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The Intersection of Housing and Policy: Evaluating Welfare Effects of Carbon and Fiscal Strategies

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Resumen

Más allá de ser una necesidad básica, la vivienda desempeña un papel clave en el funcionamiento de las economías modernas al estar estrechamente vinculada al sistema financiero, al mercado laboral y a la estabilidad macroeconómica. Su doble naturaleza como bien de consumo y activo colateral convierte al mercado inmobiliario en un canal fundamental de transmisión entre los activos financieros, las condiciones de crédito y los resultados macroeconómicos. Esta doble función la posiciona como un componente esencial para comprender los efectos de las políticas fiscales, ambientales y monetarias. Sin embargo, esa misma interconexión introduce vulnerabilidades sistémicas: las fluctuaciones en los precios de la vivienda pueden amplificar choques macroeconómicos, profundizar desigualdades y dificultar la implementación de políticas eficaces. Comprender estas dinámicas resulta esencial para diseñar políticas que promuevan la estabilidad, sostenibilidad y equidad, especialmente frente a dos desafíos cruciales de nuestra era: las secuelas de la crisis financiera global de 2008 y la transición hacia emisiones netas cero.

Esta tesis examina los mecanismos mediante los cuales la vivienda, utilizada como colateral hipotecario, modela las fricciones financieras en la economía e impacta los resultados económicos. La caída en los precios de la vivienda reduce el valor colateral disponible para hogares con restricciones crediticias, limitando su capacidad de endeudamiento y gasto. Este efecto de acelerador financiero intensifica los impactos de distintos tipos de choques, ya sean de origen político, ambiental o macroeconómico. Para los responsables de política económica, comprender estos mecanismos es crucial para evaluar la eficacia y los efectos distributivos de sus decisiones. En concreto, cada uno de esto capítulo tiene como objetivo lo siguiente:

- Capítulo 2: El papel del gasto público en el mercado de la vivienda y sus efectos sobre la inversión y precios residenciales.

- Capítulo 3: El análisis del impacto distributivo y macroeconómico de los impuestos al carbono, diferenciando sectores y tipos de hogares.

- Capítulo 4: El desarrollo de una función de pérdida de bienestar para evaluar políticas en economías condicionadas por preocupaciones climáticas.

A lo largo de estos tres capítulos, este trabajo explica, mediante modelos neokeynesianos de Equilibrio General Dinámico, la interacción entre los mercados de la vivienda, las fricciones financieras y las externalidades ambientales con políticas macroeconómicas clave: fiscal, ambiental, monetaria y macroprudencial. Los tres capítulos se articulan en torno a un marco analítico común que sitúa la heterogeneidad de los hogares, la garantía inmobiliaria y las interdependencias sectoriales en el centro de la evaluación de políticas, ofreciendo una perspectiva detallada sobre el bienestar, la estabilidad y los objetivos climáticos.

Las contribuciones principales de esta tesis en el campo de la macroeconomía dinámica, y más concretamente en el análisis de decisiones políticas a través de modelos de equilibrio general neokeynesianos, son las siguientes.

- Evidencia cómo la política fiscal interactúa con el mercado de la vivienda y las restricciones de crédito en un entorno con producción residencial endógena y movilidad laboral limitada.

- Ofrece recomendaciones prácticas para gestionar la transición hacia economías de emisiones netas cero, con un enfoque particular en las políticas basadas en impuestos al carbono, subrayando la necesidad de intervenciones sectoriales específicas y estrategias de mitigación focalizadas para abordar los impactos distributivos.

- Desarrolla un marco teórico integrado basado en el bienestar, que combina las fricciones financieras con las externalidades climáticas, proporcionando a los responsables de la política económica un enfoque estructurado para evaluar las tensiones entre la estabilidad macroeconómica y los objetivos ambientales.

En definitiva, esta investigación demuestra que las dinámicas del mercado inmobiliario son determinantes en las respuestas macroeconómicas ante crisis financieras y transiciones climáticas. A través de los modelos de equilibrio general utilizados es posible considerar la vivienda en su doble papel, como bien de consumo y como fuente de colateral, y poder entender cómo las políticas públicas interactúan con la renta disponible de los hogares y las estructuras sectoriales. El análisis ofrece a los responsables de políticas herramientas concretas para mitigar los mecanismos de amplificación financiera en tiempos de crisis, y diseñar estrategias de fijación de precios al carbono que equilibren los objetivos de reducción de emisiones con la estabilidad económica y financiera. En consecuencia, la vivienda emerge como un canal de transmisión macroeconómica esencial que debe incorporarse explícitamente en toda estrategia orientada a responder simultáneamente a los desafíos combinados del endeudamiento, la desigualdad y la transición climática.

Capítulo 2: Gasto público y el dilema de la vivienda

Este capítulo analiza cómo el gasto público se propaga a través del mercado de la vivienda mediante un modelo dinámico de equilibrio general. En la década previa a la crisis financiera de 2008, el sector residencial adquirió una relevancia sin precedentes dentro de la economía española. A partir de mediados de los años 90, tanto la inversión residencial como el empleo en el sector construcción mostraron un crecimiento sostenido. En 2006, la inversión residencial llegó a representar el 12% del PIB, duplicando el valor observado una década atrás. Asimismo, la proporción de horas trabajadas en construcción sobre el total nacional pasó del 10% al 15% en el mismo período. Esta tendencia fue incluso más pronunciada que la observada en Estados Unidos durante el mismo periodo[1] y radicalmente opuesta a la estabilidad del peso de la construcción respecto al total de la producción en el resto de los países de la Eurozona.

Estos datos posicionaban al mercado inmobiliario como motor del crecimiento económico, pero también como una fuente potencial de vulnerabilidad estructural[2]. Con el estallido de la burbuja inmobiliaria y el inicio de la recesión global, el gobierno español aumentó drástica y rápidamente el gasto público, principalmente en forma de consumo de bienes y servicios. Sin embargo, esta estrategia no logró frenar el colapso del sector construcción ni restaurar el crecimiento sostenido en los precios de la vivienda, que no volvieron a sus niveles precrisis en al menos una década. El inicio de la crisis estuvo marcado por la concurrencia de múltiples factores, lo que dificulta aislar el impacto específico de las políticas expansivas de gasto público sobre el PIB y el mercado inmobiliario, así como comprender el papel que desempeñaron en la lenta recuperación del sector de la construcción en los años posteriores.

[1]Según Weinberg (2013), los precios de la vivienda en Estados Unidos se duplicaron entre 1998 y 2006, mientras que la inversión residencial aumentó de aproximadamente el 4,5% del PIB estadounidense a cerca del 6,5% durante el mismo periodo. Además, el empleo en sectores relacionados con la vivienda llegó a representar aproximadamente el 40% de la creación neta de empleo en el sector privado entre 2001 y 2005.

[2]Autores como Green (1997), Coulson & Kim (2000) y Leamer (2007) han utilizado diversos enfoques para demostrar que la inversión residencial precede al ciclo económico. Recientemente, Green (2022) señaló un debilitamiento de la inversión en vivienda, un indicador adelantado, desde la Gran Recesión.

Este escenario plantea la siguiente cuestión: en un escenario contrafactual de ausencia de expansión fiscal, ¿habrían experimentado una caída de mayor o menor magnitud el PIB, la inversión residencial y los precios de la vivienda, manteniendo constantes el resto de las condiciones? El presente capítulo tiene como objetivo ofrecer una explicación coherente a este fenómeno y, en consecuencia, proporcionar una respuesta a dicha interrogante.

En general, la literatura que recoge la evidencia empírica muestra resultados dispares respecto al impacto del gasto público sobre los precios de la vivienda y la inversión residencial. Mientras que algunos estudios encuentran efectos positivos, otros revelan efectos negativos, y otros incluso no identifican ninguna relación significativa, siendo estas diferencias atribuibles a factores como la economía estudiada, el período, las variables consideradas, los shocks analizados y la estrategia utilizada para identificar los shocks fiscales. Aunque este amplio rango de resultados refleja la capacidad del modelo empírico para capturar características económicas clave que pueden influir en el impacto de los shocks fiscales en la actividad residencial y los precios inmobiliarios, este capítulo explica además, esta discrepancia en los resultados.

En una primera aproximación de resultados, se constata un efecto positivo del consumo público sobre el PIB y la inversión residencial en España, además de un efecto adverso y persistente, aunque leve, en los precios de la vivienda. Sin embargo, estos efectos no son independientes de los cambios en determinadas características económicas, sino que efectivamente la reacción de la inversión residencial ante un estímulo fiscal depende fundamentalmente de tres aspectos: la elasticidad de sustitución del trabajo entre sectores, la elasticidad de la oferta de trabajo frente a los cambios en los salarios, y el nivel de endeudamiento de los hogares (medido por la relación préstamo-valor, y la proporción de hogares endeudados). En concreto, cuando el mercado laboral presenta una alta movilidad intersectorial y los hogares responden débilmente a cambios en salarios, los efectos negativos del gasto público sobre la inversión residencial se intensifican. Asimismo, una alta proporción de hogares endeudados acentúa la caída en consumo y crédito, agravando la contracción en el sector. Estos resultados sugieren que no existe una "regla general" sobre el efecto del gasto público: su impacto depende de la estructura institucional, laboral y financiera del país. Paralelametne, un análisis de bienestar evidencia que las políticas de gasto expansivas, implementadas para reactivar la economía tras el desplome del sector inmobiliario, resultan especialmente perjudiciales para los hogares endeudados, quienes experimentan pérdidas significativas de bienestar. En este contexto, se observa, además que, si bien las inversiones públicas en la adquisición de vivienda pueden ayudar a sostener la actividad en el sector residencial, estas políticas conllevan una caída general del bienestar.

Este trabajo constituye una contribución relevante a la literatura que incorpora el sector de la vivienda en modelos dinámicos de equilibrio general. La Gran Recesión suscitó un notable interés investigador en el análisis de las fricciones financieras a través de la introducción del mercado de la vivienda en los modelos de equilibri general. En este marco analítico, la capacidad de endeudamiento de hogares y empresas se ve condicionada por la dinámica de los precios de la vivienda y del suelo. Los modelos que integran estas fricciones financieras han sido ampliamente utilizados para examinar los mecanismos de transmisión de perturbaciones económicas y los efectos macroeconómicos de políticas específicas. Dentro de esta línea de investigación, nuestro estudio realiza dos contribuciones principales. En primer lugar, se analiza el impacto de las perturbaciones del gasto público en el producto interior bruto (PIB), el sector residencial y el bienestar de los hogares. En segundo lugar, nuestro análisis enfatiza la importancia de variables tales como la elasticidad de la oferta de trabajo, la movilidad laboral entre los sectores de la vivienda y los bienes de consumo, el nivel de endeudamiento de los hogares y los costes en las transacciones de compra-venta de vivienda.

Sorprendentemente, a pesar de su importancia, estos parámetros han recibido una escasa atención en la literatura. No obstante, se trata de factores económicos estrechamente vinculados a políticas concretas y con importantes implicaciones normativas. Por ejemplo, las regulaciones sobre la vivienda de alquiler o en propiedad, las políticas del mercado laboral, las reformas educativas, las reformas fiscales y las políticas macroprudenciales pueden afectar los valores de estos parámetros. Según Cardi & Restout (2015), las diferencias entre países en la movilidad laboral entre los sectores de bienes comerciables y no comerciables pueden oscilar entre 0,22 para los Países Bajos y 1,80 para Estados Unidos, tal como se refleja en la elasticidad de sustitución. Algo similar ocurre con los límites que se imponen en la concesión de préstamos, especialmente en lo que respecta al ratio préstamo-valor, que compara el importe del crédito con el valor del inmueble ofrecido como garantía. Este indicador es decisivo tanto para determinar el nivel de endeudamiento de los hogares, como para establecer qué proporción de ellos enfrenta restricciones crediticias. Tanto este ratio como los costes asociados a la compra o venta de una vivienda presentan una notable variabilidad entre países y a lo largo del tiempo.

En definitiva, los resultados de este capítulo ponen de relieve la necesidad de un diseño fiscal cuidadosamente calibrado en contextos de crisis vinculadas al mercado de la vivienda. Los efectos positivos de los aumentos del gasto público sobre el PIB y el empleo pueden verse neutralizados o incluso revertidos si se ignoran las fricciones del mercado laboral y las características del endeudamiento privado. Específicamente, en economías con baja movilidad sectorial, alto nivel de deuda y fuerte dependencia del valor de la vivienda como colateral, el gasto público tradicional puede actuar como un búmeran, exacerbando las crisis en lugar de mitigarlas. Políticas públicas alternativas como la de compra de vivienda por parte del gobierno, ofrecen una vía para sostener la actividad constructora, pero conllevan importantes costes en términos de bienestar agregado. Por ello, ninguna de las dos políticas ofrece una solución óptima sin condiciones. Es necesario adoptar enfoques más integrales que combinen el gasto público con medidas macroprudenciales y reformas del mercado laboral. Esto incluye: políticas que faciliten la movilidad entre sectores, ajustes en la regulación del crédito, e instrumentos fiscales que reconozcan la naturaleza dual de la vivienda como bien de consumo y activo financiero. El modelo desarrollado en este capítulo proporciona un marco sólido para evaluar distintas combinaciones de políticas en función de sus efectos macroeconómicos, sectoriales y distributivos. Además, sus resultados pueden ser generalizados a otros países que enfrentan dilemas similares: alto endeudamiento privado, dependencia del sector inmobiliario, y presiones sobre el gasto público como mecanismo de estabilización.

Capítulo 3: Precios del Carbono, Desigualdad de los Hogares y Estrategias de Compensación

Este capítulo dirige su análisis a la política medioambiental y examina los efectos macroeconómicos y redistributivos de la fijación de precios a las emisiones de carbono en los sectores residencial y no residencial. El cambio climático representa uno de los mayores desafíos del siglo XXI, con implicaciones profundas para la economía global, los sistemas sociales y la estructura productiva. Las principales economías desarrolladas, y en particular la Unión Europea, se han comprometido con ambiciosas metas de reducción de emisiones de gases de efecto invernadero. En este contexto, las políticas de precios al carbono se han consolidado como uno de los mecanismos más eficaces para internalizar las externalidades ambientales e inducir cambios de comportamiento tanto en los hogares como en las empresas.

Aunque tradicionalmente el foco se ha centrado en el sector productivo, existe una creciente preocupación por el impacto que estos impuestos al carbono tienen en las economías cuando se aplican al sector residencial, especialmente en áreas como la calefacción, el consumo energético doméstico y el uso de combustibles. Sin embargo, hasta el momento, las políticas orientadas a reducir las emisiones en este sector han sido menos frecuentes y han recibido menor atención por parte de la comunidad investigadora. Tal como se evidenció en Francia durante el denominado movimiento de los “chalecos amarillos”, la aceptación pública de los impuestos al carbono es un factor clave en la viabilidad de estas medidas. Dicha aceptación está estrechamente vinculada a los efectos distributivos de los impuestos, que son políticamente sensibles y objeto de un amplio debate entre las autoridades europeas (European Environmental Bureau (2022); European Commission (2021); Marcu et al. (2023)). Para alcanzar el objetivo de emisiones netas cero en 2050, es fundamental diseñar políticas de fijación de precios del carbono eficaces en el sector residencial, lo cual requiere de una comprensión profunda tanto de sus efectos macroeconómicos como de sus implicaciones distributivas durante la transición. Estos efectos varían dependiendo de la estructura de ingresos, riqueza inmobiliaria, acceso al crédito y capacidad de ahorro de los hogares, generando tensiones en torno a la equidad y la aceptación política de las medidas climáticas. Conocer las implicaciones de estas políticas ambientales permitirá desarrollar estrategias que reduzcan los impactos redistributivos sin comprometer el bienestar social.

A lo largo de este capítulo se profundiza en los aspectos críticos de las políticas de transición, examinando los efectos macroeconómicos y redistributivos derivados de la incorporación del sector residencial en los objetivos de cero emisiones de la Unión Europea para 2050. Nuestra investigación contribuye a ampliar la literatura existente mediante el empleo de modelos de Equilibrio General Dinámico Ambiental, proporcionando un análisis cuantitativo detallado de los impactos del cambio climático y de las políticas de mitigación implementadas. El modelo desarrollado permite la integración de fricciones económicas, lo que redunda en una mejor comprensión de los canales de transmisión a corto plazo (Fischer & Springborn (2011), Heutel (2012), Annicchiarico & Di Dio (2015), Coenen et al. (2018), Ferrari & Landi (2022)) e introduce una heterogeneidad limitada de los hogares utilizando el marco de los modelos neokeynesianos de dos agentes (TANK), diferenciando entre hogares con restricciones de crédito y aquellos sin dichas limitaciones financieras.[3] Nuestros resultados revelan efectos distributivos significativos de las políticas ambientales entre estos grupos. En concreto, los prestatarios experimentan mayores pérdidas de bienestar que los prestamistas tras la transición a una economía de emisiones netas cero, lo que pone de manifiesto un efecto regresivo. Este efecto distributivo fluctúa a lo largo de la transición: inicialmente disminuye debido a las reducciones de impuestos de suma fija procedentes de los ingresos del carbono, pero aumenta una vez que se alcanzan los objetivos climáticos intermedios. Esta dinámica pone de evidencia la necesidad de políticas de mitigación específicas, particularmente en las fases avanzadas de la transición, donde la asignación de los ingresos derivados

[3]Para una análisis detallado sobre las implicaciones agregadas de los modelos TANK versus los modelos neokeynesianos de agentes heterogéneos (HANK), véase Debortoli & Galí (2024).

del impuesto al carbono entre los distintos tipos de hogares adquiere una importancia crucial.

La integración de hogares heterogéneos dentro de un marco de equilibrio general dinámico marca un avance significativo en la literatura sobre política ambiental. Tradicionalmente, las evaluaciones de los impactos distributivos de los impuestos ambientales sobre los grupos de ingresos de los hogares recurren a análisis input-output para determinar las variaciones de precios en diversos bienes de consumo, calculando posteriormente las cargas fiscales mediante datos de encuestas microeconómicas de hogares. Estos estudios suelen adoptar un enfoque de equilibrio parcial, centrándose en los patrones de gasto de los consumidores (un análisis desde la perspectiva de la demanda). (Dinan & Rogers (2002), Metcalf et al. (2008), Hassett et al. (2009), Fullerton & Heutel (2007), Araar et al. (2011), Rausch et al. (2011), Mathur & Morris (2014)). Sin embargo, estos estudios a menudo omiten los efectos de la heterogeneidad de los hogares en los resultados de equilibrio, una cuestión abordada específicamente por Rausch & Schwarz (2016). En su trabajo, los autores analizan cómo los efectos de equilibrio general de un impuesto al carbono se ven condicionados por la diversidad de ingresos y preferencias de los hogares, demostrando que ignorar esta heterogeneidad puede distorsionar significativamente la incidencia de estos impuestos ambientales, especialmente en los rendimientos de los factores de producción.

Nuestro estudio amplía la literatura existente sobre los efectos de los impuestos al carbono durante periodos de transición mediante su incorporación en un modelo de equilibrio general ambiental con producción endógena de vivienda, abordando así la externalidad global de las emisiones de carbono. Al considerar dos sectores productivos con sustitución imperfecta de la mano de obra entre ellos —específicamente los sectores residencial y de producción de bienes—, nuestro modelo permite evaluar las interacciones provocadas por las políticas ambientales. Además, al considerar tanto a los hogares sin restricciones crediticias como a aquellos cuya capacidad crediticia se ve afectada por las propiedades sujetas a restricciones de emisiones, nuestro estudio aborda elementos críticos de los impactos distributivos del impuesto: la incidencia en los patrones de gasto del consumidor (análisis del lado de la demanda), el precio de los factores de producción (análisis del lado de la oferta) y la capacidad de obtener crédito a partir de garantías inmobiliarias.

Nuestros resultados apuntan a que las políticas de precios del carbono generan efectos diferenciados en la economía, en función de si están dirigidos hacia el sector residencial o el no residencial. En el sector no residencial, la fijación de precios del carbono principalmente eleva los costes de producción, lo que conlleva una disminución de la demanda de mano de obra, los salarios y los beneficios, y, por consiguiente, una reducción del consumo y la actividad económica. En contraposición, la fijación de precios del carbono en el sector residencial produce efectos más complejos. Concretamente, provoca una disminución de los precios de la vivienda, lo que reduce el valor del colateral del que disponen los hogares con restricciones de crédito, y por lo tanto, limitando su capacidad de endeudamiento y sus posibilidades de consumo. Estos resultados ponen de relieve la importancia de desarrollar estrategias de fijación de precios del carbono específicas para cada sector, con el fin de mitigar los impactos distributivos adversos.

Por otra parte, se observan claras diferencias en la respuesta macroeconómica por unidad de emisión reducida entre los distintos sectores. Un aumento del precio de las emisiones tiene un impacto notable en la inversión en vivienda, los precios de la vivienda y el crédito por unidad de carbono reducida, efectos que se vuelven más pronunciados cuando el impuesto se aplica al sector residencial en lugar del sector no residencial. A modo de ejemplo, el impuesto al carbono disminuye el crédito en una proporción aproximada de 3 a 1 por unidad de emisión reducida en el sector residencial en comparación con el sector no residencial. En términos generales, los precios del carbono en el sector residencial generan menores pérdidas de bienestar por unidad de emisión reducida que las medidas equivalentes implementadas en el sector no residencial.

Desde una perspectiva de bienestar, nuestro análisis muestra que el coste total a largo plazo, medido en términos de consumo, supera el 10% bajo una política combinada de precios del carbono en ambos sectores, afectando a prestatarios y prestamistas casi por igual. No obstante, al considerar los impactos específicos de cada sector, emergen diferencias significativas. La fijación de precios del carbono en el sector no residencial genera una pérdida de bienestar para los prestatarios que excede en 0,5 puntos porcentuales a la de los prestamistas, lo que refleja la mayor sensibilidad de los prestatarios a las disminuciones en la demanda de mano de obra y la actividad económica general. En contraste, la aplicación del impuesto al sector residencial conlleva una pérdida de bienestar para los prestamistas que casi duplica la experimentada por los prestatarios. Este resultado se produce a pesar de que los prestamistas realizan un menor esfuerzo de reducción de emisiones, dado que el impuesto al carbono residencial afecta de manera desproporcionada la inversión en vivienda y el parque inmobiliario, activos predominantemente propiedad de los prestamistas.

La distribución de los ingresos generados por los impuestos al carbono es crucial para mitigar los efectos distributivos de las políticas de reducción de emisiones. Inicialmente, la distribución equitativa de dividendos de carbono entre los hogares ayuda a amortiguar el impacto del aumento de los costes energéticos, especialmente para los prestatarios, y estabiliza su consumo. Sin embargo, a medida que las emisiones se reducen gradualmente y la base de ingresos de los impuestos al carbono disminuye, la efectividad de estas transferencias se desvanece, lo que lleva a una creciente desigualdad. Nuestro análisis del coeficiente de Gini del consumo identifica los períodos críticos donde la desigualdad es más aguda, enfatizando la necesidad de políticas específicas para mitigar estos efectos a lo largo de la transición hacia las emisiones netas cero.

Las principales conclusiones de este capítulo resaltan la heterogeneidad de los efectos de la fijación de precios del carbono. Se evidencia que los impuestos al carbono residenciales impactan negativamente la inversión y el valor de los activos de la vivienda, perjudicando especialmente a los prestamistas. En contraste, la fijación de precios en el sector no residencial afecta principalmente a los prestatarios a través de la disminución de sus ingresos laborales. Este análisis subraya una importante contrapartida: si bien los impuestos residenciales pueden ser más eficaces para la protección del medio ambiente, conllevan un mayor riesgo de inestabilidad financiera. Por otro lado, los impuestos no residenciales, aunque más equitativos en su distribución, exigen aumentos de precios más pronunciados para alcanzar los objetivos ambientales. En consecuencia, el capítulo concluye que una estrategia óptima requiere una fijación de precios del carbono diferenciada por sectores, complementada con mecanismos de compensación dinámicos que permitan mitigar los efectos desiguales entre hogares y sectores económicos.

Capítulo 4: La función de pérdidas bajo consideraciones climáticas

En este último capítulo se desarrolla un marco analítico basado en una función de pérdidas de bienestar, para evaluar políticas en economías con externalidades climáticas. La función derivada integra rigideces nominales, fricciones financieras (modeladas como restricciones de crédito sujetas al valor de la vivienda) y externalidades climáticas, creando un enfoque unificado para analizar los desafíos que enfrentan las políticas en economías donde la vivienda desempeña un papel fundamental. Los esfuerzos por limitar el calentamiento global por debajo de los 2°C, tal y como se establece en el Acuerdo de París, plantean retos significativos para la estabilidad macroeconómica y financiera. Estas preocupaciones han cobrado gran relevancia entre los bancos centrales, avivando el debate sobre las implicaciones de las políticas climáticas en la estabilidad monetaria y financiera (Lagarde (2021), Mann (2023), Powell (2023)). Si bien no existe un consenso claro sobre el papel que deben desempeñar los bancos centrales en la lucha contra el cambio climático, hay un punto indiscutible: cuando el cambio climático y las políticas ambientales impactan la estabilidad macroeconómica —ya sea a través de shocks físicos derivados de fenómenos meteorológicos extremos y el aumento de las temperaturas, o de los riesgos asociados a la transición hacia una economía de emisiones netas cero—, las autoridades monetarias y macroprudenciales deben incorporar estas dinámicas dentro de su mandato de estabilidad de precios y financiera. El capítulo aborda dos cuestiones fundamentales ¿Introducen los esfuerzos de mitigación de emisiones nuevos riesgos para la estabilidad de precios y financiera? ¿Existe una contrapartida inherente entre los objetivos principales de los bancos centrales y los objetivos climáticos?

Para responder a estas cuestiones, derivamos una función de pérdida basada en el bienestar, ofreciendo a los responsables de la política monetaria un marco para la evaluar el impacto de sus decisiones. Siguiendo el enfoque propuesto por Rotemberg & Woodford (1998), incorporamos explícitamente las distorsiones derivadas de las emisiones de carbono en la producción, las rigideces nominales y las fricciones financieras. A diferencia del enfoque de Ramsey, que obtiene soluciones óptimas resolviendo el problema de optimización del planificador central, nuestro enfoque aproxima la política óptima mediante una función de pérdida cuadrática. Esta función actúa como referencia para evaluar la efectividad de las políticas y proporciona información sobre cómo las reglas de política pueden equilibrar los objetivos de estabilización dentro de las restricciones institucionales. Concretamente, la función de pérdidas captura las siguientes ineficiencias en la economía:

- Las emisiones de gases de efecto invernadero aumentan de forma proporcional a la producción (véase Annicchiarico & Di Dio (2015), Heutel (2012)) y, al acumularse en la atmósfera, reducen la productividad de las empresas. A pesar de ello, las empresas no internalizan los costes de reducción asociados y, por tanto, no las disminuyen. Para corregir este fallo de mercado, el gobierno implementa impuestos a las emisiones que internalizan su coste social y fomentan las inversiones en tecnologías limpias.

- Las fricciones financieras, modeladas como restricciones crediticias (véase Kiyotaki & Moore (1997), Iacoviello (2005)), limitan la capacidad de endeudamiento de los hogares impacientes a una fracción de su riqueza inmobiliaria. Este endeudamiento que se financia a partir de los ahorros de los hogares pacientes, crea interacciones entre los mercados financieros y los resultados macroeconómicos.

- Las rigideces nominales, introducidas a través de los costes de ajuste de precios (Rotemberg (1982b)), otorgan a la política monetaria un papel activo en la estabilización de la economía.

Este trabajo realiza importantes contribuciones a diversas ramas de la literatura. En primer lugar, mediante nuestro enfoque basado en el bienestar, ampliamos el estudio de las políticas monetarias y macroprudenciales óptimas en contextos de fricciones financieras. Siguiendo los trabajos previos de Andrés et al. (2013) y Ferrero et al. (2018), que integran fricciones financieras y el papel de los activos como garantía dentro del marco de la función de pérdidas de Rotemberg & Woodford (1998), extendemos este marco teórico al incorporar explícitamente las externalidades climáticas. Considerando en nuestro modelo la acumulación de carbono en la atmósfera como un factor que afecta negativamente la producción y, por ende, las dinámicas macroeconómicas, nuestro enfoque ofrece una base teórica sólida para analizar las interacciones entre la política monetaria, la regulación macroprudencial y las políticas medioambientales.

En segundo lugar, este trabajo hace aportaciones significativas a una línea de investigación que ha cobrado relevancia en los últimos años: las interacciones entre la política monetaria y la fiscalidad ambiental, particularmente en relación con los impuestos al carbono. Olovsson & Vestin (2023) encuentran que un aumento abrupto del impuesto al carbono puede generar presiones inflacionarias e ineficiencias económicas, mientras que una transición gradual conlleva efectos más leves. En esta situación, la política monetaria debería acomodar la transición centrándose en la inflación subyacente, asegurando que los efectos inflacionarios se mantengan en niveles modestos y mitigando las preocupaciones en torno a la "greenflation". De manera similar, Del Negro et al. (2023) demuestran que los impuestos al carbono no necesariamente generan una inflación persistente, sino que introducen un compromiso a corto plazo entre la estabilización de la inflación y el mantenimiento de la producción. Por consiguiente, los bancos centrales podrían permitir presiones inflacionarias durante la transición a economías de emisiones cero para evitar recesiones profundas, lo que sugiere que los efectos inflacionarios asociados pueden gestionarse dentro de los marcos estándar de la política monetaria. A diferencia de nuestro trabajo, estos estudios no analizan las políticas óptimas ni proporcionan una métrica para la pérdida de bienestar asociada a ellas.

Más estrechamente relacionado con nuestro estudio, el trabajo de Annicchiarico & Di Dio (2015) analiza los efectos de diversas políticas ambientales sobre las fluctuaciones macroeconómicas en un modelo neokeynesiano con rigideces nominales. Bajo la asunción de que un planificador Ramsey determina la política ambiental de manera óptima, los autores derivan prescripciones políticas considerando diferentes regímenes de política monetaria. Annicchiarico & Di Dio (2017) extienden este marco para examinar el diseño óptimo de las políticas monetarias y ambientales, ilustrando cómo la fiscalidad del carbono y el control de emisiones interactúan con la dinámica de la inflación. Sus hallazgos ponen de relieve que la rigidez de los precios y la competencia imperfecta modulan el compromiso entre la estabilización de la inflación y las políticas ambientales, influyendo así en la respuesta óptima de la política monetaria ante las perturbaciones. A diferencia de estos trabajos, el nuestro incorpora las distorsiones generadas por las fricciones, lo que permite realizar un análisis comparativo de diferentes regímenes de política basado en pérdidas de bienestar, y examinar la coordinación de políticas entre responsables de la política independientes, como los bancos centrales y los reguladores macroprudenciales.

En un trabajo más reciente, Nakov & Thomas (2023) desarrollan un modelo de política monetaria óptima de Ramsey en un marco neokeynesiano. Su investigación se centra en determinar si los bancos centrales se enfrentan a disyuntivas entre el control de la inflación y los objetivos climáticos, especialmente cuando se aplican políticas de precios al carbono. Un resultado clave de su análisis es que, cuando los impuestos al carbono se establecen de manera óptima, la política monetaria puede estabilizar completamente tanto la inflación como la brecha de producción sin generar distorsiones significativas. No obstante, si el precio del carbono se establece inicialmente de manera subóptima y se incrementa gradualmente, la política monetaria podría encontrar tensiones a corto plazo, aunque la respuesta óptima sigue siendo priorizar la estabilidad de precios. En relación con Nakov & Thomas (2023), nuestro estudio no solo adopta un enfoque de función de pérdida basado en el bienestar, lo que permite la evaluación de reglas de política tanto óptimas como subóptimas, sino que también considera el esfuerzo de inversión necesario para reducir las emisiones como una variable de decisión endógena.

Nuestro trabajo revela resultados clave para comprender los desafíos de la política macroeconómica en una economía con restricciones derivadas de una externalidad climática. En primer lugar, cuando la reducción de carbono conlleva un coste, el impuesto óptimo al carbono equivale a una fracción del consumo agregado. Este impuesto no solo internaliza el coste social directo de las emisiones, sino que también considera el efecto indirecto de la reducción de las emisiones futuras derivada de una menor productividad. En segundo lugar, nuestra función de pérdida muestra que las distorsiones relacionadas con el daño climático introducen un término de varianza adicional que captura las fluctuaciones en la fiscalidad del carbono. Este término refleja la brecha entre la producción y el consumo creada por los costes de reducción de emisiones. En tercer lugar, el peso de las desviaciones de la producción y la desigualdad en consumo en el bienestar dependen no solo de parámetros estándar en la literatura, sino también de la relación entre los ingresos del carbono y el consumo en estado estacionario. La urgencia de estabilizar la producción disminuye a medida que aumenta (disminuye) la relación entre los ingresos del impuesto al carbono y el consumo (coste marginal relacionado con el trabajo), lo que refleja la menor relevancia de las fluctuaciones de la producción cuando se dedica una mayor proporción del tiempo al ocio. Sin embargo, a medida que los costes asociados a la reducción de emisiones se incrementan, la importancia relativa del término de varianza del impuesto aumenta. Finalmente, demostramos que la convexidad de la función de pérdida —condición crítica para asegurar un óptimo único y estable— viene garantizada siempre que el ratio consumo-producción en estado estacionario sea superior a la relación ingresos del carbono-producción, una condición generalmente satisfecha en la práctica. En definitiva, estos hallazgos ponen de manifiesto la importancia de coordinar las políticas monetarias, fiscales y ambientales para equilibrar la estabilización de precios a corto plazo con la sostenibilidad a largo plazo.

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Chapter 1

Introduction

HOUSING is more than a basic human need; it is a pillar of modern economies, closely related to financial systems, labor markets, and macroeconomic stability. Its dual function as both shelter and collateral makes housing a key transmission channel between asset markets, credit conditions, and aggregate outcomes. This centrality makes it crucial for understanding the impacts of fiscal, environmental, and monetary policies. However, the same interconnectedness introduces systemic vulnerabilities: housing market fluctuations can amplify macroeconomic shocks, deepen inequality, and complicate policy implementation. Understanding these dynamics is essential for designing policies that foster stability, sustainability, and equity, especially in light of two defining challenges of our time: the aftermath of the 2008 Global Financial Crisis (Chapter 2) and the transition to net zero emissions (Chapter 3).

This thesis explores how housing collateral mechanisms interact with financial frictions to shape economic outcomes. Declining home prices reduce the collateral value available to credit-constrained households, limiting their ability to borrow and spend. This financial accelerator effect can intensify various economic shocks, whether from policy changes, environmental transitions, or broader macroeconomic disturbances. For policymakers, understanding these dynamics is crucial as they shape the effectiveness and distributional consequences of fiscal, environmental, and monetary policies.

Chapter 2 investigates how government spending shocks propagate through housing markets using a Dynamic General Equilibrium model. Focusing on Spain's pre-2008 economy, where housing played an unusually prominent role, this chapter investigates whether the fiscal response to the sharp contraction in the GDP unintentionally slowed the recovery of the construction sector. The central research question is: in the absence of the fiscal expansion, would GDP, residential investment, and housing prices have declined more or less under the same economic conditions?

The findings demonstrate that increased government consumption stimulates the production of tradable goods by increasing labor demand and wages. However, this positive effect comes with a sectoral trade-off, as resources shift away from construction, reducing housing-related employment and investment. The magnitude of this sectoral reallocation depends critically on labor mobility and household debt levels, which can amplify worker displacement and limit access to credit. As a result, indebted households experience significant welfare losses. Although redirecting government spending toward housing can sustain construction activity, it also intensifies the overall decline in welfare.

This chapter makes two key contributions to the economic literature. First, it helps resolve longstanding empirical ambiguities regarding fiscal policy's housing market impacts by identifying the key structural parameters that drive divergent outcomes. Second, it advances theoretical modeling by systematically incorporating labor market frictions, sectoral reallocation dynamics, and credit constraints. The results underscore that the effectiveness of fiscal policy in housing markets depends crucially on the underlying conditions of the labor and financial markets, not just on spending levels.

Chapter 3 shifts the focus to environmental policy analysis, examining the macroeconomic and distributional effects of carbon pricing in both the residential and nonresidential sectors. Using an Environmental Dynamic General Equilibrium (E-DGE) model that incorporates price rigidities, sectoral labor adjustments, and housing-collateral-based financial frictions, the analysis addresses three core questions: How do carbon pricing policies impact labor markets, credit availability, and household welfare? What are the sector-specific transmission mechanisms driving these effects? And how can compensation mechanisms, such as carbon dividends, mitigate adverse distributional consequences?

The results reveal sharply divergent effects between the borrowers and the lenders. Nonresidential carbon pricing reduces labor demand and wages, disproportionately impacting borrowers who depend heavily on wage income. Residential carbon pricing, in contrast, primarily affects housing prices, tightening credit constraints for borrowers while imposing greater welfare costs on asset-holding lenders. These distributional patterns intensify with increasing labor mobility and higher proportions of borrowing households. Although equally distributed carbon dividends help stabilize borrower consumption in the short term, their effectiveness declines over time as emissions, and thus carbon tax revenues,decrease.

The aggregate level of household debt emerges as a critical factor in the effectiveness of carbon pricing. Greater proportions of credit-constrained households make residential carbon taxation more effective at driving abatement investments, but at the cost of significantly sharper credit contractions, three times more severe per emissions unit reduced than nonresidential pricing. This finding reveals a fundamental policy trade-off between environmental effectiveness and financial stability. In contrast, nonresidential carbon pricing exhibits the opposite pattern, requiring progressively higher carbon prices to achieve equivalent emissions reductions as the share of borrowers increases.

This analysis advances the research field in two main ways. First, it pioneers the integration of household heterogeneity and endogenous housing production into environmental dynamic general equilibrium modeling, capturing how carbon policies interact with credit constraints to produce distributional effects. Second, it provides policymakers with a comprehensive assessment of macroeconomic and welfare impacts of carbon pricing, highlighting the need for sector-specific implementation strategies and adaptable compensation mechanisms to maintain both efficiency and equity during the net zero transition.

Chapter 4 develops an analytical framework for evaluating policy responses in climate-constrained economies through a derived welfare loss function. This function integrates nominal rigidities, financial frictions —modeled as credit constraints linked to housing collateral —and climate externalities, creating a unified approach to assess policy trade-offs in an economy where housing plays a central role. As debate continues about central banks' appropriate role in addressing climate change, the analysis proceeds from the clear premise that climate-related disruptions —whether physical or transition-related— pose material risks to macroeconomic and financial stability that fall squarely within traditional policy mandates. The chapter tackles two fundamental questions: Do emission mitigation efforts introduce new risks to price and financial stability? Is there an inherent trade-off between central banks' core objectives and climate goals?

The analysis presents critical insights into macroeconomic policy trade-offs in a climate-constrained economy. The analysis yields several crucial insights about policy design in climate-constrained economies. When abatement proves costly, optimal carbon taxation should reflect a fraction of aggregate consumption—internalizing both immediate and future climate costs. The derived welfare-based loss function reveals that climate policy introduce a carbon-taxation variance term. Furthermore, the weights assigned to output stabilization and inequality in the loss function depend crucially on the ratio of carbon revenues to consumption, while greater abatement costs heighten sensitivity to tax volatility. Importantly, the function maintains the convexity—ensuring unique optimal solutions—when consumption-to-output ratios exceed carbon revenue-to-output ratios, a condition typically met in practice. These findings collectively demonstrate the necessity of coordinated monetary, fiscal, and environmental policies to achieve both near-term stability and long-term sustainability.

This chapter contributes to economic research by developing a unified analytical framework that integrates climate externalities with monetary policy and financial stability considerations. Methodologically, it introduces a second-order welfare approximation that captures the interplay between carbon taxation, inflation stabilization, and macroprudential regulation. It also derives convexity conditions based on fundamental macroeconomic ratios and shows how abatement costs affect price dynamics through the Phillips curve. The resulting framework combines theoretical rigor with practical relevance, offering valuable tools for assessing real world policy trade-offs.

The thesis makes three overarching contributions to economic understanding and policy design. First, it uncovers how fiscal policy interacts with housing markets and borrowing constraints in a setting with endogenous residential production and imperfect labor mobility across sectors. By incorporating these structural features, it highlights the role of housing collateral channels in shaping the macroeconomic and welfare effects of fiscal interventions. Second, it offers actionable policy insights by highlighting the importance of sector-specific interventions and targeted mitigation strategies to address distributional impacts. Third, it advances economic theory by developing integrated frameworks that combine financial market imperfections with climate externalities, offering policymakers a structured approach to assess and manage the complex trade-offs between macroeconomic stability and environmental objectives.

This research demonstrates that housing market dynamics play a central role in shaping macroeconomic responses during both financial crises and climate policy transitions. By modeling housing dual function— as both a consumption good and a source of collateral— the thesis highlights how fiscal, environmental, and monetary policies interact with household balance sheets and sectoral structures. The analysis offers policymakers concrete frameworks to: (1) mitigate financial amplification mechanisms during downturns, and (2) design carbon pricing strategies that balance emissions reduction goals with macroeconomic and financial stability. Together, these findings position housing as a key transmission channel that must be explicitly accounted for in macroeconomic stabilization strategies addressing the intertwined challenges of debt, inequality, and decarbonization.

Chapter 2

Government Expenditure and the Housing Puzzle

Shocks in housing demand can trigger economic downturns, prompting governments to boost public consumption to mitigate its impact. The literature has found conflicting effects of government spending on residential investment and housing prices. We hypothesize that differences in the construction sector's responses may be due to factors unaccounted for in the estimations, such as different labor response to wage changes, labor reallocation, and household debt. Using a dynamic general equilibrium model, we demonstrate that government consumption boosts tradable goods production, resulting in increased labor demand and wages. However, this diverts workers away from construction, potentially deepening the wound in this sector. Specific labor market attributes and economic indebtedness can catalyze worker displacement from the construction sector, thus adversely affecting residential investment and overall credit. As a result, borrowers face considerable welfare losses. Redirecting government spending toward housing sustains residential activity while exacerbating the overall welfare decline. Herein, we provide plausible explanations for the disparate empirical evidence on the impact of government spending on the construction sector.

2.1 Introduction

In the decade preceding the financial crisis, housing sector's importance in the Spanish economy increased substantially. For example, Figure 2.1 shows that residential investment's share of gross domestic product (GDP) increased steadily from 6% to 12% in 2006. Similarly, the proportion of building work hours to total work hours increased from 10% to 15% by 2007. This trend was even more pronounced than that observed in the United States during the same period. [1] However, this contrasted with the relatively stable weight of the Eurozone's housing sector as a whole. The economic recession, which began in the second half of 2008, was led by a drop in the relative levels of residential investment and hours worked in the construction sector. [2] These two variables fell to one-third and more than half of their peak values in just 6 years, respectively.

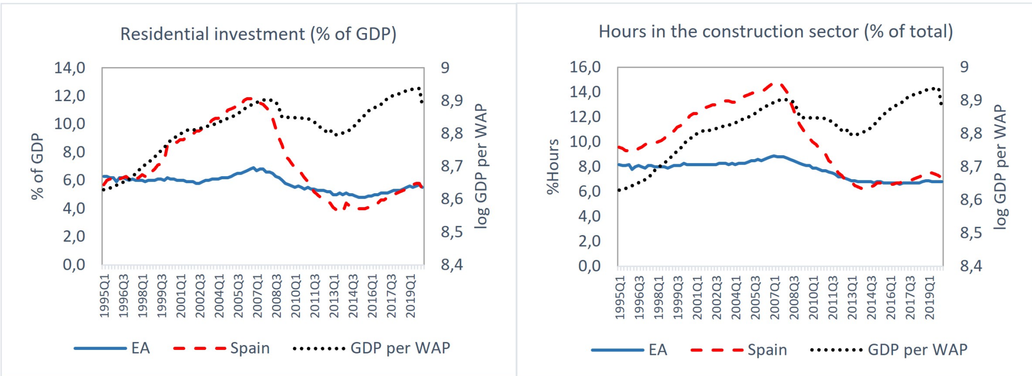


Figure 2.1: Residential investment and employment in Spain and the Euro Area. *Notes*: The graph on the left compares the GDP weight of residential investment in Spain and the Euro Area (EA). The graph on the right compares the hours worked in the construction industry to the total hours worked in Spain and EA. The weight of residential investment in GDP and the percentage of hours worked in this sector are on the left scale, whereas the logarithm of GDP per working-age population is on the right. *Source*: EUROSTAT.

At the onset of the crisis, the fiscal policy response was to expand public consumption to counteract the fall in GDP and unemployment, but this did not prevent the construction sector from shrinking. Moreover, the two years of significant fiscal stimulus beginning in 2008 coincided with an unprecedented drop in house price growth, which did not recover to trend levels until 10 years later (Figure 2.2). Based on the raw data, the observed relationship

[1] According to Weinberg (2013), home prices in the United States more than doubled between 1998 and 2006, while residential investment increased from about 4.5% of US GDP to about 6.5% during the same period. In addition, employment in housing-related sectors accounted for approximately 40% of net private sector job creation between 2001 and 2005.

[2] Authors such as Green (1997), Coulson & Kim (2000), and Leamer (2007) have used various approaches to demonstrate that residential investment leads the business cycle. Green (2022) recently reported that housing investment, a leading indicator, has weakened since the Great Recession. Figure 2.1 shows that in Spain, after the collapse, the weight of construction in the economy has not recovered with GDP

between the variables shown in Figures 2.1 and 2.2 is inconclusive[3]. Many factors may have been at play simultaneously, making it difficult to isolate the true effect of government spending on GDP and real estate. For instance, would GDP, residential investment, and housing prices have fallen more or less with the same factors at work, except for the fiscal expansion? One of the primary objectives of this chapter is to provide an answer to this question. In a preview of our results, for a benchmark calibration, we find a positive effect of public consumption on GDP and residential investment in Spain. We also detect an adverse but mild effect on housing prices that is very persistent. However, these effects are not independent of changes in specific key economic characteristics. More specifically, we show that the baseline effects on housing investment and prices can be reversed when it is easier to reallocate employment across sectors, when labor supply is more insensitive to wages, or when households have high debt.

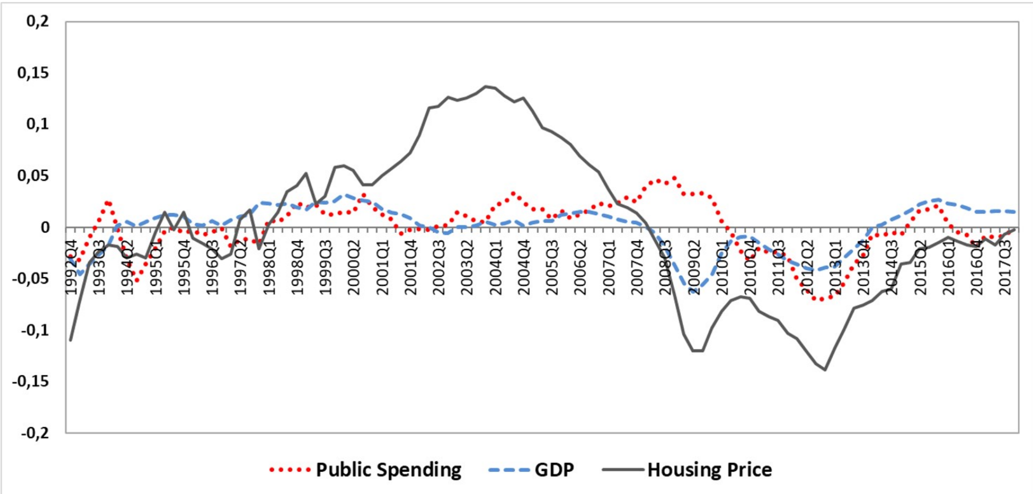


Figure 2.2: GDP growth rates, public consumption, and housing prices in Spain. *Notes*: Year-on-year growth rates (relative to the working-age population for GDP and government spending) detrended according to the mean of each variable over the period 1992Q4–2017Q4. *Source*: BDREMS.

There is no empirical consensus on how government spending affects house prices and residential investment. Regarding house prices, the results of Agnello & Sousa (2013), Andrés et al. (2015), and Miles (2021) suggest that increased government spending has a negative effect on house prices. In contrast, Afonso & Sousa (2012) and Khan & Reza (2017) find an increase

[3]The data used to create Figures 2.1 and 2.2 can be downloaded from `https://data.mendeley.com/datasets/fwg2m5sfyf/1`.

in housing prices, whereas Afonso & Sousa (2012) report that house prices rise in some countries (e.g., the United Kingdom) but drop in others (e.g., Germany). However, Aye et al. (2014) and Ruiz & Vargas-Silva (2016) found that government spending shocks do not significantly impact residential property prices. Interestingly, Gupta et al. (2014) observe a time-varying effect that is negative in the 1970s and 1980s and positive in the first decade of the 2000s.

