4.90	30.15	15.19	9.19	5.76	1.1821
	3. $-\ AF_{67:64}^{JAc1\boldsymbol{d_4}a}$	24.23	4.90	19.33	10.46	5.44	3.42	1.4466
$(\boldsymbol{d_2d_2})$	1. $-\ AF_{67:64}^{LCAc1\boldsymbol{d_2d_2}}$	41.67	2.32	39.35	5.65	10.94	22.75	1.4054
	3. $-\ AF_{67:64}^{JAc1\boldsymbol{d_2d_2}}$	28.60	2.32	26.28	4.09	7.12	15.07	1.4567
$(\boldsymbol{d_4d_4})$	1. $-\ AF_{67:64}^{LCAc1\boldsymbol{d_4d_4}}$	34.32	0.37	33.95	3.94	11.04	18.97	1.1575
	3. $-\ AF_{67:64}^{JAc1\boldsymbol{d_4d_4}}$	18.88	0.37	18.51	2.23	6.11	10.17	1.8182

Source: Own
NB: The totals will not necessarily equal the sums of the rounded components

hanced" JAc1 depending on the couple's initial health state. For all the cases analysed, adding LTC coverage would be more expensive than the benchmark (25.98%).

When the insurer/sponsor does not use the remaining life expectancy until the death of the second life to compute the annuity rate for the JAc1, the variation in the premiums is much lower than before. The savings accumulated at the inception time of the LCAc1 would need to be between 15.75% and 40.54% higher than for the JAc1, depending on the couple's initial health state. In only one case (d_2d_2) would adding the LTC coverage be more expensive (40.54%) than the benchmark (25.98%).

4.2.3. The impact of the indexation policy and the uplift structured used

Another important issue is the indexation policy used. This is because the growth of benefits in real terms (covered wage bill indexation, $F_1 = 1$) increases the cost of adding a couple's LTC coverage, given that it benefits females insofar as they are more likely to reach greater ages in a worse disability state than males.[12]

Checking the robustness of the uplift structure used is also important, given that it could explain much of the cost of converting a joint annuity with survivor payout rule (JAc1) into a graded joint life care annuity for couples with a last survivor payout rule (LCAc1). It is safe to say that the higher the ratio between the uplifts assigned to the most and least severe states (ξ_{ad_4}/ξ_{ad_1}), the higher the cost of converting the annuity for couples. This is only to be expected, given that higher uplifts in the more severe states mainly benefit females due to their higher life expectancy and morbidity rates.[13]

4.2.4. Joint LCA versus a combination of individual LCAs

Following Brown and Poterba's (2000) reasoning regarding the relationship between a joint annuity with a last survivor payout rule (JAc) and a single life annuity, a joint LCA can be structured to perfectly replicate a combination of individual LCAs by adjusting the survivor benefit ratios (Appendix 4, supplementary material).

So what would make it preferable to follow a couples approach rather than an individual approach when acquiring an LCA for couples?

The presence of loading for expenses (one of the premium components) would convert the combination of individual LCAs into a more expensive way of replicating a joint LCA. Standard insurance practice conventionally assumes that a joint life policy can be more affordable for two people than purchasing two separate policies. According to Houis (2019), three quarters of the LTCI policies issued in France provide a "couple advantage" in two forms: Either a reduction on each premium (generally 10% and not more than 20%); or a reduction on a single premium (20%).

According to Olivieri and Pitacco (2015), expenses can be included in the premium via an appropriate loading. They can be classified into three main groups: acquisition (the most important within this type being agents' commission and also for an LCA the medical examination, given that entitlement to LTC benefits depends on a mandatory medical assessment when one member of the couple makes a transition to a better/worse health state), collection and administration. Annuities typically use an upfront commission system. Agent commissions can vary depending on total contract value and annuity type, but they usually range between 1% and 7%. Commission is often higher for more complex annuities.[14]

For the benchmark couple, the lump-sum premium for an LCAc1 paying out a monthly amount of 2,000$ at the onset is compared with the sum of the premiums payable if they purchased LCAs separately (Each individual LCA pays out 1,000$ per month for each member). We consider three scenarios for quantifying the expenses loading (1.-Low, 2.-Base and 3.-High). It is assumed that the profit/safety loading is already included in the premium via mortality/morbidity biometric data and the discount factor.

