6. Financial Asset Risk and Return

Departament de finances empresarials

VNIVERSITAT DE VALÈNCIA

TABLE OF CONTENTS

- This unit's slides are based on chapter 10, "Risk and return in capital markets", of the book *Fundamentals of Corporate Finance* by Jonathan Berk et al. (Pearson 2019).

1. The Relationship between Risk and Return

- Consider how an investment would have grown if it had been placed in each of the following from the end of 1929 until the beginning of 2015:
    - Standard & Poor's 500 (S&P 500)
    - Small stocks
    - World portfolio
    - Corporate bonds
    - Treasury bills

1. The Relationship between Risk and Return

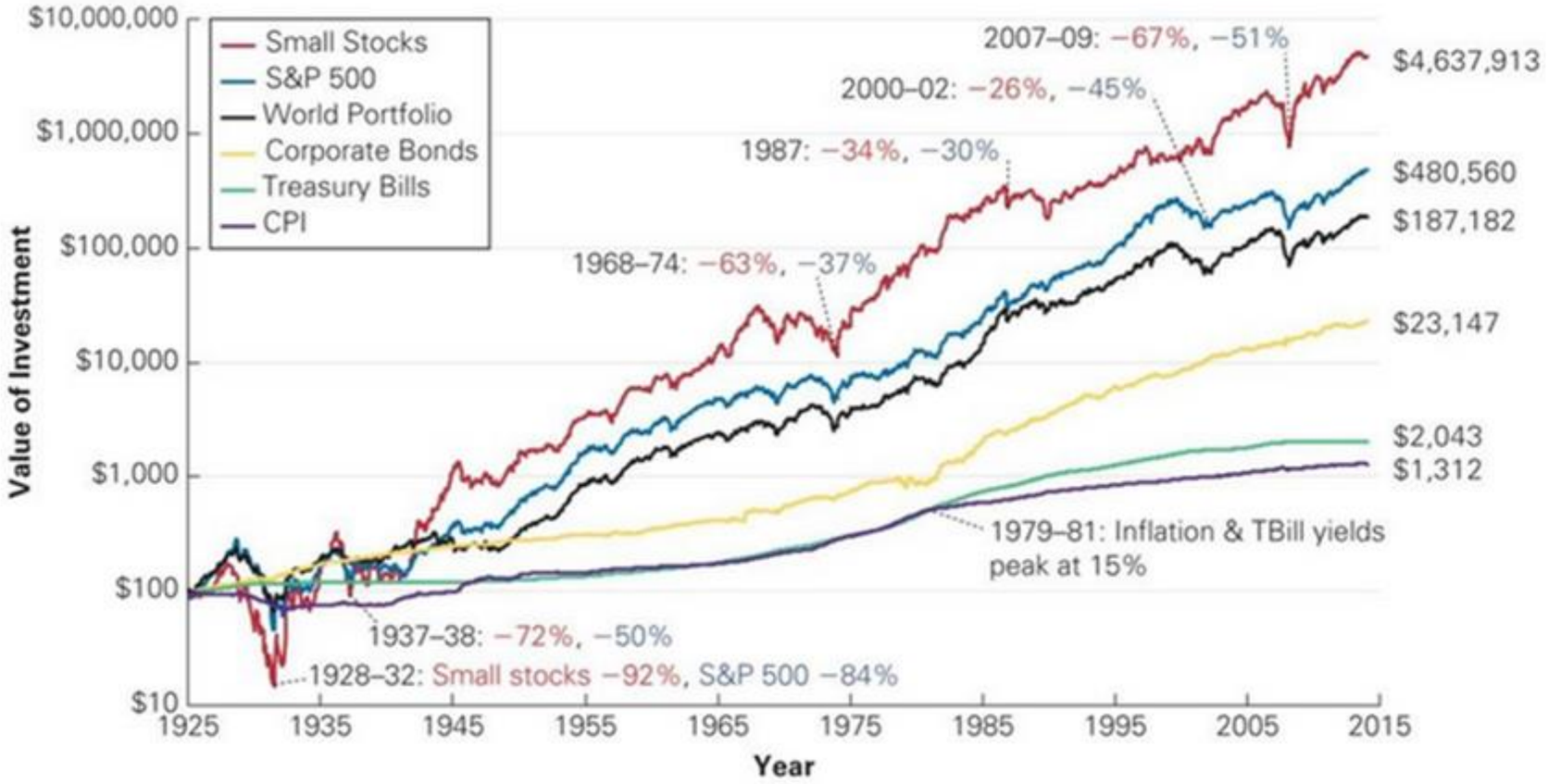


Figure 1.

Value of $100 invested at the end of 1925 in US large stocks (S&P 500), small stocks, world stocks, corporate bonds and treasury bills

1. The Relationship between Risk and Return

Year	Small Stocks	S&P 500	Corp Bonds	Treasury Bills
1926	1.09	11.14	6.29	3.30
1927	31.37	37.13	6.55	3.15
1928	65.36	43.31	3.38	4.05
1929	–43.08	–8.91	4.32	4.47
1930	–44.70	–25.26	6.34	2.27
1931	–54.68	–43.86	–2.38	1.15
1932	–0.47	–8.85	12.20	0.88
1933	216.14	52.88	5.25	0.52
1934	57.20	–2.34	9.73	0.27
1935	69.11	47.22	6.86	0.17

Source: Global Financial Data.

Table 1.

Realised returns (in percentages) for small stocks, the S&P 500, corporate bonds and treasury bills at year end, 1925–1935

1. The Relationship between Risk and Return

- Small companies (in terms of market capitalisation) have achieved the highest historical returns, due to having greater headroom for growth than companies with larger market capitalisation.