The same pattern of mixed empirical evidence applies to real estate investment. The seminal empirical analysis of Ramey & Shapiro (1998) shows a substantial decline in residential investment in response to military buildups in the United States. In his response to the previous article, Eichenbaum (1998) provides additional evidence of a decline in residential investment and wages in the residential sector following an increase in government spending. Edelberg et al. (1999), Mountford & Uhlig (2009), Ruiz & Vargas-Silva (2016), and Miles (2021) all confirmed the same behavior. On the contrary, Mihov et al. (2001) show that increased public spending leads to a significant increase in residential investment and a positive reaction in housing wages. Using various identification strategies, Caldara & Kamps (2008) find a small and generally nonstatistically significant effect on residential investment.

The previous two paragraphs summarize the literature and highlight differences in results based on the studied economy, period, variables, shocks analyzed, and the strategy used to identify fiscal shocks. The range of the results reflects the empirical model's ability to capture key economic characteristics that may influence the impact of fiscal shocks on residential activity and real estate prices. This chapter examines the effects of an increase in public consumption in a model economy. It shows that factors such as the ability to reallocate labor across sectors, the responsiveness of labor supply to wages, and the capacity to borrow from the private sector are critical to understand the quantitative and qualitative effects of public spending on residential property prices and real estate investments.

Our findings are relevant to the literature on incorporating the housing sector into dynamic stochastic general equilibrium (DSGE) models. Following the Great Recession, research emerged that focused on analyzing the financial frictions by including the housing market in DSGE models. In these models, the ability of households or firms to borrow is affected by housing or land prices. Models with such financial frictions have been used to analyze shock transmission mechanisms and the macroeconomic effects of specific policies [4]

[4]Iacoviello & Neri (2010) studied the impact of the housing market on cyclical fluctuations in the economy, whereas Liu et al. (2013) explained macroeconomic fluctuations using the comovements of land prices and business investment. Gete (2020) assumed a high reallocation of labor between construction and tradable goods to generate the

In comparison to the previous literature, we make two main contributions. First, we analyze the impact of public spending shocks on GDP, the residential sector, and household welfare. Second, our analysis highlights the significance of variables such as labor supply elasticity, labor mobility between the housing and tradable goods sectors, and household debt. This study thus bridges the gap in the literature between Khan & Reza (2017) claim that "properly accounting for the empirical evidence on government spending shocks and house prices using a DSGE model remains a significant challenge" and Aspachs-Bracons & Rabanal (2010) assertion of the existence of "very decisive [Bayesian] evidence in favor of the [two-sector] model with costly labor reallocation." Indeed, Khan & Reza (2017) find no mechanism that, when incorporated into a DSGE model, allows them to replicate their evidence that house prices rise in response to increased public spending when incorporated into a DSGE model. However, in their model without fiscal policy, Aspachs-Bracons & Rabanal (2010) find that removing barriers to labor mobility between their durable and consumer goods sectors results in a substantial decrease in the likelihood of the estimated model.

We identify a group of parameters that play a crucial role in reconciling the mixed empirical evidence on the impact of public spending shocks on the residential sector. These parameters include labor supply elasticity, labor mobility across sectors, the intensive and extensive margin of total private indebtedness, and the adjustment cost of housing demand. Surprisingly, despite their importance, these parameters have received little attention in the literature. However, they are economic factors that are inextricably linked to specific policies and have significant normative consequences [5]. For example, regulations on rental or ownership housing, labor market policies, educational reforms, tax reforms, and macroprudential policies can all impact the values of these parameters. According to Cardi & Restout (2015), cross-country differences in labor mobility between traded and nontraded sectors can range from 0.22 for the Netherlands to 1.80 for the United States as captured by the elasticity of substitution. Moreover, loan-to-value (LTV) ratios in credit decisions and the proportion of credit-constrained households can differ significantly across countries and periods, as can transaction costs

observed correlations between housing variables and the current account. Liu et al. (2016) and Pinter (2019) analyze the relationship between mortgage-backed credit and the labor market. Rubio & Comunale (2018) compared the effects of various macroprudential policy implementation methods on financial stability, whereas Rubio & Yao (2020) studied the role of macroprudential policies in a low interest–rate environment. Furthermore, within this framework, Rabanal (2018) and Bielecki & Stähler (2022) highlighted the macroeconomic impact of property taxes.

[5] See Kambourov (2009).

in the housing market.

Our study is closely related to the work of Cardi et al. (2020), who examine the sectoral effect of public spending shocks on labor mobility. However, there are several important differences between our study and theirs. First, our focus lies on the housing market, a type of nontradable good that accumulates over time. Our primary interest is in the reaction of housing prices and investment. Second, by examining both lenders and borrowers and the role of household debt, we emphasize the significance of limited household heterogeneity in our model economy. Lastly, our study highlights the role of monetary policy in a monetary union economy. This point is especially important in analyzing the implications of a closed economy managing its monetary policy.

This Chapter is organized as follows. Section 2.2 elucidates the model. Section 2.3 describes the calibration process and the parameter values used to obtain the numerical solution. Section 2.4 offers a comprehensive overview of the main findings. Finally, Section 2.5 concludes our study and presents nuanced policy implications, emphasizing the need for careful consideration.

2.2 The model

Given our interest in Spain, we consider a small open economy in a monetary union. There are two production sectors. One sector produces goods for consumption and investment that can be traded across the union. The other sector produces housing, which is not tradable internationally.

The economy is made up of two types of households, classified based on their subjective discount rates. Those with a lower discount rate (patient households) become savers. Households with the higher discount rate (impatient households) become borrowers. Both savers and borrowers consume various domestic and foreign consumer goods, demand domestic real estate, and supply labor to the tradables and housing industries. The hours worked are not perfectly interchangeable between sectors, and the model's parameter captures the degree of substitutability. The greater the substitutability, the lower the cost in terms of utility when switching from one sector to another, and the easier it is to reallocate labor between sectors.

The flow of financial funds from savers to borrowers (the financial market) is hampered by lending constraints, as borrowers have a credit limit based on the expected real value of their real estate. The endogenous labor supply response of borrowers and lenders is different. Movements in the housing market have a direct impact on borrowers' borrowing capacity and thus their consumption, investment, and labor supply. However, changes in the production conditions of tradable goods can influence the relative labor supply of different sectors, resulting in an effect on housing production that feeds back into credit availability.

A domestic fiscal authority (government) affects the economy by purchasing tradable goods. To keep the results as clean as possible of financing alternatives, we assume that government consumption is financed by a lump-sum tax paid equally per capita by lenders and borrowers. A central bank conducts monetary policy by setting the nominal reference interest rate for the entire area using a standard Taylor-type feedback rule that responds to deviations in inflation from the monetary union's long-term target. Next, we present the model, highlighting the most relevant features.

2.2.1 Households

Following Iacoviello (2005), this study has two representative households types; N_t^l of them are patient (lenders) and N_t^b are impatient (borrowers). Total population is given by $N_t = N_t^l + N_t^b$. We call $1 - \tau^b = \frac{N_t^l}{N_t}$ and $\tau^b = \frac{N_t^b}{N_t}$ the exogenous proportion of lenders and borrowers in the population[6]. In the following two subsections, we characterize their choice decisions.

Patient households

Patient households or lenders select consumption, housing, and labor to maximize the following schedule (real variables are expressed in per capita terms for the specific type of household).

$$\max_{c_t^l, h_t^l, n_{ct}^l, n_{ht}^l, b_t^l, b_t^{l,eu}, k_t^l} \quad \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^l)^i \left(\ln c_{t+i}^l + \phi_{j,t} \gamma_h \ln h_{t+i}^l - \frac{(n_{t+i}^l)^{1+\eta}}{1+\eta} \right) \tag{2.1}$$

subject to the real flow of funds expressed in terms of the consumption price index P_t^c (the *numeraire*) that takes into account the price of imported goods,

$$c_t^l + q_t(h_t^l - (1-\delta^h)h_{t-1}^l) - b_t^l - \frac{b_t^{l,eu}}{\phi_{bt}} + p_t^k(k_t^l - (1-\delta^k)k_{t-1}^l) + \frac{\Phi}{2}(k_t^l - k_{t-1}^l)^2 =$$
$$w_{ct} n_{ct}^l + w_{ht} n_{ht}^l - \frac{r_{t-1}}{\pi_t^c} b_{t-1}^l + d_t - tr_t - \frac{r_{t-1}^{eu}}{\pi_t^c} b_{t-1}^{l,eu} + r_t^k k_{t-1}^l \tag{2.2}$$

[6]This differs from other approaches, which weight household types is based on wage share (Iacoviello (2005); Iacoviello & Neri (2010)). We consider that our form of weighting decouples the size of representative households from the wage they receive. Moreover, it allows us to explore naturally the effect of the extensive margin of indebtedness, τ^b, on the results.

On the utility side, $\beta^l \in (0,1)$ represents the inverse of the patient discount factor, c_t^l is real consumption per capita, h_t^l is lenders' housing tenure, and n_t^l is labor time per person. The parameter γ_h captures housing preference, the inverse of η relates to labor elasticity with respect to wages, and $\phi_{j,t}$ is a housing preference driver that influences housing preferences. Its initial value is 1.

As in Iacoviello & Neri (2010), we assume imperfect substitutability of labor across sectors, so that n_t^l is a constant elasticity of substitution (CES) composite of lenders' labor supply in the tradable consumption sector n_{ct}^l and in the housing sector n_{ht}^l,

$$n_t^l = [\theta^{\frac{1}{\varepsilon_n}} (n_{ct}^l)^{\frac{1+\varepsilon_n}{\varepsilon_n}} + (1-\theta)^{\frac{1}{\varepsilon_n}} (n_{ht}^l)^{\frac{1+\varepsilon_n}{\varepsilon_n}}]^{\frac{\varepsilon_n}{1+\varepsilon_n}} \tag{2.3}$$

where $\theta > 0$ is a weight parameter that relates to the disutility of working in the consumption sector relative to the housing sector, and affects the proportion of patient households' working hours in each sector. $\varepsilon_n \geq 0$ is the elasticity of substitution between labor supply in the two sectors, capturing the ease of movement across sectors. When ε_n approaches infinity, then $n_t^l = n_{ct}^l + n_{ht}^l$, and hours worked are completely interchangeable between sectors. We refer to this as the perfect labor mobility case. If ε_n approaches zero, workers do not change the working hours across sectors regardless of relative wage changes. We label this the completely immobile labor case. In between, there is a range of cases[7].

Regarding the budget constraint, patient households lend in real terms $-b_t^l$ to impatient households (where b_t^l is negative) and $-b_t^{l,eu}$ to the rest of the union. They receive back $-\frac{r_{t-1}}{\pi_t^c} b_{t-1}^l$ and $-\frac{r_{t-1}^{eu}}{\pi_t^c} b_{t-1}^{l,eu}$ from the previous period's loans, where $\frac{r_{t-1}}{\pi_t^c}$ and $\frac{r_{t-1}^{eu}}{\pi_t^c}$ are the gross real interest-rate factors. To ensure the stationarity of net foreign assets, we assume a risk premium, ϕ_{bt}, that increases with the country's net foreign debt (Schmitt-Grohé & Uribe (2003))

$$\ln \phi_{bt} = \phi_b(\exp(b_t^{eu}) - 1) \tag{2.4}$$

These households buy houses $(q_t(h_t^l - (1-\delta^h)h_{t-1}^l))$, invest in new productive capital $(p_t^k(k_t^l - (1-\delta^k)k_{t-1}^l))$, and incur capital adjustment costs $(\frac{\Phi}{2}(k_t^l - k_{t-1}^l)^2)$. They receive rents from capital $(r_t^k k_{t-1}^l)$ and labor $(w_{ct} n_{ct}^l + w_{ht} n_{ht}^l)$. They also receive profits, d_t, from firms operating in noncompetitive markets, and pay lump-sum taxes, tr_t, in terms of consumption goods. The solution of the above optimization schedule generates the following first order conditions:

[7] For a full discussion of this CES function, see Cardi & Restout (2015).

$$\lambda_t^l = \frac{1}{c_t^l} \tag{2.5}$$

$$\frac{\phi_{j,t}\gamma_h}{h_t^l} = \lambda_t^l q_t - \beta^l \mathbb{E}_t(\lambda_{t+1}^l q_{t+1}(1-\delta^h)) \tag{2.6}$$

$$\frac{w_{ct}}{c_t^l} = (n_t^l)^\eta (\theta \frac{n_{ct}^l}{n_t^l})^{\frac{1}{\varepsilon_n}} \tag{2.7}$$

$$\frac{w_{ht}}{c_t^l} = (n_t^l)^\eta ((1-\theta) \frac{n_{ht}^l}{n_t^l})^{\frac{1}{\varepsilon_n}} \tag{2.8}$$

$$\lambda_t^l = \beta^l \mathbb{E}_t \lambda_{t+1}^l \frac{r_t}{\pi_{t+1}^c} \tag{2.9}$$

$$\lambda_t^l = \phi_{bt} \beta^l \mathbb{E}_t \lambda_{t+1}^l \frac{r_t^{eu}}{\pi_{t+1}^c} \tag{2.10}$$

$$\lambda_t^l (p_t^k + \Phi(k_t^l - k_{t-1}^l)) = \beta^l \mathbb{E}_t \lambda_{t+1}^l (p_{t+1}^k (1-\delta^k) + r_{t+1}^k + \Phi(k_{t+1}^l - k_t^l)) \tag{2.11}$$

Equation (2.6) represents the current demand for housing, which depends positively on housing preferences and the expected value of houses in the future, and negatively on the current price of houses.

Equations (2.7) and (2.8) represent the labor supply of patient workers to the tradable goods and housing sectors. The higher the value of $\frac{1}{\eta}$, the more sensitive the labor supply to both sectors is to a change in wages. Likewise, the higher the value of the elasticity of substitution, ε_n, the more likely workers are to shift their labor supply to the sector that has experienced an increase in wages relative to the other.

Combining Equations (2.9) and (2.10) yields the interest rate parity condition, which, in a monetary union, directly relates domestic and foreign interest rates through the risk premium.

$$r_t = \phi_{bt} r_t^{eu} \tag{2.12}$$

Working with expression (2.11), the gross investment $(k_t^l - k_{t-1}^l)$ can be expressed as a negative function of the adjustment cost Φ, as well as of current and expected real interest rates, while expected gains from the appreciation of capital assets and expected increases in the rental rate of capital push investment upward, which is the basis of Tobin's q theory of investment.

Impatient households

Impatient households have a high discount rate, which is the inverse of the discount factor $\beta^b < \beta^l$. The impatient household optimizes the following utility function

$$\max_{c_t^b, h_t^b, n_{ct}^b, n_{ht}^b, b_t^b} \quad \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^b)^i \left(\ln c_{t+i}^b + \phi_{j,t} \gamma_h \ln h_{t+i}^b - \frac{(n_{t+i}^b)^{1+\eta}}{1+\eta} \right) \tag{2.13}$$

where c_t^b is borrowers' real consumption, h_t^b denotes borrowers' holding of housing, and n_t^b is a composite of labor supply to the consumption sector n_{ct}^b and to the housing sector n_{ht}^b represented by the following CET function,

$$n_t^b = [\theta^{\frac{1}{\varepsilon_n}} (n_{ct}^b)^{\frac{1+\varepsilon_n}{\varepsilon_n}} + (1-\theta)^{\frac{1}{\varepsilon_n}} (n_{ht}^b)^{\frac{1+\varepsilon_n}{\varepsilon_n}}]^{\frac{\varepsilon_n}{1+\varepsilon_n}} \tag{2.14}$$

Impatient households become borrowers, so in addition to the budget constraint (2.15), they face a financial constraint on the maximum amount of credit they can obtain. In particular, Equation (2.16) limits the amount of borrowing, b_t^b, to a fraction κ of the expected resale value of the house held by the household.

$$c_t^b + q_t(h_t^b - (1-\delta^h)h_{t-1}^b) - b_t^b = w_{ct} n_{ct}^b + w_{ht} n_{ht}^b - \frac{r_{t-1}}{\pi_t^c} b_{t-1}^b - tr_t \tag{2.15}$$

$$b_t^b \leq \phi_{\kappa,t} \kappa \mathbb{E}_t \frac{q_{t+1} \pi_{t+1}^c h_t^b}{r_t} \tag{2.16}$$

where κ represents the LTV and $\phi_{\kappa,t}$ is a factor to exogenously change the LTV. The first order conditions for the utility maximization problem of the impatient household are the following:

$$\lambda_t^b = \frac{1}{c_t^b} \tag{2.17}$$

$$\frac{\gamma_h \phi_{j,t}}{h_t^b} = \lambda_t^b q_t - \mathbb{E}_t(\beta^b \lambda_{t+1}^b q_{t+1}(1-\delta^h) + \phi_{\kappa,t} \kappa \mu_t^b q_{t+1} \pi_{t+1}^c) \tag{2.18}$$

$$\frac{w_{ct}}{c_t^b} = (n_t^b)^\eta \left(\theta \frac{n_{ct}^b}{n_t^b} \right)^{\frac{1}{\varepsilon_n}} \tag{2.19}$$

$$\frac{w_{ht}}{c_t^b} = (n_t^b)^\eta \left((1-\theta) \frac{n_{ht}^b}{n_t^b} \right)^{\frac{1}{\varepsilon_n}} \tag{2.20}$$

$$\lambda_t^b = \beta^b \mathbb{E}_t \lambda_{t+1}^b \frac{r_t}{\pi_{t+1}^c} + \mu_t^b r_t \tag{2.21}$$

where μ_t^b is the Lagrange multiplier on the collateral constraint. Under the assumption that the constraint is binding $\mu_t^b > 0$ is an indicator of the tightness of the credit markets for borrowers. This new variable introduces a wedge in the current demand for impatient housing and consumption relative to the patient.

Aggregation

Since our economy is composed of two representative types of household, the value in per capita terms of the aggregate variables can be affected by the weight of each type of household, τ^b. Debt issued by borrowers is fully purchased by patient households

$$\tau^b b_t^b = -(1-\tau^b) b_t^l \tag{2.22}$$

External debt and private physical capital involve exclusively patient households

$$b_t^{eu} = (1-\tau^b) b_t^{l,eu} \tag{2.23}$$

$$k_t = (1-\tau^b) k_t^l \tag{2.24}$$

Net per capita investment is defined as

$$j_t = k_t - (1-\delta^k) k_{t-1} \tag{2.25}$$

Aggregate per capita consumption and housing stock depends on the mix of borrowers and lenders

$$c_t = \tau^b c_t^b + (1-\tau^b) c_t^l \tag{2.26}$$

$$h_t = \tau^b h_t^b + (1-\tau^b) h_t^l \tag{2.27}$$

We also consider different measures of working hours. Aggregate hours per worker in the consumption goods sector

$$n_{ct} = \tau^b n_{ct}^b + (1-\tau^b) n_{ct}^l \tag{2.28}$$

Aggregate hours per worker in the housing sector

$$n_{ht} = \tau^b n_{ht}^b + (1-\tau^b) n_{ht}^l \tag{2.29}$$

Aggregate hours per worker in the economy

$$n_t = \tau^b (n_{ct}^b + n_{ht}^b) + (1-\tau^b)(n_{ct}^l + n_{ht}^l) \tag{2.30}$$

2.2.2 Producers of tradable goods

There is continuum of firms indexed by $j \in [0, 1]$. Each firm j produces a differentiated good y_{jt} at the nominal price P_{jt} in a monopolistically competitive market. This product is suitable for both consumption and investment. We assume two levels of production. At the top level, there is a competitive bundler that mixes the output of these firms y_{jt} at the prices P_{jt} into a single product, y_t, which is sold to households (for consumption and investment) and the government. Assuming constant returns to scale technology like Dixit & Stiglitz (1977) to compose the bundle, we calculate the aggregate production, aggregate Producer Price Index (PPI), and total demand for each variety, respectively, as

$$y_t = \left(\int_0^1 (y_{jt})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.31}$$

$$P_t = \left(\int_0^1 (P_{jt})^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.32}$$

$$y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} y_t \tag{2.33}$$

At the bottom level, each variety is produced using the same technology. The variety's producer rents physical capital, k_{jt}, and hires labor, n_{jct}, which are combined according to a Cobb-Douglas production function,

$$y_{jt} = \phi_{c,t} z^c (k_{jt-1})^{\alpha} (n_{jct})^{1-\alpha} \tag{2.34}$$

where z^c is the steady-state productivity factor common to all firms and $\phi_{c,t}$ is a driver that exogenously changes the productivity of the sector. The optimal factor demands for production are determined by solving the cost-minimization problem faced by each variety producer.

$$w_{ct} = mc_{jt}(1-\alpha)\frac{y_{jt}}{n_{jct}} \tag{2.35}$$

$$r_t^k = mc_{jt}\alpha\frac{y_{jt}}{k_{jt-1}} \tag{2.36}$$

where mc_{jt} is the Lagrange multiplier for the production constraint, which is interpreted as the increase in production cost after a unit increase in production. Thus, mc_{jt} is real marginal cost. Combining (2.35) and (2.36) the marginal cost takes the following form

$$mc_{jt} = \frac{1}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \frac{r_{tk}^{\alpha} w_{ct}^{(1-\alpha)}}{z_t} \tag{2.37}$$

which means that the marginal cost is common to all firms.

Price setting

The prices of tradable domestic goods are sticky and can be adjusted using the Calvo staggered price setting. Each firm resets its price with a $1-\omega$ probability in each period. Firms have no bearing on this likelihood. Therefore, it can be defined as the proportion of firms in each period that optimize prices. The following indexation rule determines the proportion ω of firms that do not reprice optimally in t

$$P_{jt} = (\bar{\pi})^{1-\zeta}(\pi_{t-1})^{\zeta}P_{jt-1} \tag{2.38}$$

where ζ stands for the degree of indexation and π_{t-1} is the one-period lag of domestic PPI inflation. According to this rule, among firms that do not optimize, a proportion $1-\zeta$ of firms keep the rate of price growth equal to the steady-state inflation, $\bar{\pi}=1$, whereas a proportion ζ takes into account lagged PPI inflation.

The expression gives the value of the optimal price $p_t{}^*$, which is common to all firms that reoptimize the price

$$p_t{}^* = \frac{\varepsilon}{\varepsilon-1}\frac{V_t}{F_t} \tag{2.39}$$

and

$$V_t = \lambda_t^l mc_t y_t p_t^{\varepsilon} + \mathbb{E}_t\beta^l\omega\left(\frac{(\bar{\pi})^{1-\zeta}(\pi_t)^{\zeta}}{\pi_{t+1}^c}\right)^{-\varepsilon} V_{t+1} \tag{2.40}$$

$$F_t = \lambda_t^l y_t p_t^{\varepsilon} + \mathbb{E}_t\beta^l\omega\left(\frac{(\bar{\pi})^{1-\zeta}(\pi_t)^{\zeta}}{\pi_{t+1}^c}\right)^{1-\varepsilon} F_{t+1} \tag{2.41}$$

where $\frac{\varepsilon}{\varepsilon-1}$ represents the steady-state markup, and the optimal price depends on the current and expected future evolution of the real marginal cost.

Equation (2.32) expresses the value of the aggregate price index P_t in the tradable goods sector as a function of nominal optimal price P_t^* and the past nominal production price P_t and inflation,

$$P_t = [\omega(\bar{\pi}^{1-\zeta}\pi_{t-1}^{\zeta}P_{t-1})^{1-\varepsilon} + (1-\omega)(P_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{2.42}$$

Therefore, the price of tradable goods in terms of the consumer price (our numeraire) equals

$$p_t = \left(\frac{1-\omega\left(\frac{(\bar{\pi})^{1-\zeta}(\pi_{t-1})^{\zeta}}{\pi_t}\right)^{1-\varepsilon}}{1-\omega}\right)^{1-\varepsilon} p_t^* \tag{2.43}$$

π_t refers to the inflation of domestically produced tradable goods prices. From this, we can calculate the Consumer Price Index (CPI) inflation, $\pi_t^c = P_t^c / P_{t-1}^c$, which evolves according to

$$\pi_t^c = \pi_t \frac{p_{t-1}}{p_t} \tag{2.44}$$

and which will enter the policy rule of the central bank.

Given that all firms use the same technology, production can be aggregated as

$$y_t = \phi_{c,t} z^c (k_{t-1})^\alpha (n_{ct})^{1-\alpha} \tag{2.45}$$

2.2.3 Housing construction firms

In a perfectly competitive environment, numerous small firms owned by patient households construct identical houses. Aspachs-Bracons & Rabanal (2010) find that the prices of durable consumer goods in Spain are more flexible than those of nondurable consumer goods. We assume that housing price, q_t, is perfectly flexible, and firms cannot influence it (Iacoviello & Neri (2010)). Furthermore, because building and residential investment are relatively labor intensive (Davis & Heathcote (2005)), we simply assume that residential investment in the economy I_t^h occurs under constant returns to labor according to the production function

$$I_t^h = \phi_{h,t} z^h n_{ht} \tag{2.46}$$

where z^h is the steady-state technology factor common to all firms and n_{ht} is the labor hours in the sector. $\phi_{h,t}$ acts as a technology driver in housing construction, shifting the technology away from its initial steady state. The profit maximization problem of the representative firm yields the labor demand in the construction sector

$$w_{ht} = q_t \phi_{h,t} z_t^h \tag{2.47}$$

Considering a constant depreciation rate δ^h for the stock of houses, the total supply of houses h_t evolves according to

$$h_t = I_t^h + (1 - \delta^h) h_{t-1} \tag{2.48}$$

2.2.4 Monetary and fiscal policy

The European Central Bank manages monetary policy following a Taylor's interest-rate rule that responds to deviations of Eurozone inflation π_t^{eu} from its long-run target ($\bar{\pi} = 1$). The rule takes the form:

$$r_t^{eu} = \phi_{r,t}\bar{r}^{1-\rho}(r_{t-1}^{eu})^{\rho}(\pi_t^{eu})^{\psi(1-\rho)} \tag{2.49}$$

where $\bar{r}$ is the steady-state level of the gross interest rate, ρ is a parameter that controls the persistence of the interest rate, ψ represents the weight given by the European Central Bank (ECB) to inflation targeting and $\phi_{r,t}$ is an exogenous interest rate shifter.

Based on its economic size, the Spanish economy is assumed to contribute to Euro area inflation.

$$\pi_t^{eu} = (\pi_t^{reu})^{(1-\omega_{Sp})}(\pi_t^{c})^{\omega_{Sp}} \tag{2.50}$$

where ω_{Sp} is the weight of the Spanish economy in the Eurozone, and π_t^{reu} is the average inflation in the rest of the Eurozone.

The real exchange rate between Spain and the rest of the Eurozone is defined as $p_t^{reu} = \frac{P_t^{reu}}{P_t^c}$, where P_t^{reu} is the nominal price of goods produced in the rest of the Eurozone.

The real exchange rate can also be expressed in terms of inflation rates

$$p_t^{reu} = p_{t-1}^{reu}\frac{\pi_t^{reu}}{\pi_t^c} \tag{2.51}$$

An increase in p_t^{reu} is interpreted as an increase in competitiveness.

For the fiscal policy, to isolate the mechanisms driving our results from government funding sources and distortionary taxes, we assume the government's balanced budget in which government spending in tradable consumption goods equals total lump-sum transfers to households. Thus, an increase (decrease) in government spending corresponds to an increase (decrease) in lump-sum taxes of the same magnitude, affecting both patient and impatient households by the same amount per capita.

$$\phi_{g,t}\bar{g}_t = tr_t \tag{2.52}$$

Government spending $\bar{g}$ is determined exogenously. $\phi_{g,t}$ is a fiscal shifter that captures the exogenous changes in government consumption in the economy.

2.2.5 External sector

Tradable goods are produced for both consumption and investment. The home country trades both consumption and investment goods with the rest of the monetary union. Aggregate consumption is a CES composite basket of domestically and internationally produced consumer goods, given by

$$c_t = ((1-\omega_c)^{\frac{1}{\sigma_c}}(c_{ht})^{\frac{\sigma_c-1}{\sigma_c}} + \omega_c^{\frac{1}{\sigma_c}}(c_{ft})^{\frac{\sigma_c-1}{\sigma_c}})^{\frac{\sigma_c}{\sigma_c-1}} \tag{2.53}$$

where c_{ht} and c_{ft} are consumption of domestically produced goods and foreign goods respectively. σ_c is the elasticity of substitution between domestic and foreign consumption and ω_c is a weight parameter in the CES aggregator for the consumption of nondurable goods. Each period, the optimal allocation of consumption between domestic and foreign goods results from minimizing production costs subject to technological constraint (2.53). The first order conditions for the cost minimization problem leads to demand functions of domestic and foreign consumption goods

$$c_{ht} = (1-\omega_c)(p_t)^{-\sigma_c} c_t \tag{2.54}$$

$$c_{ft} = \omega_c (p_t^{reu})^{-\sigma_c} c_t \tag{2.55}$$

Similarly, aggregate investment is a CES composite basket of domestically and internationally produced goods,

$$j_t = ((1-\omega_j)^{\frac{1}{\sigma_j}} (j_{ht})^{\frac{\sigma_j - 1}{\sigma_j}} + \omega_j^{\frac{1}{\sigma_j}} (j_{ft})^{\frac{\sigma_j - 1}{\sigma_j}})^{\frac{\sigma_j}{\sigma_j - 1}} \tag{2.56}$$

where j_{ht} and j_{ft} denote capital investment using domestic and imported goods respectively, σ_j is the elasticity of substitution between the domestic and foreign investment good and ω_j is the weight parameter in the CES investment aggregator. The demand for domestically and foreign produced investment goods are

$$j_{ht} = (1-\omega_j)\left(\frac{p_t}{p_t^j}\right)^{-\sigma_j} j_t \tag{2.57}$$

$$j_{ft} = \omega_j \left(\frac{p_t^{reu}}{p_t^j}\right)^{-\sigma_j} j_t \tag{2.58}$$

The aggregate nominal cost of investment can be written as $p_t^k j_t = P_t j_{ht} + P_t^{reu} j_{ft}$, and substituting domestic and imported investment goods for their demand functions yields the expression for the investment price index (in terms of consumption goods),

$$p_t^k = ((1-\omega_j)p_t^{1-\sigma_j} + \omega_j (p_t^{reu})^{1-\sigma_j})^{\frac{1}{1-\sigma_j}} \tag{2.59}$$

The total imports of the economy are

$$im_t = c_{ft} + j_{ft} \tag{2.60}$$

We assume an export function based on foreign (exogenous) income and relative price elasticity.

$$ex_t = s^x \left(\frac{p_t}{p_t^{reu}}\right)^{-\sigma_x} \bar{y}_t^{reu} \tag{2.61}$$

where s^x is a driver representing changes in foreign preferences or income.

2.2.6 Total resource constraint and GDP

The aggregation of household budget constraints leads to

$$c_t + tr_t + q_t(h_t - (1-\delta^h)h_{t-1}) - \frac{b_t^{eu}}{\phi_{bt}} + p_t^k j_t + \frac{\Phi}{2}\frac{(k_t - k_{t-1})^2}{(1-\tau^b)} = \\ w_{ct}n_{ct} + w_{ht}n_{ht} + (1-\tau^b)d_t - \frac{r_{t-1}^{eu}}{\pi_t^c}b_{t-1}^{eu} + r_t^k k_{t-1} \tag{2.62}$$

Using the equality between production and income

$$p_t y_t = w_{ct}n_{ct} + (1-\tau^b)d_t + r_t^k k_{t-1} \tag{2.63}$$

$$q_t I_t^h = w_{ht}n_{ht} \tag{2.64}$$

Private consumption and investment in domestic goods, public consumption, exports, and capital adjustment costs all reduce total tradable goods production.

$$y_t = c_{ht} + j_{ht} + \frac{\Phi}{2}(k_t - k_{t-1})^2 + tr_t + ex_t \tag{2.65}$$

Equations (2.62) to (2.65) provide the law of motion for the home country's net foreign asset position,

$$\frac{b_t^{eu}}{\phi_{bt}} = p^{reu} im_t - p_t ex_t + \frac{r_{t-1}^{eu}}{\pi_t^c}b_{t-1}^{eu} \tag{2.66}$$

where $\frac{b_t^{eu}}{\phi_{bt}}$ is negative when the country is a net lender.

Incorporating (2.63), (2.64) and (2.66) into (2.62) yields another expression for total tradable goods production in terms of consumption and investment.

$$p_t y_t + q_t I_t^h = c_t + \phi_{g,t}\bar{g}_t + q_t(h_t - (1-\delta^h)h_{t-1}) + \\ p_t^k j_t + \frac{\Phi}{2}\frac{(k_t - k_{t-1})^2}{1-\tau^b} + p_t ex_t - p_t^{reu} im_t \tag{2.67}$$

Then, GDP in per capita terms is defined as spending on tradable goods and new housing

$$GDP_t = p_t y_t + q_t I_t^h \tag{2.68}$$

2.3 Calibration

This section describes the parameter values used to obtain the model's numerical solution. The calibration strategy entails setting the parameters based on previous studies in the literature, primarily referring to the Spanish economy, and setting target values for some steady-state equations of the model to obtain the remaining parameters. Working with the static version of the model allows us to reduce the entire system to only four central equations before recursively searching for the remaining solution.

2.3.1 Parameters recovered from previous studies

Table 2.1 shows the first set of parameters. We borrow some of the parameters from the estimated model of Aspachs-Bracons & Rabanal (2010) for the Spanish economy. In particular, the values of the elasticity of transformation between labor types ε_n, the inverse of the elasticity of labor to wages η, price rigidity ω, and inflation indexation ζ. We calculated the value of financially constrained households τ^b based on Kaplan et al. (2014) definition of financially fragile households in Spain. For the LTV parameter κ, we reference the models estimated for Spain by Boscá et al. (2020) and Commission et al. (2014), considering that constrained households can borrow up to 70% of the value of their home.

Patient households' steady-state time preference rate is 0.5% per quarter, equivalent to a 2% annual interest rate. Impatient households have a higher discount rate, $\frac{1}{\beta^b}$, which we set at approximately 2.5 percentage points per quarter. The depreciation rates for housing stock δ^h and private capital δ^k are consistent with the literature (Iacoviello & Neri (2010); Commission et al. (2012); Commission et al. (2014); Boscá et al. (2020)). The capital adjustment cost is based on Cardi et al. (2020).[8]. According to Boscá et al. (2020), the value of the elasticity of substitution between final goods ε implies a markup of 17% in the steady state.

Boscá et al. (2020) provided the following values for the foreign sector: substitution elasticity between domestic and foreign investment σ_j, the elasticity of substitution of consumption between domestic and foreign goods σ_c, the export function factor s^x, the long-run price elasticity of exports σ_x, the risk premium parameter ϕ_b, and the Taylor rule parameters ρ and ψ. These parameters suggest that domestic and foreign investment goods have a

[8]From the expression of the adjustment cost of capital, we can easily establish a relationship between our parameter value Φ and the value Φ_{CRC} calculated by Cardi et al. (2020). This relationship can be expressed as $\Phi = \frac{\Phi_{CRC}}{k_{t-1}}$. Similarly, the capital adjustment cost of Iacoviello & Neri (2010) yields a similar value.

Table 2.1: Parameters from previous studies

	Description	Value
β^l	Lenders' discount rate	0.995
β^b	Borrowers' discount rate	0.975
η	Inverse elasticity of labor supply	0.88
ε_n	Elasticity of transformation between types of work	0.78
Φ	Capital adjustment cost	2.9
κ	Loan-to-value	0.7
τ^b	Fraction of borrowers	0.4
ω	Calvo parameter	0.85
ε	Elasticity of substitution in the composite of tradables	6.88
ρ	Interest-rate smoothing	0.75
ψ	Interest-rate reaction to inflation	1.98
ζ	Inflation indexation	0.52
δ^k	Capital depreciation rate	0.025
δ^h	Housing depreciation rate	0.01
ϕ_b	Risk premium	0.001
s^x	Scale factor in the export function	0.023
σ_c	Elasticity between foreign and domestic consumption	0.857
σ_j	Elasticity between foreign and domestic investment	1.016
σ_x	Price elasticity of exports	0.651
ω_{Sp}	Weight of Spanish economy in the EMU	0.10

slightly higher substitution elasticity than consumer goods. The interest-rate inertia, ρ, is fairly standard, however, the interest reaction to inflation ψ is higher than in other papers (Aspachs-Bracons & Rabanal (2010); Boscá et al. (2011); Commission et al. (2012); Commission et al. (2014)). The weight of the Spanish economy in the Eurozone, ω_{Sp}, is set at 10%.

2.3.2 Parameters recovered from the model solution

Eliminating the time subscript t, reduces the static equation system to a core subsystem of four equations involving four central variables: total factor productivity in the tradable goods sector, z^c; the ratio of borrowers' to lenders' consumption c^b/c^l; the housing preference parameter in the utility function, γ_h; and the weight parameter in the labor disutility composite, θ. The remaining variables in the complete system can be obtained as a function of these four main variables (see online Appendix A.1 for more details).

Table 2.2: Steady-state ratios and endogenous variables normalized

	Description	Model	Data
$\frac{qI^h}{GDP}$	Residential investment/GDP	7.3%	7.3%
$\frac{p^j j}{GDP}$	Nonresidential Investment/GDP,	16.0%	16.0%
$\frac{n_h}{n_h+n_c}$	Share of labor in the construction industry	12.8%	12.8%
$\frac{pc_h}{c}$	Share of domestic goods in consumption	70.7%	70.7%
$\frac{pj_h}{p^j j}$	Share of domestic goods in investment	78.3%	78.3%
GDP	Gross Domestic Product	1	
q	Relative housing price	1	
p	Relative production price	1	
p^j	Relative investment price	1	
π	PPI	1	

To calibrate the model, we compare the economic aggregate model's steady-state ratios to GDP with Spain's corresponding average historical ratios. The ratios are based on Eurostat quarterly data from 1995Q1 to 2020Q1. Residential and nonresidential investments have steady-state ratios to GDP of 7.3% and 16%, respectively. In the steady state, government spending (which is exogenously determined in our model) accounts for 18% of GDP. The weight of the labor force working in the residential sector is targeted at 12.8%. For the foreign sector, we obtain data from BDREMS[9] with the coefficients for the share of domestically produced goods in consumption and investment being 70.68% and 78.26%, respectively. Furthermore, we normalize to 1 the steady-state value of national GDP, the relative prices of production of housing, tradable goods, and nonresidential investment, and the PPI. Table 2.2 provides information on the target ratios and normalized variables that the static model solution targets, whereas Table 2.3 summarizes the nine parameter values (and exogenous variables) that result from this calibration strategy. [10]

[9]Quarterly data base of the Spanish economy available in BDREMS or at the following url: `http://www.sepg.pap.hacienda.gob.es/sitios/sepg/en-GB/Presupuestos/DocumentacionEstadisticas/Documentacion/Paginas/BasedatosmodeloREMS.aspx`

[10]Both the core and the overall system are highly nonlinear. We use a Matlab solver that implements a Trust Region Reflective algorithm to obtain the solution. All Matlab codes for calibrating and solving the static and dynamic versions of the model are available to researchers.

Table 2.3: Parameters and exogenous variables from steady state

	Description	Value
γ_h	Preference for housing	0.206
θ	Weight parameter in the labor composite	0.145
α	Elasticity of production to capital	0.24
ω_c	Weight parameter in the consumption composite	0.293
ω_j	Weight parameter in the investment composite	0.217
z^c	Productivity in the tradable goods sector	0.254
z^h	Productivity in the housing sector	0.163
y^{reu}	Aggregate consumption in the rest of the EU	8.99
π^{reu}	Inflation in the rest of the EU	1

2.4 Results

This section shows the simulation results for our model[11]. First, Section 2.4.1 examines the model's ability to reproduce key macroeconomic features during Spain's Great Recession. We simulated a series of housing demand shocks to compare with the observed dynamics of residential and nonresidential investment, housing prices, consumption, and GDP in Spain during this period. Our findings show that a housing preference shock is sufficient to explain the movements of residential investment, housing prices, and labor substitution between the housing and nonhousing sectors[12]. Next, we examine the effects of government intervention via an increase in government consumption during a housing shock. Section 2.4.2, shows how the macroeconomic impact of the fiscal shock on the housing sector interacts with labor market characteristics. Meanwhile, we analyze the interplay between the fiscal shock and the economy's debt stance in Section 2.4.3. Our results show that household heterogeneity reinforces the labor market's effects via extensive and intensive margins of indebtedness. In Section 2.4.4, we conduct a welfare analysis to determine the primary causes of welfare effects. Finally, Section 2.4.5 discusses an alternative fiscal strategy: the government purchases real estate assets rather than allocating funds to consumption goods.

[11]We utilize Dynare 5.4 (Adjemian et al. (2022)) to derive the nonlinear model's deterministic perfect-forecast solutions.

[12]In the online Appendix A.2 we compare the effects of four exogenous changes that could impact the residential sector: a productivity shock, a monetary policy shock on the interest rate, a housing preference shock, and an LTV shock.

2.4.1 Housing demand shocks and the role of government consumption expansion

During Spain's financial crisis, the housing sector was critically important. As shown in Figure 2.2 in the Introduction, house price growth rates began to fall sharply below trend in the second quarter of 2008, reaching a trough in 2009Q3, with the annual growth rate plummeting below trend. This shift in house prices coincided with a decline in the GDP growth rate below its long-term trend to a trough of -6%, and a 2 percentage point decrease in the weight of residential investment in GDP between 2008Q2 and 2010Q3.

The burst of the housing bubble

In this subsection, we first examine the model's behavior concerning the observed variables in the housing sector. Our goal is to show that the model can approximate, in the presence of a few shocks, the primary dynamics of housing prices, construction investment, nonresidential investment, consumption, GDP, the ratio of nonhousing to construction investment, and the share of workers in the construction sector. We apply a sequence of unexpected negative shocks to housing preferences, $\epsilon_{j,t}$, lasting nine quarters. This corresponds to the period covering the first recession in Spain, between 2008Q2 and 2010Q3, after global financial contagion and before the sovereign debt crisis. In this case, we incorporate a persistence parameter to the dynamics of $\phi_{j,t}$, such that we consider[13]

$$\phi_{j,t} = \rho_j \phi_{j,t-1} + \epsilon_{j,t} \tag{2.69}$$

This approach is similar to Rognlie et al. (2018) initial exercise, which involved an unexpected and permanent shock to household preferences for housing to explain the observed macroeconomic dynamics of the US economy during the Great Recession.

Figure 2.3 presents the results. The plots on the left side of the figure depict the simulated response of the variables (conditioned on a housing shock) compared to the observed data on the right. The figure demonstrates that a single sequence of shocks directly impacting the housing sector, such as those affecting house preferences, can replicate the fluctuations in house prices, residential investment, and labor substitution between the construction and nonconstruction sectors observed between 2008Q2 and the 2010Q3. However, contrary to the observed behavior, nonresidential investment increases,

[13]We adopt the estimate of Boscá et al. (2020) of the persistence coefficient $\rho_j = 0.9994$ for a housing preference shock in the Spanish economy.

according to the findings of Rognlie et al. (2018). Additionally, the effects on GDP and consumption are moderate, with both returning to their original levels too quickly compared to the observed ones.

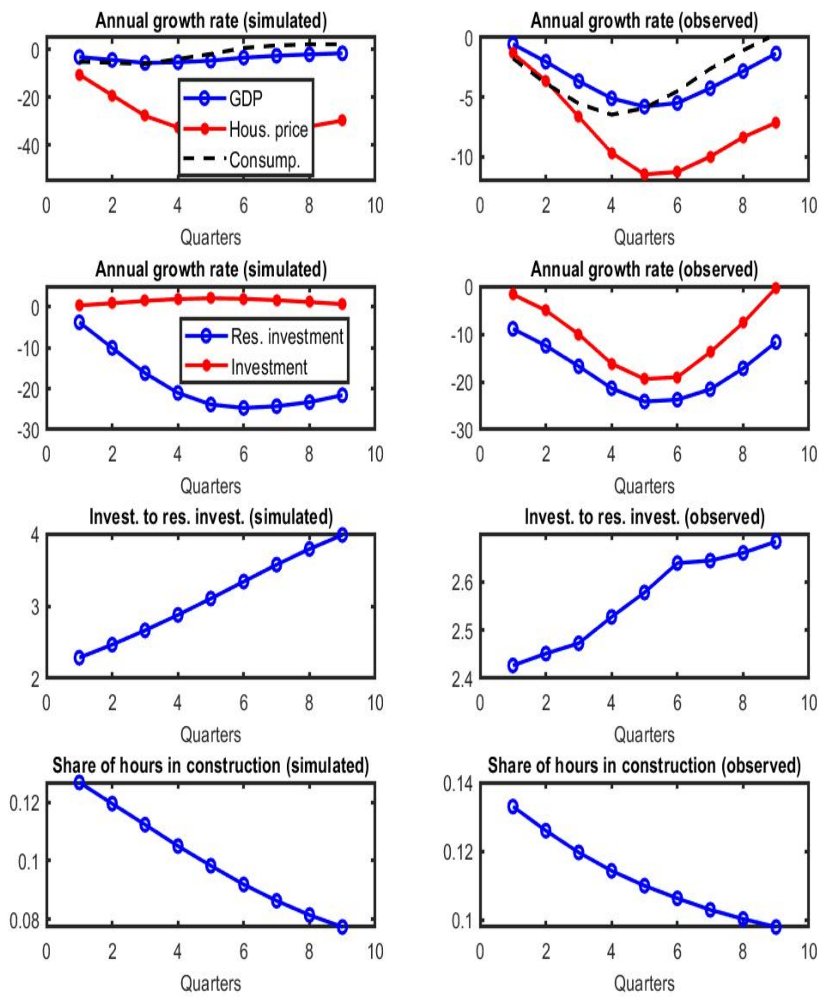


Figure 2.3: The observed and simulated behavior of GDP and its components, including the construction industry. Simulations are based on unexpected negative housing preference shocks (2008Q2–2010Q3).

To improve these counterfactual results, and following Rognlie et al. (2018), we add a risk premium shock to the housing preference shock sequence. Figure 2.4 presents the results. The inclusion of a risk premium shock, similar to what the Spanish economy experienced at the time, contributes to a pronounced fall in nonresidential investment, aggregate consumption, and GDP. Furthermore, the evolution of the residential-to-nonresidential investment ratio, as well as the decline in the share of working hours in the residential sector, closely matches the observed pattern. The only caveat is an overreaction in housing prices, in which the decline exceeds the observed one.

After confirming that the impact of the two main shocks on the model adequately accounts for the dynamics of GDP and its components, including residential investment, we move on to investigate the effects of a single fiscal shock. This shock was the most prominent tool at the time for countering the drop in economic activity.

To begin, Figure 2.5 provides a broader perspective on the general equilibrium effects of a transitory negative shock that affects housing demand and decreases the marginal utility of owning a home. In contrast to the previous sequence of shocks, we focus solely on a single housing demand shock. We introduce a one-time unanticipated change in $\epsilon_{j,t}$, leading the model to predict an immediate 6.6% decline in housing prices. This decline is consistent with the observed price drop in 2008Q4, corresponding to the peak increase in public consumption at the start of the financial crisis. Therefore, this point serves as an appropriate reference for investigating the effect of fiscal policy in mitigating a housing demand shock, which is the primary goal of our chapter.

Similar to what was observed in the aftermath of the crisis, the model predicts a significant drop in residential investment and GDP following a sharp drop in property prices (big-dotted blue line). The housing preference shock causes a fall in housing demand, resulting in a steady decline in housing prices. Housing production reacts endogenously, resulting in a decline in residential investment. Furthermore, the decline in house prices and housing production has important implications for other macroeconomic variables.

The decrease in housing production negatively affects employment demand in the housing sector. Given that home construction is heavily labor intensive (in the model, houses are produced entirely by labor), the responses of production and employment in the housing sector are interchangeable. The decrease in labor demand in the housing sector causes a decrease in wages, prompting a reallocation of labor supply to the trading sector. This shift accelerates as the wage gap between the tradable consumption and residential sectors widens over time. Given the complementary nature of labor and capital factors and the forward-looking nature of investment, capital investment in the tradable goods sector will rise as hours increase and prices recover.