The results are shown in Table 10. We can see that the total of the lump-sum premiums for LCAs purchased separately by both members of the couple is between 1.27% (6,047$) and 3.36% (17,846$) more expensive than for an LCAc1 for couples.

The resulting upfront expenses loadings for each scenario are a combination of several types of expenses (Olivieri and Pitacco, 2015) (See Table 4.12, Appendix 4, supplementary material).

In short, under very conservative assumptions about current LTC commission rates and loading expenses,[15] the price of pur-

[12] Details can be found in the supplementary material, Appendix 4 Table 5.9.
[13] Details can be found in the supplementary material, Appendix 4 Tables 4.10 and 4.11.
[14] https://www.annuityexpertadvice.com/annuity-cost/.
[15] https://www2.crumplifeinsurance.com/docLibrary/axaCommissionSchedulesLTC.pdf.

Table 10
Lump sum premiums in $ for an LCAc1 for the benchmark couple (x=67, y=64)). Uplifts are determined according to Table 5 for singles and the combination for the couple.

Items	Scenario 1: Low			Scenario 2: Base		Scenario 3: High	
	Premiums		Expenses	Premiums	Expenses	Premiums	Expenses
	Gross	Net	% Upfront	Gross	% Upfront	Gross	% Upfront
x	202,170	189,859	6.48%	213,103	12.24%	224,634	18.32%
y	275,427	258,338	6.62%	290,666	12.51%	306,737	18.73%
Couple	471,550	448,197	5.21%	492,096	9.79%	513,524	14.58%
Saving	6,047 (1.27%)			11,673 (2.32%)		17,846 (3.36%)	
Source: Own							

Table 11
The couple's willingness to choose an LCAc1 in the case of $\boldsymbol{\gamma = 2}$.

Couple $(x,\ y)$	1	2	3	4	5	6	7	8			
	$AF_{x:y}^{LCAc1}$	$AF_{x:y}^{JAc1}$	$F_{x:y}$	Q^*	$\hat{F}_{x:y}$	5-3	$\frac{Q^*}{CR_t^{LCAc1}}$	Critical values			
								ρ	γ	α_1	α_2
(70,65)	35.18	27.60	7.58	1.292	8.05	0.464	1.0132	0.0322	2.63	−0.22	0.16
(67,67)	35.66	28.04	7.62	1.291	8.16	0.540	1.0151	0.0343	2.78	−0.25	0.18
(67,64)	37.35	29.65	7.70	1.279	8.27	0.569	1.0152	0.0334	2.80	−0.23	0.17
(67,62)	38.48	30.73	7.75	1.270	8.31	0.562	1.0146	0.0324	2.76	−0.21	0.15
(65,60)	40.73	32.89	7.84	1.258	8.47	0.626	1.0154	0.0325	2.86	−0.20	0.15
Source: Own											

chasing an LCA for couples would be up to about 3.36% lower than the price of the same annuities purchased separately.

4.2.5. The couple's willingness to choose an LCAc1

To conclude the numerical example, Table 11 ($\gamma = 2$) depicts some outcomes as regards the couple's willingness to choose an LCAc1 according to the model seen in Section 3.3. Table 5.13 (Appendix 4, supplementary material) also shows results for the case of $\gamma = 0.25$.