- Fixed-income assets, corporate bonds and treasury bills have posted lower historical returns.
- Assets with higher potential returns carry higher risk.

1. The Relationship between Risk and Return

- Computing historical returns

Individual investment realised returns

- The realised return from investment in a stock from t to t+1 is:

$$R_{t+1} = \frac{Div_{t+1} + P_{t+1} - P_t}{P_t} = \frac{Div_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

= Dividend yield + Capital gains yield

1. The Relationship between Risk and Return

PROBLEM

- Microsoft paid a one-time special dividend of $3.08 on 15 November 2022. Suppose you bought Microsoft stock for $28.08 on 1 November 2022 and sold it for $27.39 immediately after the dividend was paid. What was your realised return from holding the stock?

1. The Relationship between Risk and Return

CALCULATION

- The return from 1 November 2022 to 15 November 2022 is equal to:

$$R_{t+1} = \frac{Div_{t+1} + P_{t+1} - P_t}{P_t} = \frac{3.08 + (27.39 - 28.08)}{28.08} = 0.0851\ (8.51\%)$$

- This 8.51% can be broken down into dividend yield and capital gains yield:

$$\text{Rentab. div.} = \frac{3.08}{28.08} = 0.1097\ (10.97\%)$$

$$\text{Plus. cap.} = \frac{27.39 - 28.18}{28.08} = -0.0246\ (-2.46\%)$$

1. The Relationship between Risk and Return

INTERPRETATION

These returns include both capital gains (or, in this case, a capital loss) and the yield generated by dividends. Dividends and capital gains both contribute to the return (ignoring either would create a highly misleading impression of Microsoft’s performance).