Impatient households use home equity as collateral to secure loans, which they then use to buy consumer goods or real estate. However, as the price and supply of housing fall due to the real estate shock, so does their ability

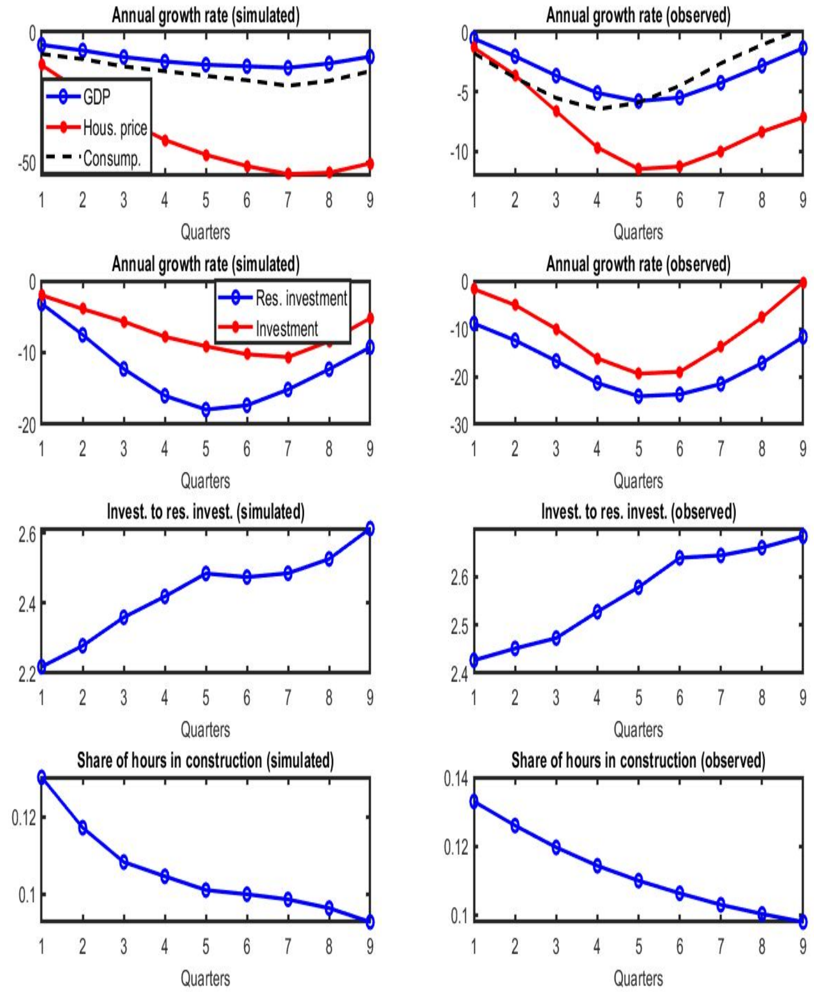


Figure 2.4: The observed and simulated behavior of GDP and its components, including the construction industry. Simulations are based on unexpected negative housing preference shocks and risk premium shocks (2008Q2–2010Q3).

to obtain credit against their collateral. As a result, credit decreases by about 20% on impact. Given that credit plays an important role in borrowers' consumption, their spending is substantially declined. Meanwhile, lenders use lower prices and shorter credit terms to increase consumption and property acquisitions. Nevertheless, despite the higher proportion of patient house-

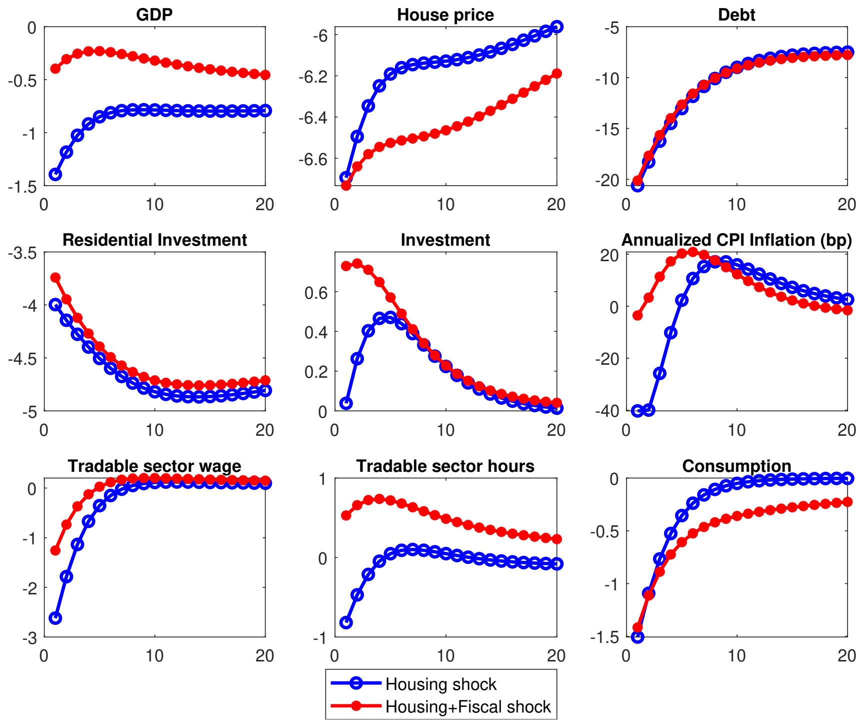


Figure 2.5: Macroeconomic response to a housing shock and countercyclical fiscal policy.

holds, their increased consumption is insufficient to offset the significant drop in borrowers' consumption caused by the housing downturn. As a result, aggregate consumption decreases by 1.5%, as shown in Figure 2.6 by the responses of lenders and borrowers.

The decrease in wages is a result of lower consumption demand and an increase in labor supply in the tradable sector due to an improvement in relative wages compared to the housing sector. Thus, a shock that begins at the housing level spreads through the general equilibrium mechanism to other sectors of the economy, resulting in a generalized drop in activity, employment, and wages. These indirect effects do not exist in Aspachs-Bracons & Rabanal (2010), which does not account for financial frictions. In their model, the housing preference shock causes a positive comovement between house prices and residential investment, but, as they point out, the spillover effects on the rest of the economy are minimal.

An expansionary fiscal policy at the time of a housing shock

In this section, we analyze the economic consequences of expanding government consumption to alleviate the adverse effects of a negative real estate shock. We assume that the shift of public spending shows persistence, so that

$$\phi_{g,t} = \rho_g \phi_{g,t-1} + \epsilon_{g,t} \tag{2.70}$$

where $\rho_g = 0.967$ is set according to the estimation of Boscá et al. (2020) for the Spanish economy.

We exogenously provide a one-shot increase in $\epsilon_{g,t}$ for public consumption to rise by 4.8% on impact from its steady-state level. This value corresponds to Spain's observed increase in per capita public spending (after deducting the year-on-year growth rate) during 2008Q4. Therefore, we incorporate the aforementioned negative housing demand shock into the model alongside the positive public spending shock, which accounts for 0.864% of GDP. Figure 2.5 shows the results in small red dotted lines.[14]

The expansionary fiscal policy slows the decline in GDP. In particular, it reduces the initial drop in GDP on impact by 1%, slightly more than the share of GDP equivalent to the initial boost in government spending. The positive effect of government spending on tradable consumer goods is primarily responsible for improving GDP. Expansionary fiscal policy raises the expectation of an increase in the return on capital among tradable goods producers. Patient households that own businesses decide to increase their physical capital investments. Because capital and labor are cooperating factors, increasing the capital stock improves labor productivity and raises labor demand in the tradable goods sector. The increased demand for labor helps to raise wages in the tradable sector.

Expansionary fiscal policy also increases consumer prices and domestic interest rates, negatively affecting the real value of housing assets and lowering the real cost of liabilities. As shown in Figure 2.6, patient households reduce their demand for consumer goods and housing due to a negative wealth effect caused by the expectation of lower asset prices and and higher taxes to fund government spending. Borrowers' wealth effect consists of two components. First, the Fisher effect (inflation reducing the real debt burden) boosts net wealth. However, as house prices fall, they experience a negative effect on net wealth. Unlike lenders, impatient households increase their demand for housing because houses also serve as collateral for credit in addition to providing utility. This increase in fresh credit is channeled into consumption.

[14]Because the two shocks are uncorrelated, the effects of a fiscal shock in isolation equal the difference in response functions between the combined housing and fiscal shocks and the housing shock alone.

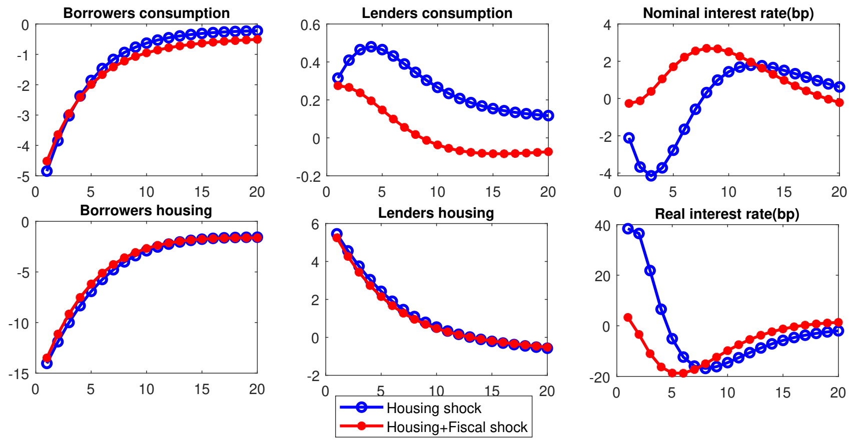


Figure 2.6: Households' purchasing behavior in response to a housing shock and countercyclical fiscal policy. The response of real and nominal interest rates to the fiscal shock in basic points (bp)

Despite lower housing prices, borrowers' assets increase as their demand for housing increases. In our model calibration, adding assets and the Fisher effect result in increased net wealth for borrowers. Furthermore, the fiscal shock has a positive income effect because it raises wages in the nondurable consumer goods sector. Impatient households quickly apply the positive effects of income, new credit, and wealth to current consumption and housing. However, for patient households, the negative wealth effect outweighs the increase in labor income, and consumption decreases. In general, higher borrower demand for housing and more persistent wage declines contribute to an increase in residential investment following the fiscal shock, as shown in Figure 2.6.

As a result, in our baseline model economy, the expansionary fiscal shock stimulates both the tradable goods and construction sectors, though house prices fall more. However, the modest impact movements in residential investment and house prices that we find depend on some labor market features and the resulting changes in the purchasing power of indebted owner households, which is highly dependent on credit conditions. We study these factors in the following subsections.

2.4.2 Fiscal policy: the role of the labor market

We now shift our focus to fiscal policy and investigate how the results of fiscal policy change with different labor market features. Labor market institutions can vary significantly between countries and within them over time. Suppose these institutions have the potential to influence the effect of fiscal policies. In that case, the diversity of labor market features has the potential to explain the seemingly contradictory results in the literature regarding the impact of government spending on housing construction and prices.

In this section, we examine two parameters of the model that relate to the (inverse) willingness to work when wages change (η) and the degree to which labor substitutes between the tradable and home building sectors (ε_n). Factors such as the generosity of unemployment benefits or the design of the tax system are obvious candidates to determine the value of η, while firing and hiring costs or business dynamism clearly influence the value of ε_n. To better understand the interactions between labor market characteristics and the impact of fiscal policy, we focus on residential investment, house prices, and GDP.We plot, for a range of values of the parameters of interest, the impact reaction (the effect at the time the shock occurs) to an expansionary fiscal shock of 1% of GDP in Equation (2.70). A nonflat line indicates a link between labor market features and the impact of government spending on the residential sector. Figure 2.7 shows the results.

The labor market characteristics significantly impact the size and direction of the response of residential investment and housing prices to fiscal shocks. We show that the disparities in empirical results found in the literature can be attributed to two deep parameters affecting the labor market. Higher values of two key parameters, the substitution elasticity of substitution between sectors (ε_n) and the inverse Frisch elasticity of labor supply (η), can exacerbate the housing sector's challenges. Residential investment decreases as workers' mobility between sectors increases (as indicated by higher values of the elasticity of substitution between sectors, ε_n), and working hours are less responsive to wage changes (as suggested by higher values of the inverse Frisch elasticity of labor supply, η). Housing prices, as shown in the right column of the subplots, react differently to changes in housing supply. They show a slight negative reaction when housing production increases and a positive response when residential investment decreases the most.

In general, our simulations reveal different combinations of results:

(a) Low values of ε_n and η lead to increased residential investment and housing supply, but decreased housing prices. When labor supply is highly responsive to wages (low η), fiscal shocks have a milder impact on consumption goods wages, mitigating the crowding effect (partial equilibrium) on housing workers and residential investment. Furthermore, when the substitution elasticity is less than one, labor supply in the housing sector supplements labor in the tradable goods sector. In this case, the fiscal shock causes a (partial equilibrium) increase in labor supply in the housing sector, lowering wages while stimulating house construction.

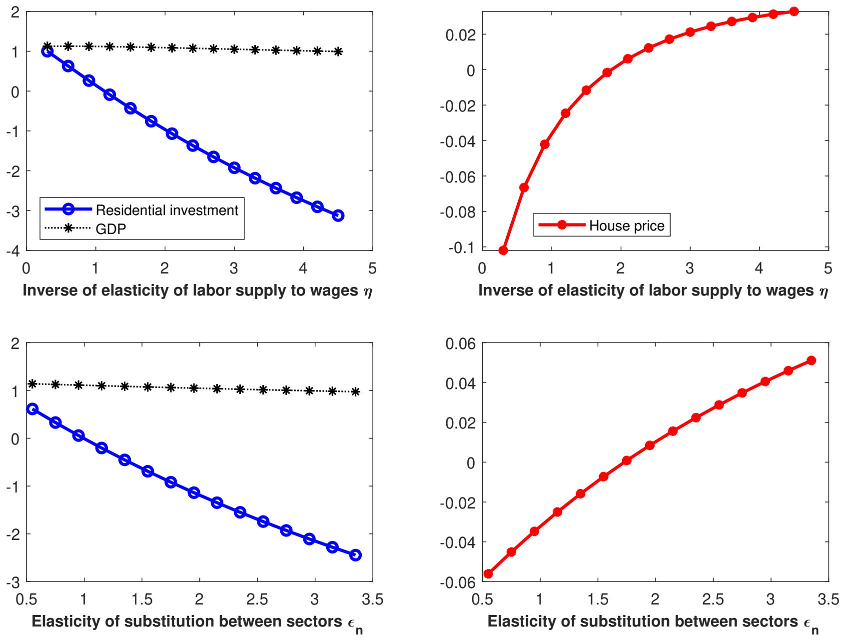


Figure 2.7: Impact responses (in %) to a public spending shock of 1% of GDP for various labor market parameters: GDP, residential investment, and house prices. *Note*: Residential investment and GDP are on the left scale, and house prices are on the right scale.

(b) For intermediate values of these parameters, both residential investment and housing decline.

(c) For high parameter values, residential investment declines while prices rise. Following the fiscal shock, the highly inelastic labor supply results in a significant wage gap between both sectors, intensifying worker reallocation and exacerbating the decrease in labor supply in the construction sector. Moreover, a high elasticity of substitution facilitates the reallocation of workers between the two sectors, yielding a similar outcome in housing production.

As a result, under certain conditions, a fiscal policy of increasing government consumption can be especially harmful to housing production and employment. However, this does not significantly affect GDP because an increase in consumption and investment in the tradable sector offsets a further decline in residential investment.

Elasticity of labor supply to wages

The top panel of Figure 2.7 shows how the government consumption shock impacts residential investment more negatively (less positively) as the value of η, the inverse of the elasticity of labor supply to wages, increases. In fact, this labor supply feature significantly impacts real estate investment.

Increased aggregate demand due to increased government spending encourages the production of tradable goods and fosters labor demand in the tradable sector. The higher the η parameter the less prone workers are to increase working hours. Therefore, producers of nondurable goods must pay higher wages. As a result, production costs rise and investment falls compared to the baseline scenario.

The wage differential in the tradable sector is more pronounced than in the residential sector (see Figure 2.8). Higher wages attract the flow of workers from the housing sector into the tradable goods sector, which disincentivizes employment for residential construction and causes residential investment to fall.

Elasticity of substitution between labor types

The bottom panel of Figure 2.7 shows the effects of varying labor mobility between production sectors. For low elasticity of substitution, ε_n (low labor mobility across sectors), the fiscal shock encourages residential investment. As ε_n increases, the response of residential investment falls sharply. More specifically, housing production begins to fall when elasticities exceed the vicinity of 1 (our benchmark is 0.78). However, GDP is largely unaffected.

The variables' response mechanism is as follows. Government spending increases demand for consumer goods, which positively affects firm output and increases demand for labor and wages in the tradable goods sector. When the substitution elasticity between types of labor, ε_n, is low, few workers move from the housing sector to the tradable goods sector. Relative wages remain high, and housing construction capitalizes on the increased demand for housing as labor income rises. However, when ε_n is large, workers migrate from the housing sector to the tradable goods sector, where they are better compensated. Firms in the tradable sector respond to the increase in labor supply by increasing output. The movement of workers from construction to tradable goods production reduces employment in the housing sector and the construction of new homes. Unlike in the case of labor supply elasticity, the impact on relative wages falls as there is a greater reallocation of workers between sectors, and wages tend to equalize (Figure 2.8) [15].

[15] In the limit, when ε_n tends to infinity, the relative wage is 1

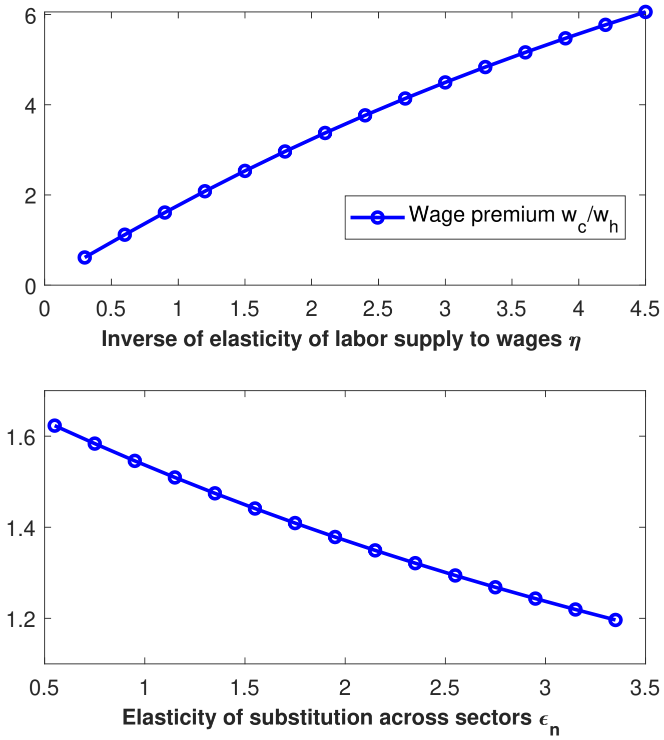


Figure 2.8: Impact responses (in %) of the wage premium to a 1% of GDP public spending shock for various values of the inverse of the Frisch elasticity of labor supply (upper panel) and the elasticity of substitution between sectors (lower panel).

Our benchmark model for Spain is based on an open economy within a monetary union. However, investigating the implications of closing the economy is worthwhile. The goal of this analysis is to identify the specific mechanisms that are driving the outlined results. In Appendix A.3, we demonstrate how the reaction of monetary policy influences the quantitative response of fiscal policy. However, our qualitative findings are largely unchanged.

2.4.3 Fiscal policy: the role of household debt

The housing sector's development in an economy is closely related to the dynamics of the financial system and the mortgage market. Financial regulations governing LTV ratios and policies promoting the housing rental market impact both the intensive and extensive margins of household debt. The parameter κ controls the ability of households to leverage home equity in loans, representing the intensive margin in the model. The extensive margin is the proportion of households with mortgages, represented by the parameter τ^b. Policies make these two characteristics of the country's household debt dependent on time, which may shape the impact of government spending on the housing sector. We use the model to shed light on the scope of this influence and the various channels through which it can be generated.

We simulate the response to a government spending shock of 1% of GDP, examining its impact as a function of the intensive and extensive margins of borrowing for different values of κ and τ^b. The three-dimensional graphs in Figure 2.9 show the results. The plots in the left column correspond to the benchmark parameterization, while those in the right column repeat the exercise but include adjustment costs for housing demand. These adjustment costs are an approximation of all costs associated with the purchase of a home other than the purchase price. They may include notary fees, specific property transfer taxes, search costs, and other costs that increase with certain regulatory requirements (such as immatriculation, segregation, municipal permits, etc.).

The results show that increasing both the intensive and extensive margins has a smaller (larger) positive (negative) impact on residential investment. Both margins reinforce each other, resulting in a nonlinear effect that increases substantially for high values of κ and τ^b. The intuition for understanding the effect of debt on housing construction is based on the Equations (2.21) and (2.26).

After a government spending shock, the variable μ_t^b decreases, indicating that the borrowing constraint becomes less tight, causing the marginal propensity to consume to fall and increasing borrowers' consumption of tradable goods (contrary to the effect of government spending on lenders' consumption). Interestingly, the decrease in μ_t^b is more pronounced the higher the LTV, as government spending eases the constraint to a greater extent for those households that can borrow more at the margin of home equity. This consumption mechanism increases the relative wage of the tradable sector compared to the housing sector, causing workers to shift away from the latter. Furthermore, the labor supply mechanism influences both sectors via the LTV. As borrowers' consumption rises with the LTV, so does their demand for leisure, resulting in a smaller increase in labor supply. Consequently, residential investment, which is labor intensive, decreases.

A higher value of τ^b assigns more weight to borrowers' consumption in aggregate consumption. Consequently, the shock's impact on aggregate consumption intensifies with a higher value of τ^b. With an increase in aggregate consumption, the demand for workers in the tradable sector also

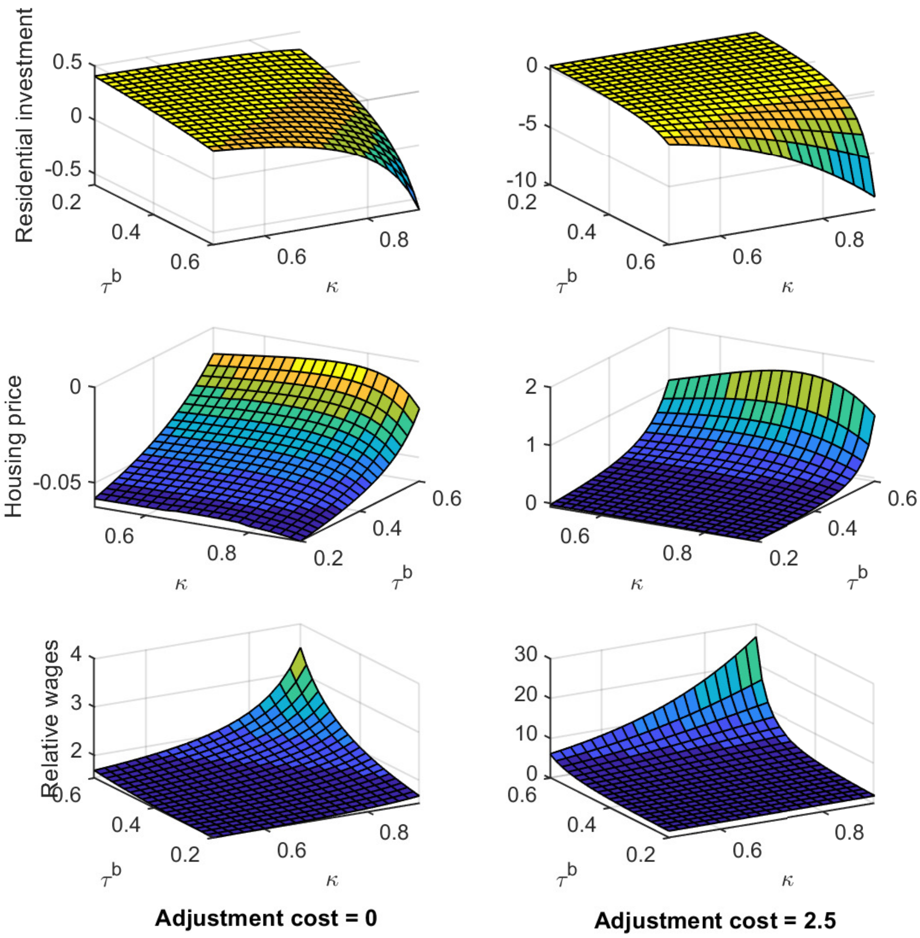


Figure 2.9: Impact responses (in %) to a public spending shock of 1% of GDP as a function of the share of borrowers, τ^b, and the LTV, κ

rises, as does the wage premium. This encourages workers to migrate from the housing sector to the tradable sector, resulting in decreased home construction and increased house prices. Moreover, lenders' consumption falls more sharply as the government spending shock absorbs consumption goods. This suggests that lenders will be more willing to provide labor than borrowers. Therefore, in an economy with a higher proportion of borrowers, the government spending shock will cause a smaller increase in labor supply, lowering residential investment.

The second column shows the importance of considering housing demand adjustment costs in determining the magnitude of the public spending impact on housing sector variables. These adjustment costs involve incorporating an additional cost of changing the housing stock in the budget constraint given by Equations (2.2) and (2.15). Following Iacoviello (2005), we have written it as $\varepsilon^a_{h,t} = \phi_h(\Delta h^a_t / h^a_{t-1})^2 q_t h^a_{t-1}/2$ (where $a = l$, b stands for lenders and borrowers), and where ϕ_h is set to 2.5, coinciding with the parameter for adjustment costs of productive capital[16]. In the same vein, housing adjustment costs significantly increase the effect of household debt on government spending.

Interactions between household debt and the labor market

The previous sections have emphasized the importance of labor market characteristics (worker mobility across sectors and wage elasticity of labor supply) and household debt (intensive and extensive margin) in determining the magnitude of fiscal shocks' effects on the housing market. In this subsection, we study the interactions between household debt and the labor market.

Figure 2.10 shows the response of residential investment to an increase in government spending for three different values of the inverse Frisch elasticity (first column) and the elasticity of substitution of labor supply across sectors (second column). The first row displays changes in the LTV ratio, whereas the second row displays changes in the proportion of borrowers. The remaining parameters not appearing in the different graphs are set to their reference values.

The lines shift downward as η and ϵ_n increase, as stated in Section 2.4.2. More interestingly, the lines all curve downward as the intensive and extensive margins of household borrowing increase, with the steeper the curve indicating greater worker mobility between sectors and lower labor elasticity concerning wages. This means that household debt interacts with the two labor market characteristics, exacerbating the negative impact on residential investment, particularly above a certain level.[17]

[16] Iacoviello (2005) and Iacoviello & Neri (2010) set the value of ϕ_h to 0, similar to our benchmark calibration. They state that preliminary attempts to estimate this parameter result in a value of 0, despite some difficulties in estimating it and some theoretical support for expecting it to be different from 0. Indeed, Flavin & Nakagawa (2008) show that housing adjustment costs are extremely important to the dynamics of nondurable consumption.

[17] Khan & Reza (2017) did not find a plausible mechanism in the DSGE models to produce the empirically observed increase in house prices following a government spending shock. They attribute the DSGE model's inability to explain their empirical findings to its property of maintaining an approximately constant shadow value of housing for lenders, which results in a counterfactual decline in house prices following a government spending shock. We can replicate the consistency of the shadow value of housing in an economic environment characterized by (a) a relatively high response of labor supply to wages; (b) low worker mobility between sectors; (c) a small share of borrowers in the population; (d) a low LTV ratio; and (e) low adjustment costs in housing demand. However, when we

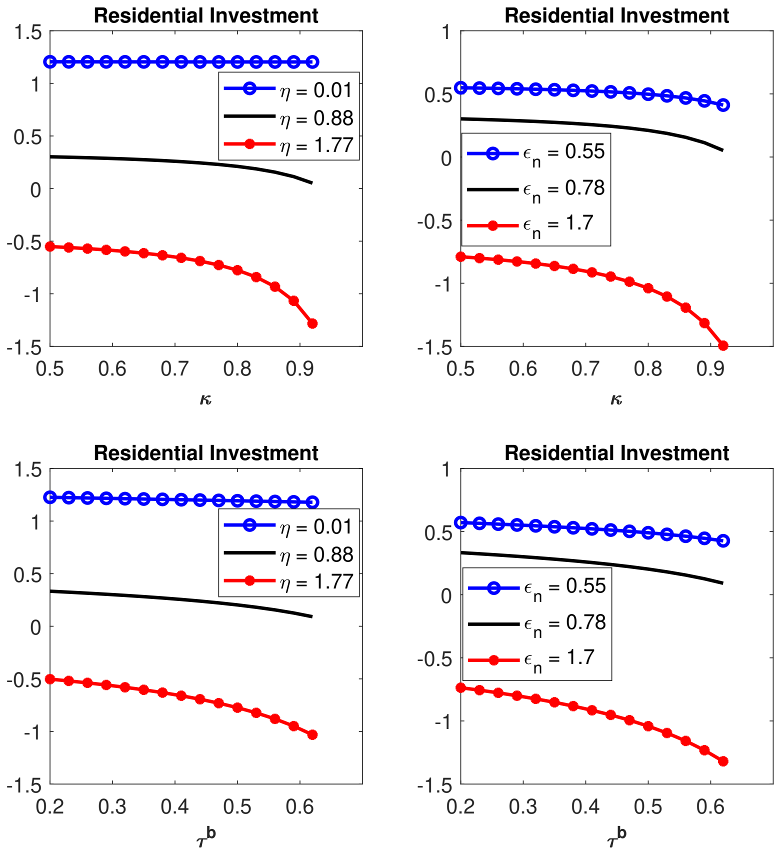


Figure 2.10: Impact responses on residential investment to a public spending shock of 1% of GDP for different values of the elasticity of substitution (ϵ_n), the inverse of the Frisch elasticity (η), the LTV (κ), and the share of borrowers (τ^b).

2.4.4 Housing demand shocks and fiscal policy: a welfare analysis

The equilibrium levels of consumption flows and housing stocks may differ significantly across household types, and the effects of a housing shock and fiscal expansion may produce interesting distributional outcomes in terms

move to a different economic environment, corresponding to a different configuration of the parameters η, ε_n, τ^b, κ, and ϕ_h, the shadow value of housing persistently increases. This is due to lenders' significantly different behavior toward housing, which raises the present value of the marginal utility of houses. As a result, house prices increase, resolving one aspect of Khan and Reza's puzzle.

of welfare. This section evaluates the effect of these two shocks on both aggregate welfare and the welfare of each household type.

We use utility-based welfare calculations, so household welfare represents the discounted sum of utilities derived over infinite periods. In our model, household utility is additive to private consumption, housing, and labor. Thus, we can define the expected welfare of household a at time t as

$$W_t^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln c_t^a + \phi_{j,t}\gamma_h \ln h_t^a - \frac{(n_t^a)^{1+\eta}}{1+\eta} \right) \tag{2.71}$$

Note that Equation (2.71) can be represented recursively as

$$W_t^a = U_t^a + \beta^a \mathbb{E}_0 W_{t+1}^a \tag{2.72}$$

We define social welfare as the weighted sum of individual welfare across different types of households. The weight of household utilities is represented by Ω_a, for which we choose the values defined by Mendicino et al. (2004), that is, Ω_a equals $(1-\beta^l)$ for lenders and $(1-\beta^b)$ for borrowers. Therefore, we express social welfare in period t as

$$W_t^s = \mathbb{E}_0 \sum_{a=l,b} \Omega_a \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln c_t^a + \phi_{j,t}\gamma_h \ln h_t^a - \frac{(n_t^a)^{1+\eta}}{1+\eta} \right) \tag{2.73}$$

or more compactly as

$$W_t^s = (1-\beta^l)W_t^l + (1-\beta^b)W_t^b \tag{2.74}$$

Under the assumption of perfect foresight, Figure 2.11 shows the percentage change in welfare over time relative to the steady state in the presence of the same temporary housing shock and expansionary fiscal shock as in Figure 2.5. Figure 2.11 shows the welfare response for each agent in the economy and the aggregate and the relative contributions of consumption, housing, and employment to welfare. The results show some interesting patterns.

First, the negative shock to housing demand negatively affects social welfare, as one might expect. However, borrowers suffer far more than lenders regarding welfare in the early stages. The welfare of lenders deteriorates over time, whereas that of borrowers improves. Credit-constrained households are less likely to smooth consumption, and when house prices fall, they can less compensate for the loss of wealth. Following the shock, both house prices and residential investment fall, and borrowers' creditworthiness deteriorates, affecting consumption and demand for housing. Lenders also suffer from falling house prices, which reduce the value of their durable assets. However,

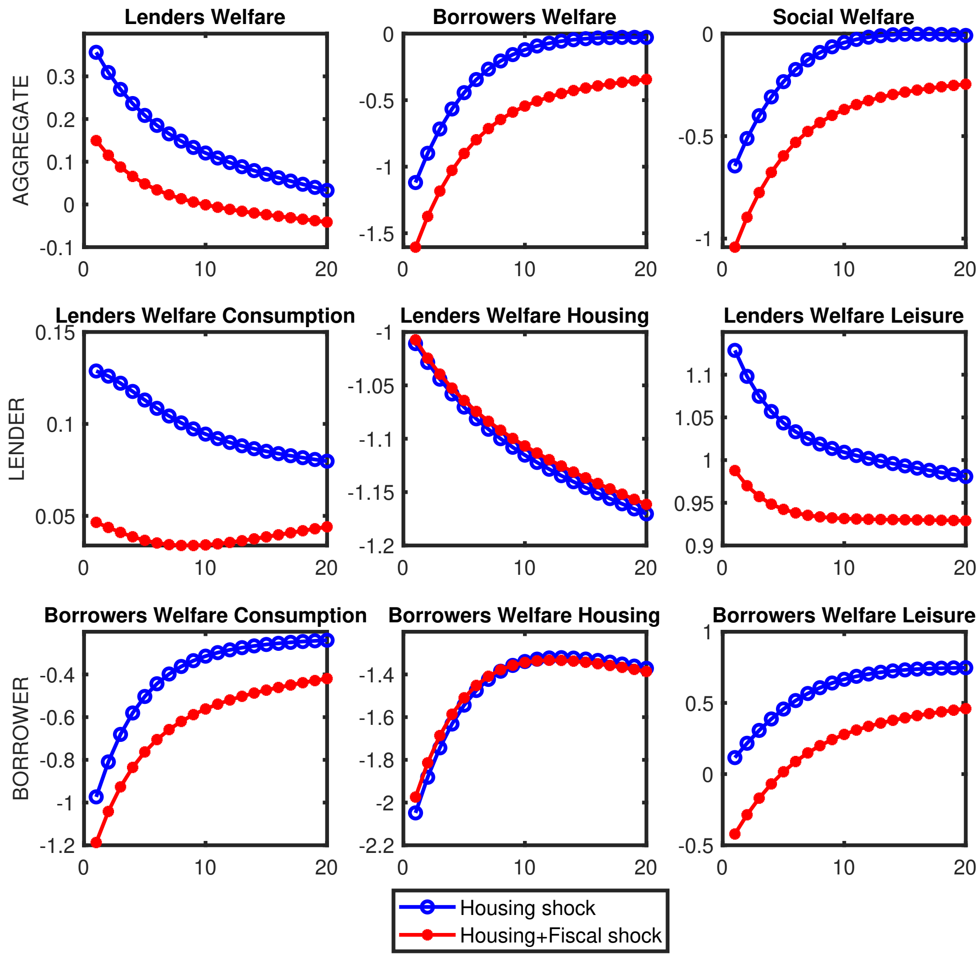


Figure 2.11: Welfare response to a real estate shock and countercyclical fiscal policy.

because they are not credit-constrained and lending falls, they actually increase consumption.

Second, and related to the previous point, variation in housing demand has the greatest negative effect on welfare, particularly for borrowers. Indeed, it is the only component that reduces lender welfare, whereas consumption also reduces borrower welfare. However, because the housing shock reduces working hours, the effects of leisure partially offset the negative welfare effects of other components.

Third, implementing an expansionary public spending policy in an environment of low housing demand reduces the welfare of both types of households. Interestingly, the policy's negative welfare difference is primarily due to variations in consumption and leisure, with virtually no effect from housing property. These findings are consistent with Sims and Sims & Wolff (2018), who found that, while government consumption positively affects aggregate output, its welfare multiplier is negative on average.

2.4.5 Government housing purchases in economic downturns

So far, we have assumed that the government responds to a negative housing shock by increasing public consumption. However, the government may take alternative measures if a housing market downturn causes an economic recession. One such strategy involves the government purchasing real estate assets rather than allocating funds for consumption goods. This policy is more likely to slow the decline in the housing market[18].

Here, we assume that the government buys houses in the market using lump-sum taxes and then distributes them to households. Houses purchased by the government enter the utility function alongside houses owned by households. However, because houses remain public property, they cannot be used as collateral for new credit. The distribution of public houses among lenders and borrowers is based on a strict population criterion, i.e., their population weights.

More specifically, we change the household utility function as follows:

$$W_t^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln c_t^a + \phi_{j,t}\gamma_h \ln(h_t^a + h_t^g) - \frac{(n_t^a)^{1+\eta}}{1+\eta} \right) \tag{2.75}$$

where $a = l, b$ and h_t^g are houses bought by the public sector.

Because we have previously extensively studied the mechanisms driving the effects of standard fiscal policy, our focus here is primarily on explaining the macroeconomic and welfare effects of government housing purchases. For comparison, we consider the effects of traditional public spending.

Figure 2.12 compares the responses of key macroeconomic variables to various public spending policies. In both cases, the increase in public expenditure remains constant at 1% of GDP and follows a similar downward trend over time. However, when the government focuses on purchasing homes to stimulate the economy, residential investment and house prices skyrocket compared to government spending on tradable goods consumption. This substantial increase in the price of durable assets has important implications for

[18]Egan & Bergin (2023) find that government spending directed specifically toward the housing sector has a greater effect on increasing dwelling numbers and improving housing affordability than a broad-based government spending shock throughout the economy.

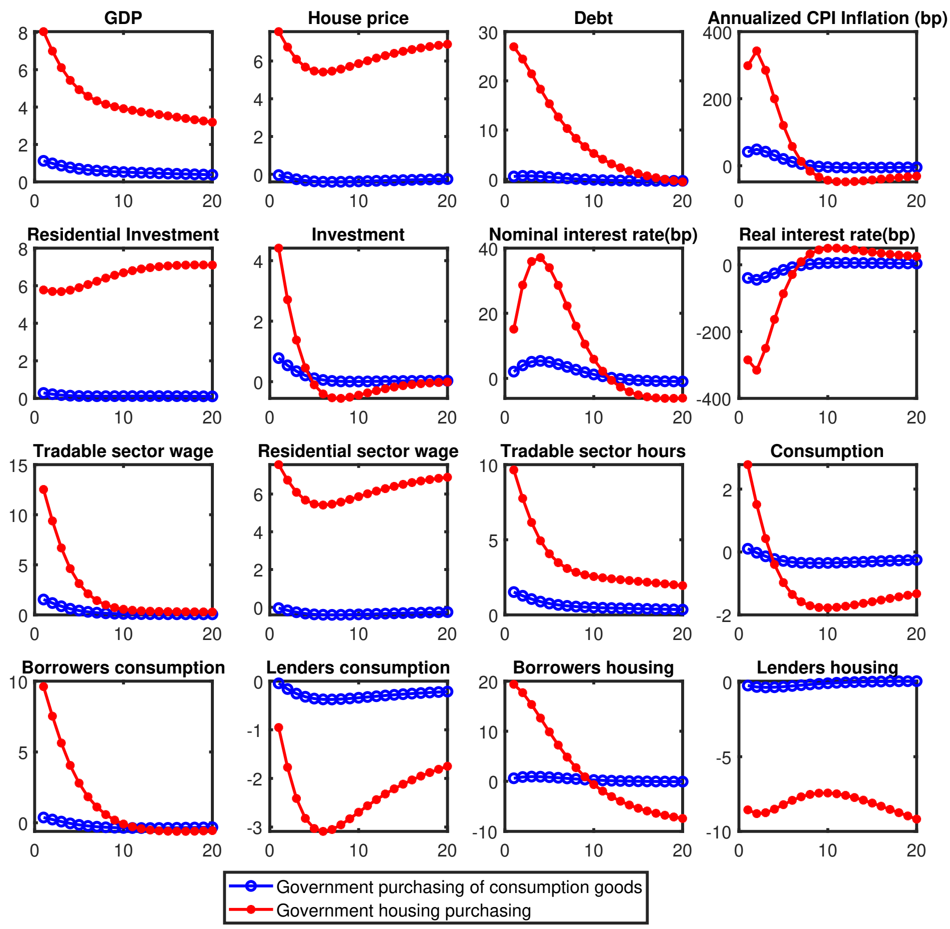


Figure 2.12: Macroeconomic response to a government purchasing of housing.

borrowers: the value of their collateral increases, increasing their borrowing capacity. As a result, their consumption grows, increasing the overall demand for tradable goods.

Increased demand for durable goods drives the CPI inflation. In response, the ECB raised interest rates to combat inflation within the global monetary union, but this was insufficient to reduce inflation in Spain in the short term. This results in a significant drop in domestic real interest rates relative to the benchmark policy of the government purchasing consumption goods. The drop in real interest rates further alleviates borrowers' financial constraints, increasing their nondurable consumption.

Unlike borrowers, who value houses for their intrinsic utility and function as collateral, lenders tend to decrease their demand for housing as residential prices rise. Instead, they prefer to increase their savings and invest in borrowers' debt. The rebound in real interest-rate mitigates the response in borrowers' demand. However, the ongoing negative impact on lender consumption, combined with the persistence of this effect, causes the aggregate consumption response to fall into negative territory after a few quarters.

In Figure 2.13, we show how macroeconomic effects translate into welfare outcomes. In the first row of subgraphs, we show the cumulative effect over time on the welfare of lenders and borrowers and the aggregate effect on social welfare. The second and third rows of the subplots show how lenders and borrowers' welfare changes due to consumption, housing, and leisure.

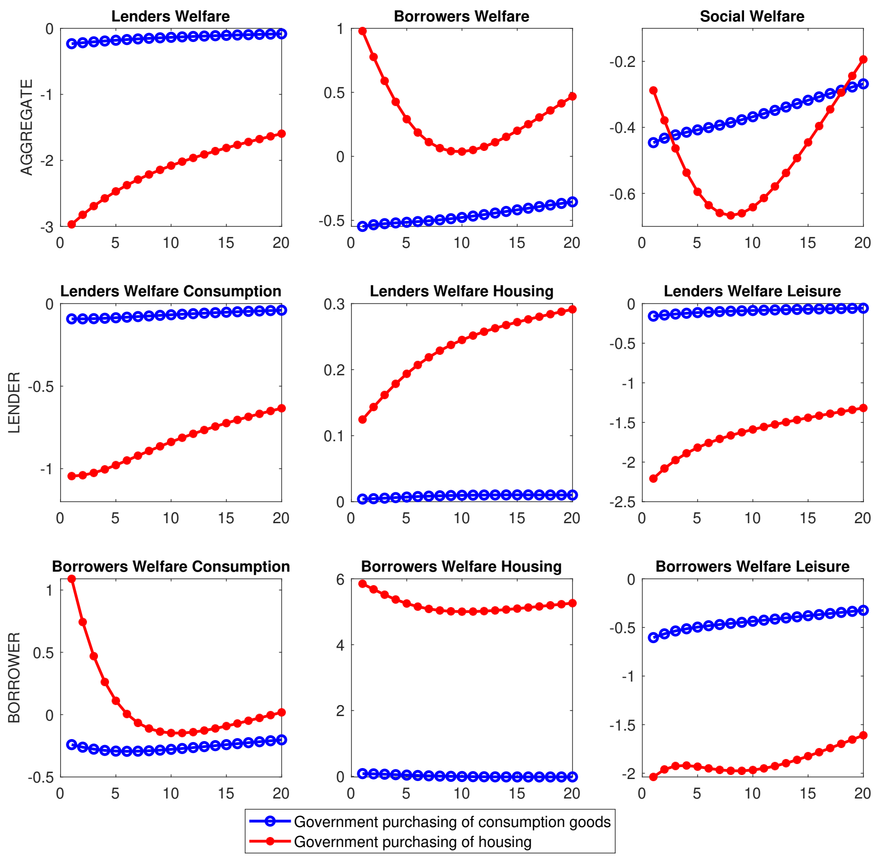


Figure 2.13: Welfare response to government housing purchases (in deviations from steady state).

Despite the positive impact on the housing market when compared to a standard government policy of increasing current consumption goods, government housing purchases are detrimental to lenders due to the dragging effect on consumption and leisure. However, this policy has the potential to reverse the welfare effect on borrowers seen in Figure 2.11, owing to a significant increase in available housing for borrowers. However, the wage increase encourages a positive response in labor supply, which decreases utility by reducing leisure. However, the (presumably positive) distributional impact on welfare is evident.[19]

In general, this policy has a negative and U-shaped effect on social welfare. When compared to the standard government policy, social welfare is lower for most of the 20 periods shown in Figure 2.13, except for the first and last.

2.5 Conclusion

The empirical literature presents mixed findings about how residential investment and house prices respond to increases in government spending. A two-sector dynamic general equilibrium model, which considers financial and labor frictions and includes the production of tradable consumer and investment goods alongside nontradable housing, can forecast the various reactions in residential investment and house prices.

Our model demonstrates that following a fiscal shock, the increase in government consumption can alleviate the adverse effects caused by the decrease in housing demand. However, unlike overall output, the response of residential investment and housing prices to the fiscal shock can be significantly influenced by characteristics of the labor market, household indebtedness, and the presence of adjustment costs in housing demand, which greatly magnifies the effect of private indebtedness. Additionally, an increase in government consumption may worsen the adverse welfare effects of a housing shock. The negative impact of fiscal policy primarily arises from shifts in consumption and working hours.

One key insight from the chapter is the complex balance and trade-offs involved in the interaction of fiscal policy, the residential sector, and societal well-being during a housing crisis. Although boosting the economy through government spending on consumption goods may inadvertently harm the residential sector, using fiscal policy to buy homes could prevent a drop in housing construction. However, this strategy results in significant reductions in societal well-being and a redistribution of consumption between lenders and borrowers.

[19]The borrowers in the model have lower levels of consumption and income prior to government housing purchases.

These observations can draw some normative implications, but only with caution. For example, if the economy is hit by a shock that severely impacts the construction sector and reverberates through the credit market, using public consumption to offset it may be counterproductive. The housing sector's negative effect could exacerbate credit market problems and further harm the economy. Although our model may not fully capture this effect, it could be better understood in an environment featuring a banking system heavily concentrated in the housing sector and vulnerable to credit defaults. In contrast, during rapid credit expansion and increased construction activity, fiscal policy can serve as macroprudential policy, addressing financial imbalances without jeopardizing economic growth.

Our results apply to other economies where the government may face a trade-off between stimulating GDP and construction investment. They indicate that the likely effects on the real estate sector as a result of fiscal policy would be complex and depend on specific aspects of the economy, such as the labor market's job flexibility, responsiveness of working hours to wages, ease of credit access, total amount of private debt, or range of home purchase costs. Therefore, an empirical program to estimate the influence of such factors on how fiscal policy affects the housing sector in various economic contexts is an immediate complement to our results.

Chapter 3

Carbon Pricing, Household Inequality and Compensation Strategies

This chapter analyzes the macroeconomic and distributional impacts of carbon pricing policies targeting both residential and nonresidential sectors. Using a model that incorporates nominal price rigidities, sectoral labor adjustments, and financial frictions tied to housing collateral, we uncover critical transmission mechanisms affecting household welfare. Our analysis highlights the distinct effects on borrowers and lenders: carbon pricing in the nonresidential sector reduces labor demand and wages, disproportionately impacting borrowers, while residential carbon pricing lowers housing prices, tightening credit constraints for borrowers but imposing higher welfare costs on lenders who own more housing assets. We explore compensation strategies, particularly the allocation of carbon dividends. An equal distribution initially stabilizes consumption for borrowers but becomes less effective as emissions decline and carbon revenues shrink. Robustness checks show that factors like labor mobility, firm markups, and debt distribution significantly influence inequality. Flexible prices can mitigate early consumption inequality, while a higher share of debt among borrowers affects policy efficiency and the required carbon price levels. Our findings underscore the need for sector-specific carbon pricing and adaptive compensation mechanisms to balance efficiency and equity. This research provides policymakers with insights for achieving environmental targets while addressing socioeconomic disparities.

3.1 Introduction

The fight against climate change is one of the most pressing challenges facing the world today. Leading developed economies, spearheaded by the European Union, have committed to ambitious emission reduction targets. These targets are crucial to mitigating the adverse effects of climate change and to progress toward a net zero emissions (NZE) economy by 2050. As efforts toward climate neutrality continue, it is vital to recognize that the transition to carbon neutrality will profoundly shape macroeconomic dynamics for decades. In this context, carbon pricing policies have emerged as a key strategy to promote emission reductions in various sectors.