Columns 1 and 2 for each couple show the value of the actuarial factors for an LCA for couples (LCAc1) and a joint annuity with survivor payout rule (JAc1). Column 3 shows the cost of adding the LTC coverage. The Q^* in column 4 is the value that fulfils $\mathbb{E}[U_{x:y}^{LCAc1}] \equiv \mathbb{E}[Q \cdot U_{x:y}^{JAc1}]$ and which, according to its conceptual interpretation, provides the nexus between the couple's assessment of LCAc1 and the actuarially fair price of JAc1. Column 5, $\hat{F}_{x:y}$, is the extra premium the couple would be willing to pay to add LTC coverage to the JAc1.

Columns 6 and 7 show the values for the decision rule, the results of the equivalent formulas (15) and (16). For the base scenario, at least in theory, all the couples would be willing to purchase/choose an LCAc1 given that the indicator ($\frac{Q^*}{CR_t^{LCAc1}}$) is greater than 1, i.e. they would be willing to add the LTC coverage as long as the welfare gains were higher than the incremental cost of purchasing LTC. For the benchmark couple $(x, y) = (67, 64)$, we get that $Q^* = 1,279$, i.e. the couple is indifferent to choosing between receiving $Q^*\cdot$ JAc1 benefits – $Q^* \cdot b = 1,279 \cdot 2 = 2,558$ monetary units for the rest of their joint life and $Q^* \cdot b' = Q^* \cdot b'' = 1,279 \cdot 1 = 1,279$ u.m. in a widowhood state – and receiving the benefits promised by the LCAc1. As seen in Section 3.3, an alternative to the decision rule ($Q^*/CR_t^{LCAc1} \geq 1$) is $\hat{F}_{x:y} - F_{x:y} = Q^* \cdot AF_{x:y}^{\text{JAC1}} - AF_{x:y}^{LCAc1} \geq 0$; that result in our numerical illustration example: $\hat{F}_{x:y} - F_{x:y} = 0.569$.

The first two columns of main column 8, ρ and γ, show the critical values for the subjective rate of time preference along with the risk aversion coefficient that would make couples indifferent to purchasing LTC coverage, i.e. ($\frac{Q^*}{CR_t^{LCAc1}} = 1$). The value used for ρ is 0.02 (Table 6).

The result for ρ makes sense, since a higher subjective discount rate could be interpreted in such a way that the couple would be less patient about the future, i.e. the couple reduces the weighting of future transactions in the utility as a whole. Consequently, and because dependence on care is more likely to occur at greater ages, the willingness to choose LTC coverage decreases the higher the couple's subjective discount rate. The same result was achieved by Chen et al. (2021) for individuals.

If the annuitants are less risk averse, their willingness to add LTC coverage increases and thus choosing an LCAc1 becomes more probable. Following the reasoning made by Chen et al. (2021) for individuals, this outcome makes sense because, compared to a JAc1, an LCAc1 provides higher benefits in dependent states but at the same time involves a less smooth pattern. A more risk averse couple would prefer a smoother consumption pattern, and the consequence of this is that their corresponding willingness to choose the LCAc1 diminishes.

Finally, for reasons of completeness and given that the indexation rate for benefits in payment (α) is a key element in most social insurance schemes, the last two columns of main column 8 in Table 11 provide the sensitivity analyses for this parameter for the base scenario.

The wide range between the upper (α_2) and lower (α_1) bounds means that the validity of the results shown in Table 11 depends very little on this parameter.

Therefore, as anticipated in Section 3.3. and under the assumption that the couple bases its decisions on the (discounted) expected lifetime utility – which is to a large extent unrealistic according to Boyer et al. (2019), Lambregts and Schut (2020) and Tennyson et al. (2022), given that most individuals (couples) are not well informed about their individual (joint) LTC risks, which makes it difficult for them to make the right LTC insurance decisions – a great number of parameters influence the couple's willingness to buy (choose) an LCAc1. The most important of these are risk aversion, transition probabilities, the design of the uplifts, the indexation rule for benefits in payment, the rate of time preference, the technical interest rate, and the value of the payment weighting factors.