1. The Relationship between Risk and Return

- Computing historical returns

Individual investment realised returns

- For quarterly returns (or for any four compounding periods that make up an entire year), the annual realised return, R annual, is found by compounding:

$$1 + R_{annual} = (1 + R_1)(1 + R_2)(1 + R_3)(1 + R_4)$$

1. The Relationship between Risk and Return

The following table includes a share's return for each period:

Date	Price	Dividend	Return
01 Nov 2018	28.08		
15 Nov 2018	27.39	3.08	8.51%
15 Feb 2019	25.93	0.08	-5.04%
16 May 2019	25.49	0.08	-1.39%
15 Aug 2019	27.13	0.08	6.75%
31 Oct 2019	25.7		-5.27%

1. The Relationship between Risk and Return

To calculate the annual return:

- (1+ R annual) = (1+ R1)(1+ R2)(1+R3)(1+ R4)(1+ R5)
- (1+ R annual) = (1+ 0.0851)(1-0.0504)(1-0.0139)(1+ 0.0675)(1-0.0527)
- R annual = 1.0275-1= 0.0275 (2.75%)

1. The Relationship between Risk and Return

INTERPRETATION

We have calculated the returns obtained by an investor with this stock in each period and, on that basis, the return for one year. In this exercise, we can see that the returns are volatile. The stock fluctuated up and down throughout the year and finished only slightly up (2.75%) overall.

1. The Relationship between Risk and Return

- Average annual returns:

- Average annual return of a security (average return over a certain number of past years)