Despite significant attention to emissions from the production sector, there remains a notable oversight of emissions from the residential sector, where heating, cooling, and energy use contribute substantially to total emissions. Policies directly targeting these residential emissions are less prevalent and less explored. The public acceptance of such policies is crucial, largely hinging on their distributional effects, which are politically sensitive and widely debated (European Environmental Bureau (2022); European Commission (2021); Marcu et al. (2023)). Effective carbon pricing policies in the residential sector, essential for achieving the 2050 net-zero target, depend critically on a thorough understanding of both macroeconomic and distributional impacts during the transition. This understanding is vital for developing policies that mitigate redistributive impacts while minimizing welfare costs.

This study delves into the critical aspects of transition policies, examining the macroeconomic and redistributive effects of incorporating the residential sector into the EU's 2050 zero-emission goals. Our research extends the body of knowledge using Environmental Dynamic General Equilibrium (E-DGE) models to provide a detailed quantitative analysis of the impacts of climate change and mitigation policies. This model allows for the integration of economic frictions, enhancing our understanding of the transmission channels in the short term (Fischer & Springborn (2011), Heutel (2012), Annicchiarico & Di Dio (2015), Coenen et al. (2018), Ferrari & Landi (2022)).

We introduce limited household heterogeneity in our model using the framework of Two-Agent New Keynesian (TANK) models, differentiating between credit-constrained and financially unconstrained households.[1] Our findings reveal significant distributional effects of environmental policies between these groups. Borrowers experience greater welfare losses than lenders following the transition to a net-zero emissions (NZE) economy, highlighting a

[1]For a discussion on the aggregate implications of TANK models versus Heterogeneous Agent New Keynesian (HANK) models, see Debortoli & Galí (2024).

regressive effect. Notably, this distributional effect fluctuates throughout the transition—initially decreasing due to lump-sum tax reductions from carbon revenues but increasing once intermediate climate targets are reached. This dynamic emphasizes the need for targeted mitigation policies, particularly in later stages of the transition, where the distribution of carbon tax revenues among household types becomes critical.

The integration of household heterogeneity within a dynamic general equilibrium framework marks a significant advancement in the environmental policy literature. Traditionally, assessments of the distributional impacts of environmental taxes on household income groups utilize input-output analysis to determine price changes for various consumer goods, subsequently calculating tax burdens through micro-household survey data. These studies generally take a partial equilibrium approach, concentrating on consumer spending patterns (a use-side analysis). Conversely, other research employs numerical general equilibrium models to evaluate the impacts of taxes on consumer goods and factor prices (Dinan & Rogers (2002), Metcalf et al. (2008), Hassett et al. (2009), Fullerton & Heutel (2007), Araar et al. (2011), Rausch et al. (2011), Mathur & Morris (2014)). However, these often fail to account for the effects of household heterogeneity on equilibrium outcomes, an issue specifically addressed by Rausch & Schwarz (2016). In their study, they investigate how the general equilibrium effects of a carbon tax are shaped by the diverse incomes and preferences of heterogeneous households, illustrating that ignoring this heterogeneity can greatly skew the incidence of carbon taxes, especially impacting returns to factors of production.

Thus, our study enriches the body of literature on the transitional effects of carbon pricing policies by incorporating these policies into an E-DGE model with endogenous housing production, addressing the global externality of carbon emissions. By considering two productive sectors with imperfect labor substitution between them, specifically the residential and goods production sectors, our model assesses the interactions provoked by environmental policies. Moreover, by accounting for both unconstrained households and those whose credit capacity is impacted by properties affected by emission restrictions, our study addresses critical elements of the distributional impacts of the tax: the incidence on consumer spending patterns (use-side analysis), the price of production factors (source-side analysis), and the capacity to extract credit from housing collateral.

Our findings indicate that carbon pricing policies have diverse effects on the economy depending on whether they target residential or nonresidential sectors. In the nonresidential sector, carbon pricing mainly drives up production costs, leading to a decrease in labor demand, wages and profits, which in turn suppresses consumption and economic activity. In contrast, carbon pricing in the residential sector has more complex effects. Specifically, it triggers a decrease in housing prices, reducing the collateral value available to credit-constrained households. This drop in collateral limits their borrowing capacity and further restricts consumption. These contrasting outcomes highlight the importance of developing sector-specific carbon pricing strategies to mitigate unfavorable distributional impacts.

Reducing emissions in the residential sector can lead to very different macroeconomic responses per unit of abated emissions compared to the nonresidential sector. Specifically, raising the emission price has a notable impact on housing investment, housing prices, and credit per unit of carbon abated, with these effects being more pronounced when the tax targets the residential sector rather than the nonresidential sector. For example, the carbon tax reduces credit by a ratio of approximately 3 to 1 per unit of emissions abated in the residential sector compared to the nonresidential sector. Overall, carbon pricing policies in the residential sector result in lower welfare costs per unit of abated emissions than equivalent measures in the nonresidential sector.

From a welfare perspective, our analysis shows that the total long-term welfare cost, in terms of consumption, exceeds 10% under a combined carbon pricing policy, which affects both borrowers and lenders almost equally. However, significant differences arise when we consider sector-specific impacts. Carbon pricing in the nonresidential sector results in a welfare loss for borrowers that is 0.5 percentage points higher than that for lenders, reflecting borrowers' heightened sensitivity to declines in labor demand and overall economic activity. In contrast, targeting the residential sector causes lenders to experience nearly double the welfare loss compared to borrowers. This outcome occurs even though lenders exert less effort to reduce carbon emissions, as the residential carbon tax disproportionately affects housing investment and housing stock, which are assets predominantly owned by lenders.

The distribution of debt within the economy affects the efficiency of carbon pricing policies. When the proportion of credit-constrained borrowers is higher, carbon pricing in the residential sector becomes more effective. This is because borrowers, facing a greater utility loss from reducing their housing stock, are incentivized to exert higher abatement efforts. In contrast, lenders, with their larger housing portfolios and greater financial flexibility, experience a lower marginal utility loss and are less motivated to engage in abatement. However, in the nonresidential sector, higher carbon prices are needed when a larger share of the population consists of borrowers. The nonresidential carbon tax influences lenders' income through its impact on firm profits, creating a direct incentive for emissions reduction. As the proportion of lenders decreases, this incentive weakens, necessitating higher carbon prices to maintain the same level of emissions reduction.

The way carbon tax revenues are distributed plays a vital role in addressing these distributional effects. Initially, distributing carbon dividends evenly across households helps cushion the impact of higher energy costs, especially for borrowers, and stabilizes their consumption. However, as emissions are gradually reduced and the revenue base from carbon taxes diminishes, the effectiveness of these transfers wanes, leading to growing inequality. Our analysis of the consumption Gini coefficient identifies critical periods where inequality is most acute, emphasizing the need for targeted policies to mitigate these effects throughout the transition to net zero emissions (NZE).

The chapter is structured as follows: Section 3.2 introduces the model, detailing the economic structure, household heterogeneity, and production sectors under environmental regulation. Section 3.3 discusses the model calibration, focusing on aligning key parameters with empirical data. Section 3.4 presents the simulation results, divided into four subsections: 4.1 examines the macroeconomic impacts of carbon pricing on GDP, consumption, and investment; 4.2 explores the differential effects on borrowers and lenders, emphasizing the dynamics of inequality during the transition to net-zero emissions; 4.3 investigates the robustness of inequality outcomes under various parameter settings and policy designs; and 4.4 evaluates compensation mechanisms, such as the distribution of carbon dividends, to mitigate adverse distributional impacts. Finally, Section 5 provides a discussion of policy implications and suggests directions for future research.

3.2 The Model

The model portrays the European Union as a unified economic region with some degree of limited heterogeneity in both the population and the production side. The population consists of two distinct household types differentiated by their subjective discount rates, reflecting varying levels of impatience. Households with lower discount rates are more patient and tend to save, while those with higher rates are more impatient and typically become borrowers. Both classes consume goods and services, engage in the real estate market, and contribute labor to the production of consumer goods and construction.

The economy encompasses two primary production sectors. The first is responsible for producing consumer and investment goods (also named "consumer production goods" or "nonresidential sector" through the chapter), and the second focuses on housing construction. Firms in the consumer goods sector emit greenhouse gases during production, with emissions proportionally increasing with output levels. Residential activities, such as heating and cooking, also generate carbon emissions, varying by household due to differences in real estate ownership and the associated services houses provide.

Accumulated emissions adversely affect the economy's productivity but do not inherently drive households or firms to reduce their carbon footprint due to the associated costs. In the absence of regulation, minimal effort is made to mitigate emissions.

To counteract this, the government implements carbon pricing policies to internalize the social costs of emissions, thereby encouraging investment in emissions reduction. These policies apply to both production and residential emissions, prompting firms and households to adjust their behaviors. The government redistributes any additional revenue from these policies back to households and can offset the financial burden on households by adjusting the lump-sum taxation scheme to manage distributional impacts. The financing for government expenditures is sourced from general taxation and the revenues generated from carbon taxes.

3.2.1 Households

As in Iacoviello (2005), there are two representative households types; N_t^l of them are patient (lenders) and N_t^b are impatient (borrowers). Total population is given by $N_t = N_t^l + N_t^b$. We call $1 - \tau^b = \frac{N_t^l}{N_t}$ and $\tau^b = \frac{N_t^b}{N_t}$ the exogenous proportion of lenders and borrowers in the population.

Patient households

Patient households, or lenders, choose consumption, housing services[2] and labor to maximize the following schedule (real variables are expressed in per capita terms of the specific type of household).

$$\max_{c_t^l, h_t^l, n_{ct}^l, n_{ht}^l, b_t^l, k_t^l, j_t^l, u_{ht}^l} \quad \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^l)^i \left(\ln c_{t+i}^l + \gamma_h \ln h_{t+i}^l - \frac{(n_{t+i}^l)^{1+\eta}}{1+\eta} \right) \tag{3.1}$$

subject to the real flow of funds expressed in terms of consumption goods price, P_t, (the *numeraire*),

$$\begin{aligned} c_t^l + q_t(h_t^l - (1-\delta^h)h_{t-1}^l) - b_t^l + j_{ct}^l + j_{ht}^l + \tau_{ht} e_{ht}^l + z_{ht}^l = \\ w_{ct} n_{ct}^l + w_{ht} n_{ht}^l - \frac{r_{t-1}}{\pi_t} b_{t-1}^l + d_t - tr_t^l + r_{kt} k_{ct-1}^l + r_{ht} k_{ht-1}^l \end{aligned} \tag{3.2}$$

[2]We make the standard assumption that the flow of residential services is proportional to the stock of housing.

On the utility side, $\beta^l \in (0,1)$ represents the inverse of the patient discount factor, c_t^l is real consumption per capita, h_t^l is lenders' housing units, and n_t^l is labor time per person. The parameter γ_h captures housing preference and the inverse of η relates to the elasticity of labor supply to wages.

We assume imperfect substitutability, as in Iacoviello & Neri (2010), of labor across sectors, so that n_t^l is a CES composite of lenders' labor supply in the consumer goods industry n_{ct}^l and in the housing sector n_{ht}^l,

$$n_t^l = [\theta^{\frac{1}{\varepsilon_n}} (n_{ct}^l)^{\frac{1+\varepsilon_n}{\varepsilon_n}} + (1-\theta)^{\frac{1}{\varepsilon_n}} (n_{ht}^l)^{\frac{1+\varepsilon_n}{\varepsilon_n}}]^{\frac{\varepsilon_n}{1+\varepsilon_n}} \tag{3.3}$$

where $\theta > 0$ is a weight parameter that relates with the disutility of working in the consumption sector relative to the housing sector, and affects the proportion of patient households working hours in each sector. $\varepsilon_n \geq 0$ is the elasticity of substitution between labor supply in the two sectors, capturing the easiness of movement between sectors [3].

As for the budget constraint, patient households lend in real terms $-b_t^l$ to impatient households (where b_t^l is negative). They receive back $-\frac{r_{t-1}}{\pi_t} b_{t-1}^l$ from the previous period's loans, where $\frac{r_{t-1}}{\pi_t^c}$ is the gross real interest rate factor. They invest in new productive capital in both sectors j_{ht}^l and j_{ct}^l and receive rents from capital $(r_{kt} k_{ct-1}^l + r_{ht} k_{ht-1}^l)$ and labor $(w_{ct} n_{ct}^l + w_{ht} n_{ht}^l)$. Since they are the owners they receive profits, d_t, from firms operating in noncompetitive markets, and pay lump-sum taxes, tr_t^l, in terms of consumption goods.

Households incur investment adjustment costs $\frac{\Phi^s}{2}\left(\frac{j_{st}^l}{j_{st-1}^l} - 1\right)^2$ where $s = c,\ h$ denote the consumer goods production sector and housing sector. The law of motion of private physical capital in both sectors is given by,

$$k_{ct}^l = (1-\delta^{kc}) k_{ct-1}^l + \left[1 - \frac{\Phi^c}{2}\left(\frac{j_{ct}^l}{j_{ct-1}^l} - 1\right)^2\right] j_{ct}^l \tag{3.4}$$

$$k_{ht}^l = (1-\delta^{kh}) k_{ht-1}^l + \left[1 - \frac{\Phi^h}{2}\left(\frac{j_{ht}^l}{j_{ht-1}^l} - 1\right)^2\right] j_{ht}^l \tag{3.5}$$

These households incur a cost of $q_t(h_t^l - (1-\delta^h) h_{t-1}^l)$ for the acquisition of new housing services in period t and for the depreciation of their housing stock. Living in house produces carbon emissions which may be subject to taxation by the government at a rate denoted as τ_{ht}. Houses release carbon emissions into the atmosphere which are proportional to the housing stock, but abatement effort, denoted as u_{ht}^l, reduces emissions,

$$e_{ht}^l = (1 - u_{ht}^l) \varphi_{1h} (h_t^l)^{1-\varphi_{2h}} \tag{3.6}$$

[3]For a full discussion of this CES function see Cardi & Restout (2015).

The parameter φ_{1h} represents the carbon emissions intensity per unit of dwelling services, indicating the fossil fuel usage by housing services. The elasticity, denoted as $1-\varphi_{2h}$, relates to the energy efficiency of buildings. A higher exponent indicates lower efficiency, as the rate of growth of carbon emissions increases more significantly with the growth in residential services.

Each period, households have the option to invest in their houses to reduce a fraction of their emissions. However, this action entails a cost proportional to the amount and quality of their residential holdings,

$$z^l_{ht} = \theta_{1h}(u^l_{ht})^{\theta_{2h}} h^l_t \tag{3.7}$$

where θ_{1h} and θ_{2h} are technological parameters of the abatement cost function.

The solution to this optimization problem yields the following first-order conditions:

$$\lambda^l_t = \frac{1}{c^l_t} \tag{3.8}$$

$$\lambda^l_t q_t = \frac{\gamma_h}{h^l_t} - \lambda^l_t(\tau_{ht}(1-u^l_{ht})\varphi_{1h}(1-\varphi_{2h})(h^l_t)^{-\varphi_{2h}} + \theta_{1h}(u^l_{ht})^{\theta_{2h}} + \\ + \beta^l \mathbb{E}_t(\lambda^l_{t+1} q_{t+1}(1-\delta^h)) \tag{3.9}$$

$$\frac{w_{ct}}{c^l_t} = (n^l_t)^{\eta}(\theta \frac{n^l_{ct}}{n^l_t})^{\frac{1}{\varepsilon_n}} \tag{3.10}$$

$$\frac{w_{ht}}{c^l_t} = (n^l_t)^{\eta}((1-\theta) \frac{n^l_{ht}}{n^l_t})^{\frac{1}{\varepsilon_n}} \tag{3.11}$$

$$\lambda^l_t = \beta^l \mathbb{E}_t \lambda^l_{t+1} \frac{r_t}{\pi_{t+1}} \tag{3.12}$$

$$\frac{\lambda_{ct}}{\lambda^l_t} = \beta^l \mathbb{E}_t \frac{\lambda^l_{t+1}}{\lambda^l_t}\left[r^k_{t+1} + \frac{\lambda_{ct+1}}{\lambda^l_{t+1}}(1-\delta^{kc})\right] \tag{3.13}$$

$$\frac{\lambda_{ht}}{\lambda^l_t} = \beta^l \mathbb{E}_t \frac{\lambda^l_{t+1}}{\lambda^l_t}\left[r^h_{t+1} + \frac{\lambda_{ht+1}}{\lambda^l_{t+1}}(1-\delta^{kh})\right] \tag{3.14}$$

$$1 = \frac{\lambda_{ct}}{\lambda^l_t}\left[1 - \Phi^c\left(\frac{j^l_{ct}}{j^l_{ct-1}}\right)\left(\frac{j^l_{ct}}{j^l_{ct-1}} - 1\right) - \frac{\Phi^c}{2}\left(\frac{j^l_{ct}}{j^l_{ct-1}} - 1\right)^2\right] + \\ + \beta^l \Phi^c \mathbb{E}_t \frac{\lambda_{ct+1}}{\lambda^l_{t+1}} \frac{\lambda^l_{t+1}}{\lambda^l_t}\left[\left(\frac{j^l_{ct+1}}{j^l_{ct}} - 1\right)\left(\frac{j^l_{ct+1}}{j^l_{ct}}\right)^2\right] \tag{3.15}$$

$$1 = \frac{\lambda_{ht}}{\lambda_t^l}\left[1 - \Phi^h\left(\frac{j_{ht}^l}{j_{ht-1}^l}\right)\left(\frac{j_{ht}^l}{j_{ht-1}^l} - 1\right) - \frac{\Phi^h}{2}\left(\frac{j_{ht}^l}{j_{ht-1}^l} - 1\right)^2\right] + \\ + \beta^l \Phi^h \mathbb{E}_t \frac{\lambda_{ht+1}}{\lambda_{t+1}^l}\frac{\lambda_{t+1}^l}{\lambda_t^l}\left[\left(\frac{j_{ht+1}^l}{j_{ht}^l} - 1\right)\left(\frac{j_{ht+1}^l}{j_{ht}^l}\right)^2\right] \tag{3.16}$$

$$u_{ht}^l = \left(\frac{\tau_{ht}\varphi_{1h}}{\theta_{1h}\theta_{2h}(h_t^l)^{\varphi_{2h}}}\right)^{\frac{1}{\theta_{2h}-1}} \tag{3.17}$$

Equation (3.9) represents the intertemporal condition for housing demand, requiring the lender to balance the marginal utility of current consumption against the marginal benefit of housing, offset by the marginal costs resulting from carbon taxation. These costs arise as long as the government imposes a carbon price on housing emissions, negatively affecting housing demand. From the equation, it is clear that the marginal cost of an additional unit of housing consists of two environmental components: the costs associated with emissions $\lambda_t^l \tau_{ht}(1 - u_{ht}^l)\varphi_{1h}(1 - \varphi_{2h})(h_t^l)^{-\varphi_{2h}}$, which depend on the carbon tax on residential emissions, and the costs associated with abatement in housing emissions $\lambda_t^l \theta_{1h}(u_{ht}^l)^{\theta_{2h}}$. The marginal benefit of housing, typical in models à la Iacoviello, stems from both the direct utility gain from an additional unit of housing and the expected utility from the possibility of increasing future consumption through the resale value of real estate holdings.

Equations (3.10) and (3.11) represent the labor supply of patient workers to the manufacturing and construction sectors, respectively. Equation (3.17) delineates the optimal abatement path and is influenced by the residential carbon tax and the housing stock. The optimal household abatement equates the cost savings from reduced emissions (left-hand side) to its marginal cost (right-hand side).

Impatient households

Impatient households are characterized by having a relatively high discount rate, so the inverse of the discount factor $\beta^b < \beta^l$. Similarly to their patient counterparts, impatient household maximizes the following utility function,

$$\max_{c_t^b, h_t^b, n_{ct}^b, n_{ht}^b, b_t^b, u_{ht}^b} \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^b)^i \left(\ln c_{t+i}^b + \gamma_h \ln h_{t+i}^b - \frac{(n_{t+i}^b)^{1+\eta}}{1+\eta}\right) \tag{3.18}$$

where c_t^b is borrowers' real consumption, h_t^b denotes borrower's holding of housing units, and n_t^b is a composite of labor supply to the consumption goods sector n_{ct}^b and to the housing sector n_{ht}^b represented by the following CES function,

$$n_t^b = [\theta^{\frac{1}{\varepsilon_n}}(n_{ct}^b)^{\frac{1+\varepsilon_n}{\varepsilon_n}} + (1-\theta)^{\frac{1}{\varepsilon_n}}(n_{ht}^b)^{\frac{1+\varepsilon_n}{\varepsilon_n}}]^{\frac{\varepsilon_n}{1+\varepsilon_n}} \quad (3.19)$$

Impatient households become borrowers, so in addition to the budget constraint (3.20), they face a financial constraint on the maximum amount of credit they can obtain. In particular, expression (3.21) limits the amount of borrowing, b_t^b, to a fraction κ of the expected resale value of housing held by household.

$$c_t^b + q_t(h_t^b - (1-\delta^h)h_{t-1}^b) - b_t^b + \tau_{ht}e_{ht}^b + z_{ht}^b = w_{ct}n_{ct}^b + w_{ht}n_{ht}^b - \frac{r_{t-1}}{\pi_t}b_{t-1}^b - tr_t^b \quad (3.20)$$

$$b_t^b \leq \kappa \mathbb{E}_t \frac{q_{t+1}\pi_{t+1}h_t^b}{r_t} \quad (3.21)$$

Similar to lenders, homes owned by borrowers emit carbon emissions e_{ht}^b through the combustion of fossil fuels used in cooking and heating and cooling systems. These emissions depends on total housing services h_t^b and are subject to the same price τ_{ht}, making them costly. To mitigate the expenses incurred by residential carbon pricing policy, impatient households can opt for abate emissions, u_{ht}^b. The investment in emissions reduction incurs a cost z_{ht}^b, which is contingent upon their dwelling stock. The emission and abatement cost functions for borrowers are as follows,

$$e_{ht}^b = (1-u_{ht}^b)\varphi_{1h}(h_t^b)^{1-\varphi_{2h}} \quad (3.22)$$

$$z_{ht}^b = \theta_{1h}(u_{ht}^b)^{\theta_{2h}}h_t^b \quad (3.23)$$

The first order conditions derived from maximizing the utility function provide the equations governing consumption, housing investment, labor supply, borrowing, and emissions abatement for impatient households,

$$\lambda_t^b = \frac{1}{c_t^b} \quad (3.24)$$

$$\lambda_t^b q_t = \frac{\gamma_h}{h_t^b} - \lambda_t^b(\tau_{ht}(1-u_{ht}^b)\varphi_{1h}(1-\varphi_{2h})(h_t^b)^{-\varphi_{2h}} + \theta_{1h}(u_{ht}^b)^{\theta_{2h}} + \\ + \beta^b\mathbb{E}_t(\lambda_{t+1}^b q_{t+1}(1-\delta^h) + \kappa\mu_t^b q_{t+1}\pi_{t+1}) \quad (3.25)$$

$$\frac{w_{ct}}{c_t^b} = (n_t^b)^\eta\left(\theta\frac{n_{ct}^b}{n_t^b}\right)^{\frac{1}{\varepsilon_n}} \quad (3.26)$$

$$\frac{w_{ht}}{c_t^b} = (n_t^b)^\eta\left((1-\theta)\frac{n_{ht}^b}{n_t^b}\right)^{\frac{1}{\varepsilon_n}} \quad (3.27)$$

$$\lambda_t^b = \beta^b \mathbb{E}_t \lambda_{t+1}^b \frac{r_t}{\pi_{t+1}} + \mu_t^b r_t \tag{3.28}$$

$$u_{ht}^b = \left(\frac{\tau_{ht}\varphi_{1h}}{\theta_{1h}\theta_{2h}(h_t^b)^{\varphi_{2h}}} \right)^{\frac{1}{\theta_{2h}-1}} \tag{3.29}$$

Equation 3.25 is the inter-temporal condition for borrowers' housing demand, indicating that, similar to lenders, housing demand is negatively affected by emissions and abatement costs when environmental policies come into play. Housing demand for borrowers differs from that of lenders in the term $\beta^b \mathbb{E}_t \kappa \mu_t^b q_{t+1} \pi_{t+1}$, which represents the marginal utility of relaxing the borrowing constraint. This can be influenced by environmental policies through various means, such as the impact on the housing price q_{t+1} or the tightness of the credit constraint μ_t^b, which introduces a differential in the consumption and housing demand dynamics between impatient and patient households.

3.2.2 Aggregation

Since our economy is composed of two representative households, the value in per capita terms of the aggregate variables may be affected by the weight of each type of household, τ_b. Aggregate per capita values are calculated as follows.

Debt issued by borrowers is fully purchased by patient households

$$\tau^b b_t^b = -(1-\tau^b) b_t^l \tag{3.30}$$

Private physical capital inn both sectors involve exclusively patient households

$$k_{ct} = (1-\tau^b) k_{ct}^l \tag{3.31}$$

$$k_{ht} = (1-\tau^b) k_{ht}^l \tag{3.32}$$

Net per capita investment is defined for each sector as

$$j_{ct} = (1-\tau^b) j_{ct}^l \tag{3.33}$$

$$j_{ht} = (1-\tau^b) j_{ht}^l \tag{3.34}$$

Aggregate per capita consumption and housing stock depends on the borrowers and lenders mix

$$c_t = \tau^b c_t^b + (1-\tau^b) c_t^l \tag{3.35}$$

$$h_t = \tau^b h_t^b + (1-\tau^b) h_t^l \tag{3.36}$$

Likewise, aggregate per capita residential emissions and total households abatement cost are defined as,

$$e_{ht} = \tau^b e^b_{ht} + (1-\tau^b)e^l_{ht} \tag{3.37}$$

$$z_{ht} = \tau^b z^b_{ht} + (1-\tau^b)z^l_{ht} \tag{3.38}$$

Total lump-sum transfers paid by households becomes,

$$tr_t = \tau^b tr^b_t + (1-\tau^b)tr^l_t \tag{3.39}$$

We also consider different measures of working hours. Aggregate hours per worker in the consumption goods sector

$$n_{ct} = \tau^b n^b_{ct} + (1-\tau^b)n^l_{ct} \tag{3.40}$$

Aggregate hours per worker in the housing sector

$$n_{ht} = \tau^b n^b_{ht} + (1-\tau^b)n^l_{ht} \tag{3.41}$$

Aggregate hours per worker in the economy

$$n_t = \tau^b(n^b_{ct} + n^b_{ht}) + (1-\tau^b)(n^l_{ct} + n^l_{ht}) \tag{3.42}$$

3.2.3 Producers of consumer goods

In our model, we distinguish between two levels of production. At the top level, there is a competitive bundler that aggregates the output of intermediate-good firms y_{jt}, priced at P_{jt}, into a single composite product y_t. Although we use the term 'consumer goods' for simplicity, this composite product is sold to households for both consumption and investment purposes, as well as to the government.

Aggregation Technology We assume a constant returns to scale technology à la Dixit & Stiglitz (1977) for the aggregation of the output, described by the following expressions for the aggregate production, the aggregate production price index (PPI), and the total demand for each variety:

$$y_t = \left(\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.43}$$

$$P_t = \left(\int_0^1 P_{jt}^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \tag{3.44}$$

$$y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} y_t \tag{3.45}$$

Production at the Firm Level At the bottom level, a continuum of firms indexed by $j \in [0, 1]$ produces a differentiated good y_{jt} in a monopolistically competitive market. Each firm employs a uniform production technology, specifically a Cobb-Douglas production function, where physical capital k_{jt-1} and labor n_{jct} are the inputs:

$$y_{jt} = (1 - D(x_t))a_t^c k_{jt-1}^{\alpha_c} n_{jct}^{1-\alpha_c} \tag{3.46}$$

where a_t^c represents a technological factor common across all firms. Following Heutel (2012), we introduce a negative externality where the stock of pollutants x_t adversely affects productivity via the damage function:

$$D(x_t) = d_0 + d_1 x_t + d_2 x_t^2 \tag{3.47}$$

Emissions and Abatement Following Annicchiarico & Di Dio (2015), firms emit greenhouse gases (GHGs) as a byproduct of their production processes. The volume of emissions depends on the production level and the firm's abatement efforts. Although emissions can be reduced, the associated cost is a function of the firm's abatement effort relative to its output. The emissions and abatement cost functions for firm j at time t are modeled as follows:

$$e_{jct} = (1 - u_{jct})\varphi_{1c}(y_{jt})^{1-\varphi_{2c}} \tag{3.48}$$

$$z_{jct} = \theta_{1c}(u_{jct})^{\theta_{2c}} y_{jt} \tag{3.49}$$

Similarly to the case of housing services, in these equations, the parameter φ_{1c} denotes the carbon intensity of production, quantifying emissions per unit of output. The exponent $1 - \varphi_{2c}$ reflects the elasticity of emissions with respect to changes in output, indicating how emissions increase with a rise in production. For the abatement costs, θ_{1c} represents the baseline cost of reducing emissions, while θ_{2c} captures the degree of cost increase as the effort in abatement intensifies. These parameters are uniformly applied across all firms, consistent with the model's assumption of homogeneous abatement technologies within the production sector.

Firm's Optimization Problem

Intermediate-good firms, operating under monopolistic competition, adjust their pricing in response to the demand dynamics dictated by the final goods firm. Following Rotemberg (1982a), firms incur a nominal adjustment cost AC_{jt} when they deviate from the target inflation rate $\bar{\pi}$. This adjustment cost is influenced by the nominal production in the sector, represented by $P_t y_t$ (with P_t serving as the *numeraire*),

$$AC_{jt} = \frac{\Phi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi}\right)^2 P_t y_t$$

Firms aim to set optimal prices at period t that maximize their discounted expected profits, taking into account both the demand for their goods (3.45) and their technological capabilities (3.46). The optimization problem in real terms is formulated as:

$$\max_{P_{jt}, n_{jct}, k_{jct-1}, u_{jct}} \mathbb{E}_t \sum_{t=0}^{\infty} (\beta^l)^t \frac{\lambda_t^l}{\lambda_0} \left(\frac{P_{jt}}{P_t} y_{jt} - w_{ct+i} n_{jct+i} - r_{t+i}^k k_{jct-1+i} - \right.$$
$$\left. \tau_{ct}(1 - u_{jct})\varphi_{1c}(y_{jt})^{1-\varphi_{2c}} - \theta_{1c}(u_{jct})^{\theta_{2c}} y_{jt} - \frac{\Phi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi}\right)^2 y_t \right)$$

The resulting first-order conditions dictate the optimal choices for capital, labor, abatement efforts, and pricing for each firm. In a symmetric equilibrium, where firms choose identical prices, inputs, and outputs, the conditions simplify as follows:

$$w_{ct} = mc_t(1 - \alpha_c)\frac{y_t}{n_{ct}} \tag{3.50}$$

$$r_{kt} = mc_t \alpha_c \frac{y_t}{k_{ct-1}} \tag{3.51}$$

$$u_{ct} = \left(\frac{\tau_{ct}\varphi_{1c}}{\theta_{1c}\theta_{2c}(y_t)^{\varphi_{2c}}}\right)^{\frac{1}{\theta_{2c}-1}} \tag{3.52}$$

$$\pi_t(\pi_t - \bar{\pi}) = \beta^l \frac{\lambda_{t+1}^l}{\lambda_t^l} \pi_{t+1}(\pi_{t+1} - \bar{\pi}) \frac{y_{t+1}}{y_t} +$$
$$+ \frac{\varepsilon}{\Phi_p}\left[\frac{1-\varepsilon}{\varepsilon} + (1 - \varphi_{2c})\tau_{ct}\varphi_{1c}(1 - u_{ct})y_t^{-\varphi_{2c}} + \theta_{1c}(u_{ct})^{\theta_{2c}} + mc_t\right] \tag{3.53}$$

The variable mc_t denotes the Lagrange multiplier related to the production constraint common to all firms, and it reflects the incremental production costs associated with a unit increase in production (marginal cost).

Given a carbon tax τ_{ct} on emissions from consumer goods production, equation (3.52) equates the value of the marginal product of abatement, $\tau_{ct}\varphi_{1c}(y_t)^{1-\varphi_{2c}}$, to its marginal cost, $\theta_{1c}\theta_{2c}(u_{ct})^{\theta_{2c}-1}y_t$. In the absence of τ_{ct}, firms have no incentive to abate emissions, and $u_{ct} = 0$. This equation implies that the abatement effort is uniform across all firms; hence, emissions and abatement costs are also uniform and can be aggregated as follows:

$$e_{ct} = (1 - u_{ct})\varphi_{1c}(y_t)^{1-\varphi_{2c}} \tag{3.54}$$

$$z_{ct} = \theta_{1c}(u_{ct})^{\theta_{2c}} y_t \tag{3.55}$$

With all firms using identical technology, aggregate production can be expressed as:

$$y_t = (1 - D(x_t))a_t^c (k_{ct-1})^{\alpha_c} (n_{ct})^{1-\alpha_c} \tag{3.56}$$

Equation (3.53) represents the nonlinear New Phillips Curve, which links inflation to both current and future real marginal costs, with $\frac{\varepsilon}{\varepsilon-1}$ denoting the desired markup in equilibrium. In our environmental model, the marginal costs of producing an additional unit of output include mc_t, as well as costs related to pollution. In the absence of a carbon tax, the New Phillips Curve simplifies to the standard formulation.

Real profits for intermediate firms in a symmetric equilibrium are obtained using Equations (3.50) and (3.51), expressed as follows:

$$(1-\tau^b)d_t = y_t \left[1 - mc_t - \tau_{ct}(1 - u_{ct})\varphi_{1c}(y_t)^{-\varphi_{2c}} - \theta_{1c}(u_{ct})^{\theta_{2c}} - \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2\right] \tag{3.57}$$

3.2.4 Housing construction firms

There are many small firms owned by patient households that build identical houses in a perfectly competitive environment. The price of housing, q_t, is perfectly flexible, and firms cannot influence it (see Monacelli et al. (2006) and Iacoviello & Neri (2010)). Construction firms produce houses using labor and capital according to a Cobb-Douglas function,

$$IH_t = (1 - D(x_t))a_t^h (k_{ht-1})^{\alpha_h} (n_{ht})^{1-\alpha_h} \tag{3.58}$$

where a_t^h is the steady-state technology factor common to all firms, and $D(x_t)$ is the damage function that positively depends on the emissions stock, as defined in (3.47). The economic objective of the representative firm is to maximize profits subject to the production function

$$\max_{n_{ht}, k_{ht-1}} \sum_{i=0}^{\infty} \left(q_{t+i} IH_{t+i} - w_{ht+i} n_{ht+i} - r_{t+i}^h k_{ht-1+i}\right)$$

which yields the labor and capital demand in the construction sector,

$$w_{ht} = q_t(1 - \alpha_h)\frac{IH_t}{n_{ht}} \tag{3.59}$$

$$r_{ht} = q_t\alpha_h\frac{IH_t}{k_{ht-1}} \tag{3.60}$$

Considering a constant depreciation rate δ^h for the stock of houses, the total supply of houses h_t evolves according to,

$$h_t = IH_t + (1 - \delta^h)h_{t-1} \tag{3.61}$$

3.2.5 Monetary and carbon policies

The European authority sets carbon pricing for both the residential and production sectors, denoted as τ_{ht} and τ_{ct} respectively, in line with EU carbon emissions reduction targets. It collects carbon revenues and decides whether to allocate them to lump-sum tax reductions. We assume that the government maintains a constant and exogenous per capita level of consumption $\bar{g}$. In the absence of any environmental policy, public consumption is entirely financed through lump-sum taxes tr_t, which are initially distributed equally on a per capita basis between lenders and borrowers. When carbon taxes are introduced, government consumption are financed by lump-sum taxes and carbon revenues.

$$\bar{g} = tr_t + \tau_{ht}e_{ht} + \tau_{ct}e_{ct} \tag{3.62}$$

Monetary policy is managed by the European Central Bank following a Taylor's interest rate rule that respond to deviations of euro-zone inflation π_t from its long-run target ($\bar{\pi} = 1$). The rule takes the form:

$$r_t = \bar{r}^{1-\rho}(r_{t-1})^{\rho}(\pi_t)^{\psi(1-\rho)} \tag{3.63}$$

where $\bar{r}$ is the steady state level of the gross interest rate, ρ is a parameter that controls the persistence of the interest rate and ψ represents the weight given by the ECB to inflation targeting.

3.2.6 Pollution Dynamics

The economic losses resulting from climate change are incorporated through the damage function (3.47), which reduces productivity in both sectors and thus dampens total production. The damage function captures the relationship between the level of carbon in the atmosphere and economic damages, measured as a percentage of final-good output. In addition, to include the reverse effects, i.e., the impact of economic activity on emissions and carbon accumulation in the atmosphere, we adopt a simplification based on models such as those of Heutel (2012) and Annicchiarico & Di Dio (2015), which assume that the concentration of carbon in the atmosphere x_t follows the following process:

$$x_t = (1-\delta^x)x_{t-1} + e_{ct} + e_{ht} + e^{row} \tag{3.64}$$

where, in each period, emissions from production of consumer goods, e_{ct}, and from combustion of fossil fuels in houses, e_{ht}, feed the stock of carbon. Additionally, we assume that emissions from the rest of the world, e^{row}, also impact productivity in the European region and remain constant. The parameter δ^x measures the fraction of carbon that naturally decays in each time period.

3.2.7 Total resource constraint and GDP

The aggregation of household budget constraints leads to

$$c_t + tr_t + q_t(h_t - (1-\delta^h)h_{t-1}) + j_{ct} + j_{ht} + \tau_{ht}e_{ht} + \tau^b z^b_{ht} + \\ (1-\tau^b)z^l_{ht} = w_{ct}n_{ct} + w_{ht}n_{ht} + (1-\tau^b)d_t + r_{kt}k_{ct-1} + r_{ht}k_{ht-1}$$

Using the equality between production and income in both sectors[4] and considering the fiscal budget condition (3.62) while defining total aggregate production ($\tilde{y}^A_t$) as the sum of consumer goods production and housing production, $y_t + q_t IH_t$, the resource constraint of the economy can be expressed as:

$$\tilde{y}^A_t = c_t + \bar{g} + q_t(h_t - (1-\delta^h)h_{t-1}) + j_{ct} + j_{ht} + z_{ct} + z_{ht} + \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2 y_t \tag{3.65}$$

GDP can be expressed as:

$$GDP_t = c_t + \bar{g} + q_t(h_t - (1-\delta^h)h_{t-1}) + j_{ct} + j_{ht} \tag{3.66}$$

These equalities highlight the inherent limitations within the economy, indicating that all available resources are allocated to production, which in turn is used for consumption, investment, housing production, and to cover the costs associated with emissions abatement and price adjustments. Therefore, our model explores how environmental policies, particularly those aimed

[4]The equality between production and income in consumption goods sector is given by $y_t = w_{ct}n_{ct} + (1-\tau^b)d_t + r_{kt}k_{ct-1} + \tau_{ct}e_{ct} + z_{ct} + \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2 y_t$ and in house construction sector by $q_t IH_t = w_{ht}n_{ht} + r_{ht}k_{ht-1}$

at reducing carbon emissions, affect resource allocation across the economy. Specifically, the resources allocated to mitigate the environmental impacts of carbon emissions are represented by abatement costs in both production (z_{ct}) and housing (z_{ht}). As the government implements carbon pricing policies to curb emissions, a significant portion of the economy's resources is redirected towards investments in emissions reduction. This shift within the resource constraints leads to a decrease in both investment and consumption and, hence, on GDP.

3.3 Calibration

We calibrate the model to replicate certain macroeconomic and environmental ratios of the EU-27 economy for 2019, the year preceding the COVID-19 crisis. While most parameters utilized in the calibration are drawn from previous literature or derived from the model's steady-state equations, we uniquely estimate the elasticity of residential emissions to housing stock using data specific to the EU-27. Our model incorporates the emission functions proposed by Heutel (2012) and Annicchiarico & Di Dio (2015). However, what distinguishes our work in the literature on E-DGE models is our consideration of how residential emissions are contingent upon the housing stock owned by households. Estimating this parameter specifically for the EU-27 ensures a more accurate representation of the dynamics of residential emissions and their relationship with housing characteristics. Below, we provide detailed information on the strategy followed to calibrate the parameters in the model.

3.3.1 Estimating the elasticity of residential emissions

To set the elasticity parameter φ_{1h} in the emission function, we study how carbon dioxide emissions from dwellings vary with the housing stock. Specifically, we estimate the following regression model:

$$\log(e_{hit}) = \beta_0 + \beta_1 \log(h_{it}) + \gamma_i + \gamma_t + u_{it} \tag{3.67}$$

where γ_i and γ_t represent country and time fixed effects, respectively, and β_1 is our parameter of interest.

We use annual country-level data on residential emissions and house floor area for the 27 countries of the European Union plus the United Kingdom over the period 1990–2017. More particularly, our dependent variable is direct emissions produced by the combustion of fossil fuels in homes, principally by heating and cooling systems. Emissions data are taken from national residential emissions reported to the UNFCCC and the EU Greenhouse Gas

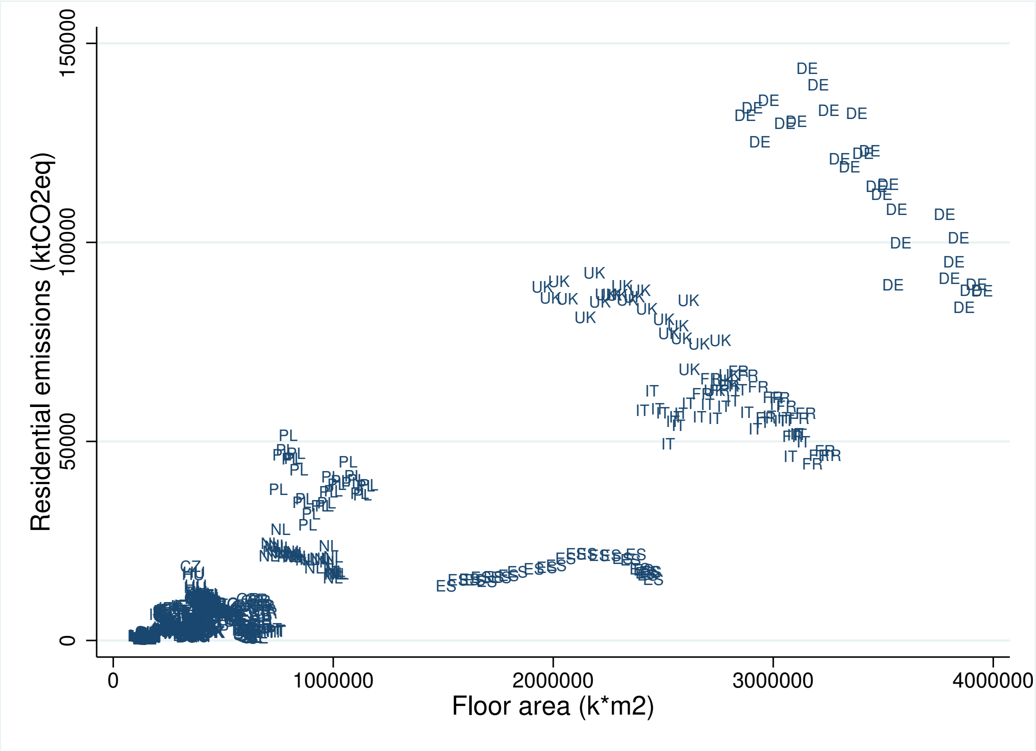


Figure 3.1: Annual emissions over the period 1990:2017 and total floor area for housing

Monitoring Mechanism (link), and residential floor area by country is sourced from the EEA (link) and the EU Building Stock Observatory (link).

Figure 3.1 presents the relationship between annual CO_2 emissions from dwellings and total residential floor area across all countries during the sample period. It can be observed that, in the pooled scenario, residential emissions increase with floor area over the period considered. However, it is evident from the figure that this relationship varies significantly by country.

Table 3.1 displays the estimations. The Newey-West estimator addresses potential autocorrelation and heteroskedasticity in the error terms. We present the estimation without fixed effects, followed by models with country and time fixed effects, respectively. The no-fixed-effects estimation produces a coefficient substantially larger than those obtained with fixed effects, highlighting the influence of cross-country differences in residential floor area on carbon emissions. In column (2), after introducing country fixed effects, the coefficient becomes negative, indicating that once we control for time-invariant, country-specific characteristics, emissions per unit of floor area have decreased in Europe over the past 30 years. When time fixed effects are

Table 3.1: Estimation of emission elasticity with respect to floor area

	(1) $log(e_h)$	(2) $log(e_{hi})$	(3) $log(e_{hit})$
	1.0656***	-0.4296***	0.4688**
	(0.03826)	(0.12833)	(0.19248)
Standard errors	Newey-West	Newey-West	Newey-West
Observations	655	655	655
Country fixed effects	No	Yes	Yes
Year fixed effects	No	No	Yes

Standard errors in parenthesis. $* \; p < 0.1$; $** \; p < 0.05$; $*** \; p < 0.01$.

added to control for common trends affecting carbon emissions over time, the coefficient remains significant, suggesting that emissions increase by 0.4688% for every 1% increase in residential floor area[5]. Overall, our findings emphasize the importance of technological developments that reduce emissions per unit of dwelling floor area (column 2). Including time effects partially controls for technological progress, reversing the sign of the elasticity. However, given the importance of this parameter, we will include it in our sensitivity analysis in the simulations results later.

In a study by Cong et al. (2015), the authors find that, depending on the region, the elasticity of carbon emissions from buildings per square meter in China varies from 0.3382 to 0.6175, an interval that includes the value obtained in our preferred estimation. When the elasticity is measured in emissions per square meter on a per capita basis, it varies from 0.418 to 0.8098. Brounen et al. (2012) find that the elasticity of residential gas consumption per square meter in the Netherlands is 0.295, accounting for housing construction period and thermal characteristics. This value is slightly lower than ours and falls outside the range reported by Cong et al. (2015), but it exclusively accounts for residential gas emissions[6]. Also focusing exclusively on gas consumption, Alberini et al. (2011) report that the elasticity of residential heating gas consumption per square foot in the US is 0.22. In view of the previous results

[5]An alternative specification with robust standard errors yields similar results

[6]As indicated in the Guidelines for National Greenhouse Gas Inventories Intergovernmental Panel on Climate Change (IPCC) (2006), emissions of each greenhouse gas from stationary sources are calculated by multiplying fuel consumption by the corresponding emission factor. Therefore, the emissions of a fossil fuel are directly proportional to its consumption, making the elasticity of emissions per area straightforwardly comparable to the elasticity of consumption per area.

in the literature, we set the value of the elasticity φ_{2h} to 0.4688, according to our estimation in column (3).

3.3.2 Parameters and targeted ratios

This section describes the values of the parameters used to obtain the numerical solution. The model is calibrated to the EU-27 setting by using parameters from previous studies and by fixing target values for the steady-state values of selected model variables to determine the remaining parameters.

General Parameters and Target Steady-State Values. We take some of the parameters from the European Central Bank's New Area-Wide Model II Coenen et al. (2018) used for policy analysis. Specifically, we set the inverse elasticity of labor to wages to $\eta = 2$, the elasticity of substitution among final goods to $\varepsilon = 3.85$, price adjustment costs in consumption goods to $\Phi^p = 71.56$, capital adjustment costs in manufacturing and construction to $\Phi^c = 10.78$ and $\Phi^h = 10.78$, the capital depreciation rate in consumer goods production to $\delta^{kc} = 0.025$, and the parameters of the Taylor rule ρ and ψ to 0.93 and 2.74, respectively. Following Iacoviello & Neri (2010), we set the housing stock depreciation rate to $\delta^h = 0.04$ per year.

The steady-state rate of time preference for patient households is set at 1% per quarter, corresponding to an annual interest rate of 4%. For impatient households, the discount rate $\frac{1}{\beta^b}$ is set roughly at 3 percentage points per quarter. Following Bielecki & Stähler (2022), we set the loan-to-value ratio to the standard value $\kappa = 0.75$ and the share of constrained households to $\tau^b = 0.40$. For the elasticity of substitution between labor types ε_n and the elasticity of housing production to capital α^h, we use the estimated values for the EU-27 from Hinterlang et al. (2022), fixing them at 1 and 0.255, respectively.