Finally, as we have seen in this paper, an LCAc is very a complex annuity and, as stated by Brown et al. (2017, 2021), the complexity of the annuity decision reduces people's ability to value it. While this complexity can be somewhat lessened by presenting the annuity information more transparently (in our case we have tried to do so by showing the joint and survivor life expectancy of active couples disaggregated into healthy and unhealthy life years), most of the complexity stems from having to consider how the annuity would alter consumption streams in different states of the world, an inherently complex task.

5. Conclusion and future research

In this paper, which helps to fill a gap in the literature, we have developed a methodology for valuing a synthetic joint LCA in which the health dynamics rely on the illness-death multistate framework. Transitions were modelled from the initial healthy state of both members of the couple to the absorbing death state. The impact of introducing the LTC contingency on the couple's annuity has been assessed by comparing the initial benefits in both cases. We have also analysed the couples' willingness to choose this type of LCA for couples. We have extended to the case of LCAs for couples the methodology used by Chen et al. (2021) to determine the policyholder's willingness to pay for the care option.

Using Australian LTC transition probability data for a realistic calibration and assuming independence of the risks involved, we have numerically illustrated the model and the theoretical findings involved. Two different LCAs for couples have been explored: a joint life care annuity with a last survivor payout rule (LCAc1) and a life care annuity with a reversionary payout rule (LCAc2). We have argued that LCAc1 is the most suitable annuity for dual-earner couples. Given that there is clear evidence that retirement decisions within couples are interdependent and that dual-earner couples are on the rise in developed countries, the availability of both types of LCAc may complete the market over and above the use of products for singles alone.

One important aspect of our model is that it enables us to express the actuarial factor for both types of LCAc by using the joint and survivor life expectancy of active couples disaggregated into healthy and unhealthy life years. The paper has highlighted the importance of reporting the expected years both spouses will be alive (joint life expectancy) and the expected years the surviving spouse will be a widow(er) (survivor life expectancy) broken down by health state, given that this information makes the computation of the actuarial factors transparent and provides highly useful information to help the couple understand the need to be protected against the cost of LTC services. This is in line with the proposal to increase the transparency of complex products and strategies during the decumulation phase (Bär and Gatzert, 2022).

We have also shown that a great number of parameters influence the couple's willingness to buy (choose) an LCAc1, the most important of these being risk aversion, transition probabilities, the design of the uplifts, the indexation rule of benefits in payment, the rate of time preference, the technical interest rate, and the value of the payment weighting factors. As a general rule, those couples that are more impatient to consume and more averse to risk will be less willing to choose an LCAc.

Finally, based on the model presented in this paper, at least two directions for future research can be identified:

- ✓ To introduce the more realistic assumption that there is dependence between two lifetimes. The impact of this assumption on the annuity rate depends on several factors: the data set, the ages of both members of the couple, the technical interest rate, the structure of the benefits, the type of dependence between lifetimes … etc. To make it possible to introduce the dependence between mortality risks with any degree of coherence, we would have to abandon the Markov model and change to another type (semi-Markov (Ji and Zhou, 2019) or Marshall-Olkin (Gobbi et al., 2019).
- ✓ To analyse the issue of gender redistribution when using gender-neutral rates to compute the initial LCA benefit for couples. Generally speaking, converting retirement benefit into an LCA with graded benefits within a pre-existing public pay-as-you-go pension system using gender-neutral annuity factors involves a large increase in ex-ante gender redistribution compared to a system without LTC coverage (Vidal-Meliá et al., 2019). It would be useful to analyse the effect of using gender-neutral rates within the couple.

CRediT authorship contribution statement

Manuel Ventura-Marco: Conceptualization, Methodology, Software, Validation, Formal analysis.
Carlos Vidal-Meliá: Conceived the research and was in charge of overall direction and planning, Supervision, Formal analysis, Writing- Original draft preparation. Writing - Review & Editing.
Juan Manuel Pérez-Salamero: Visualization, Investigation.
All authors contributed to the interpretation of the results and have read and agreed to the published version of the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Supplementary material

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