$$\bar{R} = \frac{R_1 + R_2 + R_3 + \cdots + R_t}{t}$$

2. Estimating Risk and Return based on Historical Data

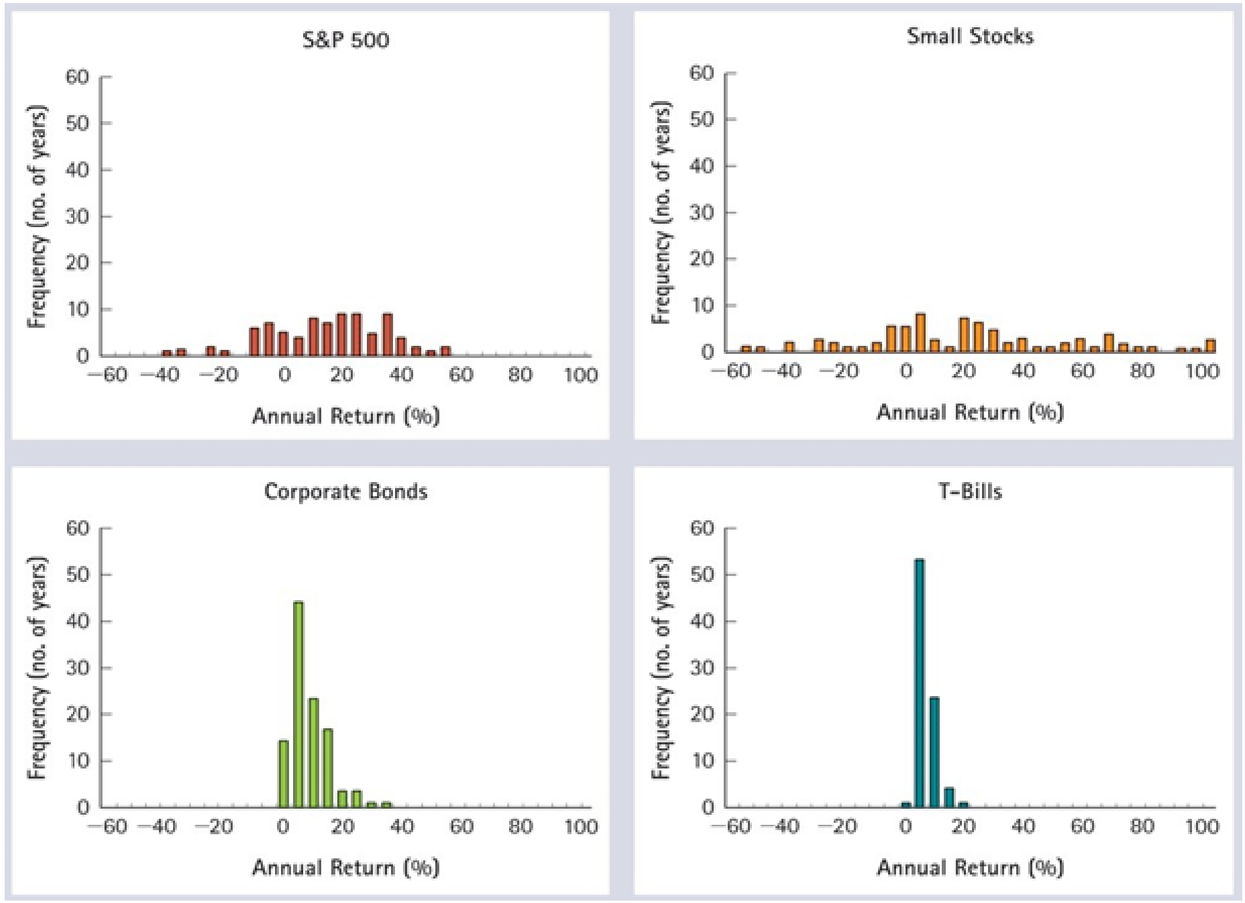


Figure 2.

Distribution of annual returns of US large stocks (S&P 500), small stocks, corporate bonds and treasury bills, 1926–2015