To calibrate the model, we match the steady-state ratios of economic aggregates to GDP with the corresponding values for the EU-27 in the year 2019. The ratios are obtained from Eurostat. The steady-state ratios of residential investment and nonresidential investment to GDP are 5.2% and 18.9%, respectively. In the residential sector, corporate gross fixed capital formation is equivalent to 0.604% of GDP. Government spending (which is determined exogenously in our model) represents 21.4% of GDP in steady state. Additionally, the weight of the labor force working in the residential sector is targeted at 7.2%. For simplicity, we assume that lump-sum taxes are initially distributed equally on a per capita basis between lenders and borrowers, such that $tr_t^l = tr_t^b = tr_t$. Table 3.2 shows the general parameters

Table 3.2: Macroeconomic parameters

	Description	Value	
β^l	Lenders' discount rate	0.99009	Annual interest 4%
β^b	Borrowers' discount rate	0.97009	8pp less anually
γ_h	Preference for housing	0.1984	Calibration
θ	Weight parameter in CES	0.0648	Calibration
η	Inverse elasticity of labor supply	2	Coenen et al. (2018)
ε_n	Elasticity of labor between sectors	1	Hinterlang et al. (2022)
κ	Loan-to-value	0.75	Bielecki & Stähler (2022)
τ^b	Fraction of borrowers	0.4	Bielecki & Stähler (2022)
ε	Elasticity of final goods	3.8571	Coenen et al. (2018)
Φ^c	Manufacturing capital adjustment cost	10.78	Coenen et al. (2018)
Φ^h	Housing capital adjustment cost	10.78	Coenen et al. (2018)
δ^{kc}	Manufacturing capital depreciation rate	0.025	Coenen et al. (2018)
δ^{kh}	Construction capital depreciation rate	0.0084	Calibration
Φ^p	Pricing adjustment cost	71.56	Coenen et al. (2018)
α^c	Elasticity of output to capital	0.3648	Calibration
α^h	Elasticity of housing to capital	0.255	Hinterlang et al. (2022)
δ^h	Housing depreciation rate	0.01	Iacoviello & Neri (2010)
ρ	Interest rate smoothing	0.93	Coenen et al. (2018)
ψ	Interest rate reaction to inflation	2.74	Coenen et al. (2018)
a_t^c	TFP consumption goods sector	0.2001	Calibration
a_t^h	TFP housing sector	0.1433	Calibration

and exogenous variables that are set and calibrated.

Environmental Parameters. Most of the environmental parameters are taken from Gibson & Heutel (2023) and adjusted to the units of the model, as total GDP is normalized to 1 (in millions of euros), and therefore, most of the variables are interpreted in terms of million euros of total production. Moreover, we impose values for emissions in both sectors e_c, e_h, and the atmospheric carbon dioxide stock x. According to data from the Global Monitoring Laboratory, the carbon stock in the atmosphere in 2019 is set at $x_{2019} = 876.84$ GtC. The steady-state ratios of the EU-27 annual emissions in 2019 for consumption goods firms and the residential sector are obtained from the UNFCCC and the EU Greenhouse Gas Monitoring Mechanism. The steady-state values of annual emissions in the manufacturing and residential sectors are 3284.548 and 317.035 MtCO_2 equivalent, respectively.

From Gibson & Heutel (2023), we take the elasticity of emissions to output in manufacturing, $1-\varphi_{2c} = 0.6$. We also use their value for the pollution stock decay rate $1 - \delta^x$, which assumes a half-life of atmospheric carbon dioxide of 50 years, as well as their coefficients of the damage and abatement cost

Table 3.3: Environmental parameters

	Description	Value	
$1-\delta^x$	Pollution decay rate	0.9965	Gibson & Heutel (2023)
d_2	Damage quadratic coefficient	1.17E-07	Gibson & Heutel (2023)
d_1	Damage linear coefficient	2.70E-05	Gibson & Heutel (2023)
d_0	Damage constant	-0.0076	Gibson & Heutel (2023)
φ_{1h}	Emission intensity housing	0.0034	Calibration
$1-\varphi_{2h}$	Emissions elasticity housing	0.4689	Authors' calculations
φ_{1c}	Emission intensity in production	0.0692	Calibration
$1-\varphi_{2c}$	Emissions elasticity in production	0.6	Gibson & Heutel (2023)
θ_{1h}	Housing abatement cost coefficient	0.074	Gibson & Heutel (2023)
θ_{2h}	Housing abatement cost exponent	2.6	Nordhaus (2018)
θ_{1c}	Production abatement cost coefficient	0.074	Gibson & Heutel (2023)
θ_{2c}	Production abatement cost exponent	2.6	Nordhaus (2018)
e^{row}	rest-of-the-world emissions	0.85 ktC/mill€	Calibration

functions. From Equation (3.64) we obtain e^{row}. The atmospheric carbon stock is measured in gigatons (GtC) per million euros of GDP (in constant 2015 euros). After scaling the damage function coefficients with respect to the EU-27 GDP in 2019 to match the production losses reported by Gibson & Heutel (2023), we impose $d_2 = 1.17\times10^{-7}$, $d_1 = 2.70\times10^{-5}$, and $d_0 = -0.0076$. In the abatement cost function, we choose $\theta_{1c} = 0.074$, corresponding to a total share of abatement costs over production of 7.4% when the emission reduction share is 100%. Similarly, θ_{1h} is calibrated to ensure that the cost of abating 100% of residential emissions in the steady state is equivalent to 7.4% of housing production. Specifically, $z_h = 0.074 \times IH$, which in the steady state simplifies to $z_h = 0.074 \times h \times \delta^h$. According to equations (3.7), (3.23), and (3.38), under full abatement, $z_h = \theta_{1h} \times h$, meaning that $\theta_{1h} = \delta^h \times 0.074$ $= 0.0074$.

The cost elasticities to abatement, θ_{2c} and θ_{2h}, are set to 2.6 from Nordhaus (2018). Finally, the calibration of the elasticity of carbon emissions to the housing stock $1 - \varphi_{2h}$ is based on the estimation of the log of residential emissions on the log of dwelling floor area, as shown in Section 3.3.1.

Table 3.3 summarizes the environmental parameters recovered from previous studies and those that result from the calibration strategy.

3.4 Results

The adoption of the European Climate Act (European Commission, 2021) transformed the political commitment of the 2019 Green Deal (European Commission, 2019) into a legal obligation. EU countries are thus required to reduce greenhouse gas emissions by at least 55% below 1990 levels by 2030, with the goal of achieving climate neutrality by 2050. The building sector is a critical area for addressing climate and environmental challenges; therefore, the EU will extend the existing EU Emissions Trading Scheme to include emissions from the residential sector. The objective is to achieve a 43% reduction in emissions from buildings by 2030 compared to 2005 levels. While the introduction of a carbon price on residential emissions rightly encourages cost-effective emissions reductions and incentivizes behavioral change, it also imposes a burden on consumers.

In this section, we present simulation results on the macroeconomic effects of policies that require households to pay for their housing emissions while also pricing carbon emissions from the production of other consumption goods and services. After analyzing these macroeconomic impacts, we focus on the differential effects between household types to uncover the inequality effects on key components of household welfare: consumption, housing holdings, and leisure time. Furthermore, we explore government strategies to alleviate the effects of carbon pricing policies during the transition to climate neutrality. Specifically, we evaluate measures to lessen the burden on households by utilizing carbon revenues from environmental policies to address redistributive issues.

3.4.1 Carbon pricing

The transition to zero emissions is simulated by imposing a linear increase in the carbon price in both the residential and consumer goods production sectors. Starting from an initial steady state where no environmental policy is anticipated, an emissions pricing path is suddenly announced, and agents respond with perfect foresight until the new steady state is reached. We designate period 1 as 2019Q4, when the government introduces emission prices. The carbon price for residential emissions (τ_{ht}) and production emissions (τ_{ct}) increases linearly over 120 quarters, with a discontinuity before the halfway point, ensuring that emission reduction targets are met by 2030 and climate neutrality is achieved by 2050. Our intermediate targets include a 43% reduction in housing emissions by 2030 relative to 2005 levels and a 55% reduction in carbon emissions from the nonresidential sector by 2030 compared to 1990 levels. Considering that the absorption rate used in the calibration implies that terrestrial ecosystems are able to absorb 25% of global emissions,[7] achieving NZE by 2050 requires a 75% reduction in both residential and nonresidential emissions by 2050 relative to 2019 levels.

[7]This figure is consistent with observations over the past 50 years (see Brienen et al. (2020)

Macroeconomic impacts

Our simulations isolate the gross impacts of carbon pricing policies on greening the economy, without accounting for the effects of technological improvements on energy use or carbon emissions per unit of energy. In our model, agents adjust their allocation of resources for emission reduction in response to increases in carbon prices. Consequently, the carbon prices depicted in Figure 3.2 represent an upper bound of those required to meet emission reduction targets.

In the absence of technological change, the carbon price would need to increase by approximately €460 per ton of CO_2 in the nonresidential sector, and by €260 per ton of CO_2 in the residential sector. This increase would be accompanied by a more pronounced acceleration in the carbon price in the residential sector after 2030.

The environmental policy reduces atmospheric carbon stock by 1.36% by 2050. [8] However, the additional tax burden on households and firms during the transition period, leads to an approximate 7.45% decrease in aggregate GDP. This decline in GDP is accompanied by substantially lower levels of investment and consumption.

Firms in the consumer sector, faced with the additional costs of the carbon price, τ^c, reduce their demand for labor and capital, thereby decreasing production. Capital in the nonresidential sector declines by over 10%. Similarly, housing production is adversely affected, primarily due to the demand-side effects of carbon pricing. The carbon price targeting emissions from housing deters housing demand by increasing the costs associated with emissions pricing and abatement, leading to a 3% reduction in residential investment and a significant 8.7% decrease in housing prices. The adjustment in housing prices reflects not only weakened demand but also an oversupply of capital and labor resulting from reduced overall production. Idle factors of production experience a decrease in their prices, which firms pass on as lower product prices, particularly in the construction sector. Consequently, price behavior differs notably between the two production sectors. While inflation in consumer goods production remains relatively stable—since the cost factors pushing prices upward are offset by weaker demand—housing prices decline, primarily due to the negative demand impact on this sector.

Aggregate consumption is adversely affected by declining incomes, abatement costs incurred by households during the transition, and deteriorating credit conditions. In our baseline calibration, consumption decreases by 8% by 2050, with abatement costs representing 3.4% of GDP upon achieving NZE.

[8] This figure reflects only Europe achieving NZE, without contributions from other countries.

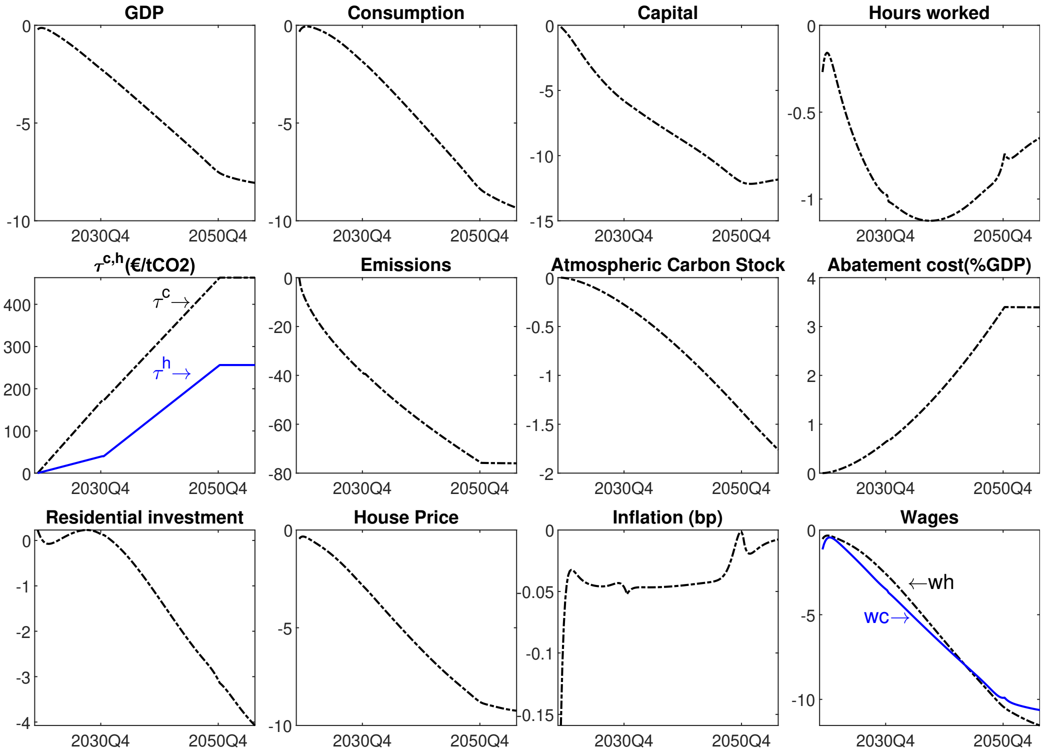
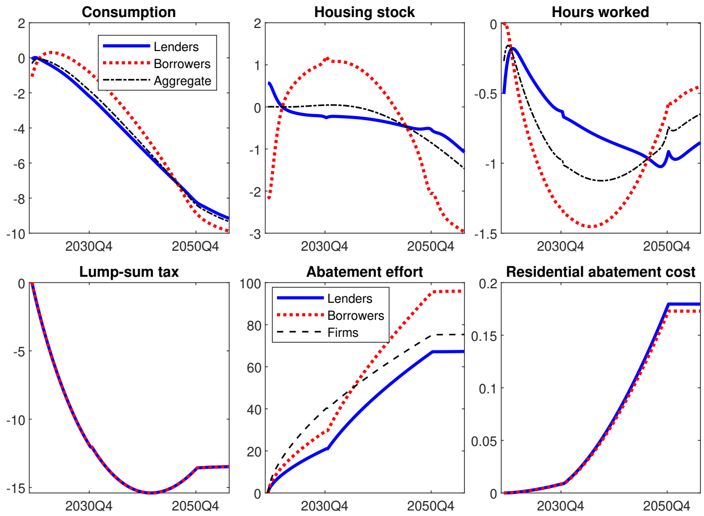


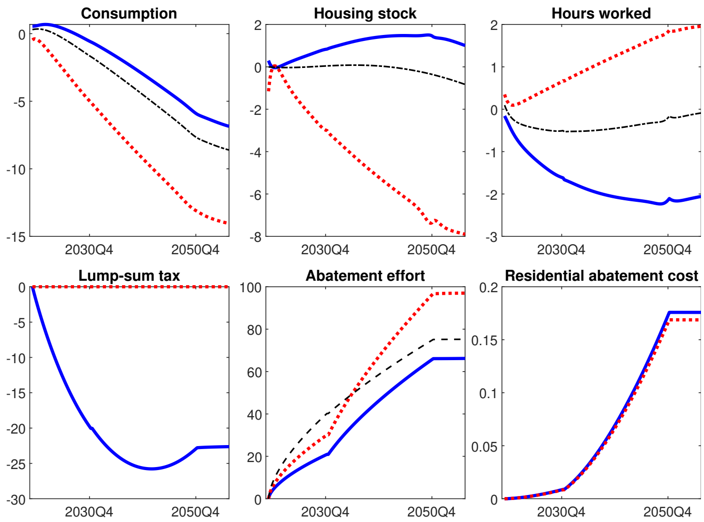
Figure 3.2: Macroeconomic Impacts of Carbon Pricing Policies. The results are presented as percentage deviations from their expected trajectories in the absence of environmental policies, except for emission prices, which are denominated in euros, inflation, measured in basis points, and abatement costs, expressed as percentage points of GDP.

Since changes in housing prices are not incorporated into the Central Bank's policy framework, monetary policy does not respond to declines in housing prices as it does to consumer goods inflation. Consequently, the monetary policy tool is less effective in mitigating the adverse aggregate effects on GDP resulting from reduced consumption by borrowers, which is further exacerbated by declining housing prices and the resulting deterioration in credit conditions.

To further investigate the impact of carbon pricing on household consumption and other economic decisions, the next subsection disaggregates the effects by household type, analyzing the responses of borrowers and lenders to the emissions reduction policy.



(a) Equitable distribution of carbon dividends across the population



(b) Allocation of all carbon dividend to patient households

Figure 3.3: Impact of carbon pricing policy by household type. Variables are presented as percentage deviations from their expected trajectories in the absence of environmental policy, except for abatement and abatement costs. Abatement is expressed as a percentage of emissions, and residential abatement costs are expressed as a percentage of GDP.

Impact by household type

Figure 3.3 displays the percentage deviations of household-related variables relative to the no-policy scenario, highlighting the differential impacts of the carbon pricing policy across household types. In our benchmark scenario, depicted in Figure 3.3a, government carbon revenues are distributed equally among household types as a carbon dividend that offsets their tax burden.

Credit-constrained households exhibit the highest abatement effort, measured as the percentage of residential emissions reduced each period. This behavior results from the optimal abatement path, as detailed in equations (3.17) and (3.29), which indicate that abatement effort is inversely related to housing stock. The intuition behind this outcome is that households invest in abatement to mitigate the negative impact of increased carbon prices on their housing stock, as reducing housing serves as an alternative to offsetting higher carbon costs. Since borrowers own fewer houses than lenders, reducing housing would result in a greater utility loss for borrowers than for lenders due to the decreasing marginal utility of housing. Consequently, borrowers find it optimal to exert greater abatement effort. In contrast, despite lower abatement efforts by lenders, they incur slightly higher abatement costs because they own more houses.

The first row of plots in Figure 3.3a illustrates the evolution of key variables influencing household welfare: consumption, housing property, and leisure (working hours). Significant disparities emerge in household responses concerning these variables. Borrowers, because they have a higher preference rate for the present against the future, are more responsive to government transfers (tax reductions) than lenders. As shown in the figure, lump-sum taxes decrease in a convex manner due to two opposing factors. First, the carbon tax increases over time, raising government revenues that, when rebated to households, result in lower taxes. Second, emissions decline over time, reducing the tax base. Consequently, the carbon dividend reaches an upper limit before the end of the transition period.

In the very first periods, borrowers' consumption falls compared to lenders', primarily due to the reduction in credit caused by the decline in the value of collateral housing through decreasing house prices (see Figure 3.4). Borrowers demand houses not only for the housing services they provide but also because they can be used as collateral to obtain credit. As a result, borrowers' housing demand decreases in the initial periods. Over time, the dynamic choices of both household types diverge significantly.

Patient households bear a greater tax burden from residential carbon taxes because they own more houses per capita. Additionally, they experience a reduction in profits from firm ownership as emissions prices increase, which adversely affects their consumption prospects. In contrast, impatient households face a relatively lower environmental tax burden and thus benefit relatively more from the carbon dividend, which is distributed evenly among households. After the initial negative impact of the policy announcement, this dividend offsets their residential emissions payments and increases their disposable income. Consequently, borrowers respond by expanding their housing stock, helping to sustain their consumption.

However, as emissions are progressively reduced, the diminishing carbon dividend leads to reduced government transfers, and the significant decline in house prices weakens borrowers' ability to use their properties as collateral, explaining the hump-shaped trajectory of houses owned by borrowers. This reduction in collateral limits both consumption and housing demand. Labor demand decreases with economic activity, driving down wages and hours. As consumption continues to deteriorate, borrowers are compelled to increase their labor supply over time in an effort to restore income (and consumption) stability. Eventually, the increase in labor supply compensates for the decline in labor demand, as reflected in the U-shape of hours worked.

Despite the differing transition dynamics, both borrowers and lenders conclude the transition period with substantial declines in consumption, decreasing by 9% and 8%, respectively. Housing demand decreases by 2% for borrowers compared to 0.5% for lenders, while working hours decline by 0.6% for borrowers and 0.9% for lenders.

Under an alternative distribution of carbon dividends, household responses exhibit significant changes. For illustrative purposes, Figure 3.3b demonstrates the impact of the carbon pricing policy on consumption, housing stock, and hours worked when credit-constrained households do not receive carbon revenues, thus maintaining their initial lump-sum taxes at a constant level. In this scenario, lenders fully receive the carbon dividend. They allocate a portion of their additional income to sustain consumption and private bonds during the initial periods. The rebound of debt in lenders' hands observed in Figure 3.4 helps support borrowers' consumption in the early stages of the transition to NZE. However, as the transition progresses, lenders benefit from an increasing share of their income from government transfers, while the income of impatient households deteriorates without the support of those transfers. Consequently, borrowers' consumption and credit demand shrink, and lenders take advantage of declining house prices to redirect their savings toward housing investments.

In general, this results in a markedly different trajectory for consumption, housing holdings, and working hours throughout the transition period for borrowers compared to the carbon dividend distribution of the benchmark illustrated in Figure 3.3a. Specifically, previously observed hump-shaped

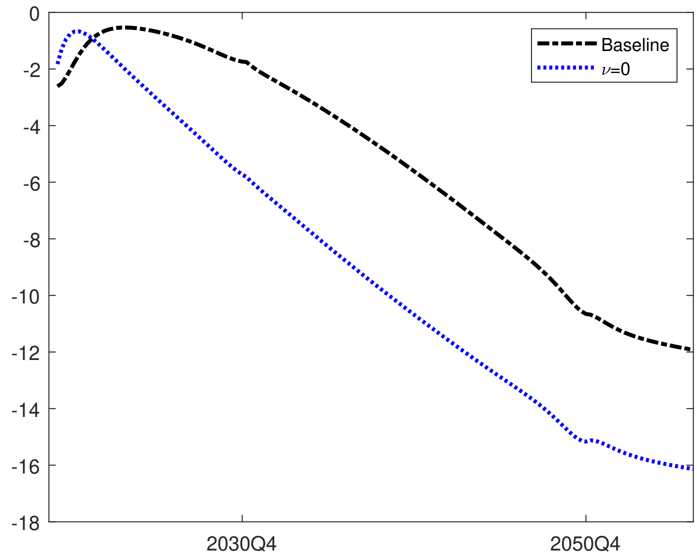


Figure 3.4: One-period debt held by borrowers. Two scenarios are displayed: (a) equitable distribution of carbon dividends, and (b) allocation of all carbon dividends to patient households ($\nu = 0$)

patterns in borrowers' consumption and housing stock, as well as the U-shaped pattern in working hours, disappear. Despite the significant distributional effects of allocating government revenues from carbon pricing, the overall consumption dynamics remains largely unaffected. Therefore, focusing solely on aggregate results can obscure substantial distributional impacts.

From the above discussion, we identify two key insights. First, the carbon dividend allocation method significantly affects household consumption, housing, and labor responses, with borrowers being more sensitive to the specific dividend distribution scheme. Second, despite owning fewer houses, borrowers undertake higher abatement efforts because reducing their housing stock would result in a greater utility loss compared to lenders.

3.4.2 Sector-specific and combined carbon pricing policies

So far, we have examined the effects of simultaneously increasing carbon prices in both the residential and nonresidential sectors. This section investigates the distinct economic, distributional, and welfare implications of implementing

Table 3.4: Impact of carbon pricing under three scenarios: combined, production-only, and residential-only policies

	No policy	$\tau_c + \tau_h$	Variation (%)	τ_c	Variation (%)	τ_h	Variation (%)
Macroeconomic							
GDP	1.00	0.918	(-8.23%)	0.919	(-8.05%)	0.998	(-0.18%)
c	0.545	0.490	(-10.00%)	0.491	(-9.88%)	0.544	(-0.13%)
x	262.60	246.62	(-6.09%)	248.02	(-5.55%)	261.21	(-0.53%)
z	0.00	3.39%	-	3.21%	-	0.18%	-
IH	0.0520	0.0501	(-3.75%)	0.0508	(-2.24%)	0.0513	(-1.40%)
q	1.00	0.913	(-8.70%)	0.922	(-7.79%)	0.991	(-0.90%)
b^b	1.990	1.731	(-13.02%)	1.785	(-10.32%)	1.936	(-2.74%)
Households							
c^l	0.687	0.619	(-9.91%)	0.620	(-9.74%)	0.686	(-0.18%)
c^b	0.332	0.298	(-10.30%)	0.298	(-10.32%)	0.332	(0.02%)
h^l	6.880	6.639	(-3.49%)	6.735	(-2.11%)	6.791	(-1.29%)
h^b	2.680	2.554	(-4.73%)	2.607	(-2.75%)	2.631	(-1.85%)
n^l	0.844	0.839	(-0.59%)	0.838	(-0.65%)	0.844	(0.05%)
n^b	1.213	1.208	(-0.38%)	1.209	(-0.33%)	1.212	(-0.05%)
Welfare							
W^s	-0.716	-0.826	(-15.26%)	-0.821	(-14.68%)	-0.720	(-0.54%)
CE^l	0.00	10.23%	-	9.77%	-	0.47%	-
CE^b	0.00	10.55%	-	10.3%	-	0.27%	-

Note: Abatement costs z are expressed as percentage of GDP. Equivalent consumption CE^l and CE^b represent percentage of no-policy consumption that a representative consumer would forgo to achieve the same welfare level under carbon emissions policies.

sector-specific policies within the broader framework of achieving NZE targets.

Long-run effects

Table 3.4 compares scenarios where carbon prices increase simultaneously in both residential and nonresidential sectors with those where only the nonresidential or only the residential carbon price is elevated.[9]

The table is organized into three sections: first, it presents the effects on a set of aggregate macroeconomic variables; second, it analyzes household-type-specific variables; and third, it provides welfare indicators. This table focuses on the long run, with the columns representing steady-state levels with and without the policy. Differences in levels are shown in brackets as percentage deviations from the nonpolicy values.

The analysis reveals that, unsurprisingly given the volume of emissions in both sectors, carbon pricing in the nonresidential production sector is

[9]In all cases, the carbon price trajectories follow those depicted in Figure 3.2.

the primary driver of economic changes. In contrast, residential carbon pricing generally has relatively modest effects on overall economic indicators. However, significant long-term effects are observed in residential investment, housing stock, and new credit, specifically due to residential emissions pricing, both in absolute and relative terms.

Residential emissions account for 8.3% of total carbon emissions in the model. However, as shown in the upper section of Table 3.4, the impact of a residential-specific policy on GDP or consumption is significantly lower than this share when compared to a combined policy aimed at achieving general NZE across all emission sources. This indicates that pricing residential emissions adversely affects aggregate activity much less per unit of emissions reduction than pricing nonresidential emissions.[10] Consumption exhibits a similar pattern, decreasing by 10% under dual pricing and 9.88% under single nonresidential pricing.

The impact of τ^h becomes more pronounced when examining variables directly related to the residential sector and credit. Residential investment declines by 3.75% due to the combined environmental policies aimed at achieving climate neutrality. While general equilibrium effects from taxing carbon emissions in the nonresidential sector account for a 2.24% reduction in residential investment, Table 3.4 indicates that 1.40% decline can be attributed to residential carbon pricing policies in isolation. Additionally, housing prices fall by approximately 0.9% under τ^h, representing over 10% of the decline caused by the combined policy of $\tau^h + \tau^c$.

In our model, debt (the private asset issued by borrowers) expires each period and can be interpreted as fresh credit. This credit is adversely affected by the residential carbon tax, decreasing by 2.74% in the long run. Compared to the nonresidential carbon policy, taxing residential emissions reduces credit by a ratio of approximately 3 to 1 per unit of emissions reduced. Thus, the impact on housing investment, housing prices, and credit per unit of carbon reduction is significantly more pronounced for τ^h than for τ^c.

In the middle section of the table, we assess the impact of carbon pricing on different household types by focusing on key determinants of utility: consumption, housing, and leisure. The findings are as follows: first, in the long term, the combined policy adversely affects borrowers' consumption and housing more than lenders', and borrowers experience a smaller increase in

[10]For example, using the figures from table row GDP, considering that residential emissions represent 8.3% of total emissions, and that the long-run rate of reduction in emissions is the same in both sectors (equal to 75%), we can calculate $\left(\frac{-0.18}{-8.05}\right)\left(\frac{1-0.083}{0.083}\right) \approx \frac{1}{4}$, indicating that the GDP cost of reducing emissions in the residential sector is about 1/4 of that in the nonresidential sector.

leisure. Overall, this indicates a relative long-term deterioration of borrowers' welfare compared to lenders'. Second, the residential emissions pricing policy has minimal long-term effects on consumption and working hours (leisure) across all household types. Third, the residential emissions policy exerts a relatively greater negative impact on borrowers' housing compared to lenders', likely further exacerbating the long-term decline in borrowers' welfare.

Next, we will examine the welfare implications of the policies in more detail. To this end, we will not only compare static solutions but also consider the full dynamics implied by the policies.

Welfare effects

In the bottom section of Table 3.4, we translate the previous results into welfare changes, this time incorporating all transition dynamics. The first row, denoted as W^s, represents a direct measure of social welfare in terms of utility, defined as the weighted sum of individual utility across household types. It is given by

$$W^s = (1 - \tau^b)(1 - \beta^l)W^l + \tau^b(1 - \beta^b)W^b,$$

where W^l and W^b correspond to the present expected value of lifetime welfare of patient and impatient households, respectively, as shown by expressions (3.1) and (3.18) at $t = 0$. The weights in the functions have been chosen so that for the same constant flow of final consumption, patients and inpatients would receive the same level of utility (see Lambertini et al. (2013) or Notarpietro & Siviero (2015))

The following two rows present household-specific changes in welfare, measured in terms of consumption equivalent variation, CE^l and CE^b. More generally, the post-policy welfare of household a (where $a = l, b$)—measured as the expected present value—is given by

$$\widetilde{W}^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln \tilde{c}_t^a + \gamma_h \ln \tilde{h}_t^a - \frac{(\tilde{n}_t^a)^{1+\eta}}{1+\eta} \right), \tag{3.68}$$

The discounted sum of utility for a representative household a, assuming the economy is at the steady state in period 0 and remains constant throughout is

$$\overline{W}^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln (\bar{c}_t^a) + \gamma_h \ln \bar{h}_t^a - \frac{(\bar{n}_t^a)^{1+\eta}}{1+\eta} \right), \tag{3.69}$$

The consumption equivalent measures the percentage of consumption in the no-policy scenario that a representative consumer would be willing to forgo to achieve the same level of welfare under the carbon emissions policies. A positive value indicates a welfare loss, suggesting that the carbon pricing policy is less desirable from a welfare perspective. To determine CE^a, we solve

$$\widetilde{W}^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln \left(\bar{c}_t^a (1 - CE^a) \right) + \gamma_h \ln \bar{h}_t^a - \frac{(\bar{n}_t^a)^{1+\eta}}{1+\eta} \right), \tag{3.70}$$

from which we obtain

$$CE^a = 1 - \exp \left[(1 - \beta^a)(\widetilde{W}^a - \overline{W}^a) \right]. \tag{3.71}$$

Achieving NZE targets through a combined increase in carbon pricing across both sectors results in a generalized welfare loss of -15.3% relative to the no-policy scenario in utility units. In contrast, the welfare simulation for carbon pricing in the residential sector alone yields a modest welfare effect of -0.54%. Considering the residential sector's share of emissions, this result indicates that reducing a unit of carbon emissions in the residential sector is less than half as costly in welfare terms as reducing emissions in the nonresidential sector.

In terms of consumption equivalent, the long-run welfare cost in the benchmark model represents a decline of more than 10% in consumption. This cost is relatively evenly distributed across households under the combined policy. However, significant differences emerge when comparing the impacts of taxing carbon emissions in the residential versus nonresidential sectors. Specifically, the welfare cost in terms of consumption is 0.5 percentage points higher for borrowers than for lenders when the carbon price increase applies only to the nonresidential sector. Conversely, when only the residential sector is subject to the carbon price increase, the welfare cost for lenders, though modest, is nearly twice that of borrowers. Therefore, a more in-depth welfare analysis contradicts the steady-state intuition that housing carbon pricing would further harm borrowers through a reduction in housing properties. When all transitional dynamics are considered from the outset of the policy, and with an equitable distribution of the carbon dividend, borrowers benefit from increased consumption, housing, and leisure during certain periods of the transition (see Figure 3.3a).

Overall, our findings indicate that while carbon pricing in the nonresidential sector is the primary driver of economic changes and welfare losses, residential carbon pricing—despite its modest impact on aggregate indicators—significantly affects the residential sector by reducing investment, housing stock, and credit availability. Notably, reducing emissions in the residential sector is less costly in welfare terms per unit of emissions reduced compared to the nonresidential sector. Furthermore, the differential impacts on borrowers and lenders highlight important distributional effects: borrowers experience greater declines in consumption and housing under combined policies, whereas lenders bear a relatively higher welfare cost when only residential emissions are priced. These insights suggest that carbon pricing policies can have significant implications for inequality, emphasizing the importance of accounting for these differential impacts during the transition to NZE. This objective is addressed in the next subsection.

3.4.3 Carbon pricing and inequality

The burden of carbon pricing is not distributed equally across households. If it disproportionately affects less advantaged households, it can lead to social resistance against policies. In addition, distributional impacts can change throughout the transition. This section examines how these evolving distributional effects influence inequality between household types.[11]

We present in Figure 3.5 two policy scenarios: a combined increase in both residential and nonresidential carbon prices, and a single increase in the nonresidential carbon price. The difference between these scenarios approximates the inequality impact of the residential carbon policy. We maintain the benchmark of an equal per capita distribution of the carbon dividend, with the pre-policy value shown as a large solid dot at 2019.

In the first subplot of Figure 3.5, we assess inequality based on consumption using the Gini coefficient. Given that $(1-\tau^b)$ represents the share of high-income households who consume a fraction $(1-\tau^b)c_t^l/c_t$ of total consumption, the Gini coefficient is calculated as the difference between their implied percentage of total consumption and their percentage in population, that is, $(1-\tau^b)(\frac{c_t^l}{c_t}-1)$.[12]

The figure shows an initial increase in inequality that soon starts to decrease during the transition period as lump-sum tax reductions, which primarily benefit borrowers' consumption, are implemented (discussed in Section 3.4.1). From 2019 until 2030, coinciding with the achievement of intermediate climate targets, the Gini coefficient decreases by 0.45 percentage points. This reduction is approximately half of the decline in the Gini coefficient of disposable income observed in the EU between 2013 and 2024,

[11]While carbon pricing policies introduce economic adjustments, their long-term benefits—ranging from reduced climate damages and health improvements to economic growth through innovation—can outweigh the short-term costs. These long-run effects are beyond the scope of this work, which focuses primarily on the realized costs during the transition.

[12]The same statistics is used by Rubio & Unsal (2017)

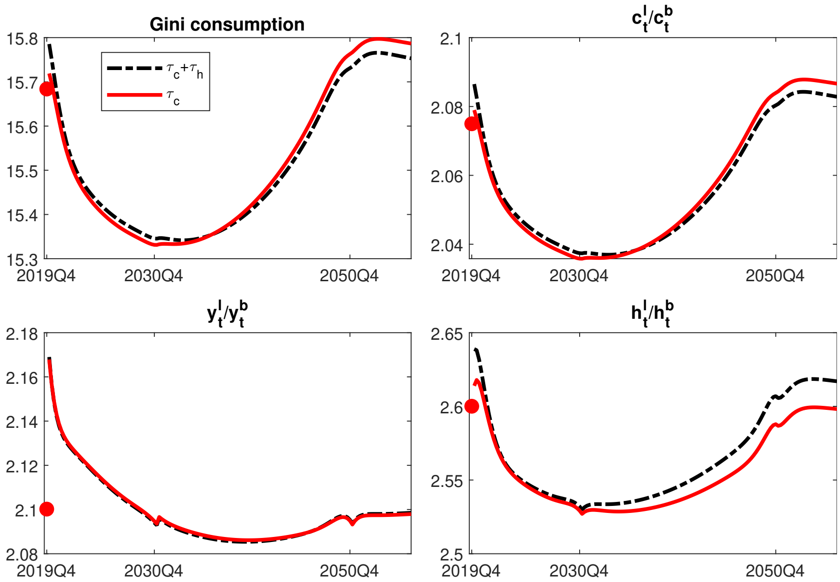


Figure 3.5: Inequality in the transition to NZE. Dashed line: Combined carbon pricing scenario; Solid line: Only carbon pricing on the nonresidential sector

according to Eurostat[13]. However, after reaching a minimum around 2030, the Gini coefficient begins to rise steadily, reversing the gains in consumption distribution and resulting in the highest level of inequality when the economy reaches NZE by 2050. As previously discussed, the reduction in the carbon dividend, as carbon target gaps close, and the decline in the value of borrowers' collateral contribute to this outcome.

Using only the ratio between lenders' and borrowers' consumption in the second subplot of the first row provides a similar picture to that obtained with the Gini coefficient. Similarly, in the second row of Figure 3.5, we present the ratios of incomes—excluding financial incomes—and housing between the two household types. Unlike the U-shaped transition dynamics observed for consumption and housing inequality, income inequality displays an L-shaped pattern, decreasing and remaining lower throughout the entire transition period. We interpret the differences between the dynamics of consumption inequality and income inequality as indicative of the significance of wealth and financial channels, which adversely affect borrowers after 2030.

The comparison of the two policy scenarios reveals that residential carbon pricing, despite its relatively moderate impact, further increases consumption

[13] Eurostat (2024)

inequality for approximately half of the transition period up to 2030, before helping to mitigate the rise in inequality in the later stages. The dynamics differ when considering the ratio of housing, as the residential carbon tax induces a shift in the distribution of houses owned by households, becoming more skewed toward lenders, especially as the tax accelerates after 2030. To explain this outcome, we show in Figure 3.6 the response of several variables.

Borrowers demand houses not only for housing services, as lenders do, but also because they use them as collateral for obtaining credit. The increasing carbon price leads to a decline in house prices, reducing the value of the collateralized asset, which in turn negatively impacts credit availability and consumption. Regarding housing demand, borrowers must balance the marginal disutility of reducing their housing stock to cope with higher housing costs—which is greater for them than for lenders—against the loss of capacity to convert houses into consumption through credit, a constraint that lenders do not face. The results indicate that the latter effect dominates, causing borrowers to reduce their housing stock more than lenders. However, the abatement effort is much higher for borrowers than for lenders, suggesting that, without the disutility effect mentioned before, the reduction in borrowers' housing demand would have been even more pronounced.

Because borrowers are more impatient than lenders, they tend to convert changes in income into contemporaneous changes in consumption at a higher rate than lenders. Consequently, as the carbon dividend increases, borrowers are more likely to immediately allocate it to consumption. Over time, this leads to a substitution effect where borrowers shift from housing to consumption, a dynamic that is not observed for lenders.

Our results suggest that consumption Gini dynamics vary depending on whether taxes are applied to the residential or nonresidential sector. In any case, the time-dependent response of the Gini coefficient for consumption highlights periods when inequality mitigation policies are most needed. In a realistic scenario that combines both residential and nonresidential taxes, such policies are particularly crucial in both the early and later stages of the transition, especially after reaching the intermediate 2030 targets and advancing toward NZE. When the policy on residential emissions is considered in isolation, inequality relief policies are primarily needed during the initial years.

Robustness of inequality dynamics

We investigate the impact of several factors on inequality dynamics, including the proportion of borrowers in the economy, the intensive margin of borrowing, the distribution of the carbon dividend, labor mobility flexibility between

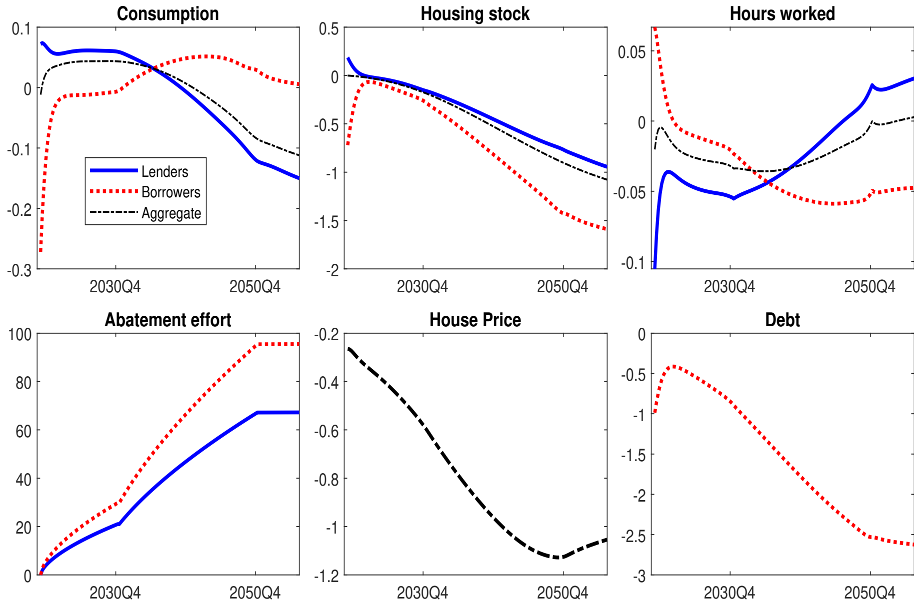


Figure 3.6: Residential carbon price effects. Variables are presented as percentage deviations from their expected trajectories in the absence of environmental policy, except for abatement, which is expressed as a percentage of emissions.

sectors, labor supply elasticity with respect to wages, the intensity of emissions response to housing demand and to goods and services production, and the cost of abatement in both sectors.

Figure 3.7 presents the Gini coefficients across different parameter settings, with the benchmark result included for comparison. In each subplot, only one model parameter is altered, while the growth rate of carbon emissions prices remains constant across all simulations, ensuring that any changes in inequality dynamics throughout the transition are directly attributable to specific parameter variations. However, adjustments in parameters may alter the initial equilibrium. Additionally, Figure 3.8 illustrates how carbon emissions dynamics respond to changes in economic conditions. Together, Figures 3.7 and 3.8 offer a comparative assessment of the impacts on inequality (via Gini coefficients) and efficiency (via emissions reduction) of carbon pricing policies under different settings.

The overall picture that emerges is that, in general, the U-shaped pattern in inequality dynamics observed in the benchmark calibration holds across a wide range of different settings. This means that consumption inequality increases immediately after the policy announcement, then begins to decline,

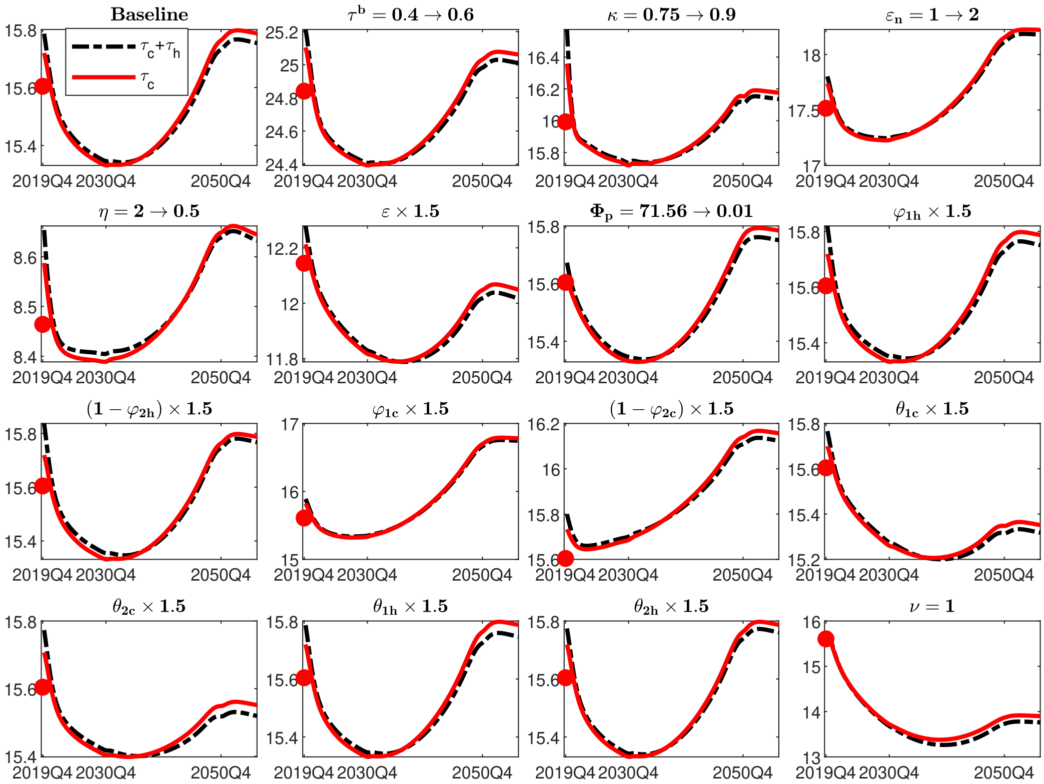


Figure 3.7: Consumption Gini dynamics under different settings. Carbon pricing is implemented under two scenarios: (a) simultaneously in both the nonresidential and residential sectors, and (b) exclusively in the nonresidential sector.

eventually recovering at some point. The isolated effect of residential carbon pricing also appears to remain relatively unaffected across the various subplots. However, a closer examination reveals that certain factors within the economy induce notable differences.

Parameters related to carbon emissions in the production sector significantly influence the trajectory of inequality over time. Economies with industries characterized by higher carbon intensity (φ_{1c}) experience a marked increase in inequality during the later stages of the transition, missing out on the benefits of early-period inequality reductions. The increased production costs from carbon pricing in more carbon-intensive economies result in higher abatement effort (as Figure 3.8 shows, emissions fall by more for a given carbon price) reduced economic activity, lower wages, and decreased government transfers to households, disproportionately affecting borrowers—whose

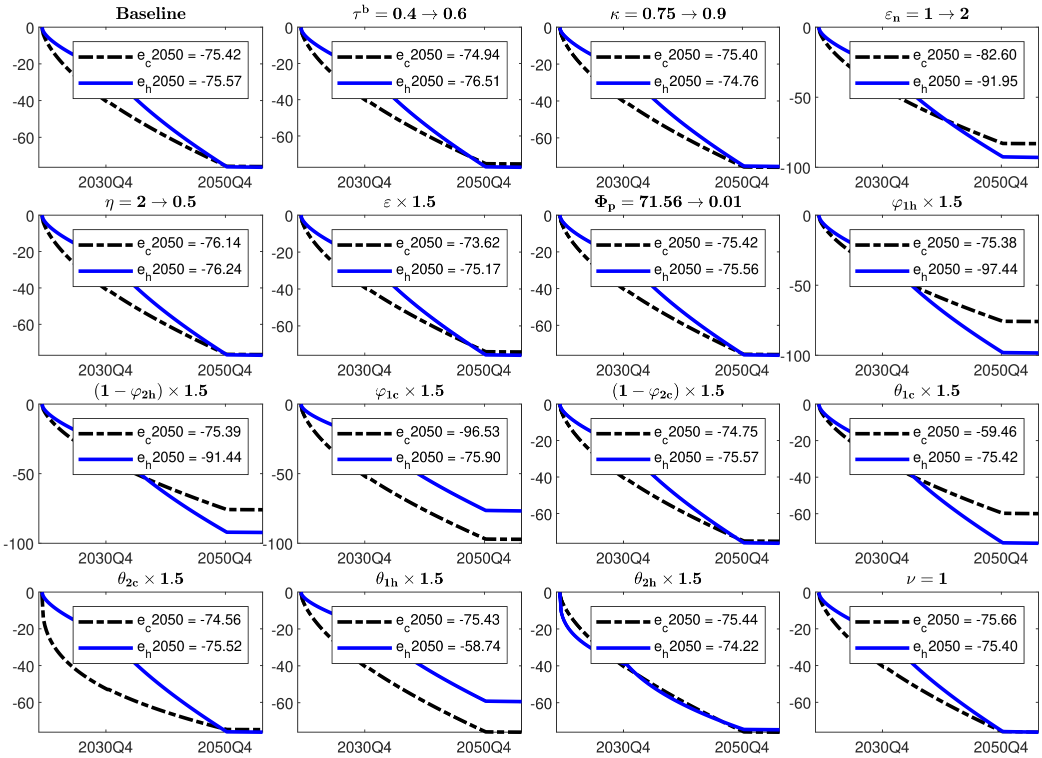


Figure 3.8: Emissions reduction dynamics under different settings. Carbon pricing is implemented simultaneously in both the nonresidential and residential sectors. Dash lines represent the evolution of emission s in the nonresidential sector, and the solid line reflects the evolution of emissions in the residential sector. Values in the legend indicate emissions reduction by 2050.

consumption is more dependent on current income—and widening the consumption gap between households. Consequently, technological advances that reduce φ_{1c} are crucial not only for bridging the gap with NZE, but also for mitigating the inequality impacts of carbon emissions pricing.

Labor mobility between sectors, represented by the parameter ε_n, also significantly impacts inequality. We represent the case of an increase in ε_n. In this case, workers move from the nonresidential sector, where wages experience a greater decline, to the residential sector, amplifying the drop in activity within the nonresidential sector. Reduced activity in this sector lowers emissions by more, subsequently decreasing the carbon dividend, which disproportionately harms the consumption of borrower households. The positive aspect is that the accelerated decline in nonresidential activity facilitates achieving the NZE target, as shown in the corresponding subplot in Figure 3.8. Thus, higher labor mobility between sectors creates a trade-off between efficiency in terms of emissions and consumption inequality.

An increase in the number of credit-constrained households (τ^b) makes the economy more unequal but does not significantly alter the dynamics of the Gini index during the transition. However, increasing the capacity to take credit (a rise in κ) initially worsens inequality more than in the benchmark when environmental measures are first announced. The expected negative impact of the policy on housing prices has a greater effect on borrowers' consumption when the loan-to-value ratio is higher.

Increasing the cost of abatement in the nonhousing sector (θ_{1c}) makes it more difficult to reduce emissions for a given carbon price. Hence, the reduction in the tax base is mitigated as we move along the transition period, keeping carbon dividends higher and benefiting borrowers relative to lenders. Consequently, this scenario improves consumption inequality at the cost of worsening emissions reduction. In the same vein, a higher cost of abatement in the housing sector (θ_{1h}) reduces the inequality impact of the carbon pricing policy compared to the benchmark when targeted at this sector. This effect is evident in the widening gap between the solid and dashed lines in the final years of the period.

Flexible prices ($\Phi_p = 0.01$) mitigate the initial increase in consumption inequality, though they do not alter the subsequent dynamics of the consumption Gini coefficient compared to the baseline scenario. In contrast, a lower initial firm markup (higher ε) contributes to a more pronounced reduction in consumption inequality.

In a scenario where $\nu = 1$, meaning borrowers receive the entire carbon dividend, consumption inequality decreases from the outset, with the Gini coefficient showing the most pronounced change. Furthermore, this positive impact on inequality is achieved without compromising the effectiveness of the taxes in reducing emissions. The importance of ν will be explored in greater depth in Section 3.4.4.

Overall, these findings underscore the significant role that specific economic characteristics play in shaping inequality under carbon pricing policies. Some parameters that influence the behavior of the model economy can have a sizable effect on inequality, whether through increased lump-sum transfers or wage changes. While, in some cases, changes in the economic environment improve both the efficiency of the carbon pricing policies and their effects on consumption inequality, in other cases, the alternative settings can alter emissions reduction targets, necessitating increases in carbon pricing. The destination of the carbon dividend emerges as the most effective tool for managing inequality, highlighting the critical importance of carefully designing carbon dividend distribution and emissions pricing policies to mitigate short and long-term adverse distributional impacts.

Population Composition and Carbon Pricing

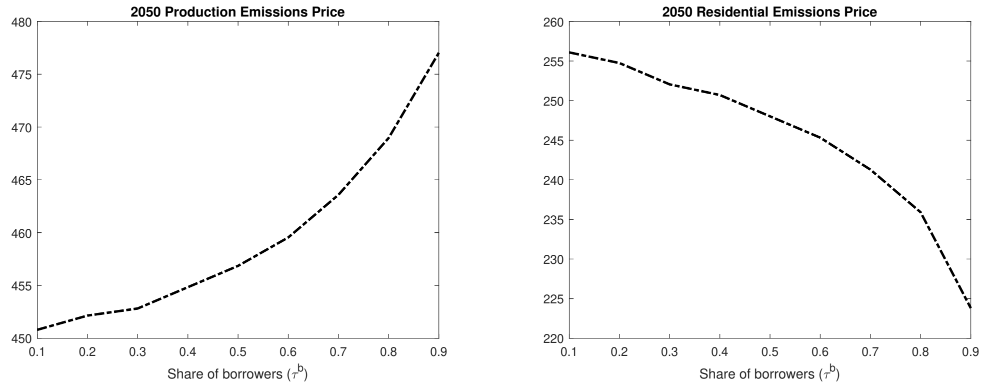
In this subsection, we explore whether changes in population composition can affect the carbon pricing levels needed to achieve climate goals. In the subplot corresponding to τ^b in Figure 3.8, we do not detect significant differences in the effect on emissions when the share of borrowers increases, provided that the pricing pace for both residential and nonresidential carbon emissions is maintained at the baseline. However, this result may obscure differences in the tax required to achieve NZE separately in each of the two sectors.

Figure 3.9 illustrates how, under a linear policy scheme, the carbon prices needed in 2050 to satisfy the NZE for both the residential and nonresidential sectors vary in different directions as the share of borrowers in the population, τ^b, changes. As the share of borrowers increases, the effectiveness of the emissions policy in the residential sector improves, allowing for a reduction in carbon prices. As explained earlier in this chapter, borrowers exert more effort than lenders in abating residential emissions due to the greater disutility associated with reducing residential demand as an alternative. Conversely, as the number of borrowers in the population increases, a higher carbon price is needed in the nonresidential sector to achieve the NZE target. Nonresidential carbon emissions prices directly impact lenders' income through the effect on firm profits, which provides an incentive to abate emissions. As the share of lenders shrinks, this incentive diminishes, and carbon prices need to increase further to achieve the same effect.

3.4.4 Equitable compensation mechanism

The preceding analysis highlights the negative consequences that carbon pricing policies can have on variables crucial for household welfare, emphasizing the need for well-designed compensation schemes, such as carbon dividends, to alleviate these adverse impacts. While offsetting undesired welfare effects is a vital objective, it is not the only consideration for effective climate policy. Achieving a balance among environmental goals, economic efficiency, and social equity is crucial. In this section, we explore the use of carbon dividends—lump-sum transfer rebates distributed to households—as a mechanism to compensate those affected by carbon pricing, ensuring that the financial burden of the transition is distributed more equitably.

Recall that the per capita lump-sum taxes paid by borrowers and lenders are denoted as tr_t^b and tr_t^l, respectively. The aggregate (weighted average)



(a) Variations in τ_c, with τ_h set to the baseline.

(b) Variations in τ_h, with τ_c set to the baseline.

Figure 3.9: Carbon emission price projections to meet NZE by 2050

of these taxes is represented by tr_t. After the implementation of the carbon dividend policy, the lump-sum taxes for each household type equal the uniform lump-sum tax minus the household-specific carbon dividend rebate. The amount of carbon revenue that the government can allocate for tax reductions, referred to as the carbon dividend, is given by s_t and depends on the environmental policy:

$$s_t = \tau_{ht} e_{ht} + \tau_{ct} e_{ct}.$$

The adjusted lump-sum taxes for each household type, following the distribution of the carbon dividend, are expressed as:

$$tr_t^l = \overline{tr} - (1 - \nu)\frac{s_t}{1 - \tau^b}$$

$$tr_t^b = \overline{tr} - \nu\frac{s_t}{\tau^b}$$

where $\overline{tr}$ is the steady-state value of lump-sum taxes required to uniformly fund government spending $\bar{g}$ in the pre-policy scenario. Hence, ν represents the proportion of the carbon dividend allocated to borrowers, while $1 - \nu$ denotes the share allocated to lenders. When $\nu = \tau^b$, the carbon dividend is distributed equally on a per capita basis between the two groups, which is our benchmark assumption. The government's budget constraint, incorporating the post-policy lump-sum taxes tr_t^l and tr_t^b, is given by:

$$\bar{g} = \tau^b tr_t^b + (1 - \tau^b) tr_t^l + \tau_{ht} e_{ht} + \tau_{ct} e_{ct}.$$

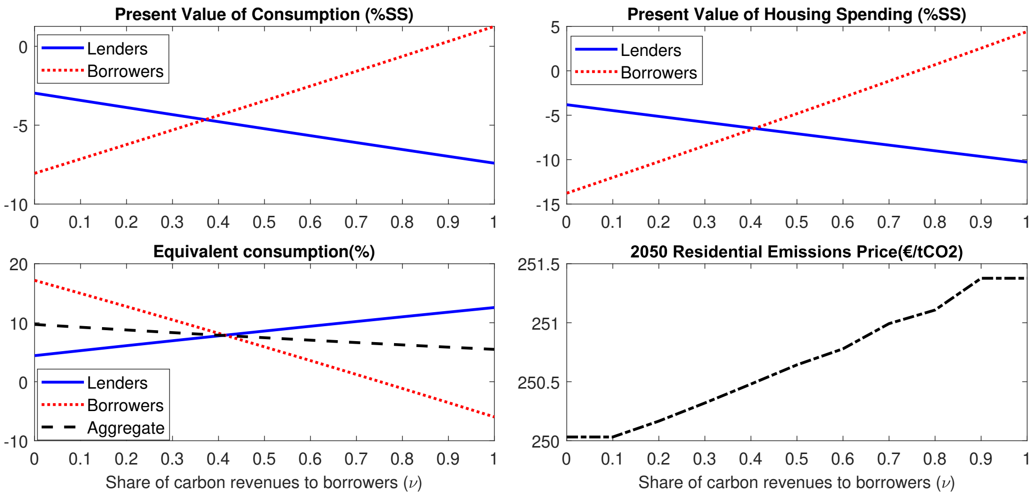


Figure 3.10: Impact on consumption, housing, welfare, and carbon emission prices when carbon revenues are redirected to reduce household taxes. Carbon pricing is implemented simultaneously in both the nonresidential and residential sectors. Present values in percentage deviation from the path they would have followed without any environmental policy.

Figure 3.10 illustrates how different schemes in the distribution of carbon revenues among households impact both equity and efficiency during the transition to a NZE economy. Specifically, we analyze the present value of consumption and housing demand, along with welfare effects and the projected residential carbon price for 2050 required to achieve NZE. By adjusting the parameter ν, the government shifts the allocation of tax relief between lenders and borrowers, thereby altering their disposable income.

The first row in Figure 3.10 shows the present value of consumption and housing spending as a percentage deviation from the path they would have followed without any environmental policy, where consumption and housing demand are constant and equal to their steady-state values. The present value analysis demonstrates that the carbon dividend has a greater impact on borrowers than on lenders and affects housing demand more significantly than consumption spending. This asymmetry poses a challenge for policymakers in designing compensation mechanisms, as no Pareto optimal policy exists. Redirecting revenues to lenders leads to substantial losses for credit-constrained households, with housing spending falling by as much as -15% and consumption decreasing by up to -8% compared to a scenario without carbon pricing. Conversely, lenders experience maximum losses of -10% in housing and -8% in consumption when all carbon revenues are directed to borrowers. Additionally, allocating more of the carbon dividend to borrowers raises the residential carbon emissions price projection for 2050, slightly reducing the policy's efficiency, though this effect is relatively minor.

The left subplot at the bottom captures the welfare effect, including variations in consumption, housing, and working hours, expressed in terms of equivalent variations. Interestingly, the most equitable welfare impact of the environmental policy on households occurs when the carbon dividend is distributed fairly evenly. This is indicated by the intersection of the dotted and solid lines at a value for ν close to τ^b. However, the steeper slope of the line representing the welfare effects for borrowers, compared to that for lenders, suggests that starting from a homogeneous distribution of the carbon dividend, further biasing the dividend towards borrowers more than compensates for the welfare loss experienced by lenders. Under certain reasonable welfare aggregations, this adjustment could lead to an improvement in overall social welfare.

3.5 Conclusion

This chapter provides a comprehensive analysis of the macroeconomic and distributional effects of carbon pricing policies, with a focus on the differential impacts when these policies target residential versus nonresidential emissions. Using an Environmental Dynamic General Equilibrium (E-DGE) model, we account for critical transmission mechanisms, including nominal price rigidities and mark-ups, sectoral labor adjustments, and financial frictions tied to housing collateral. A key innovation of our approach lies in the integration of borrower-lender heterogeneity within a dynamic general equilibrium framework that features two distinct production sectors. This modeling approach distinguishes our work from much of the existing literature. Unlike previous studies that often rely on static or partial equilibrium models to evaluate distributional impacts, our framework captures the dynamic interplay between climate policies and the broader economy over time.

Our findings reveal sector-specific welfare differences. While the overall long-term welfare cost of combined carbon pricing policies is significant—exceeding 10% in terms of consumption and distributed relatively evenly between borrowers and lenders—the impacts vary notably when emissions from each sector are taxed separately. Carbon pricing in the nonresidential sector imposes a greater welfare burden on borrowers due to their increased vulnerability to declines in labor demand and economic output. In contrast, residential carbon pricing disproportionately affects lenders, resulting in nearly twice the welfare loss compared to borrowers, despite lenders' lower abatement efforts. This outcome is driven by a more concentrated reduction in housing investment and property values, which are predominantly owned by lenders. Moreover, the efficiency of emissions reduction is higher in the residential sector, making it more cost-effective in welfare terms per unit of emissions reduced compared to the nonresidential sector.

The chapter also highlights the role of debt distribution in shaping the efficiency of carbon pricing policies. The effectiveness of carbon pricing in the residential sector increases when there is a higher proportion of credit-constrained borrowers, as these households are more inclined to invest in abatement to avoid severe welfare losses. However, the opposite occurs in the nonresidential sector: a larger share of borrowers necessitates higher carbon prices to achieve emissions reductions because the income-driven incentives for lenders to abate diminish. These findings emphasize the need for sector-specific and well-calibrated carbon pricing strategies.

Our analysis of inequality dynamics, including robustness tests, shows that carbon pricing policies can have varying impacts depending on key economic parameters. The consumption Gini coefficient generally follows a U-shaped pattern during the transition, initially decreasing due to carbon dividends but then rising as these revenues dwindle. Sensitivity analysis reveals that factors such as labor mobility and the share of credit-constrained households significantly affect the inequality trajectory. For instance, higher labor mobility between sectors can exacerbate inequality, while increasing the proportion of borrowers makes the economy more unequal but does not alter the general pattern of inequality over time. These insights underline the importance of adaptive policy measures that can address distributional challenges, especially after intermediate climate targets are reached.

This study sets the stage for future research in several compelling areas. One promising direction is to delve deeper into the role of the labor market in shaping the effectiveness of environmental policies. This includes exploring within a dynamic general equilibrium framework labor mobility between sectors and wage dynamics, building on previous work like Fullerton & Monti (2013) and Aubert & Chiroleu-Assouline (2019). Gaining a better understanding of how shifts in employment across sectors influence the outcomes of carbon pricing policies could provide valuable insights into labor market dynamics that may either amplify or buffer the economic impacts of these policies. Another crucial avenue for research involves examining financial frictions, specifically the behavior of endogenous loan-to-value ratios under the influence of macroprudential tools. Investigating how these financial mechanisms interact with carbon pricing policies could shed light on the trade-offs between environmental goals and financial stability.

Additionally, expanding the model to incorporate a more diverse range of heterogeneous households, similar to the work of Andres et al. (2022), would enable a more granular analysis of distributional effects. Such a framework would better capture the varied economic realities across different household groups, helping policymakers design strategies that more effectively balance economic efficiency, equity, and environmental sustainability, and paving the way for more robust and inclusive strategies in the global transition to a low-carbon economy.

In conclusion, our work contributes to the literature by integrating household heterogeneity and sector-specific dynamics within an E-DGE framework, providing a more nuanced understanding of the economic and distributional consequences of carbon pricing. Policymakers must balance environmental goals with economic and social equity, carefully considering the differential impacts on borrowers and lenders and the diminishing effectiveness of carbon dividends over time. By designing carbon pricing schemes that incorporate differentiated impacts on borrowers and lenders and distributing carbon dividends in a way that addresses long-term inequality, it is possible to achieve emissions reduction targets in a manner that minimizes economic disruption and promotes social fairness.

Chapter 4

A Loss Function Approach under Climate Concerns

We derive a welfare-based loss function within a dynamic general equilibrium framework incorporating climate externalities, financial frictions, and nominal rigidities. This setup enables the evaluation of optimal and suboptimal policy responses in a climate-constrained economy. Emissions from aggregate production accumulate in the atmosphere, lowering productivity and output. Carbon pricing policies, by incentivizing abatement, create a wedge between production and consumption and introduce new trade-offs for policymakers. We show that when abatement is costly, the optimal carbon tax is a fraction of aggregate consumption, capturing both the direct social cost of emissions and the indirect productivity benefits of future mitigation. Our welfare loss function incorporates abatement fluctuations due to carbon tax volatility and reveals that a higher tax-to-consumption ratio reduces the relative weight on output stabilization and inequality, while greater abatement costs heighten sensitivity to tax volatility. These findings underscore the trade-offs in climate policy and the need for coordinated monetary, macroprudential, and environmental strategies to ensure both stabilization and sustainability.

4.1 Introduction

Climate change is one of the most pressing challenges of the 21st century. Since 1982, the Earth's surface has been warming at an accelerating pace, with 2023 recorded as the hottest year since global temperature measurements began in 1850 (Figure 4.1). The rapid increase in temperatures and the growing frequency of extreme weather events are mainly attributed to greenhouse gas emissions from human activity, as documented by the Intergovernmental

Panel on Climate Change (IPCC). Efforts to limit global warming to well below 2°C, as outlined in the Paris Agreement, pose significant challenges to macroeconomic and financial stability. These concerns have gained increasing attention from central banks, sparking intense debate over the implications of climate policies for monetary and financial stability (Lagarde (2021), Mann (2023), Powell (2023)).

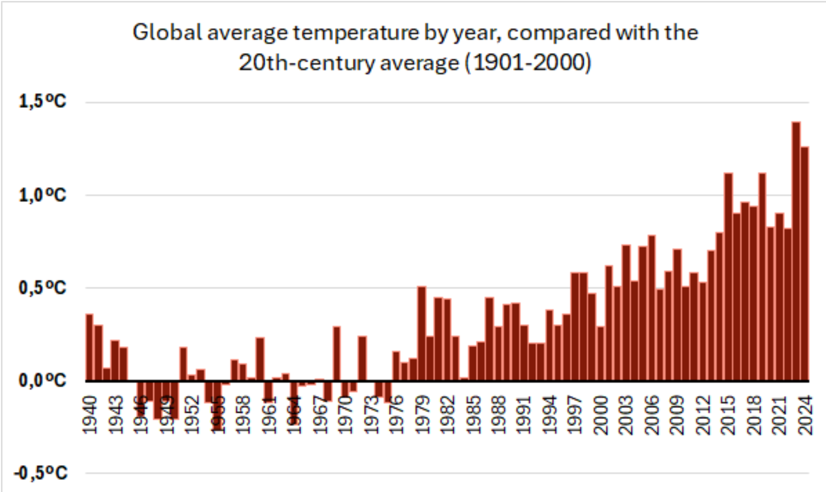


Figure 4.1: Yearly surface temperature from 1940–2024 compared to the 20th-century average (1901-2000). Data: NOAA Climate.gov https://www.ncdc.noaa.gov/cag/global/time-series

Do emission mitigation efforts introduce new risks to price and financial stability? Is there an inherent trade-off between central banks' core objectives and climate goals? Although there is no clear consensus on the precise role central banks should play in addressing climate change, one point is indisputable: When climate change and mitigation policies affect macroeconomic stability - whether through physical shocks from extreme weather and rising temperatures or transition risks linked to the shift toward a net-zero economy -monetary and macroprudential authorities must account for these dynamics within their price and financial stability mandate. Climate-related disruptions can influence inflation, output volatility, and credit conditions, making them a relevant concern for stabilization purposes, even under a narrow interpretation of policy makers' responsibilities.

In this study, we address these questions by deriving a welfare-based loss function to evaluate the policy trade-offs faced by monetary policymakers. Following the welfare-based approach pioneered by Rotemberg & Woodford (1998), we explicitly account for distortions arising from carbon emissions in production, nominal rigidities, and financial frictions. Unlike the Ramsey approach, which derives policy prescriptions by solving the constrained optimization problem of the social planner, this approximates the optimal policy problem through a quadratic loss function that serves as a benchmark to evaluate the effectiveness of policy, providing insights into how policy rules can balance stabilization objectives while respecting institutional constraints.

Our welfare loss function captures three key distortions affecting the economy. First, greenhouse gas emissions, which scale proportionally with output, arise from aggregate production (Annicchiarico & Di Dio (2015), Heutel (2012)). Accumulated emissions in the atmosphere reduce productivity, but firms lack incentives to curb emissions due to abatement costs. To correct this market failure, the government implements carbon pricing policies that internalize the social costs of emissions and encourage abatement investments. Second, financial frictions, modeled as collateral constraints (Kiyotaki & Moore (1997), Iacoviello (2005)), limit the borrowing capacity of impatient households to a fraction of their housing wealth. Borrowing is financed by patient households' savings, creating interactions between financial markets and macroeconomic outcomes. Finally, nominal rigidities, introduced through price adjustment costs (Rotemberg (1982b)), give monetary policy an active role in stabilizing the economy.

Our work builds on the contributions of Andrés et al. (2013) and Ferrero et al. (2018), who introduce financial frictions and the role of assets as collateral in credit markets to obtain a welfare loss function. We extend this framework by explicitly incorporating climate externalities, modeling carbon accumulation as a factor that damages output and influences macroeconomic trade-offs. By integrating these elements, our approach provides a theoretical foundation for analyzing the interactions between monetary policy, macroprudential regulation, and environmental policy.

This study contributes to several strands of literature. First, as discussed above, our welfare-based approach builds on research examining the optimality of macroprudential and monetary policies under financial frictions. Andrés et al. (2013), using a DSGE model with entrepreneur credit constraints as in Iacoviello (2005), demonstrate that welfare optimization requires stabilizing inflation, the output gap, and the consumption and housing gaps between borrowers and savers. These constraints generate trade-offs between stabilization objectives. Similarly, Rubio & Yao (2020) show that optimal coordination between monetary and macroprudential policies enhances welfare, particularly in a low-interest rate environment. Additionally, Cúrdia & Woodford (2016) analyze optimal monetary policy in the presence of financial frictions, modeling them through an interest rate spread between savers and borrowers. Their findings suggest that cyclical fluctuations in credit spreads have a limited impact on the overall design of optimal monetary policy, which is well-approximated by a flexible inflation target. However, their framework does not incorporate collateral constraints and, therefore, fails to capture the financial accelerator mechanism, wherein changes in asset prices directly influence borrowing capacity and amplify macro-financial fluctuations.

Our analysis also contributes to the literature linking monetary and carbon tax policies. Olovsson & Vestin (2023) find that an abrupt carbon tax increase can generate inflationary pressures and economic inefficiencies, whereas a gradual transition results in milder effects. They conclude that monetary policy should accommodate the transition by focusing on core inflation, ensuring that inflationary effects remain modest and mitigating concerns over ”greenflation”. Similarly, Del Negro et al. (2023) show that carbon taxes do not necessarily lead to persistent inflation but introduce a short-term trade-off between stabilizing inflation and maintaining output. Consequently, central banks may need to tolerate temporary inflationary pressures to prevent deep recessions, suggesting that the inflationary effects of the green transition can be managed within standard monetary policy frameworks. However, these studies do not analyze optimal policies or provide a metric for the welfare loss associated with them.

More closely related to our work, Annicchiarico & Di Dio (2015) analyze the effects of different environmental policies on macroeconomic fluctuations in a New Keynesian model with nominal rigidities. The authors derive policy prescriptions under the assumption that a Ramsey planner optimally determines environmental policy while considering different monetary policy regimes. Annicchiarico & Di Dio (2017) extend this framework to examine the optimal joint design of monetary and environmental policies in a Ramsey setting, illustrating how carbon taxation and emissions control interact with inflation dynamics. Their findings highlight that price stickiness and imperfect competition shape the trade-offs between inflation stabilization and environmental policies, thereby influencing the optimal monetary policy response to shocks. Compared to previous studies, we incorporate the distortions generated by financial frictions and collateral constraints. Additionally, the welfare-based loss function approach we present quantifies the trade-offs between multiple stabilization objectives, facilitating a comparative analysis of different policy regimes. Furthermore, it enables the examination of policy coordination among independent policymakers, such as central banks and macroprudential regulators.

More recently, Nakov & Thomas (2023) develop a Ramsey-optimal monetary policy model within a New Keynesian framework. They investigate whether central banks face trade-offs between inflation control and climate objectives when governments implement carbon pricing policies. A key finding is that when carbon taxes are optimally set, monetary policy can fully stabilize inflation and the output gap without significant distortions. However, if carbon pricing is initially suboptimal and gradually increases, monetary policy may face short-term trade-offs, yet the optimal response remains prioritizing price stability. With respect to Nakov & Thomas (2023), we not only adopt a welfare-based loss function approach, which permits the assessment of both optimal and suboptimal policy rules, but also model abatement effort as an endogenous decision margin. In contrast, the referenced paper focuses on optimal monetary policy under exogenous carbon pricing policies without explicitly accounting for abatement choices.

Regarding the link between climate policies and financial frictions, our paper relates to Diluiso et al. (2021), who examine the macro-financial risks of climate transition using a New Keynesian model with financial frictions. Their study suggests that central banks face trade-offs when responding to climate-related shocks, as stabilizing inflation may conflict with financial stability objectives. However, rather than relying on a Ramsey problem or computing a welfare-based loss function, they assess the macro-financial effects of various policy interventions, such as green quantitative easing and climate-sensitive macroprudential policies.

In our analysis, we obtain several key results that shed light on the macroeconomic policy trade-offs in a climate-constrained economy. First, we show that when carbon abatement is costly, the optimal carbon tax can be expressed as a fraction of aggregate consumption. This tax not only internalizes the direct social cost of emissions but also accounts for the indirect effect of reduced future emissions resulting from lower productivity. Second, by deriving a welfare-based loss function that explicitly incorporates climate externalities, financial frictions, and nominal rigidities, we uncover novel insights into the structure of policy trade-offs. Our second-order approximation of the loss function shows that climate-related distortions introduce an additional variance term that captures fluctuations in carbon taxation. This term reflects the wedge between output and consumption created by abatement costs. The loss function assigns weights to output deviations and consumption inequality that depend not only on standard parameters but also on the ratio of steady-state carbon revenues to consumption, or the marginal labor-related costs per unit of consumption. We show that the weight on output stabilization decreases as the ratio of carbon tax revenues (labor-related marginal cost) to consumption increases (reduces), reflecting the reduced relevance of output fluctuations when a larger share of time is devoted to leisure. However, as abatement costs grow with rising taxes, the relative importance of the tax variance term increases. Importantly, we demonstrate that the convexity of the loss function—a critical condition for ensuring a unique and stable optimum—is guaranteed if the steady-state consumption-to-output ratio exceeds the carbon revenue-to-output ratio, a condition generally satisfied in practice. Collectively, these findings underscore the importance of coordinating monetary, fiscal, and environmental policies to balance short-term stabilization with long-term sustainability.

The remainder of the chapter is organized as follows. Section 4.2 introduces the model and outlines the key distortions affecting the economy. It shows how the marginal cost in the Phillips curve depends on the abatement effort. Section 4.3 examines the efficient equilibrium, which serves as a normative benchmark for monetary policy, and derives the optimal carbon tax. Section 4.4 analyzes the policies required to deal with distortions, including the optimal carbon tax. Section 4.5 analyzes the stabilization objectives that show up in the loss function in the presence of climate-related distortions, and the weights of the different components. Finally, Section 4.6 summarizes the main findings, describing how the welfare loss function can guide climate-aware economic policies, and suggesting important next steps for research.

4.2 The Model

The economy consists of two types of households, differentiated by their subjective discount rates, following Iacoviello & Neri (2010). Patient households, with lower discount rates, act as savers, while impatient households, with higher discount rates, become borrowers. We denote by $\tau^b \in (0, 1)$ the mass of the borrowers and normalize the total size of the population to one. Both groups consume, demand domestic real estate, and supply labor. Borrowers face lending constraints as their credit is limited by the expected real value of their real estate holdings. This creates financial frictions that restrict the flow of funds between savers and borrowers.

The production sector consists of a perfectly competitive final goods industry, where firms aggregate intermediate goods into the final product. Intermediate-good producers operate in a monopolistically competitive environment and face nominal rigidities due to quadratic price adjustment costs.

We extend this model to incorporate a climate externality, following Annicchiarico & Di Dio (2015). Production generates greenhouse gas emissions proportional to output, and accumulated emissions reduce overall productivity. Firms can allocate resources to reduce emissions, but have little incentive to do so due to associated costs. In response, the government introduces carbon pricing to internalize the social costs of emissions and encourage investment in abatement efforts. Any additional revenue from these policies is redistributed back to households.

4.2.1 Patient households

Patient households, or lenders, choose consumption, housing services, and labor to maximize the following objective function. Unless otherwise specified, variables are expressed in real terms and per capita, relative to the size of the household.

$$\max_{c_t^l, h_t^l, n_t^l, b_t^l} \quad \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^l)^i \left(\ln c_{t+i}^l + \gamma_h \ln h_{t+i}^l - \frac{(n_{t+i}^l)^{1+\eta}}{1+\eta} \right) \tag{4.1}$$

subject to the real flow of funds expressed in terms of production price index P_t (the *numeraire*),

$$c_t^l + (1+\tau_h) q_t h_t^l - b_t^l = w_t^l n_t^l + q_t h_{t-1}^l - \frac{r_{t-1}}{\pi_t} b_{t-1}^l + d_t - tr_t^l \tag{4.2}$$

On the utility side, $\beta^l \in (0,1)$ represents the inverse of the discount factor for patients, c_t^l is the real consumption per capita, h_t^l is lenders' housing tenure, and n_t^l is labor time per person. The parameter γ_h captures the preference for housing and the inverse of η relates to the Frisch elasticity.

Regarding budget constraints, patient households lend in real terms $-b_t^l$ to impatient households (where b_t^l is negative) and receive back $-\frac{r_{t-1}}{\pi_t} b_{t-1}^l$ as repayment of the previous period's loans. These households buy houses $(q_t((1+\tau_h)h_t^l - h_{t-1}^l))$, earn wages from labor $(w_t^l n_t^l)$, and pay lump-sum taxes tr_t^l in terms of consumption goods[1]. As owners, they receive profits d_t, from firms operating in noncompetitive markets. The constant τ_h represents a tax on the purchase of housing by lenders, which ensures an efficient steady state in the housing market, as we will see in Section 4.3.

The intertemporal maximization of (4.1) subject to the set of constraints of the form (4.2) implies the optimality conditions:

Lenders' marginal utility of consumption:

$$\lambda_t^l = \frac{1}{c_t^l} \tag{4.3}$$

Lenders' demand for housing:

$$\frac{\gamma_h}{h_t^l} = \lambda_t^l q_t (1+\tau_h) - \beta^l \mathbb{E}_t (\lambda_{t+1}^l q_{t+1}) \tag{4.4}$$

[1]These lump-sum taxes guarantee that the government budget is balanced at any time.

Lenders' labor supply:

$$w_t^l = \frac{(n_t^l)^\eta}{\lambda_t^l} \tag{4.5}$$

Lenders' Euler condition for consumption:

$$\lambda_t^l = \beta^l \mathbb{E}_t \lambda_{t+1}^l \frac{r_t}{\pi_{t+1}} \tag{4.6}$$

4.2.2 Impatient households

Impatient households have a high discount rate, which is the inverse of the discount factor $\beta^b < \beta^l$. The impatient household optimizes the following utility function

$$\max_{c_t^b, h_t^b, n_t^b, b_t^b} \quad \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^b)^i \left(\ln c_{t+i}^b + \gamma_h \ln h_{t+i}^b - \frac{(n_{t+i}^b)^{1+\eta}}{1+\eta} \right) \tag{4.7}$$

where c_t^b is the real consumption of the borrowers, h_t^b denotes the holding of the borrowers' homes, and n_t^b are the hours worked by the borrowers.

Impatient households become borrowers, so in addition to the budget constraint (4.8), they face a financial constraint on the maximum amount of credit they can obtain. In particular, expression (4.9) limits the amount of borrowing, b_t^b, to a fraction κ of the expected resale value of housing held by household.

$$c_t^b + q_t(h_t^b - h_{t-1}^b) - b_t^b = w_t^b n_t^b - \frac{r_{t-1}}{\pi_t} b_{t-1}^b + s^b \tag{4.8}$$

$$b_t^b \leq \kappa \mathbb{E}_t \frac{q_{t+1} \pi_{t+1} h_t^b}{r_t} \tag{4.9}$$

The constant s^b in the budget constraint stands for a constant subsidy that borrowers receive to ensure an efficient allocation of aggregate consumption ($c^b = c^l = c$) in the steady state (see section 4.3).

The first order conditions for the maximization of the utility of (4.7) subject to the set of constraints (4.8) and (4.9) are the following.

Borrowers' marginal utility of consumption:

$$\lambda_t^b = \frac{1}{c_t^b} \tag{4.10}$$

Borrowers' demand for housing:

$$\frac{\gamma_h}{h_t^b} = \lambda_t^b q_t - \mathbb{E}_t(\beta^b \lambda_{t+1}^b q_{t+1} + \kappa \mu_t^b q_{t+1} \pi_{t+1}) \tag{4.11}$$

Borrowers' labor supply:

$$w_t^b = \frac{(n_t^b)^\eta}{\lambda_t^b} \tag{4.12}$$

Borrowers' Euler condition for consumption:

$$\lambda_t^b = \beta^b \mathbb{E}_t \lambda_{t+1}^b \frac{r_t}{\pi_{t+1}} + \mu_t^b r_t \tag{4.13}$$

4.2.3 Final goods producers

Production occurs at two levels, which is a standard assumption. At the top level, a competitive bundler aggregates the output of intermediate-good firms, y_{jt}, priced at P_{jt}, into a single composite product, y_t, which is sold to households for consumption.

Assuming constant returns to scale technology *'a la* Dixit & Stiglitz (1977) to compose the bundle, the aggregate production, the aggregate production price index (PPI), and the total demand for each variety are given, respectively, by

$$y_t = \left(\int_0^1 (y_{jt})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.14}$$

$$P_t = \left(\int_0^1 (P_{jt})^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \tag{4.15}$$

$$y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} y_t \tag{4.16}$$

4.2.4 Intermediate goods producers

At the base level of production, a continuum of firms indexed by $j \in [0, 1]$ operates within a monopolistically competitive market, each producing a differentiated good y_{jt}. Firms employ a uniform Cobb-Douglas production technology that uses only labor from both household type

$$y_{jt} = (1 - D(x_t)) a_t (n_{jt}^b)^\alpha (n_{jt}^l)^{1-\alpha} \tag{4.17}$$

where a_t is a random variable following an exogenous productivity process of AR(1) zero mean, and $(1 - D(x_t))$ captures the negative productivity effects of the atmospheric carbon stock x_t, measured as a percentage of the final good output.

Firms emit greenhouse gases (GHG) as a by-product of production, contributing to the accumulation of carbon stock in the atmosphere. Emissions released by firm j in period t are determined by the level of production y_{jt} and the abatement effort u_{jt}

$$e_{jt} = \varphi(1 - u_{jt})y_{jt} \tag{4.18}$$

where φ denotes the intensity of carbon production, measuring emissions per unit of output. The government may impose a carbon tax on emissions, τ_{et}, which increases the cost per unit of output by $\tau_{et}\frac{e_t}{y_t}$. To mitigate emissions, firms can invest in abatement efforts, incurring costs that depend on their abatement level and production scale:

$$z_{jt} = \theta_1(u_{jt})^{\theta_2}y_{jt} \tag{4.19}$$

Here, θ_1 represents the baseline cost of reducing emissions, while θ_2 captures the rate at which costs increase as the abatement effort increases.

Since intermediate good firms operate in monopolistic competition, they price their own variety according to the demand of the final good firm. Following Rotemberg (1982b) firms pay a cost AC_{jt} in nominal terms whenever they adjust prices. This cost depends on deviation from target inflation $\bar{\pi}$ and total production in the sector $P_t y_t$

$$AC_{jt} = \frac{\Phi_p}{2}(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi})^2 P_t y_t$$

Firms choose an optimal price in the period t to maximize their expected discounted profits (4.2.4), subject to the demand for their differentiated good (4.16) and the production technology (4.17. The problem can be expressed as follows

$$\max_{P_{jt},n^l_{jt},n^b_{jt},u_{jt}} \quad \mathbb{E}_t\sum_{t=0}^{\infty}(\beta^l)^t\frac{\lambda^l_t}{\lambda_0}\Bigg((1+\tau_p)\frac{P_{jt}}{P_t}y_{jt} - w^l_t n^l_{jt} - w^b_t n^b_{jt} - \theta_1(u_{jt})^{\theta_2}y_{jt}$$
$$- \tau_{et}\varphi(1-u_{jt})y_{jt} - \frac{\Phi_p}{2}(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi})^2 y_t\Bigg)$$

where the constant τ_p represents a production subsidy that compensates for the distortion caused by the monopoly power of the firms. The Lagrangian of the maximization problem is

$$\mathcal{L} = \mathbb{E}_t\Bigg\{\sum_{t=0}^{\infty}(\beta^l)^t\frac{\lambda^l_t}{\lambda_0}\Bigg\{(1+\tau_p)\left(\frac{P_{jt}}{P_t}\right)^{1-\varepsilon}y_t - w^l_t n^l_{jt} - w^b_t n^b_{jt}$$
$$- \theta_1(u_{jt})^{\theta_2}\left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon}y_t - \tau_{et}\varphi(1-u_{jt})\left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon}y_t - \frac{\Phi_p}{2}(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi})^2 y_t$$
$$+ psi_{jt}\Bigg((1-D(x_t))a_t(n^b_{jt})^{\alpha}(n^l_{jt})^{1-\alpha} - \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon}y_t\Bigg)\Bigg\}\Bigg\}$$

In a symmetric equilibrium, all firms set the same price, use identical inputs, and produce the same output. In addition, clearing the labor market ensures that the labor demand for each type of household matches its total supply. Within this framework, the firm's first-order maximization conditions are given by

$$w_t^l = \psi_t(1-\alpha)\frac{y_t}{\int_0^1 n_{jt}^l\, dj} \tag{4.20}$$

$$w_t^b = \psi_t\alpha\frac{y_t}{\int_0^1 n_{jt}^b\, dj} \tag{4.21}$$

$$\theta_1\theta_2(u_t)^{\theta_2-1} = \varphi\tau_{et} \tag{4.22}$$

$$\begin{aligned}\pi_t(\pi_t-\bar{\pi}) &= \beta^l\frac{\lambda_{t+1}^l}{\lambda_t^l}\pi_{t+1}(\pi_{t+1}-\bar{\pi})\frac{y_{t+1}}{y_t}\\ &+\frac{\varepsilon}{\Phi_p}\left[(1+\tau_p)\frac{1-\varepsilon}{\varepsilon}+\tau_{et}\varphi(1-u_t)+\theta_1(u_t)^{\theta_2}+\psi_t\right]\end{aligned} \tag{4.23}$$

Equations (4.20) and (4.21) define the demand for labor by firms from lenders and borrowers, shaping the labor-related component of marginal cost, ψ_t. This variable quantifies the additional labor expense required to produce one more unit of output. However, firms also face pollution-related costs arising from carbon taxation and abatement measures. Consequently, in addition to labor-related marginal costs, these environmental costs also affect inflation through the Phillips curve (4.23). We define:

$$mc_t = \tau_{et}\varphi(1-u_t)+\theta_1(u_t)^{\theta_2}+\psi_t \tag{4.24}$$

as the total marginal cost in the Phillips curve. Thus, the marginal costs associated with the firms' abatement decisions and the cost of polluting enter directly into the inflation dynamics. As shown in equation (4.22), given a carbon tax τ_{et}, firms choose their abatement effort so that the marginal cost of abatement, $\theta_1\theta_2(u_t)^{\theta_2-1}$, equals the private benefit from reducing emissions costs, $\tau_{et}\varphi$. In contrast to Golosov et al. (2014) and Nakov & Thomas (2023), where the carbon tax directly raises marginal costs by increasing fossil fuel prices, our framework introduces production costs that influence inflation through firms' endogenous abatement choices.

Moreover, the labor-related component of marginal cost can be expressed in terms of the consumption gap between borrowers and lenders, output, and the abatement effort itself. This relationship becomes more apparent when the expression is linearized:

$$\hat{\psi}_t = (\alpha-\tau^b)(\hat{c}_t^b-\hat{c}_t^l)-\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t+(\eta+1)(\hat{y}_t-a_t+\gamma x\hat{x}_t)$$

The preceding discussion of the Phillips curve allows us to establish Proposition 1.

Proposition 1. *When firms have an endogenous margin for carbon abatement effort decisions, the relationship between the inflation gap and the output gap is mediated not only by the consumption inequality gap but also by the abatement effort.*

Since all firms use the same technology and adopt a uniform level of abatement effort, we can aggregate as follows:

$$y_t = (1 - D(x_t))a_t \left(\int_0^1 n_{jt}^b \, dj \right)^{\alpha} \left(\int_0^1 n_{jt}^l \, dj \right)^{1-\alpha} \tag{4.25}$$

$$e_t = (1 - u_t)\varphi y_t \tag{4.26}$$

$$z_t = \theta_1 (u_t)^{\theta_2} y_t \tag{4.27}$$

Accordingly, aggregate firms' profits are given by:

$$\begin{aligned}(1 - \tau^b)d_t = y_t \Big[&(1 + \tau_p) - \psi_t - \tau_{et}(1 - u_t)\varphi \\ &- \theta_1 (u_t)^{\theta_2} - \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2 \Big]\end{aligned} \tag{4.28}$$

4.2.5 Damage and pollution dynamics

Following Golosov et al. (2014), we assume an exponential damage function, $D(x_t)$, defined as:

$$D(x_t) = 1 - \exp(-\gamma(x_t - \bar{x})) \tag{4.29}$$

where $\bar{x}$ denotes the reference atmospheric carbon concentration, typically set to the preindustrial level of CO_2. The parameter γ determines the percentage reduction in output per unit increase in carbon concentration relative to this reference level.

Emissions from the production of consumption goods, e_t, contribute to the accumulation of atmospheric carbon. The law of motion for the atmospheric carbon stock is given by:

$$x_t - \bar{x} = \sum_{s=0}^{t+T} (1 - \delta^x)^s e_{t-s} \tag{4.30}$$

where δ^x is the proportion of atmospheric carbon that naturally decays in each period. Therefore, $(1 - \delta^x)^s$ represents the share of emissions generated in period $t - s$ that remains in the atmosphere s periods later.

4.2.6 Government Budget Constraint

The government balances its budget by levying a lump-sum tax (or equivalently, adjusting transfers) to patient households. This is captured by the government budget constraint:

$$(1-\tau^b)tr_t^l + \tau_{et}e_t + (1-\tau^b)\tau_h q_t h_t^l = \tau_p y_t + \tau^b s^b \tag{4.31}$$

where tr_t^l is positive, if a tax, and negative, if a transfer.

4.2.7 Equilibrium

With housing supply fixed, the housing market equilibrium requires:

$$\bar{h} = \tau^b h_t^b + (1-\tau^b)h_t^l \tag{4.32}$$

In the labor market, equilibrium conditions ensure that the total labor supplied by each household type matches firms' labor demand:

$$\int_0^1 n_{jt}^l\, dj = (1-\tau^b)n_t^l \qquad \int_0^1 n_{jt}^b\, dj = \tau^b n_t^b$$

Aggregate consumption is given by:

$$c_t = \tau^b c_t^b + (1-\tau^b)c_t^l \tag{4.33}$$

Then, if we aggregate household budget constraints, use the relationship between total production and income, and incorporate the government's budget constraint (4.31), the economy's resource constraint simplifies to:

$$c_t = y_t\left(1 - \theta_1 u_t^{\theta_2} - \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2\right) \tag{4.34}$$

Emissions abatement and price adjustment costs consume production resources, creating a wedge between output and consumption that depends on abatement decisions.

Finally, in the credit market, total lending by patient households must equal total borrowing by impatient households, ensuring:

$$(1-\tau^b)b_t^l + \tau^b b_t^b = 0 \tag{4.35}$$

4.3 Efficient equilibrium

In this section, we examine the efficient equilibrium, that is, the optimal allocation of resources in the absence of market imperfections. This theoretical benchmark serves as a normative reference for policymakers. In the context of climate-related externalities, it allows us to derive the optimal carbon tax. As will be evident, consistent with the standard Pigouvian framework, the optimal tax is set equal to the marginal external damage caused by emissions.

4.3.1 Social Planner Equilibrium

The social planner maximizes a weighted average of lifetime utilities for savers and borrowers, subject to constraints imposed by the aggregate production function, market-clearing conditions for goods and housing, and the climate externality. We assume that the policymaker discounts future periods at the patient households' discount factor, β^l. If different discount factors were used, impatient households would progressively consume less relative to patient households over time. Consequently, the equilibrium allocation would depend on the amount of time elapsed since the implementation of the planner's allocation (see Andrés et al. (2013)).

The social planner's maximization problem is formalized as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^l)^t \left((1-\tau^b) U^l(c_t^{*l}, h_t^{*l}, n_t^{*l}) + \tau^b U^b(c_t^{*b}, h_t^{*b}, n_t^{*b}) \right)$$

$$\begin{aligned}
\text{s.t.:}\quad & [(1-D(x_t^*))a_t(\tau^b n_t^{*b})^\alpha((1-\tau^b)n_t^{*l})^{1-\alpha}](1-\theta_1 u_t^{*\theta_2}) = \tau^b c_t^{*b} + (1-\tau^b)c_t^{*l} \\
& \bar{h} = \tau^b h_t^{*b} + (1-\tau^b)h_t^{*l} \\
& x_t^* - \bar{x} = \sum_{s=0}^{t+T} (1-\delta^x)^s e_{t-s}^* \\
& e_t^* = \varphi(1-u_t^*)[(1-D(x_t^*))a_t(\tau^b n_t^{*b})^\alpha((1-\tau^b)n_t^{*l})^{1-\alpha}]
\end{aligned}$$

where the first constraint, representing the total resource constraint, can be written in a more compact way as

$$y_t^* - \theta_1 u_t^{*\theta_2} y_t^* = c_t^* \tag{4.36}$$

First-order conditions (FOC) with respect to consumption (c^{*l}, c^{*b}), housing (h^{*l}, h^{*b}), labor (n^{*l}, n^{*b}), abatement efforts (u^*), emissions (e^*) and atmospheric carbon (x^*) are given as follows

$$\frac{1}{c_t^{*l}} = \mu_{1t} \tag{4.37}$$

$$\frac{1}{c_t^{*b}} = \mu_{1t} \tag{4.38}$$

$$\frac{\gamma_h}{h_t^{*l}} = \mu_{2t} \tag{4.39}$$

$$\frac{\gamma_h}{h_t^{*b}} = \mu_{2t} \tag{4.40}$$

$$(n_t^{*l})^\eta = \frac{\mu_{1t}(1-\alpha)y_t^*}{(1-\tau^b)n_t^{*l}}[1 - \theta_1 u_t^{*\theta_2} - \frac{\mu_{4t}}{\mu_{1t}}\varphi(1-u_t^*)] \tag{4.41}$$

$$(n_t^{*b})^\eta = \frac{\mu_{1t}\alpha y_t^*}{\tau^b n_t^{*b}}[1 - \theta_1 u_t^{*\theta_2} - \frac{\mu_{4t}}{\mu_{1t}}\varphi(1-u_t^*)] \tag{4.42}$$

$$\frac{\mu_{4t}}{\mu_{1t}} = \frac{\theta_1\theta_2 u_t^{*\theta_2-1}}{\varphi} \tag{4.43}$$

$$\mu_{4t} = \sum_{s=0}^{\infty}(\beta^l)^s(1-\delta^x)^s\mu_{3t+s} \tag{4.44}$$

$$\mu_{3t} = \gamma y_t^*\mu_{1t}(1 - \theta_1 u_t^{*\theta_2} - \frac{\mu_{4t}}{\mu_{1t}}\varphi(1-u_t^*)) \tag{4.45}$$

where μ_{1t}, μ_{2t}, μ_{3t}, and μ_{4t} denote the Lagrange multipliers associated with the total resource constraint, the housing market constraint, the dynamics of the carbon stock, and the emissions equation, respectively.

In efficient equilibrium, consumption and housing are equalized across lenders and borrowers, as there are no inefficiencies stemming from financial frictions.

$$\begin{aligned} c_t^{*l} &= c_t^{*b} = c_t^* \\ h_t^{*l} &= h_t^{*b} = \bar{h} \end{aligned} \tag{4.46}$$

Consequently, the social planner ensures an equal distribution of aggregate consumption across households and a uniform housing allocation across both types of agents.

According to equations (4.41) and (4.42), the social cost of emissions, given by $\frac{\mu_{4t}}{\mu_{1t}}\varphi(1-u_t^*)$, and the abatement cost, $\theta_1 u_t^{*\theta_2}$, affect the optimal allocation of labor. Changes in the abatement effort generate a trade-off between these two types of costs. Intuitively, as the costs associated with abatement and environmental damage increase, the marginal contribution of labor to social welfare declines, prompting a reduction in the optimal allocation of labor time.

Condition (4.44) captures how the marginal cost of emissions, μ_{4t}, internalizes both present and future damages through their contribution to atmospheric carbon accumulation. Specifically, μ_{4t} equals the social welfare damage from pollution, represented by μ_{3t}, adjusted for carbon depreciation and the social planner's discount rate.

Using expressions (4.46) and the aggregate resource constraint, the first-order condition with respect to the carbon stock x_t yields:

$$\mu_{3t} = \gamma(1 - \mu_{4t} e_t^*) \tag{4.47}$$

Thus, social pollution damages correspond to the marginal negative effect on consumption from an additional unit of pollution stock (γ), net of the reduction in welfare damages resulting from lower emissions due to decreased production associated with carbon-related damages. From this equation, we can express (4.44) as

$$\mu_{4t} = \gamma \sum_{s=0}^{\infty} (\beta^l)^s (1 - \delta^x)^s - \gamma \sum_{s=0}^{\infty} (\beta^l)^s (1 - \delta^x)^s \mu_{4t+s} e_{t+s}^* \tag{4.48}$$

The expression (4.48) reflects that the marginal social damage of carbon emissions in the period t includes, as the first term, the well-known expression of Golosov et al. (2014). It relates to the present value of the damage on production, originated from the carbon atmosphere persistence of these emissions over time. However, in our model, emissions depend directly on aggregate production. Then, as emissions cause damage to production, they reduce production and have an effect on reducing emissions. This indirect effect is captured by the second term, which hence mitigates the marginal externality damage with respect to Golosov et al. (2014).