2. Estimating Risk and Return based on Historical Data

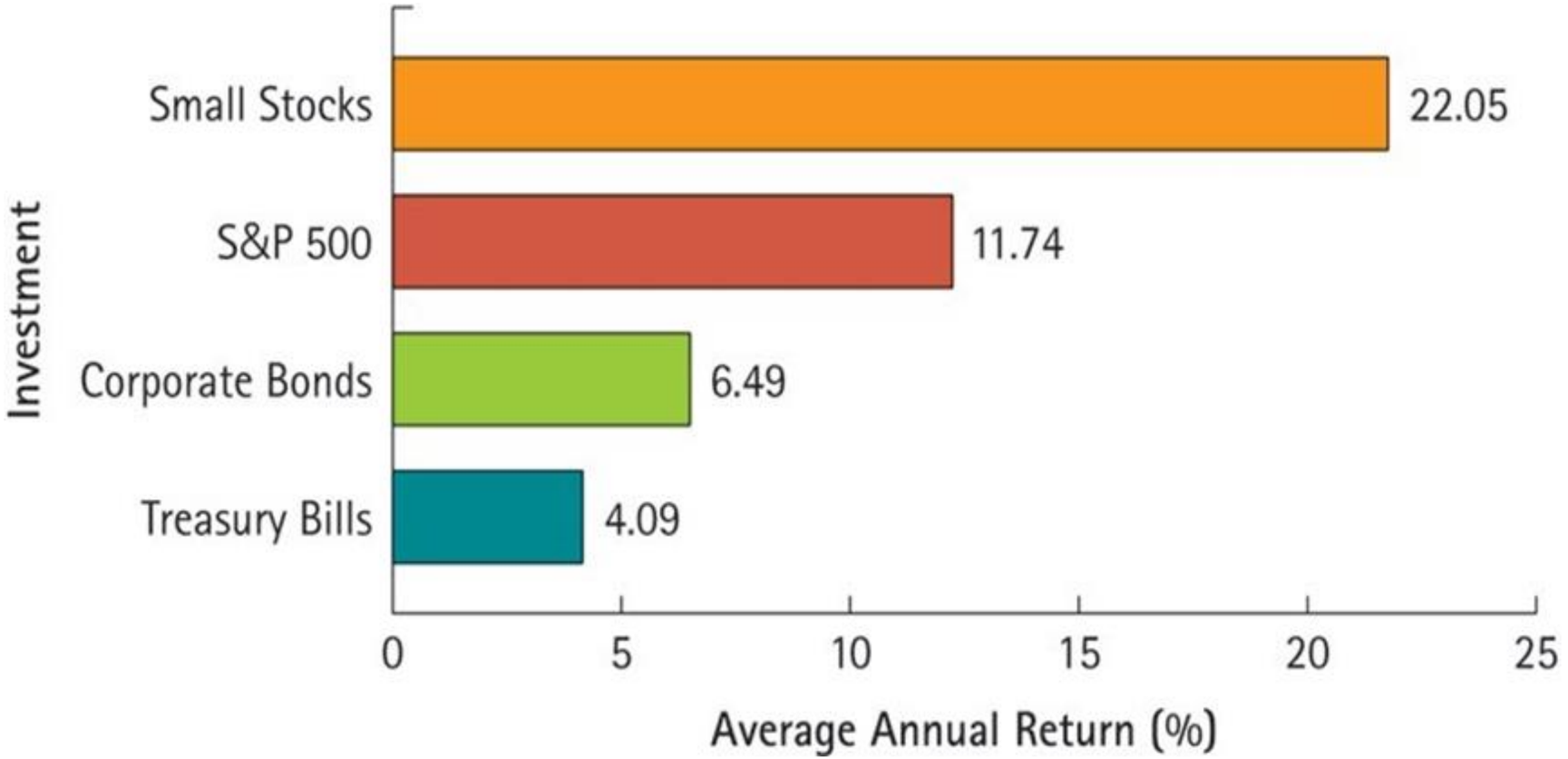


Figure 3.

Average annual returns of US small stocks, large stocks (S&P 500), corporate bonds and treasury bills, 1926–2018

2. Estimating Risk and Return based on Historical Data

- **Variance and volatility of returns:**

Variance

$$Var(R) = \frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \cdots + (R_t - \bar{R})^2}{t - 1}$$

Standard deviation

$$DS(R) = \sqrt{Var(R)}$$

2. Estimating Risk and Return based on Historical Data

EXAMPLE

•Calculate a stock's risk for the following period:

2016	2017	2018	2019	2020
4.9%	15.8%	5.5%	-37%	26.5%

Raverage=(0.049+0.158+0.055-0.37+0.265)/5 = 3.1%

$$\bar{R} = \frac{(R_1 + R_2 + \ldots + R_T)}{T}$$

$$Var(R) = \frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \ldots + (R_T - \bar{R})^2}{T-1}$$