Finally, as shown in equation (4.43), the optimal level of emissions abatement is determined by equating the marginal cost of abatement to the marginal social damage of emissions.

4.4 Dealing with distortions

Once we have obtained the market and efficient equilibrium, we are in a condition to obtain the values for τ_p, τ_h, s^b, and τ_e that guarantee that the market and efficient equilibrium coincides in the steady state (see Appendix B.2 for details).

4.4.1 Price Stickiness Distortion

We first eliminate friction related to price rigidity by setting the price adjustment cost parameter, Φ_p, to zero.[2] Under flexible prices, equation (4.23)

[2] The same outcome can be obtained by assuming monetary policy in the market economy follows a Taylor rule with a strict inflation target.

simplifies to:[3]

$$\psi_t = (1+\tau_p)\frac{\varepsilon-1}{\varepsilon} - \theta_1(u_t)^{\theta_2} - \tau_{et}\varphi(1-u_t) \tag{4.49}$$

and the total marginal cost, taking into account (4.24)

$$mc_t = (1+\tau_p)\frac{\varepsilon-1}{\varepsilon} \tag{4.50}$$

Define $\hat{mc}_t \equiv \ln(mc_t) - \ln(\bar{mc})$ as the log deviation of the marginal cost from its steady-state value. Then, the deviation of the marginal cost in the market equilibrium from the efficient level is given by

$$\hat{mc}_t - \hat{mc}_t^* \equiv \hat{mc}_t \tag{4.51}$$

Thus, the log deviation of marginal costs in the market equilibrium from their efficient level equals their log deviation from the steady state.

4.4.2 Monopolistic distortion and climate externality

Next, we use the equilibrium conditions in the labor market: for lenders, combining equations (4.5) and (4.20), and for borrowers, using equations (4.12) and (4.21). These conditions are then combined with equation (4.49) to obtain:

$$(n_t^l)^\eta = \left[(1+\tau_p)\frac{\varepsilon-1}{\varepsilon} - \theta_1(u_t)^{\theta_2} - \tau_{et}\varphi(1-u_t)\right]\frac{(1-\alpha)y_t}{(1-\tau^b)n_t^l c_t^l} \tag{4.52}$$

$$(n_t^b)^\eta = \left[(1+\tau_p)\frac{\varepsilon-1}{\varepsilon} - \theta_1(u_t)^{\theta_2} - \tau_{et}\varphi(1-u_t)\right]\frac{\alpha y_t}{\tau^b n_t^b c_t^b} \tag{4.53}$$

We determine τ_p and τ_{et} to ensure that equations (4.52) and (4.53) match their efficient counterparts (4.41) and (4.42), respectively. In doing so, we eliminate the distortions that arise in the market equilibrium from monopolistic competition and the externality of the climate. Removing the distortion caused by monopolistic competition requires setting:

$$\tau_p = \frac{\varepsilon}{\varepsilon-1} - 1$$

[3] Annicchiarico & Di Dio (2015) derive an analogous expression for real marginal costs under flexible prices in a similar framework. Their model incorporates emissions as a by-product of production and includes abatement costs, with prices adjusting according to a Calvo pricing mechanism.

Thus, the production subsidy precisely offsets the markup distortion.

With respect to climate distortion, internalizing the climate externality is equivalent to setting an emissions tax that aligns the market abatement effort with the efficient abatement effort, as can be clearly observed by comparing equations (4.43) and (4.22). Thus, the optimal emissions tax is given by:

$$\tau_{et}^{*} = \frac{\mu_{4t}}{\mu_{1t}} = \mu_{4t} c_t$$

or

$$\tau_{et}^{*} = \frac{\gamma}{1-\beta^l(1-\delta^x)} c_t - \gamma \left(\sum_{s=0}^{\infty} (\beta^l)^s (1-\delta^x)^s \mu_{4t+s} e_{t+s} \right) c_t \tag{4.54}$$

In other words, the optimal emissions tax reflects the marginal social damage of carbon emissions, measured as a fraction of consumption in each period. In our framework, unlike in Golosov et al. (2014) and Nakov & Thomas (2023), it is consumption, not production, that appears in the optimal tax formula. This distinction arises due to the wedge introduced between output and consumption by abatement costs (equation (4.36)). The first term of expression (4.54) aligns exactly with that of Golosov et al. (2014). However, as indicated in equation (4.48), carbon-related damages influence the economy's emission capacity. The second term in the expression captures this indirect effect.

The steady state version of the optimal carbon tax can be written as:

$$\tau_{e}^{*} = \left(\frac{\gamma}{1-\beta^l(1-\delta^x)+\gamma e} \right) c \tag{4.55}$$

The previous discussion leads us to the following proposition

Proposition 2. *In an economy in which the decision to abate carbon is costly, the optimal tax is a fraction of aggregate consumption.*

The New Phillips curve and the optimal emissions tax

We can now proceed to examine the implications of implementing an optimal carbon emissions tax.

Aggregate output in the economy depends on labor inputs, abatement effort, and the damage caused by the carbon stock. Thus, output can be implicitly represented as

$$y_t = h[e_t(u_t, y_t), x_{t-1}, n_t^l, n_t^b] \tag{4.56}$$

or equivalently,

$$y_t = h'[u_t, x_{t-1}, n_t^l, n_t^b] \tag{4.57}$$

Starting from the same initial conditions for the carbon stock x_{t-1}, once price rigidity distortions are corrected and the optimal emissions tax τ_{et}^* is implemented, the market equilibrium values for abatement effort u_t and labor supplies n_t^l, n_t^b coincide with their efficient equilibrium counterparts. Therefore, aggregate output y_t in the market equilibrium matches that of the efficient equilibrium. Using equations (4.36) and (4.34), aggregate consumption c_t is also identical in both equilibria. Given equation (4.26), it is clear that implementing τ_{et}^* replicates the efficient emissions trajectory.

From equations (4.23), (4.24), and (4.51), the Phillips curve can be expressed in terms of the output gap as

$$\hat{\pi}_t = \beta^l \hat{\pi}_{t+1} + \frac{\varepsilon}{\Phi_p}\Bigg[\tau_e\varphi(1-u)[\hat{\tau}_{et} - \hat{\tau}_{et}^*]$$
$$+\psi\left((\alpha-\tau^b)(\hat{c}_t^b - \hat{c}_t^l) - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}[\hat{u}_t - \hat{u}_t^*] + (\eta+1)\left([\hat{y}_t - \hat{y}_t^*] + \gamma x[\hat{x}_t - \hat{x}_t^*]\right)\right)\Bigg]$$

or alternatively, expressing the carbon tax in terms of abatement effort:

$$\begin{aligned}\hat{\pi}_t = \beta^l \hat{\pi}_{t+1} + \frac{\varepsilon\psi}{\Phi_p}\Bigg[&\left(\frac{\tau_e\varphi(1-u)(\theta_2-1)}{\psi} - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\right)[\hat{u}_t - \hat{u}_t^*] \\ &+ (\alpha-\tau^b)(\hat{c}_t^b - \hat{c}_t^l) + (\eta+1)\left([\hat{y}_t - \hat{y}_t^*] + \gamma x[\hat{x}_t - \hat{x}_t^*]\right)\Bigg]\end{aligned} \tag{4.58}$$

Notice that, when the carbon tax is optimally implemented, the gaps in abatement effort and carbon stock vanish ($\hat{u}_t - \hat{u}_t^* = 0$ and $\hat{x}_t - \hat{x}_t^* = 0$). This means that, provided that the carbon emissions tax is set optimally, and that there exists some other policy (i.e macroprudential policy) that is able to close the consumption gap between borrowers and lenders, a monetary policy of strict inflation targeting ($\hat{\pi}_t$) will be able to close the output gap relative to the efficient equilibrium.

4.4.3 Borrowing constraints

Given the existence of two household types and borrowing constraints, achieving the social planner's optimal allocation in a market equilibrium requires additional fiscal measures. Specifically, we introduce a government transfer to borrowers s^b, and a housing tax on lenders τ_h. As discussed previously, we have ensured that aggregate consumption in the market matches efficient allocation. Furthermore, since the total housing stock is exogenous, we must guarantee that, in the steady-state market equilibrium, consumption and housing allocations are equalized between borrowers and lenders, as shown in equation (4.46).

The transfer required to eliminate consumption disparities is

$$s^b = \left(1 - \frac{\alpha}{\tau^b}\right)c + \frac{1-\beta^l}{\beta^l}b^b + \frac{\alpha}{\tau^b}\tau_e e \tag{4.59}$$

The use of this subsidy addresses household consumption inequality. Notice that the fraction of borrowers appears directly in the determination of the subsidy. This distinguishes our approach from that of Iacoviello & Neri (2010) and Ferrero et al. (2018) who implicitly assume that $\alpha = \tau^b$, in which case the previous expression becomes

$$s^b = \frac{1-\beta^l}{\beta^l}b^b + \tau_e e$$

A second fundamental difference with respect to Ferrero et al. (2018) and Andrés et al. (2013) is the introduction of a new term that captures the environmental policy.

Additionally, the housing tax τ_h

$$\tau_h = (\beta^l - \beta^b)(1-\kappa) \tag{4.60}$$

ensures a uniform distribution of the housing stock between borrowers and lenders.

Together, these four instruments,τ_p, τ_e^*, s^b, and τ_h align the conditions of flexible-price equilibrium of the market economy in steady state with those of the social planner's solution, achieving an efficient and socially optimal allocation.

4.5 A welfare loss function under pollution externalities

In this section, we derive a welfare-based loss function for the market equilibrium and discuss its policy implications. The derivation follows the linear-quadratic approach (LQ) pioneered by Rotemberg & Woodford (1998) for economies with price rigidities. We extend this methodology by incorporating financial frictions as modeled in Andrés et al. (2013), Ferrero et al. (2018), and Rubio & Yao (2020), and crucially, by including climate externalities.

This extension allows our welfare loss function to explicitly capture environmental costs associated with polluting production activities. To the best of our knowledge, this represents the first attempt to derive a loss function for an economy characterized by both pollution externalities and borrowing constraints.

The log-quadratic approximation of aggregate welfare, which serves as the central bank's objective function, highlights how environmental considerations modify traditional economic trade-offs. This framework thus provides a comprehensive foundation for the optimal design of monetary and macroprudential policies in the presence of climate-related challenges and facilitates comparisons across suboptimal policy rules.

To construct the welfare-based loss function, we consider a weighted average of the lifetime utility functions of patient and impatient households. We assume that the planner discounts future periods at the savers' discrete rate β^l. The planner's objective function is given by:

$$W_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^l)^t U_t$$

where:

$$U_t = (1-\tau^b)\left(\ln c_t^l + \gamma_h \ln h_t^l - \frac{(n_t^l)^{1+\eta}}{1+\eta}\right) + \tau^b \left(\ln c_t^b + \gamma_h \ln h_t^b - \frac{(n_t^b)^{1+\eta}}{1+\eta}\right)$$

The second-order approximation of this welfare function around the efficient steady state relies on assumptions derived from our previous analysis.

Assumption 1. *The subsidy rate on the revenue of final goods producers to eliminate monopolistic distortions is given by*

$$\tau_p = \frac{\varepsilon}{\varepsilon - 1} - 1$$

Assumption 2. *The carbon tax ensures an efficient steady state solution for abatement and emissions,*

$$\tau_e^* = \left(\frac{\gamma}{1-\beta^l(1-\delta^x)+\gamma e}\right) c$$

Assumption 3. *The borrowers' subsidy to achieve an efficient steady state distribution of aggregate consumption is given by*

$$s^b = \left(1 - \frac{\alpha}{\tau^b}\right) c + \frac{1-\beta^l}{\beta^l} b^b + \frac{\alpha}{\tau^b} \tau_e^* e$$

Assumption 4. *The tax on lenders' housing that ensures an efficient steady state distribution of total housing stock is given by*

$$\tau_h = (\beta^l - \beta^b)(1 - \kappa)$$

Under the given assumptions, we perform a second-order Taylor expansion of the utility function U_t around the efficient steady state, resulting in the expression (4.61). Appendix B.3 provides the algebraic details of this derivation.

$$W_0 \approx -\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^l)^t \frac{\Omega}{2} \Big[\lambda_\pi \hat{\pi}_t^2 + (\hat{y}_t - a_t + \gamma x \hat{x}_t)^2 + \lambda_u \hat{u}_t^2 + \lambda_c (\hat{c}_t^b - \hat{c}_t^l)^2 + \lambda_h (\hat{h}_t^b - \hat{h}_t^l)^2 \Big] \tag{4.61}$$

Hatted variables denote log-deviations from the efficient steady state. The composite parameters of the loss function, normalized so that the output coefficient is unity, are:

$$\Omega \equiv (1 - \tau_e^* \mu_1 e)(1 + \eta)$$
$$\lambda_\pi \equiv \frac{\mu_1 y \Phi_p}{(1 + \eta)(1 - \tau_e^* \mu_1 e)}$$
$$\lambda_u \equiv \frac{\mu_1 y \theta_2^2 (\mu_1 y - 1)}{(1 - \tau_e^* \mu_1 e)(1 + \eta)}$$
$$\lambda_c \equiv \frac{\tau^b (1 - \tau^b)}{(1 - \tau_e^* \mu_1 e)(1 + \eta)} + \frac{\alpha(1 - \alpha)}{(1 + \eta)^2}$$
$$\lambda_h \equiv \gamma_h \frac{\tau^b (1 - \tau^b)}{(1 - \tau_e^* \mu_1 e)(1 + \eta)}$$

where variables without a time subscript denote their steady-state values. From the coefficients derived above, we establish the following proposition:

Proposition 3. *The relative weights in the loss function under climate concerns depend on the emissions tax.*

Our derived welfare loss function collapses to the standard expression commonly used in the literature when financial frictions and nominal rigidities are present, but climate externalities are absent (see, for example, the welfare loss functions derived in Andrés et al. (2013), Rubio & Yao (2020), Ferrero et al. (2018), and Millard et al. (2024)). To abstract from climate externalities, we assume that there is no climate damage, which implies setting $\gamma = 0$. Consequently, carbon taxes ($\tau_e = 0$) and emission abatement ($u = 0$) are unnecessary. Under these conditions, output equals consumption, implying $\mu_1 y = 1$, and the loss function coefficients simplify to:

$$\Omega' \equiv (1+\eta)$$
$$\lambda'_\pi \equiv \frac{\Phi_p}{(1+\eta)}$$
$$\lambda'_u \equiv 0$$
$$\lambda'_c \equiv \frac{\tau^b(1-\tau^b)(1+\eta)+\alpha(1-\alpha)}{(1+\eta)^2}$$
$$\lambda'_h \equiv \gamma_h \frac{\tau^b(1-\tau^b)}{(1+\eta)}$$

Moreover, in the absence of climate externalities, the term representing output stabilization simplifies to $(\hat{y}_t - \hat{y}_t^*)^2$, since deviations of output from its efficient level depend solely on technological shocks. The welfare loss function thus takes its conventional form:

$$W_0 \approx -\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^l)^t \frac{\Omega'}{2} \left(\lambda'_\pi \hat{\pi}_t^2 + (\hat{y}_t - \hat{y}_t^*)^2 + \lambda'_c (\hat{c}_t^b - \hat{c}_t^l)^2 + \lambda'_h (\hat{h}_t^b - \hat{h}_t^l)^2 \right)$$

For policy applications, it is preferable that the welfare loss function is not only purely quadratic but also convex. Although a nonconvex function can be meaningful under extreme economic conditions, such as severe recessions, or when policymakers exhibit strongly asymmetric preferences, ensuring convexity simplifies optimization and makes policy evaluation in nonlinear models more interpretable and implementable. Convexity guarantees a unique and stable minimum and captures the notion that larger deviations from policy targets are increasingly undesirable.

To ensure convexity, we need to determine the conditions under which the relevant coefficients are positive. Defining $s_c \equiv \frac{c}{y}$ as the steady-state consumption-to-output ratio and $s_e \equiv \frac{\tau_e^* e}{y}$ as the steady-state carbon revenue-to-output ratio, we rewrite the loss function as follows

$$W_0 \approx -\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^l)^t \frac{1}{2} \Big[\lambda''_\pi \hat{\pi}_t^2 + \lambda''_y (\hat{y}_t - a_t + \gamma x \hat{x}_t)^2 + \lambda''_u \hat{u}_t^2 + \lambda''_c (\hat{c}_t^b - \hat{c}_t^l)^2 + \lambda''_h (\hat{h}_t^b - \hat{h}_t^l)^2 \Big] \quad (4.62)$$

where

$$\lambda''_y \equiv (1+\eta)\frac{s_c - s_e}{s_c}$$
$$\lambda''_c \equiv \tau^b(1-\tau^b) + \frac{s_c - s_e}{s_c}\frac{\alpha(1-\alpha)}{(1+\eta)}$$

and the rest of the parameters in the loss function are always non negative

$$\lambda''_\pi \equiv \frac{\Phi_p}{s_c}$$
$$\lambda_u \equiv \theta_2^2 \frac{(1-s_c)}{s_c^2}$$
$$\lambda''_h \equiv \gamma_h \tau^b (1-\tau^b)$$

In steady state, aggregate production is spent on consumption and emissions abatement costs, such that $y = c + \theta_1 u^{\theta_2} y$. From this it directly follows that $s_c = 1 - \theta_1 u^{\theta_2}$, which implies that the fraction of production consumed by households decreases with the abatement effort. Given these conditions, the coefficients of the welfare loss function will always be positive as long as the term $(s_c - s_e)$ remains positive. We summarize the above discussion in the following proposition.

Proposition 4. *For the welfare loss function to be convex, it is necessary that $s_c > s_e$, where s_c represents the fraction of output consumed by households, and s_e denotes the ratio of carbon revenue to output.*

This proposition establishes that, for the loss function to be convex, the benefits of production, measured in terms of consumption, have to exceed the costs associated with carbon taxation. Given that an economy is unlikely to sustain a scenario in which more than half of its production is allocated to carbon taxes, this constraint is unlikely to be binding.

4.5.1 Stabilization objectives

The welfare loss function (4.62) identifies the key stabilization objectives of policy makers. The first term is about inflation stabilization, as the price adjustment costs faced by firms introduce inefficiencies that translate directly into welfare losses, which are proportional to the price adjustment cost parameter, Φ_p.

The second term relates to output stabilization. Inflation and output stabilization are standard objectives in new Keynesian models, as they address the key distortions introduced by nominal price rigidities. These rigidities prevent firms from optimally adjusting prices, leading to inefficient fluctuations in inflation and output. However, in the presence of climate concerns, the stabilization of the output becomes more complex due to environmental externalities and abatement efforts. Both elements affect marginal costs in the New Keynesian Phillips curve, as shown in equation (4.58), thereby altering the relationship between inflation and output gap.

Hence, climate change and environmental policies influence how policy-makers weigh stabilization objectives. In particular, expression (4.62) shows that the ratio $\frac{s_e}{s_c}$—the carbon revenue-to-consumption ratio—determines the relative importance assigned to output stabilization. As this ratio increases, the weight on output stabilization in the loss function declines. Moreover, from expressions (4.36) and (4.49), we know that $s_c - s_e$ corresponds to the steady-state value of the labor-related component of marginal cost, ψ_t. According to equations (4.52) and (4.53), a lower labor-related marginal cost implies weaker labor demand in the steady state and lower wages. As a result, households tend to substitute leisure for labor, reducing the sensitivity of their utility to output fluctuations and, hence, diminishing the welfare relevance of output stabilization.

Additionally, greater efforts by producers to abate carbon emissions require diverting more resources toward covering abatement costs, thereby increasing the wedge between consumption and output. The third term in the welfare loss function captures losses specifically associated with fluctuations in these abatement costs—losses that are not reflected in the other terms. As shown in equation (4.34), consumption possibilities are constrained by both price adjustment and abatement costs. These arise from two firm-level decision margins—price setting and emissions abatement—which introduce distinct sources of inefficiency. While the former is accounted for by the inflation term in the loss function, the latter is captured by the $\hat{u}_t^2$ component, which is ultimately influenced by the volatility of carbon taxes. Consequently, higher volatility in carbon taxation leads to greater consumption volatility for a given level of output, thereby lowering overall welfare.

The coefficient λ_u captures the weight assigned to welfare losses resulting from volatility in carbon taxes. Its value increases with the share of output allocated to abatement costs, $1 - s_c$. The policy implication is clear: the more costly it is to reduce emissions, the more important it becomes to avoid fluctuations in carbon taxation over time.

The previous discussion leads to the following proposition

Proposition 5. *In an economy where firms endogenously choose the level of emissions to abate, and where abatement entails a cost, volatility in environmental taxation contributes to welfare losses by increasing consumption volatility. Hence, carbon tax volatility directly affects the social welfare loss function, and its importance grows with the magnitude of abatement costs.*

The final two objectives, consumption and housing distribution gaps, stem from credit frictions and capture the welfare costs associated with imbalances in nondurable consumption and housing between borrowers and savers. These inefficiencies, driven by borrowing constraints faced by impatient

households, are reflected in the corresponding terms of the welfare loss function, underscoring the broader macroeconomic consequences of financial distortions. Similarly to the output component, the weight of the consumption inequality in expression (4.62) is also influenced by the ratio $\frac{s_e}{s_c}$, although to a lesser extent. This weight also depends on the distribution of borrowers and lenders in terms of population (τ^b) and labor income (α). In particular, greater asymmetries in these shares - that is, lower values of $\tau^b(1-\tau^b)$
$\alpha(1-\alpha)$ - diminish the importance that policymakers assign to closing the consumption inequality gap. Furthermore, a higher carbon revenue to consumption ratio ($\frac{s_e}{s_c}$) reduces the relevance of the labor-income component in the weight assigned to the consumption distribution gap, further shifting the focus of the policy away from distributional considerations. Moreover, a higher Frisch elasticity (η) reduces the weight on the consumption inequality in the loss function, while augmenting that of output.

As is standard in the literature (see Ferrero et al. (2018)), stronger preferences for housing amplify the importance of the housing inequality objective. However, unlike the consumption inequality gap, the weight assigned to the housing gap depends exclusively on the asymmetry in the population shares of the two household types.

4.6 Conclusion

In this chapter, we study how climate concerns influence the stabilization objectives of policymakers. To this end, we adopt the welfare-based approach of Rotemberg & Woodford (1998). Along the way, we derive a climate-adjusted version of the New Phillips Curve and characterize the optimal carbon tax when emissions abatement is costly. The model features Rotemberg-style price stickiness, housing preferences, credit constraints, greenhouse gas emissions that reduce productivity, and costly abatement efforts.

We show that, when firms have an endogenous margin for carbon abatement effort decisions, the relationship between the inflation gap and the output gap is mediated not only by the consumption inequality gap but also by the abatement effort.

Similarly to Golosov et al. (2014), the optimal carbon tax in our model is set equal to the present discounted value of climate damages. However, unlike these frameworks, it is consumption—rather than production—that enters the optimal tax formula. This reflects the wedge between output and consumption created by abatement costs. Moreover, the optimal tax expression comprises two components: The first captures the standard present value of production damages from persistent carbon emissions, while the second accounts for the indirect effect of emissions on production. Specifically, as emissions reduce productivity, they also reduce future emissions, thus mitigating marginal externality compared to Golosov et al. (2014). The steady-state version of the carbon tax reveals that in an economy where carbon abatement is costly, the optimal tax corresponds to a fraction of aggregate consumption.

We derive a welfare loss function and show that it remains convex under the condition that the fraction of output consumed by households exceeds the carbon revenue-to-output ratio. This ensures that the welfare benefits of production outweigh the costs associated with carbon taxation. Compared to a loss function without climate concerns, our formulation includes a new component that captures the variance of carbon taxation and reflects the wedge between output and consumption resulting from abatement costs. We also show that climate externalities and environmental policies reduce the weights assigned to output variability and consumption inequality in the loss function, while ambitious emissions targets with costly abatement can exacerbate economic instability by increasing the wedge between output and consumption.

Clearly, all climate-related effects on the loss function influence the design of monetary and macroprudential policy responses to economic shocks. Overall, our results highlight the need for policymakers to balance traditional stabilization goals with emerging climate-related challenges.

The derived welfare-based loss function offers a powerful analytical tool for evaluating macroeconomic policy in a climate-constrained environment. It allows for the assessment of welfare implications resulting from deviations from the optimal policy by simulating the model under alternative policy rules. Moreover, it enables the quantification of welfare losses associated with key economic distortions, including those induced by climate policies, financial frictions, and nominal rigidities. The framework also facilitates the evaluation of the benefits of aligning monetary and macroprudential policies to simultaneously pursue traditional stabilization objectives, financial stability, and emerging climate-related goals. Finally, it provides a basis for analyzing how variations in key climate policy parameters—particularly carbon taxes—affect (sub)optimal policy outcomes and the associated welfare costs.

Chapter 5

Conclusion

This thesis explores how housing markets, financial frictions, and environmental externalities may interact with key macroeconomic policies—fiscal, environmental, monetary and macroprudential—through the lens of Dynamic General Equilibrium (DGE) models. The three chapters are united by a common analytical framework that places household heterogeneity, housing collateral, and sectoral interdependencies at the core of policy evaluation, offering a refined perspective on welfare, stability, and climate goals.

Chapter 2 focuses on the macroeconomic and welfare consequences of fiscal expansion in a housing-dominant economy, using the Spanish economy before the 2008 crisis as a case study. It shows that while increased government consumption stimulates tradable goods production, it crowds out the construction sector due to resource reallocation. This effect is particularly harmful for indebted households, whose welfare declines amid limited labor mobility and credit access. The chapter underscores that fiscal interventions must account for structural features such as workers' mobility across sectors and private debt levels to avoid unintended welfare costs.

Chapter 3 extends the analysis to climate policy by evaluating the heterogeneous effects of carbon pricing across household types and economic sectors. Through an Environmental DGE (E-DGE) model, it demonstrates that residential carbon taxes depress housing investment and asset values, disproportionately harming lenders, while nonresidential carbon pricing mainly affects borrowers via labor income losses. These findings highlight a key trade-off: residential taxes may be more environmentally effective but financially destabilizing, while nonresidential taxes are more equitable but require steeper price paths. The chapter makes the case for sector-specific carbon pricing and adaptive compensation mechanisms as carbon dividends, which can buffer distributional impacts but lose effectiveness over time.

Chapter 4 integrates the insights from previous chapters into a comprehensive welfare-based loss function that accommodates climate concerns, credit constraints, and price rigidities. This analytical tool enables policymakers to assess the macroeconomic costs of deviating from optimal monetary and macroprudential policies in a climate-constrained context. The model illustrates that ambitious climate goals, particularly when abatement is costly, can amplify output-consumption gaps and exacerbate macroeconomic instability. However, by aligning monetary and macroprudential frameworks with climate objectives, the welfare losses from such trade-offs can be mitigated.

Together, the thesis contributes to our understanding of how structural frictions and distributional dynamics reshape the effectiveness of economic policy. The key connecting thread across chapters is the role of household heterogeneity—particularly in housing and debt—and its implications for the design of targeted, equitable, and stable policy responses. We think that the findings presented across chapters offer relevant insights for policymakers navigating the complex intersections of housing, finance, and climate transition, highlighting the need for policy instruments that not only achieve their intended objectives but also safeguard macroeconomic and financial stability in the presence of structural and distributional frictions.

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Appendices

A. Appendix to Chapter 2

This appendix provides supplementary material for Chapter 2, Government expenditure and the housing puzzle. It includes the steady-state calculations used for model calibration, and two additional results: structural shocks that trigger a decline in housing prices and the main findings under a closed economy framework.

A.1 Steady state

Key subvector of variables

$$\mathbf{X} = (z^c, \theta, \frac{c^b}{c^l}, \gamma_h)$$

Steady-state values imposed

Endogenous variables	Exogenous variables	Ratios
$GDP = 1$	$\frac{g}{GDP} = 0.18$	$\frac{qI^h}{GDP} = 0.073$
$q = 1$		$\frac{p^j j}{GDP} = 0.16$
$p = 1$		$\frac{n_h}{n_h+n_c} = 0.128$
$p^j = 1$		$\frac{c_h}{c} = 0.7068$
$\pi = 1$		$\frac{j_h}{j} = 0.7826$

Steady-state values which follow straightforwardly

$$p^{reu} = \left(\frac{(p^j)^{1-\sigma_j} - (1-\omega_j)p^{1-\sigma_j}}{\omega_j} \right)^{\frac{1}{1-\sigma_j}}$$

$$\pi^c = \pi$$

$$\pi^{reu} = \pi^c$$

$$\pi^{eu} = \pi^{reu}$$

$$r = \frac{\pi^c}{\beta^l}$$

$$r^{eu} = r$$

$$\phi_b = \frac{r}{r^{eu}}$$

$$b^{eu} = 0$$

$$b^{l,eu} = \frac{b^{eu}}{1 - \tau^b}$$

$$y = GDP - qI^h$$

$$h = \frac{I^h}{\delta^h}$$

$$c = GDP - g - qI^h - p^j j$$

$$k = \frac{j}{\delta^k}$$

$$k^l = \frac{k}{1 - \tau^b}$$

$$r^k = \frac{1}{\beta^l} + \delta^k - 1$$

$$mc = \frac{\varepsilon - 1}{\varepsilon}$$

$$p^* = p$$

$$tr = g$$

$$c_h = c(1 - \omega_c)(p)^{-\sigma_c}$$

$$c_f = c\omega_c(p^{reu})^{-\sigma_c}$$

$$j_h = j(1 - \omega_j)\left(\frac{p}{p^j}\right)^{-\sigma_j}$$

$$j_f = j\omega_j\left(\frac{p^{reu}}{p^j}\right)^{-\sigma_j}$$

$$im = c_f + j_f$$

$$ex = \frac{p^{reu} im}{p}$$

$$y^{reu} = \frac{ex}{s^x}\left(\frac{p}{p^{reu}}\right)^{\sigma_x}$$

Steady-state values that are a function of X

$$c^l = \frac{c}{\tau^b(1-(1-\frac{c^b}{c^l}))} \equiv \mathbf{c^l(x)}$$

$$c^b = \frac{c^b}{c^l}\mathbf{c^l(x)} \equiv \mathbf{c^b(x)}$$

$$\lambda^l = \frac{1}{\mathbf{c^l(x)}} \equiv \lambda^{\mathbf{l}}(\mathbf{x})$$

$$\lambda^b = \frac{1}{\mathbf{c^b(x)}} \equiv \lambda^{\mathbf{b}}(\mathbf{x})$$

$$\mu = \lambda^{\mathbf{b}}(\mathbf{x})(\frac{1}{r} - \beta^b) \equiv \mu(\mathbf{x})$$

$$n_c = \left(\frac{y}{z^c k^\alpha}\right)^{\frac{1}{1-\alpha}} \equiv \mathbf{n_c(x)}$$

$$w_c = \frac{mc(1-\alpha)y}{\mathbf{n_c(x)}} \equiv \mathbf{w_c(x)}$$

$$n_h = \frac{0.138}{1-0.138}\mathbf{n_c(x)} \equiv \mathbf{n_h(x)}$$

$$z^h = \frac{I^h}{\mathbf{n_h(x)}} \equiv \mathbf{z^h(x)}$$

$$w_h = q\mathbf{z^h(x)} \equiv \mathbf{w_h(x)}$$

$$n^l = \left(\frac{1}{\theta}\left(\frac{\mathbf{w_c(x)}}{\mathbf{c^l(x)}}\right)^{1+\varepsilon_n} + \frac{1}{1-\theta}\left(\frac{\mathbf{w_h(x)}}{\mathbf{c^l(x)}}\right)^{1+\varepsilon_n}\right)^{\frac{1}{\eta(1+\varepsilon_n)}} \equiv \mathbf{n^l(x)}$$

$$n_c^l = \left(\frac{\mathbf{w_c(x)}(\mathbf{n^l(x)})^{\frac{1}{\varepsilon_n}-\eta}}{\mathbf{c^l(x)}\theta^{\frac{1}{\varepsilon_n}}}\right)^{\varepsilon_n} \equiv \mathbf{n_c^l(x)}$$

$$n_h^l = \left(\frac{\mathbf{w_h(x)}(\mathbf{n^l(x)})^{\frac{1}{\varepsilon_n}-\eta}}{\mathbf{c^l(x)}(1-\theta)^{\frac{1}{\varepsilon_n}}}\right)^{\varepsilon_n} \equiv \mathbf{n_h^l(x)}$$

$$n^b = \left(\frac{1}{\theta}\left(\frac{\mathbf{w_c(x)}}{\mathbf{c^b(x)}}\right)^{1+\varepsilon_n} + \frac{1}{1-\theta}\left(\frac{\mathbf{w_h(x)}}{\mathbf{c^b(x)}}\right)^{1+\varepsilon_n}\right)^{\frac{1}{\eta(1+\varepsilon_n)}} \equiv \mathbf{n^b(x)}$$

$$n_c^b = \left(\frac{\mathbf{w_c(x)}(\mathbf{n^b(x)})^{\frac{1}{\varepsilon_n}-\eta}}{\mathbf{c^b(x)}\theta^{\frac{1}{\varepsilon_n}}}\right)^{\varepsilon_n} \equiv \mathbf{n_c^b(x)}$$

$$n_h^b = \Big(\frac{\mathbf{w_h}(\mathbf{x})(\mathbf{n^b}(\mathbf{x}))^{\frac{1}{\varepsilon_n}-\eta}}{\mathbf{c^b}(\mathbf{x})(1-\theta)^{\frac{1}{\varepsilon_n}}}\Big)^{\varepsilon_n} \equiv \mathbf{n_h^b}(\mathbf{x})$$

$$n_c = \tau^b \mathbf{n_c^b}(\mathbf{x}) + (1-\tau^b)\mathbf{n_c^l}(\mathbf{x}) \equiv \mathbf{n_c}(\mathbf{x})$$

$$n_h = \tau^b \mathbf{n_h^b}(\mathbf{x}) + (1-\tau^b)\mathbf{n_h^l}(\mathbf{x}) \equiv \mathbf{n_h}(\mathbf{x})$$

$$h^b = \frac{\gamma^h}{q\lambda^{\mathbf{b}}(\mathbf{x})(1-\beta^b(1-\delta^h)) - q\kappa\mu(\mathbf{x})} \equiv \mathbf{h^b}(\mathbf{x})$$

$$h^l = \frac{\gamma^h}{q\lambda^{\mathbf{l}}(\mathbf{x})(1-\beta^b(1-\delta^h))} \equiv \mathbf{h^l}(\mathbf{x})$$

$$h = \tau^b \mathbf{h^b}(\mathbf{x}) + (1-\tau^b)\mathbf{h^l}(\mathbf{x}) \equiv \mathbf{h}(\mathbf{x})$$

$$b^b = \kappa\frac{q\mathbf{h^b}(\mathbf{x})}{r} \equiv \mathbf{b^b}(\mathbf{x})$$

$$b^b = \frac{\mathbf{c^b}(\mathbf{x}) + q\delta^h\mathbf{h^b}(\mathbf{x}) + tr - \mathbf{w_c}(\mathbf{x})\mathbf{n_c^b}(\mathbf{x}) - \mathbf{w_h}(\mathbf{x})\mathbf{n_h^b}(\mathbf{x})}{1-r} \equiv \mathbf{b^b}(\mathbf{x})$$

Nonlinear subsystem for the steady state value of X

The steady-state value of $\mathbf{X} = (z^c, \theta, \frac{c^b}{c^l}, \gamma_h)$ is the solution to the following four-equation nonlinear subsystem:

$$\Big(\frac{y}{z^c k^\alpha}\Big)^{\frac{1}{1-\alpha}} - (\tau^b\mathbf{n_c^b}(\mathbf{x}) + (1-\tau^b)\mathbf{n_c^l}(\mathbf{x})) = 0$$

$$\frac{0.138}{1-0.138}\mathbf{n_c}(\mathbf{x}) - (\tau^b\mathbf{n_h^b}(\mathbf{x}) + (1-\tau^b)\mathbf{n_h^l}(\mathbf{x})) = 0$$

$$\frac{I^h}{\delta^h} - (\tau^b\mathbf{h^b}(\mathbf{x}) + (1-\tau^b)\mathbf{h^l}(\mathbf{x})) = 0$$

$$\kappa\frac{q\mathbf{h^b}(\mathbf{x})}{r} - (\frac{\mathbf{c^b}(\mathbf{x}) + q\delta^h\mathbf{h^b}(\mathbf{x}) + tr - \mathbf{w_c}(\mathbf{x})\mathbf{n_c^b}(\mathbf{x}) - \mathbf{w_h}(\mathbf{x})\mathbf{n_h^b}(\mathbf{x})}{1-r}) = 0$$

Rest of steady state Given the steady-state values computed so far the rest of them follow straightforwardly:

$$V = \frac{y\lambda^l mc}{1 - \omega\beta^l}$$

$$F = \frac{y\lambda^l}{1 - \omega\beta^l}$$

$$n = \tau^b(n_h^b + n_c^b) + (1 - \tau^b)(n_h^l + n_h^l)$$

$$b^l = -b^b \frac{\tau^b}{1 - \tau^b}$$

A.2 Other shocks leading to a decline in housing prices

Here we shock the economy with an unanticipated and permanent change in: (a) the productivity of the tradable goods sector, relative to the housing sector ($\phi_{c,t}$-$\phi_{h,t}$); (b) the Eurozone interest rate ($\phi_{r,t}$); (c) the housing demand preferences ($\phi_{j,t}$); (d) the LTV ratio ($\phi_{\kappa,t}$). We hold constant $\phi_{h,t}$ at the initial value of 1, and thus the change in $\phi_{c,t}$-$\phi_{h,t}$ is completely driven by $\phi_{c,t}$.

For the sake of comparison, we use the long-run percentage decline in house prices as a metric, so that all shocks produce the same drop in this variable. Table A.1 shows the long-term responses of some variables. The first row represents the percentage change from the value of 1 in each of the four shifters. The interest rate shock is equivalent to a permanent annual decrease of 30 basis points in the ECB interest rate. We also represent the long-run effects on residential investment, on hours worked in the tradable goods sector, on hours worked in the housing sector and, finally, the effect on GDP.

The long-run effects of a preference shock and an LTV ratio shock are quite similar. To produce a 1 percent reduction in housing prices, both reduce residential investment by more than 0.80%, which translates into an equivalent decrease in hours worked in this sector. However, the effect on hours worked in the tradable goods sector is negligible. Overall, GDP declines moderately, by about -0.15%. This implies high elasticities of residential property prices and residential investment to GDP of more than 6.7 and 5.6, respectively, conditional on these shocks.

A technology shock and a monetary shock produce long-term responses in the variables considered that go in the same direction. A monetary shock, however, causes a greater redistribution of labor in favor of the tradable sector. On the other hand, the shock that increases the total productivity of the tradable goods sector is the one that is most detrimental to aggregate output.

Table A.1: Long-run macroeconomic effects to different structural shocks

Description	$\phi_{c,t}$	$\phi_{r,t}$	$\phi_{j,t}$	$\phi_{\kappa,t}$
Shock size	−1.60	−0.02	−1.80	−10.4
Housing price	**-1.00**	**-1.00**	**-1.00**	**-1.00**
Residential investment	0.02	0.03	−0.80	−0.84
Hours tradable good sector	0.12	0.29	−0.02	−0.04
Hours housing sector	0.02	0.03	−0.80	−0.84
GDP	−0.77	−0.41	−0.14	−0.15
Long-run elasticity (price)	1.3	2.4	7.1	6.7
Long-run elasticity (investment)	−0.0	−0.1	5.7	5.6

Note: Long-run responses (in %) to different shocks that reduce housing prices by 1%. The shocks include changes in relative productivity ($\phi_{c,t}$-$\phi_{h,t}$), the Eurozone interest rate ($\phi_{r,t}$), housing demand preferences ($\phi_{j,t}$), and the LTV ratio ($\phi_{\kappa,t}$). The interest rate shock corresponds to a permanent 30 basis point decrease in the ECB rate.

Conditional on this shock, the long-run elasticity of real estate prices to GDP is only 1.3, one-fifth of the elasticity conditional on preference or LTV shocks, whereas the residential investment elasticity to GDP is virtually zero.

Hence, the long-run elasticities of housing prices and residential investment to GDP change dramatically depending on the type of shock that affects the economy.

A.3 Closed economy

Spain is a member of the monetary union, and our benchmark model takes this into account. However, it would be worthwhile to explore the repercussions of a recession. This allows us to identify specific mechanisms that contribute to the results. The closed economy model retains the same structural parameters as the open economy, with the exception of those relating to the external sector.

Figure A.3.1 replicates Figure 2.7 in the main text for a closed economy, demonstrating the impact responses of GDP, residential investment, and housing prices on various labor market characteristics. There are quantitative and qualitative differences between the benchmark open economy and the monetary union. In general, the main distinctions we observe are as follows: The magnitude of the effects on housing investment and GDP is lower, but the impact on house prices is greater, and, unlike in the open-economy case,

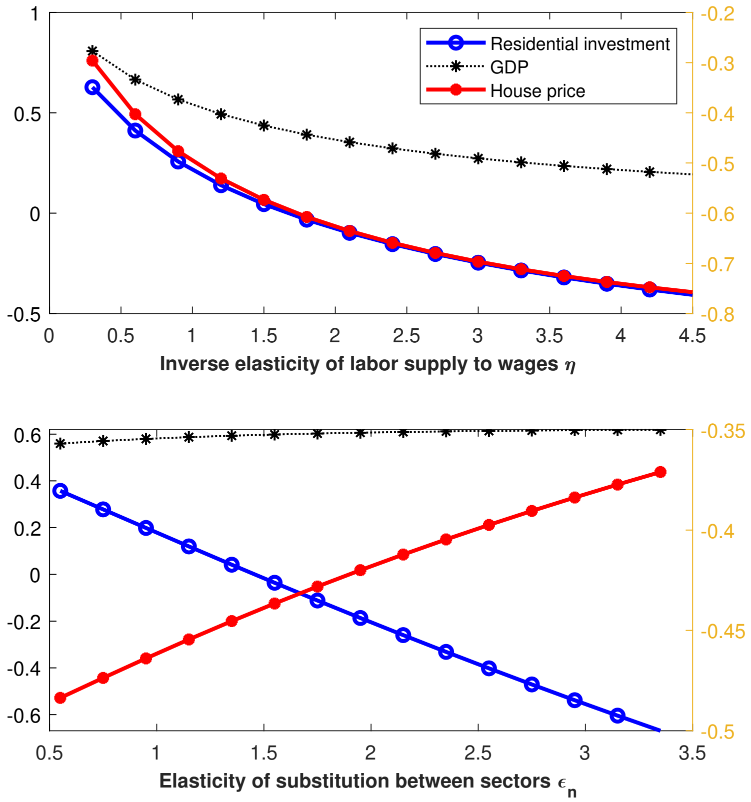
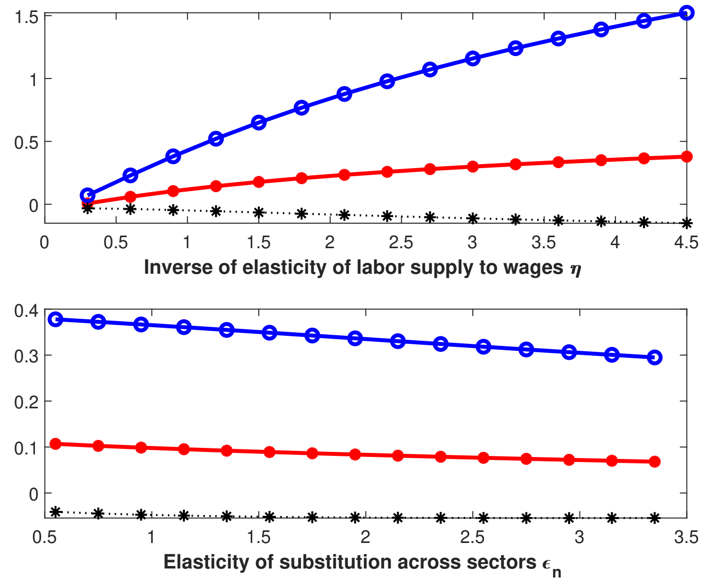


Figure A.3.1: Impact responses (in %) to a 1% public spending shock on various labor market parameters in a closed economy: GDP, residential investment, and house prices. Note: Residential investment and GDP are on the left scale, and house prices are on the right scale.

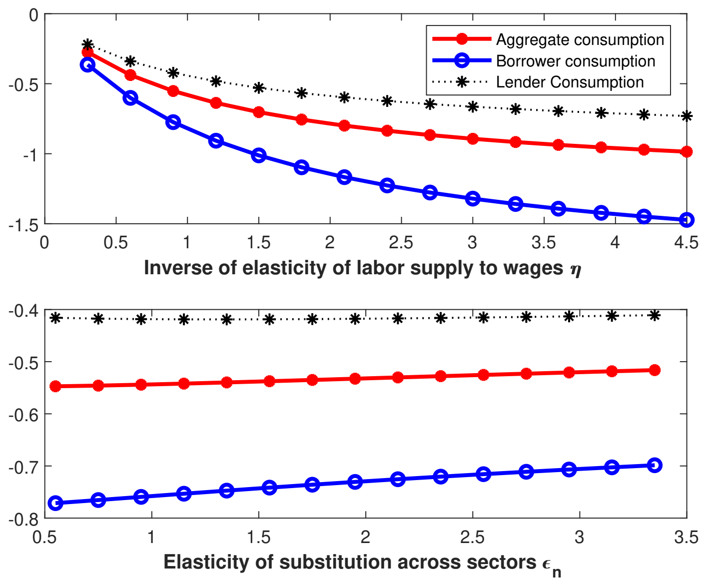
GDP, residential investment, and house prices all move in the same direction as labor supply becomes more inelastic.

In a closed economy, the central bank responds more directly to domestic inflation than in a monetary union, effectively curbing inflation to a greater extent. Consequently, the real interest-rate experiences a less significant decline in a closed economy. Real interest-rate movements are important in influencing borrowers' consumption. A decrease in the real interest rate relieves their budget constraints by increasing their home's equity and improving their credit access (Equation 2.16), while also lowering the repayment value of outstanding debt (Equation 2.15). As a result, borrowers increase their demand for consumer goods as well as for housing.

Figure A.3.2 depicts the different consumption patterns for nonresidential



(a) Open economy within a monetary union



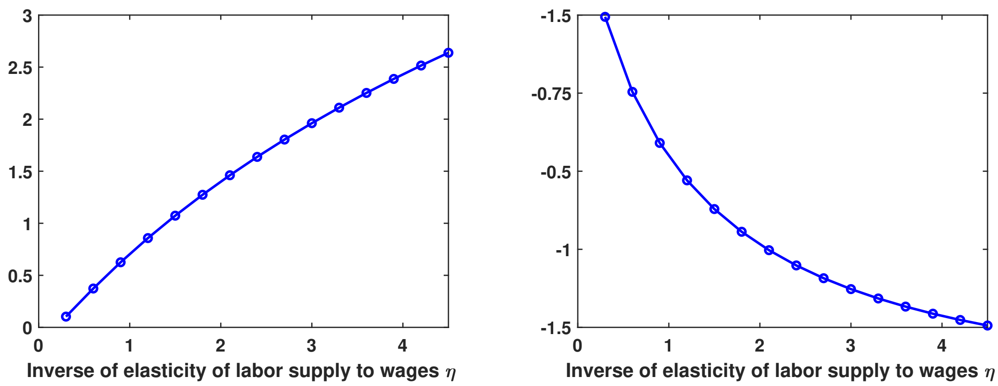
(b) Closed economy

Figure A.3.2: Impact responses (in%) to a public spending shock of 1% of GDP for different labor elasticities: Consumption goods in an open economy within a monetary union versus a closed economy. The central bank's varying ability to control the growth of consumption prices results in a completely different consumption response, primarily influenced by borrower behavior. In the benchmark open economy within a monetary union (in the closed economy), private consumption adds to (subtracts from) public consumption, amplifying (alleviating) the impact of the fiscal shock on labor market wages for consumption goods and exacerbating (mitigating) the crowding-out effect on the residential labor market and, as a result, on residential investment. This explains why the intensity of the effect that government consumption has on GDP and residential investment is lower in a closed economy than in the benchmark open economy of a monetary union.

The movement in the relative price of durables caused by the fiscal shock in the presence of borrowing constraints explains the response of housing prices. The increase in public spending decreases the relative price of durable goods and, consequently, the value of the collateral asset. As a result, borrowers reduce their borrowing. This reduction in collateral demand drives down the price of durable goods, creating a feedback loop that affects collateral value. In our benchmark open economy, a lower real interest rate increases borrowers' net worth, offsetting the declining value of collateral due to the relative price effect. In contrast, in a closed economy, the negative effect of relative prices on borrowing capacity is reinforced by the stronger response of the monetary authority, resulting in further declines in house demand. This explains why, unlike in a monetary union, house prices and residential investment respond in the same direction regardless of labor supply elasticity. When the elasticity of labor supply to wages is lower, wages increase more, and as a result, inflation increases. In a closed economy, the central bank's response to domestic inflation is stronger, so higher interest rates reduce housing demand and prices.

Impact of labor market on macroeconomic dynamics

A lower labor supply elasticity (higher η) leads to higher domestic inflation after a fiscal shock, following a higher increase in wages. The greater the difference between domestic inflation and constant foreign inflation, the greater the central bank's ability to stabilize domestic inflation within a monetary union versus a closed economy. This leads to a more pronounced fall, as η increases, of real interest rates in the benchmark open economy with respect to the closed economy. It also results in a more pronounced increase in open-economy goods consumption. In the closed economy, borrowers' private consumption and housing demand decrease, unlike in the benchmark economy where they increase with higher η. In a closed economy, the response of the ratio between the consumption goods market wage and the residential market wage becomes less pronounced as η increases, and there is a lower decrease in residential investment after the less dynamic labor reallocation caused by the fiscal shock.



(a) Open economy within a monetary union **(b)** Closed economy

Figure A.3.3: Impact responses (in%) to a public spending shock of 1% of GDP for different labor elasticities: Borrowers' Housing Demand

The differing consumption behavior in response to changes in η helps explain GDP's declining response to changes in labor supply elasticity. In contrast to the economy within a monetary union, the reaction to housing prices is primarily driven by decreasing housing demand, which intensifies with η (Figure A.3.3). This explains why the pattern observed for this variable in Figure A.3.1 differs from Figure 2.7.

B. Appendix to Chapter 4

This appendix provides supplementary material for Chapter 4, A loss function approach under climate concerns. It includes the steady-state calculations required to align the competitive steady state with the efficient steady state, and a detailed derivation of the welfare loss function.

B.1 Log-linearized equations

All variables are in log-deviations from the efficient steady state.

Aggregate demand

$$\hat{c}_t^l = E_t \hat{c}_{t+1}^l - E_t(\hat{r}_t - \hat{\pi}_{t+1})$$

$$\hat{w}_t^l = \hat{c}_t^l + \eta \hat{n}_t^l$$

$$\hat{c}_t^b = \frac{\beta^b}{\beta^l} E_t(\hat{c}_{t+1}^b + \hat{\pi}_{t+1}) - \hat{r}_t + \frac{\beta^l - \beta^b}{\beta^l} \hat{\mu}_t^b$$

$$\hat{w}_t^b = \hat{c}_t^b + \eta \hat{n}_t^b$$

$$\hat{b}_t^b = \hat{q} + \hat{\pi}_{t+1} + \hat{h}_t^b - \hat{r}_t$$

$$\hat{c}_t^b + \frac{qh}{c}(\hat{h}_t^b - \hat{h}_{t-1}^b) - \frac{b^b}{c}\hat{b}_t^b = \frac{w^b n^b}{c}(\hat{w}_t^b + \hat{n}_t^b) - \frac{b^b r}{c}(\hat{r}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1}^b)$$

Aggregate supply

$$\hat{y}_t = -\gamma x \hat{x}_t + a_t + \alpha \hat{n}_t^b + (1-\alpha)\hat{n}_t^l$$

$$\hat{\pi}_t = \beta^l \hat{\pi}_{t+1} + \frac{\varepsilon}{\Phi_p}[\tau_e^* \varphi (1-u)\hat{\tau}_{et} + \psi \hat{\psi}_t]$$

$$\hat{\psi}_t = (\alpha - \tau^b)(\hat{c}_t^b - \hat{c}_t^l) - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t + (\eta + 1)(\hat{y}_t - a_t + \gamma x \hat{x}_t)$$

$$\hat{w}_t^l = \hat{\psi}_t + \hat{y}_t - \hat{n}_t^l$$

$$\hat{w}_t^b = \hat{\psi}_t + \hat{y}_t - \hat{n}_t^b$$

$$\hat{e}_t = \hat{y}_t - \frac{u}{1-u}\hat{u}_t$$

$$\hat{\tau}_{et} = (\theta_2 - 1)\hat{u}_t$$

Pollution dynamics

$$\hat{x}_t = \frac{e}{x}\sum_{s=0}^{t+T}(1-\delta^x)^s \hat{e}_{t-s}$$

Equilibrium

$$\tau^b \hat{h}_t^b + (1-\tau^b)\hat{h}_t^l = 0$$

$$\tau^b \hat{c}_t^b + (1-\tau^b)\hat{c}_t^l = \hat{y}_t - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t$$

Carbon taxes

$$\hat{\tau}_{et}^* = \hat{c}_t - \gamma e \sum_{s=0}^{\infty}(\beta^l(1-\delta^x))^s \left((\theta_2 - 1) + \frac{y}{c}\theta_1\theta_2 u^{\theta_2} - \frac{u}{1-u}\right)\hat{u}_{t+s}$$

B.2 Efficient Steady State

In efficient equilibrium, the steady-state equations are derived as follows. The marginal utilities satisfy

$$U_c^l = U_c^b = \mu_1 \tag{B.2.1}$$

$$U_h^l = U_h^b = \mu_2 \tag{B.2.2}$$

and consequently

$$\frac{U_c^l}{U_h^l} = \frac{U_c^b}{U_h^b} = \frac{\mu_1}{\mu_2} \tag{B.2.3}$$

For labor supply, the following optimality conditions hold

$$U_n^l = -\frac{\mu_1(1-\alpha)y}{(1-\tau^b)n^l}\left[1-\theta_1 u^{\theta_2} - \frac{\mu_4}{\mu_1}\varphi(1-u)\right] = -\frac{\mu_3(1-\alpha)}{\gamma(1-\tau^b)n^l} \tag{B.2.4}$$

$$U_n^b = -\frac{\mu_1 \alpha y}{\tau^b n^b}\left[1-\theta_1 u^{\theta_2} - \frac{\mu_4}{\mu_1}\varphi(1-u)\right] = -\frac{\mu_3 \alpha}{\gamma \tau^b n^b} \tag{B.2.5}$$

From the constraints on resources and production technology at the symmetric steady state, we obtain

$$c^l = c^b = c \tag{B.2.6}$$

$$h^l = h^b = \bar{h} \tag{B.2.7}$$

and the aggregate consumption satisfies

$$c = y(1-\theta_1 u^{\theta_2}) \tag{B.2.8}$$

Moreover, the steady-state ratios involving multipliers are given by

$$\frac{\mu_4}{\mu_1} = \frac{\theta_1\theta_2 u^{\theta_2-1}}{\varphi} \tag{B.2.9}$$

and the steady-state multipliers are defined by:

$$\mu_4 = \frac{\mu_3}{1-\beta^l(1-\delta^x)} = \frac{\gamma}{1-\beta^l(1-\delta^x)+\gamma e} \tag{B.2.10}$$

$$\mu_3 = \gamma y \mu_1 \left(1-\theta_1 u^{\theta_2} - \frac{\mu_4}{\mu_1}\varphi(1-u)\right) = \gamma(1-\mu_4 e) \tag{B.2.11}$$

Finally, the atmospheric carbon stock in steady state is approximately given by

$$x = \frac{e\left[1-(1-\delta^x)^{t+T+1}\right]}{\delta^x} \approx \frac{e}{\delta^x} \tag{B.2.12}$$

B.2.1 Implementation of the Efficient Steady State

In this subsection, we provide detailed calculations for deriving the taxes and subsidies that enable the market economy to replicate the efficient steady state described earlier. By comparing equation (4.22) with its efficient counterpart, the steady-state optimal carbon tax is straightforwardly derived as

$$\tau_e^* = \frac{\mu_4}{\mu_1}$$

Next, we use the steady-state conditions from labor-market equilibrium for lenders (equations (4.5) and (4.20)) and for borrowers (equations (4.12) and (4.21)), combined with equation (4.49), to obtain

$$(n^l)^\eta = \left[(1+\tau_p)\frac{1-\varepsilon}{\varepsilon} - \theta_1 u^{\theta_2} - \tau_e \varphi(1-u)\right] \frac{(1-\alpha)y}{(1-\tau^b)n^l c^l} \tag{B.2.13}$$

$$(n^b)^\eta = \left[(1+\tau_p)\frac{1-\varepsilon}{\varepsilon} - \theta_1 u^{\theta_2} - \tau_e \varphi(1-u)\right] \frac{\alpha y}{\tau^b n^b c^b} \tag{B.2.14}$$

We determine the production subsidy τ_p by equating these conditions with their efficient counterparts (4.41) and (4.42). This yields

$$\tau_p = \frac{\varepsilon}{\varepsilon - 1} - 1$$

From borrowers' budget constraint (equation (4.8)), the subsidy required to equalize borrowers' consumption is

$$s^b = \left(1 - \frac{\alpha}{\tau^b}\right) c + \frac{1-\beta^l}{\beta^l} b^b + \tau_e e$$

Using the Euler equations for lenders and borrowers (equations (4.6) and (4.13))

$$r = \frac{1}{\beta^l}$$

$$\mu^b = \frac{\beta^l - \beta^b}{c^b}$$

and the housing demand equations for lenders and borrowers (equations (4.4) and (4.11))

$$\frac{c^b}{h^b} = \frac{q}{\gamma_h}(1 - \beta^b - \kappa(\beta^l - \beta^b))$$

$$\frac{c^l}{h^l} = \frac{q}{\gamma_h}(1 - \beta^l + \tau_h)$$

At the symmetric steady state, the consumption-to-housing ratio is equal across households, yielding:

$$\tau_h = (\beta^l - \beta^b)(1 - \kappa)$$

B.3 Derivation of the quadratic loss function

To construct the welfare-based loss function, we take a weighted-average of the per period utility functions of patient and impatient households. The planner is assumed to discount the future at the savers' discount rate, β^l. This objective is equivalent to maximizing the following welfare objective function

$$W_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^l)^t U_t$$

where

$$U_t = (1-\tau^b)\left(\ln c_t^l + \gamma_h \ln h_t^l - \frac{(n_t^l)^{1+\eta}}{1+\eta} \right) + \tau^b \left(\ln c_t^b + \gamma_h \ln h_t^b - \frac{(n_t^b)^{1+\eta}}{1+\eta} \right)$$

Following Rotemberg & Woodford (1998), we derive a second-order approximation of the welfare function around the efficient steady state defined in the previous section.

$$\begin{aligned} U_t - U \approx \\ &\omega^p[U_c^b(c_t^b - c^b) + \frac{1}{2}U_{cc}^b(c_t^b - c^b)^2] + (1-\omega^p)[U_c^l(c_t^l - c^l) + \frac{1}{2}U_{cc}^l(c_t^l - c^l)^2] \\ &+\omega^p[U_h^b(h_t^b - h^b) + \frac{1}{2}U_{hh}^b(h_t^b - h^b)^2] + (1-\omega^p)[U_h^l(h_t^l - h^l) + \frac{1}{2}U_{hh}^l(h_t^l - h^l)^2] \\ &+\omega^p[U_n^b(n_t^b - n^b) + \frac{1}{2}U_{nn}^b(n_t^b - n^b)^2] + (1-\omega^p)[U_n^l(n_t^l - n^l) + \frac{1}{2}U_{nn}^l(n_t^l - n^l)^2] \end{aligned}$$

Factoring out the marginal utility of consumption, housing and labor for each household

$$\begin{aligned} U_t - U \approx \\ &\omega^p U_c^b[(c_t^b - c^b) + \frac{1}{2}\frac{U_{cc}^b}{U_c^b}(c_t^b - c^b)^2] + (1-\omega^p)U_c^l[(c_t^l - c^l) + \frac{1}{2}\frac{U_{cc}^l}{U_c^l}(c_t^l - c^l)^2] \\ &+\omega^p U_h^b[(h_t^b - h^b) + \frac{1}{2}\frac{U_{hh}^b}{U_h^b}(h_t^b - h^b)^2] + (1-\omega^p)U_h^l[(h_t^l - h^l) + \frac{1}{2}\frac{U_{hh}^l}{U_h^l}(h_t^l - h^l)^2] \\ &+\omega^p U_n^b[(n_t^b - n^b) + \frac{1}{2}\frac{U_{nn}^b}{U_n^b}(n_t^b - n^b)^2] + (1-\omega^p)U_n^l[(n_t^l - n^l) + \frac{1}{2}\frac{U_{nn}^l}{U_n^l}(n_t^l - n^l)^2] \end{aligned}$$

A. Substituting marginal utilities with their values at the efficient equilibrium (as determined by the first-order conditions of the efficient steady state) and considering planner weights coincide with the population share

$(\omega^p = \tau^b)$,

$$
\begin{aligned}
&U_t - U \approx \\
&\quad \tau^b \mu_1[(c_t^b - c^b) + \frac{1}{2}\frac{U_{cc}^b}{U_c^b}(c_t^b - c^b)^2] + (1-\tau^b)\mu_1[(c_t^l - c^l) + \frac{1}{2}\frac{U_{cc}^l}{U_c^l}(c_t^l - c^l)^2] \\
&+\tau^b \mu_2[(h_t^b - h^b) + \frac{1}{2}\frac{U_{hh}^b}{U_h^b}(h_t^b - h^b)^2] + (1-\tau^b)\mu_2[(h_t^l - h^l) + \frac{1}{2}\frac{U_{hh}^l}{U_h^l}(h_t^l - h^l)^2] \\
&-\alpha\frac{\mu_3}{\gamma n^b}[(n_t^b - n^b) + \frac{1}{2}\frac{U_{nn}^b}{U_n^b}(n_t^b - n^b)^2] - (1-\alpha)\frac{\mu_3}{\gamma n^l}[(n_t^l - n^l) + \frac{1}{2}\frac{U_{nn}^l}{U_n^l}(n_t^l - n^l)^2]
\end{aligned}
$$

Given our preferences assumption, at the efficient steady state

$$
\frac{U_{cc}^j}{U_c^j} = -\frac{1}{c}
$$
$$
\frac{U_{hh}^j}{U_h^j} = -\frac{1}{h}
$$
$$
\frac{U_{nn}^j}{U_n^j} = \frac{\eta}{n^j}
$$

Substituting and collecting linear terms in consumption and housing,

$$
\begin{aligned}
&U_t - U \approx \\
&\quad \mu_1[\tau^b(c_t^b - c^b) + (1-\tau^b)(c_t^l - c^l)] + \mu_2[\tau^b(h_t^b - h^b) + (1-\tau^b)(h_t^l - h^l)] \\
&\quad - \mu_1\frac{1}{2c}[\tau^b(c_t^b - c^b)^2 + (1-\tau^b)(c_t^l - c^l)^2] \\
&\quad - \mu_2\frac{1}{2h}[\tau^b(h_t^b - h^b)^2 + (1-\tau^b)(h_t^l - h^l)^2] \\
&-\alpha\frac{\mu_3}{\gamma n^b}[(n_t^b - n^b) + \frac{1}{2}\frac{\eta}{n^b}(n_t^b - n^b)^2] - (1-\alpha)\frac{\mu_3}{\gamma n^l}[(n_t^l - n^l) + \frac{1}{2}\frac{\eta}{n^l}(n_t^l - n^l)^2]
\end{aligned}
$$

B. To eliminate first order terms, we approximate the market clearing conditions for goods and housing to second order.

B.1. Approximation of the housing market constraint,

$$
h = \tau^b h_t^b + (1-\tau^b)h_t^l
$$

We have,

$$
0 = \tau^b(h_t^b - h^b) + (1-\tau^b)(h_t^l - h^l)
$$

B.2. Second-order approximation of the resource constraint,

$$
\tau^b c_t^b + (1-\tau^b)c_t^l = y_t(1 - \theta_1 u_t^{\theta_2} - \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2)
$$

Let's calculate first right-hand side term $f(y_t, u_t, \pi_t) = y_t(1 - \theta_1 u_t^{\theta_2} - \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2)$

$$\frac{\partial f}{\partial y_t} = 1 - \theta_1 u_t^{\theta_2} - \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2 \qquad \frac{\partial^2 f}{\partial^2 y_t} = 0$$
$$\frac{\partial f}{\partial u_t} = -y_t\theta_1\theta_2 u^{\theta_2 - 1} \qquad \frac{\partial^2 f}{\partial^2 u_t} = -y_t\theta_1\theta_2(\theta_2 - 1)u^{\theta_2 - 2}$$
$$\frac{\partial f}{\partial \pi_t} = -y_t\Phi_p(\pi_t - \bar{\pi}) \qquad \frac{\partial^2 f}{\partial^2 \pi_t} = -y_t\Phi_p$$
$$\frac{\partial f}{\partial y_t \partial u_t} = -\theta_1\theta_2 u_t^{\theta_2 - 1} \qquad \frac{\partial f}{\partial y_t \pi_t} = -\Phi_p(\pi_t - \bar{\pi})$$
$$\frac{\partial f}{\partial u_t \partial y_t} = -\theta_1\theta_2 u^{\theta_2 - 1} \qquad \frac{\partial f}{\partial u_t \partial \pi_t} = 0$$
$$\frac{\partial f}{\partial \pi_t \partial y_t} = -\Phi_p(\pi_t - \bar{\pi}) \qquad \frac{\partial f}{\partial \pi_t \partial u_t} = 0$$

Using the partial derivatives, the approximation of $f(y_t, u_t, \pi_t)$ around the steady state becomes,

$$\begin{aligned} & y(1 - \theta_1 u^{\theta_2}) + (1 - \theta_1 u^{\theta_2})(y_t - y) - y\theta_1\theta_2 u^{\theta_2 - 1}(u_t - u) \\ + & \frac{1}{2}(-y\theta_1\theta_2(\theta_2 - 1)u^{\theta_2 - 2}(u_t - u)^2 - y\Phi_p(\pi_t - \bar{\pi})^2) \\ - & \theta_1\theta_2 u^{\theta_2 - 1}(y_t - y)(u_t - u) \end{aligned}$$

Now, by calculating the second-order approximation of the left-hand side and substituting it into the resource constraint:

$$\begin{aligned} & \tau^b c^b + (1 - \tau^b)c^l + \tau^b(c_t^b - c^b) + (1 - \tau^b)(c_t^l - c^b) = \\ & y(1 - \theta_1 u^{\theta_2}) + (1 - \theta_1 u^{\theta_2})(y_t - y) - y\theta_1\theta_2 u^{\theta_2 - 1}(u_t - u) \\ - & \frac{1}{2}y\theta_1\theta_2(\theta_2 - 1)u^{\theta_2 - 2}(u_t - u)^2 - \frac{y}{2}\Phi_p(\pi_t - \bar{\pi})^2 \\ - & \theta_1\theta_2 u^{\theta_2 - 1}(y_t - y)(u_t - u) \end{aligned}$$

Using the identity $z_t \equiv e^{\ln z_t}$ for any variable z_t, we can approximate it around $\ln z$ to the second order as

$$\frac{z_t - z}{z} \approx \hat{z}_t + \frac{1}{2}\hat{z}_t^2$$

where $\hat{z}_t \equiv \ln \frac{z_t}{z}$. Applying this, the resource constraint can be approximated to the second order as

$$\tau^b(c_t^b - c^b) + (1-\tau^b)(c_t^l - c^b) =$$
$$y(1-\theta_1 u^{\theta_2})\hat{y}_t + \frac{y}{2}(1-\theta_1 u^{\theta_2})\hat{y}_t^2 - y\theta_1\theta_2 u^{\theta_2}\hat{u}_t - \frac{y}{2}\theta_1\theta_2 u^{\theta_2}\hat{u}_t^2 - \frac{y}{2}\theta_1\theta_2(\theta_2-1)u^{\theta_2}\hat{u}_t^2$$
$$-\theta_1\theta_2 u^{\theta_2} y(\hat{y}_t\hat{u}_t + \frac{1}{2}\hat{y}_t\hat{u}_t{}^2 + \frac{1}{2}\hat{y}_t{}^2\hat{u}_t + \frac{1}{4}\hat{y}_t{}^2\hat{u}_t{}^2)$$
$$-y\frac{\Phi_p}{2}\hat{\pi}_t^2$$

Rearranging terms

$$\tau^b(c_t^b - c^b) + (1-\tau^b)(c_t^l - c^b) =$$
$$y(1-\theta_1 u^{\theta_2})\hat{y}_t + \frac{y}{2}(1-\theta_1 u^{\theta_2})\hat{y}_t^2 - y\theta_1\theta_2 u^{\theta_2}\hat{u}_t - \frac{y}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2$$
$$-\theta_1\theta_2 u^{\theta_2} y(\hat{y}_t\hat{u}_t + \frac{1}{2}\hat{y}_t\hat{u}_t{}^2 + \frac{1}{2}\hat{y}_t{}^2\hat{u}_t + \frac{1}{4}\hat{y}_t{}^2\hat{u}_t{}^2)$$
$$-y\frac{\Phi_p}{2}\hat{\pi}_t^2$$

Omitting terms of third or higher order and rearranging, the equation becomes

$$\tau^b(c_t^b - c^b) + (1-\tau^b)(c_t^l - c^b) =$$
$$y(1-\theta_1 u^{\theta_2})\hat{y}_t - y\theta_1\theta_2 u^{\theta_2}\hat{u}_t + \frac{y}{2}(1-\theta_1 u^{\theta_2})\hat{y}_t^2 - \frac{y}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2 - \theta_1\theta_2 u^{\theta_2} y\hat{y}_t\hat{u}_t$$
$$-y\frac{\Phi_p}{2}\hat{\pi}_t^2$$

Substituting second order approximation in welfare function

$$U_t - U \approx$$
$$\mu_1 y[(1-\theta_1 u^{\theta_2})\hat{y}_t - \theta_1\theta_2 u^{\theta_2}\hat{u}_t + \frac{1}{2}(1-\theta_1 u^{\theta_2})\hat{y}_t^2 - \frac{1}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2 - \theta_1\theta_2 u^{\theta_2}\hat{y}_t\hat{u}_t - \frac{\Phi_p}{2}\hat{\pi}_t^2]$$
$$-\mu_1\frac{c}{2}[\tau^b(\hat{c}_t{}^b)^2 + (1-\tau^b)(\hat{c}_t{}^l)^2] - \mu_2\frac{h}{2}[\tau^b(\hat{h}_t{}^b)^2 + (1-\tau^b)(\hat{h}_t{}^l)^2]$$
$$-\frac{\mu_3}{\gamma}\alpha[\hat{n}_t{}^b + \frac{1+\eta}{2}(\hat{n}_t{}^b)^2] - \frac{\mu_3}{\gamma}(1-\alpha)[\hat{n}_t{}^l + \frac{1+\eta}{2}(\hat{n}_t{}^l)^2]$$

Rearranging linear terms,

$$\begin{aligned}U_t - U \approx & \mu_1 y[(1-\theta_1 u^{\theta_2})\hat{y}_t - \theta_1\theta_2 u^{\theta_2}\hat{u}_t] - \frac{\mu_3}{\gamma}(\alpha\hat{n}_t{}^b + (1-\alpha)\hat{n}_t{}^l) \\ & +\mu_1 y[\frac{1}{2}(1-\theta_1 u^{\theta_2})\hat{y}_t^2 - \frac{1}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2 - \theta_1\theta_2 u^{\theta_2}\hat{y}_t\hat{u}_t - \frac{\Phi_p}{2}\hat{\pi}_t^2] \\ & -\mu_1\frac{c}{2}[\tau^b(\hat{c}_t{}^b)^2 + (1-\tau^b)(\hat{c}_t{}^l)^2] - \mu_2\frac{h}{2}[\tau^b(\hat{h}_t{}^b)^2 + (1-\tau^b)(\hat{h}_t{}^l)^2] \\ & -\alpha\frac{\mu_3}{\gamma}\frac{1+\eta}{2}(\hat{n}_t{}^b)^2 - (1-\alpha)\frac{\mu_3}{\gamma}\frac{1+\eta}{2}(\hat{n}_t{}^l)^2\end{aligned}$$

C. To eliminate first order terms, let's log-linearize the aggregate production function.

$$\hat{y}_t = -\gamma x\hat{x}_t + \alpha\hat{n}_t^b + (1-\alpha)\hat{n}_t^l$$

Replacing from this equation for the weighted average of the two types of labor supply and dropping the term related to productivity (as it is independent of policy) first line of welfare loss function reads,

$$\begin{aligned}
&\mu_1 y[(1-\theta_1 u^{\theta_2})\hat{y}_t - \theta_1\theta_2 u^{\theta_2}\hat{u}_t] - \frac{\mu_3}{\gamma}(\alpha\hat{n}_t{}^b + (1-\alpha)\hat{n}_t{}^l) \\
&= \hat{y}_t - \mu_1 y\theta_1\theta_2 u^{\theta_2}\hat{u}_t - \frac{\mu_3}{\gamma}(\hat{y}_t + \gamma x\hat{x}_t) \\
&= \hat{y}_t[1-(1-y\mu_4\varphi(1-u))] - \mu_1 y\theta_1\theta_2 u^{\theta_2}\hat{u}_t - \mu_3 x\hat{x}_t \\
&= \hat{y}_t y\mu_4\varphi(1-u) - \mu_1 y\theta_1\theta_2 u^{\theta_2}\hat{u}_t - \mu_3 x\hat{x}_t \\
&= \hat{y}_t y\mu_4\varphi(1-u) - \mu_4 yu\varphi\hat{u}_t - \mu_3 x\hat{x}_t \\
&= y\varphi\mu_4[(1-u)\hat{y}_t - u\hat{u}_t] - \mu_3 x\hat{x}_t
\end{aligned}$$

Replacing in the welfare loss function

$$\begin{aligned}
U_t - U \approx\, & y\varphi\mu_4[(1-u)\hat{y}_t - u\hat{u}_t] - \mu_3 x\hat{x}_t \\
& + \mu_1 y[\frac{1}{2}(1-\theta_1 u^{\theta_2})\hat{y}_t^2 - \frac{1}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2 - \theta_1\theta_2 u^{\theta_2}\hat{y}_t\hat{u}_t - \frac{\Phi_p}{2}\hat{\pi}_t^2] \\
& - \mu_1\frac{c}{2}[\tau^b(\hat{c}_t{}^b)^2 + (1-\tau^b)(\hat{c}_t{}^l)^2] - \mu_2\frac{h}{2}[\tau^b(\hat{h}_t{}^b)^2 + (1-\tau^b)(\hat{h}_t{}^l)^2] \\
& - \alpha\frac{\mu_3}{\gamma}\frac{1+\eta}{2}(\hat{n}_t{}^b)^2 - (1-\alpha)\frac{\mu_3}{\gamma}\frac{1+\eta}{2}(\hat{n}_t{}^l)^2
\end{aligned}$$

Taking account that in the efficient steady state $c = y(1-\theta_1 u^{\theta_2})$

$$\begin{aligned}
U_t - U \approx\, & y\varphi\mu_4[(1-u)\hat{y}_t - u\hat{u}_t] - \mu_3 x\hat{x}_t \\
& - \mu_1 y[\frac{1}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2 + \theta_1\theta_2 u^{\theta_2}\hat{y}_t\hat{u}_t + \frac{\Phi_p}{2}\hat{\pi}_t^2] \\
& - \mu_1\frac{c}{2}[\tau^b(\hat{c}_t{}^b)^2 + (1-\tau^b)(\hat{c}_t{}^l)^2 - \hat{y}_t^2] - \mu_2\frac{h}{2}[\tau^b(\hat{h}_t{}^b)^2 + (1-\tau^b)(\hat{h}_t{}^l)^2] \\
& - \frac{\mu_3}{\gamma}\frac{1+\eta}{2}[\alpha(\hat{n}_t{}^b)^2 - (1-\alpha)(\hat{n}_t{}^l)^2]
\end{aligned}$$

D. We focus on the consumption terms inside the brackets on the third line and use the log-linearized resource constraint to substitute the difference between each household type's consumption and output.

$$\begin{aligned}
&\tau^b(\hat{c}_t{}^b)^2 + (1-\tau^b)(\hat{c}_t{}^l)^2 - \hat{y}_t^2 = \\
&\qquad = \tau^b[(\hat{c}_t{}^b)^2 - \hat{y}_t^2] + (1-\tau^b)[(\hat{c}_t{}^l)^2 - \hat{y}_t^2] \\
&\qquad = \tau^b(\hat{c}_t{}^b + \hat{y}_t)(\hat{c}_t{}^b - \hat{y}_t) + (1-\tau^b)(\hat{c}_t{}^l + \hat{y}_t)(\hat{c}_t{}^l - \hat{y}_t)
\end{aligned}$$

D.1. First, we log-linearize the resource constraint:
$\tau^b c_t^b + (1-\tau^b)c_t^l = y_t(1-\theta_1 u_t^{\theta_2} + \frac{\Phi_p}{2}(\pi_t - \pi)^2)$.

$$\begin{aligned}
y_t(1-\theta_1 u_t^{\theta_2} + \frac{\Phi_p}{2}(\pi_t-\pi)^2) \approx& y(1-\theta_1 u^{\theta_2}) + (1-\theta_1 u^{\theta_2})(y_t - y) - y\theta_1\theta_2 u^{\theta_2 - 1}(u_t - u) \\
=& y(1-\theta_1 u^{\theta_2}) + (1-\theta_1 u^{\theta_2}) y\hat{y}_t - y\theta_1\theta_2 u^{\theta_2}\hat{u}_t \\
\tau^b c_t^b + (1-\tau^b)c_t^l \approx& \tau^b c^b + (1-\tau^b)c^l + \tau^b(c_t^b - c^b) + (1-\tau^b)(c_t^b - c^b) \\
=& \tau^b c^b + (1-\tau^b)c^l + \tau^b c^b \hat{c}_t^b + (1-\tau^b)c^l\hat{c}_t^l \\
\tau^b c^b \hat{c}_t^b + (1-\tau^b)c^l \hat{c}_t^l =& (1-\theta_1 u^{\theta_2}) y\hat{y}_t - y\theta_1\theta_2 u^{\theta_2}\hat{u}_t
\end{aligned}$$

Taking account that in the efficient steady state $c^l = c^b = c$, then

$$\tau^b \hat{c}_t^b + (1-\tau^b)\hat{c}_t^l = \hat{y}_t - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t$$

D.2. Second, we calculate the difference between each types consumption and output,

$$\begin{aligned}
\hat{c}_t^b - \hat{y}_t =& \hat{c}_t^b - \tau^b\hat{c}_t^b - (1-\tau^b)\hat{c}_t^l - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t \\
=& (1-\tau^b)(\hat{c}_t^b - \hat{c}_t^l) - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t \\
\hat{c}_t^l - \hat{y}_t =& \hat{c}_t^l - \tau^b\hat{c}_t^b - (1-\tau^b)\hat{c}_t^l - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t \\
=& \tau^b(\hat{c}_t^l - \hat{c}_t^b) - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t
\end{aligned}$$

Substituting,

$$\begin{aligned}
&\tau^b(\hat{c}_t{}^b)^2 + (1-\tau^b)(\hat{c}_t{}^l)^2 - \hat{y}_t^2 = \\
&\quad = \tau^b(\hat{c}_t{}^b + \hat{y}_t)(\hat{c}_t{}^b - \hat{y}_t) + (1-\tau^b)(\hat{c}_t{}^l + \hat{y}_t)(\hat{c}_t{}^l - \hat{y}_t) \\
&\quad = \tau^b(\hat{c}_t{}^b + \hat{y}_t)[(1-\tau^b)(\hat{c}_t^b - \hat{c}_t^l) - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t] \\
&\qquad + (1-\tau^b)(\hat{c}_t{}^l + \hat{y}_t)[\tau^b(\hat{c}_t^l - \hat{c}_t^b) - \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t]
\end{aligned}$$

$$\begin{aligned}
=& \tau^b(1-\tau^b)(\hat{c}_t{}^b + \hat{y}_t)(\hat{c}_t^b - \hat{c}_t^l) - \tau^b(\hat{c}_t{}^b + \hat{y}_t)\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t \\
&- \tau^b(1-\tau^b)(\hat{c}_t{}^l + \hat{y}_t)(\hat{c}_t^b - \hat{c}_t^l) - (1-\tau^b)(\hat{c}_t{}^l + \hat{y}_t)\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t \\
=& \tau^b(1-\tau^b)(\hat{c}_t^b - \hat{c}_t^l)(\hat{c}_t{}^b + \hat{y}_t - \hat{c}_t{}^l - \hat{y}_t) \\
&- \frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t(\tau^b\hat{c}_t{}^b + \tau^b\hat{y}_t + (1-\tau^b)\hat{c}_t{}^l + (1-\tau^b)\hat{y}_t)
\end{aligned}$$

$$
\begin{aligned}
=&\tau^b(1-\tau^b)(\hat{c}_t^b-\hat{c}_t^l)^2-\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t(\hat{c}_t+\hat{y}_t)\\
=&\tau^b(1-\tau^b)(\hat{c}_t^b-\hat{c}_t^l)^2-(\hat{y}_t-\hat{c}_t)(\hat{c}_t+\hat{y}_t)\\
=&\tau^b(1-\tau^b)(\hat{c}_t^b-\hat{c}_t^l)^2-(\hat{y_t}^2-\hat{c_t}^2)\\
=&\tau^b(1-\tau^b)(\hat{c}_t^b-\hat{c}_t^l)^2+(\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t)^2-2\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t\hat{y}_t
\end{aligned}
$$

Replacing in the welfare loss equation,

$$
\begin{aligned}
U_t-U\approx&y\varphi\mu_4[(1-u)\hat{y}_t-u\hat{u}_t]-\mu_3 x\hat{x}_t\\
&-\mu_1 y[\frac{1}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2+\theta_1\theta_2 u^{\theta_2}\hat{y}_t\hat{u}_t+\frac{\Phi_p}{2}\hat{\pi}_t^2]\\
&-\mu_1\frac{c}{2}[\tau^b(1-\tau^b)(\hat{c}_t^b-\hat{c}_t^l)^2+(\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t)^2-2\frac{y}{c}\theta_1\theta_2 u^{\theta_2}\hat{u}_t\hat{y}_t]\\
&-\mu_2\frac{h}{2}[\tau^b(\hat{h}_t^b)^2+(1-\tau^b)(\hat{h}_t^l)^2]\\
&-\frac{\mu_3}{\gamma}\frac{1+\eta}{2}[\alpha(\hat{n}_t^b)^2-(1-\alpha)(\hat{n}_t^l)^2]
\end{aligned}
$$

Rearranging terms

$$
\begin{aligned}
U_t-U\approx&y\varphi\mu_4[(1-u)\hat{y}_t-u\hat{u}_t]-\mu_3 x\hat{x}_t\\
&-\mu_1 y[\frac{1}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2+\frac{1}{2}\frac{y}{c}(\theta_1\theta_2 u^{\theta_2})^2\hat{u}_t^2+\frac{\Phi_p}{2}\hat{\pi}_t^2]\\
&-\mu_1\frac{c}{2}[\tau^b(1-\tau^b)(\hat{c}_t^b-\hat{c}_t^l)^2-\mu_2\frac{h}{2}[\tau^b(\hat{h}_t^b)^2+(1-\tau^b)(\hat{h}_t^l)^2]\\
&-\frac{\mu_3}{\gamma}\frac{1+\eta}{2}[\alpha(\hat{n}_t^b)^2-(1-\alpha)(\hat{n}_t^l)^2]
\end{aligned}
$$

E. Next, we will focus on terms related to working hours $\alpha(\hat{n_t}^b)^2-(1-\alpha)(\hat{n_t}^l)^2$ and reformulate them in terms of the household consumption gap. Combining labor demand and labor supply conditions

$$
\frac{w_t^b n_t^b}{w_t^l n_t^l}=\frac{c_t^b(n_t^b)^{1+\eta}}{c_t^l(n_t^l)^{1+\eta}}=\frac{(1-\tau^b)\alpha}{\tau^b(1-\alpha)}
$$

Thus, household labor can be expressed as a function of their consumption,

$$
n_t^l=n_t^b\left(\frac{c_t^b}{c_t^l}\frac{\tau^b(1-\alpha)}{(1-\tau^b)\alpha}\right)^{\frac{1}{1+\eta}}\qquad\qquad n_t^b=n_t^l\left(\frac{c_t^l}{c_t^b}\frac{(1-\tau^b)\alpha}{\tau^b(1-\alpha)}\right)^{\frac{1}{1+\eta}}
$$

Combining with the production function $y_t=(1-D(x_t))a_t(\tau^b n_t^b)^\alpha((1-\tau^b)n_t^l)^{1-\alpha}$ where $(1-D(x_t))=exp(-\gamma_t(x_t-\bar{x}))$ and log-linearizating the result

$$
\begin{aligned}
y_t =&(1-D(x_t))a_t(\tau^b)^\alpha(1-\tau^b)^{1-\alpha}\left(\frac{c_t^l}{c_t^b}\frac{(1-\tau^b)\alpha}{\tau^b(1-\alpha)}\right)^{\frac{\alpha}{1+\eta}} n_t^l \\
&\rightarrow \hat{n}_t^l = \hat{y}_t - \hat{a}_t + \gamma x \hat{x}_t - \frac{\alpha}{1+\eta}(\hat{c}_t^l - \hat{c}_t^b) \\
y_t =&(1-D(x_t))a_t(\tau^b)^\alpha(1-\tau^b)^{1-\alpha}\left(\frac{c_t^b}{c_t^l}\frac{\tau^b(1-\alpha)}{(1-\tau^b)\alpha}\right)^{\frac{1-\alpha}{1+\eta}} n_t^b \\
&\rightarrow \hat{n}_t^b = \hat{y}_t - \hat{a}_t + \gamma x \hat{x}_t - \frac{1-\alpha}{1+\eta}(\hat{c}_t^b - \hat{c}_t^l)
\end{aligned}
$$

Then, by weighting each labor type according to its participation in the economy,

$$
\begin{aligned}
&\alpha(\hat{n_t}^b)^2 + (1-\alpha)(\hat{n_t}^l)^2 = \\
&\quad =\alpha\left(\hat{y}_t - \hat{a}_t + \gamma x \hat{x}_t - \frac{\alpha}{1+\eta}(\hat{c}_t^l - \hat{c}_t^b)\right)^2 \\
&\quad +(1-\alpha)\left(\hat{y}_t - \hat{a}_t + \gamma x \hat{x}_t - \frac{1-\alpha}{1+\eta}(\hat{c}_t^b - \hat{c}_t^l)\right)^2 \\
&\quad =(\hat{y}_t - \hat{a}_t + \gamma x \hat{x}_t)^2 + \alpha(1-\alpha)\left(\frac{1}{1+\eta}(\hat{c}_t^b - \hat{c}_t^l)\right)^2
\end{aligned}
$$

Replacing in the welfare loss equation,

$$
\begin{aligned}
&U_t - U \approx \\
&\quad y\varphi\mu_4[(1-u)\hat{y}_t - u\hat{u}_t] - \mu_3 x \hat{x}_t \\
&\quad -\mu_1 y[\frac{1}{2}\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2 + \frac{1}{2}\frac{y}{c}(\theta_1\theta_2 u^{\theta_2})^2\hat{u}_t^2 + \frac{\Phi_p}{2}\hat{\pi}_t^2] \\
&\quad -\mu_1\frac{c}{2}[\tau^b(1-\tau^b)(\hat{c}_t^b - \hat{c}_t^l)^2] - \mu_2\frac{h}{2}[\tau^b(\hat{h}_t^b)^2 + (1-\tau^b)(\hat{h}_t^l)^2] \\
&\quad -\frac{\mu_3}{\gamma}\frac{1+\eta}{2}\left[(\hat{y}_t - \hat{a}_t + \gamma x \hat{x}_t)^2 + \alpha(1-\alpha)\left(\frac{1}{1+\eta}(\hat{c}_t^b - \hat{c}_t^l)\right)^2\right]
\end{aligned}
$$

F. Now we work with the final component of the welfare function, which includes the housing-related terms, $\tau^b(\hat{h}_t^b)^2 + (1-\tau^b)(\hat{h}_t^l)^2$. From the linear approximation of the housing market clearing condition, we have

$$\begin{aligned}0 =& \tau^b(\hat{h}_t^b)^2 + (1-\tau^b)(\hat{h}_t^l)^2 \\ \rightarrow \quad & \hat{h}_t^b = (1-\tau^b)(\hat{h}_t^b - \hat{h}_t^l) \\ \rightarrow \quad & \hat{h}_t^l = -\tau^b(\hat{h}_t^b - \hat{h}_t^l) \\ \tau^b(\hat{h}_t^b)^2 + (1-\tau^b)(\hat{h}_t^l)^2 =& \tau^b(1-\tau^b)^2(\hat{h}_t^b - \hat{h}_t^l)^2 + (1-\tau^b)(\tau^b)^2(\hat{h}_t^b - \hat{h}_t^l)^2 \\ =& \tau^b(1-\tau^b)(\hat{h}_t^b - \hat{h}_t^l)^2\end{aligned}$$

We can now substitute the housing term back in the welfare function and rewrite $\mu_2 h = \gamma_h$ using conditions for efficient steady state,

$$\begin{aligned}U_t - U \approx & y\varphi\mu_4[(1-u)\hat{y}_t - u\hat{u}_t] - \mu_3 x\hat{x}_t \\ & -\frac{1}{2}\Bigg[\mu_1 y\theta_1(\theta_2)^2 u^{\theta_2}\hat{u}_t^2 + \mu_1 y\frac{y}{c}(\theta_1\theta_2 u^{\theta_2})^2\hat{u}_t^2 \\ & +\mu_1 y\Phi_p\hat{\pi}_t^2 + \frac{\mu_3}{\gamma}(1+\eta)(\hat{y}_t - \hat{a}_t + \gamma x\hat{x}_t)^2 \\ & +\left(\tau^b(1-\tau^b) + \frac{\alpha(1-\alpha)\mu_3}{\gamma(1+\eta)}\right)(\hat{c}_t^b - \hat{c}_t^l)^2 + \gamma_h\tau^b(1-\tau^b)(\hat{h}_t^b - \hat{h}_t^l)^2\Bigg]\end{aligned}$$

G. Let us now turn our attention to the first line, which contains the linear terms of the derivation. Taking a first-order approximation of the equation for the accumulation of carbon in the atmosphere

$$\hat{x}_t = \frac{e}{x}\sum_{s=0}^{t}(1-\delta^x)^{t-s}\hat{e}_s$$

We are interested in the present discount value of this expression, given by

$$\sum_{t=0}^{\infty}(\beta^l)^t\mu_3 x\hat{x}_t = \mu_3 e\sum_{t=0}^{\infty}(\beta^l)^t\sum_{s=0}^{t}(1-\delta^x)^{t-s}\hat{e}_s$$

Focusing on the double sum on the right-hand side of the last expression, we can expand it as follows

$$
\begin{aligned}
&\sum_{t=0}^{\infty}(\beta^l)^t\sum_{s=0}^{t}(1-\delta^x)^{t-s}\hat{e}_s \\
&\quad= \hat{e}_0+\beta^l[(1-\delta^x)\hat{e}_0+\hat{e}_1]+(\beta^l)^2[(1-\delta^x)^2\hat{e}_0+(1-\delta^x)\hat{e}_1+\hat{e}_0]+... \\
&\quad= \sum_{s=0}^{\infty}[\beta^l(1-\delta^x)]^s\hat{e}_0+\beta^l\sum_{s=0}^{\infty}[\beta^l(1-\delta^x)]^s\hat{e}_1+(\beta^l)^2\sum_{s=0}^{\infty}[\beta^l(1-\delta^x)]^s\hat{e}_2+... \\
&\quad= \hat{e}_0\sum_{s=0}^{\infty}[\beta^l(1-\delta^x)]^s+\beta^l\hat{e}_1\sum_{s=0}^{\infty}[\beta^l(1-\delta^x)]^s+(\beta^l)^2\hat{e}_2\sum_{s=0}^{\infty}[\beta^l(1-\delta^x)]^s+... \\
&\quad= \sum_{t=0}^{\infty}(\beta^l)^t\hat{e}_t\sum_{s=0}^{\infty}[\beta^l(1-\delta^x)]^s=\frac{1}{1-\beta^l(1-\delta^x)}\sum_{t=0}^{\infty}(\beta^l)^t\hat{e}_t
\end{aligned}
$$

Therefore, we can write

$$\sum_{t=0}^{\infty}(\beta^l)^t\mu_3x\hat{x}_t=\mu_3e\frac{1}{1-\beta^l(1-\delta^x)}\sum_{t=0}^{\infty}(\beta^l)^t\hat{e}_t$$

Substituting into the welfare function and using the log-linearized expression for carbon emissions, $e\hat{e}_t=y\varphi[(1-u)\hat{y}_t-u\hat{u}_t)]$, the first can be expressed as

$$
\begin{aligned}
&y\varphi\mu_4[(1-u)\hat{y}_t-u\hat{u}_t]-\mu_3x\hat{x}_t \\
&=\mu_4e\hat{e}_t-\mu_3e\frac{1}{1-\beta^l(1-\delta^x)}\hat{e}_t \\
&=\mu_4e\hat{e}_t-\mu_4e\hat{e}_t
\end{aligned}
$$

Thus, the linear terms cancel out and disappear. Substituting the shadow price of carbon with its steady-state value, $\mu_3=\gamma(1-\mu_4e)$, and expressing μ_4 in terms of the steady state value of the optimal carbon price, $\mu_4=\tau_e^*\mu_1$ the welfare loss function can be rewritten

$$
\begin{aligned}
&U_t-U\approx \\
&-\frac{1}{2}\Bigg[\mu_1y\theta_2^2(\mu_1y-1)\hat{u}_t^2+\mu_1y\Phi_p\hat{\pi}_t^2+(1-\tau_e^*\mu_1e)(1+\eta)(\hat{y}_t-\hat{a}_t+\gamma x\hat{x}_t)^2 \\
&+\left(\tau^b(1-\tau^b)+\frac{\alpha(1-\alpha)(1-\tau_e^*\mu_1e)}{(1+\eta)}\right)(\hat{c}_t^b-\hat{c}_t^l)^2+\gamma_h\tau^b(1-\tau^b)(\hat{h}_t^b-\hat{h}_t^l)^2\Bigg]
\end{aligned}
$$

Finally, up to a second order we can rewrite the welfare objective function as

$$W_0\approx-\mathbb{E}_0\sum_{t=0}^{\infty}(\beta^l)^t\Bigg[\frac{\Omega}{2}\Bigg(\lambda_u\hat{u}_t^2+\lambda_\pi\hat{\pi}_t^2+(\hat{y}_t-a_t+\gamma x\hat{x}_t)^2+\lambda_c(\hat{c}_t^b-\hat{c}_t^l)^2+\lambda_h(\hat{h}_t^b-\hat{h}_t^l)^2\Bigg)$$

where

$$\Omega = (1 - \tau_e^* \mu_1 e)(1 + \eta)$$

$$\lambda_u = \frac{\mu_1 y \theta_2^2 (\mu_1 y - 1)}{(1 - \tau_e^* \mu_1 e)(1 + \eta)}$$

$$\lambda_\pi \equiv \frac{\mu_1 y \Phi_p}{(1 + \eta)(1 - \tau_e^* \mu_1 e)}$$

$$\lambda_y = 1$$

$$\lambda_c = \frac{\tau^b (1 - \tau^b)}{(1 - \tau_e^* \mu_1 e)(1 + \eta)} + \frac{\alpha(1 - \alpha)}{(1 + \eta)^2}$$

$$\lambda_h = \gamma_h \frac{\tau^b (1 - \tau^b)}{(1 - \tau_e^* \mu_1 e)(1 + \eta)}$$

Alternatively, expressing abatement in terms of the carbon tax

$$W_0 \approx -\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^l)^t \left[\frac{\Omega}{2} \left(\lambda_\tau \hat{\tau}_{et}^2 + \lambda_\pi \hat{\pi}_t^2 + (\hat{y}_t - a_t + \gamma x \hat{x}_t)^2 + \lambda_c (\hat{c}_t^b - \hat{c}_t^l)^2 + \lambda_h (\hat{h}_t^b - \hat{h}_t^l)^2 \right) \right.$$

where

$$\lambda_\tau = \frac{\mu_1 y \theta_2^2 (\mu_1 y - 1)}{(1 - \tau_e^* \mu_1 e)(1 + \eta)(\theta_2 - 1)^2}$$