$$= \frac{(0.049-0.031)^2 + (0.158-0.031)^2 + (0.055-0.031)^2}{5-1} +$$

$$+ \frac{(0.37-0.031)^2 + (0.265-0.031)^2}{5-1} = 0.058$$

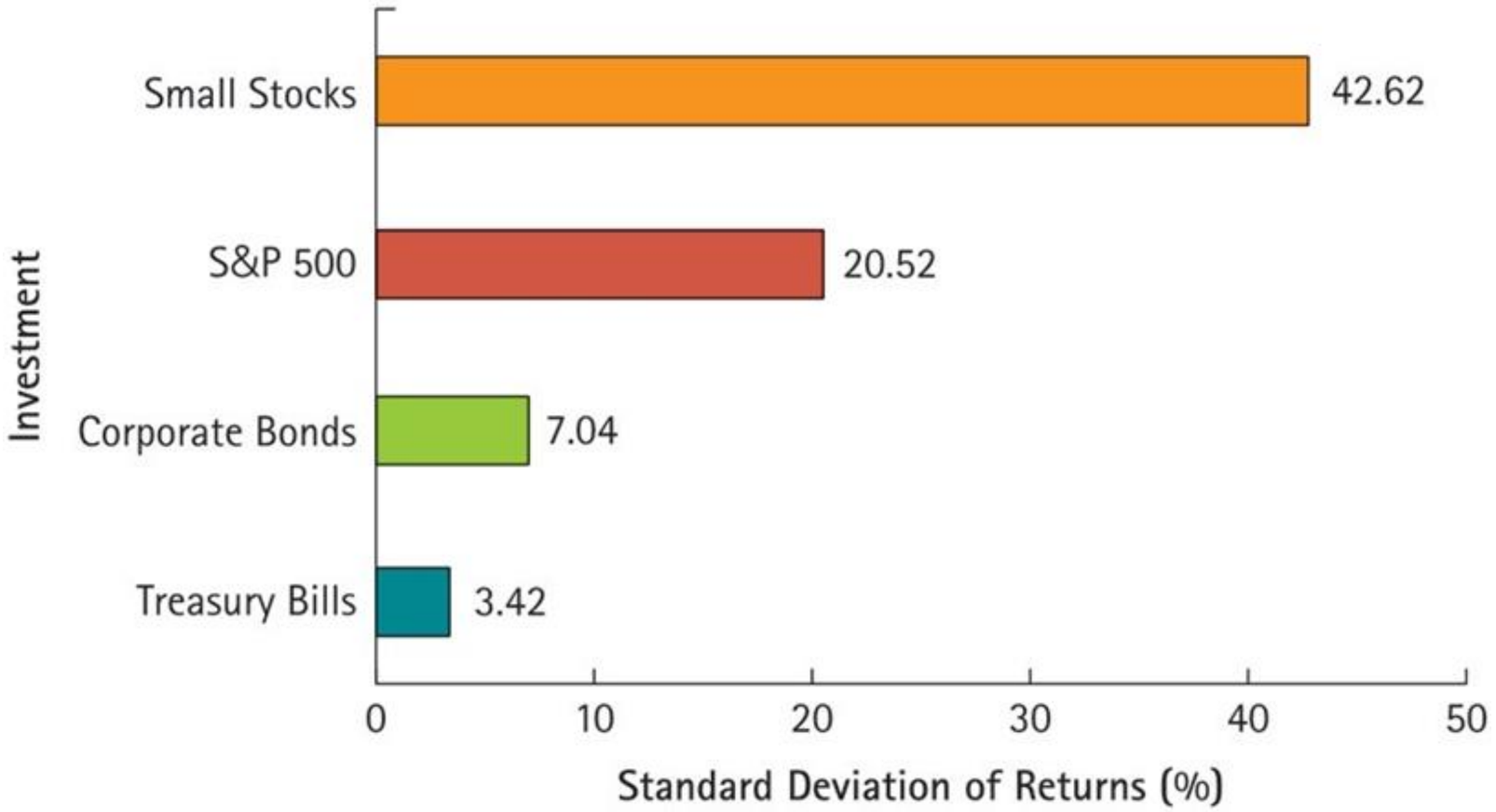
$$DS(R) = \sqrt{Var(R)} = \sqrt{0.058} = 0.241\ (24.1\%)$$

- Is the best estimate of the stock's expected return its average return of 3.1%?

NO: past returns do not guarantee future returns

- Is the expected volatility of 24.1% a good estimator of the stock's risk?
  - YES

2. Estimating Risk and Return based on Historical Data



Volatility (standard deviation) of US small stocks, large stocks (S&P 500), corporate bonds and treasury bills, 1926–2018

2. Estimating Risk and Return based on Historical Data

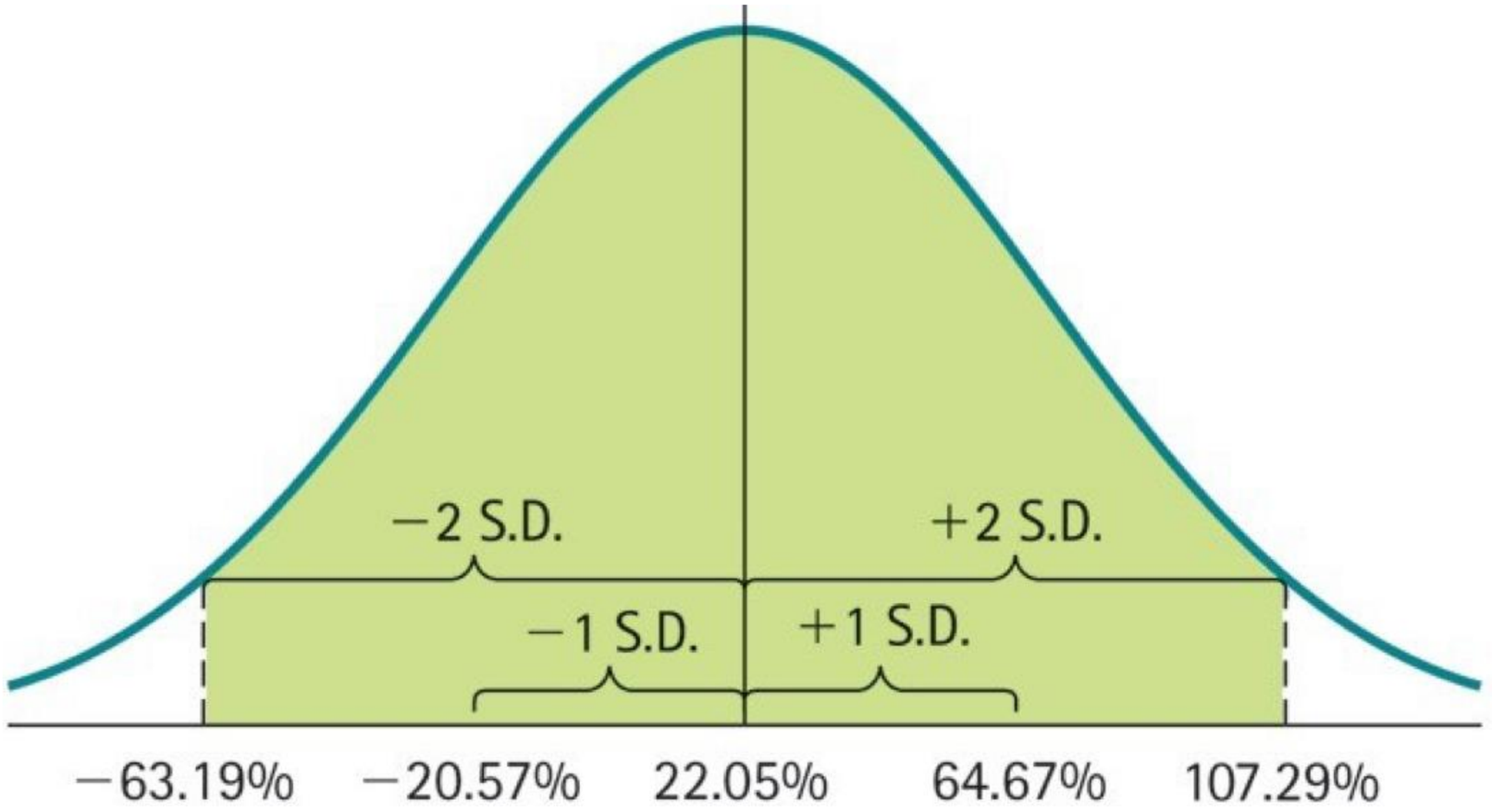
- Normal distribution:
  - 95% prediction interval

Average return +/- (2 x Standard deviation)

$$R \pm (2 \times SD(R))$$

- About two-thirds of all possible outcomes fall within one standard deviation above or below the average.
- Approximately 95% of all possible outcomes fall within two standard deviations above or below the average.

2. Estimating Risk and Return based on Historical Data



- This asset had an average return of 22.05% and its risk (standard deviation) was 42.62%, so its prediction interval was between -63.19% and 107.29%.

2. Estimating Risk and Return based on Historical Data

CALCULATION

- Taking the average return for the S&P 500 index for 2005-2023 to be 3.1% with a standard deviation of 24.1%, what is the 95% confidence interval for the 2024 return?

Average ± (2 ◊ Standard deviation) = 3.1% – (2 x 24.1%) to 3.1% + (2 x 24.1%)

= -45.1% to 51.3%

2. Estimating Risk and Return based on Historical Data

INTERPRETATION

- Despite the average return from 2005 to 2023 being 3.1%, the S&P 500 index was volatile. So, if we want to estimate the return for 2024 with a 95% confidence level, we can say that it will be between -45.1% and 51.3%.

2. Estimating Risk and Return based on Historical Data

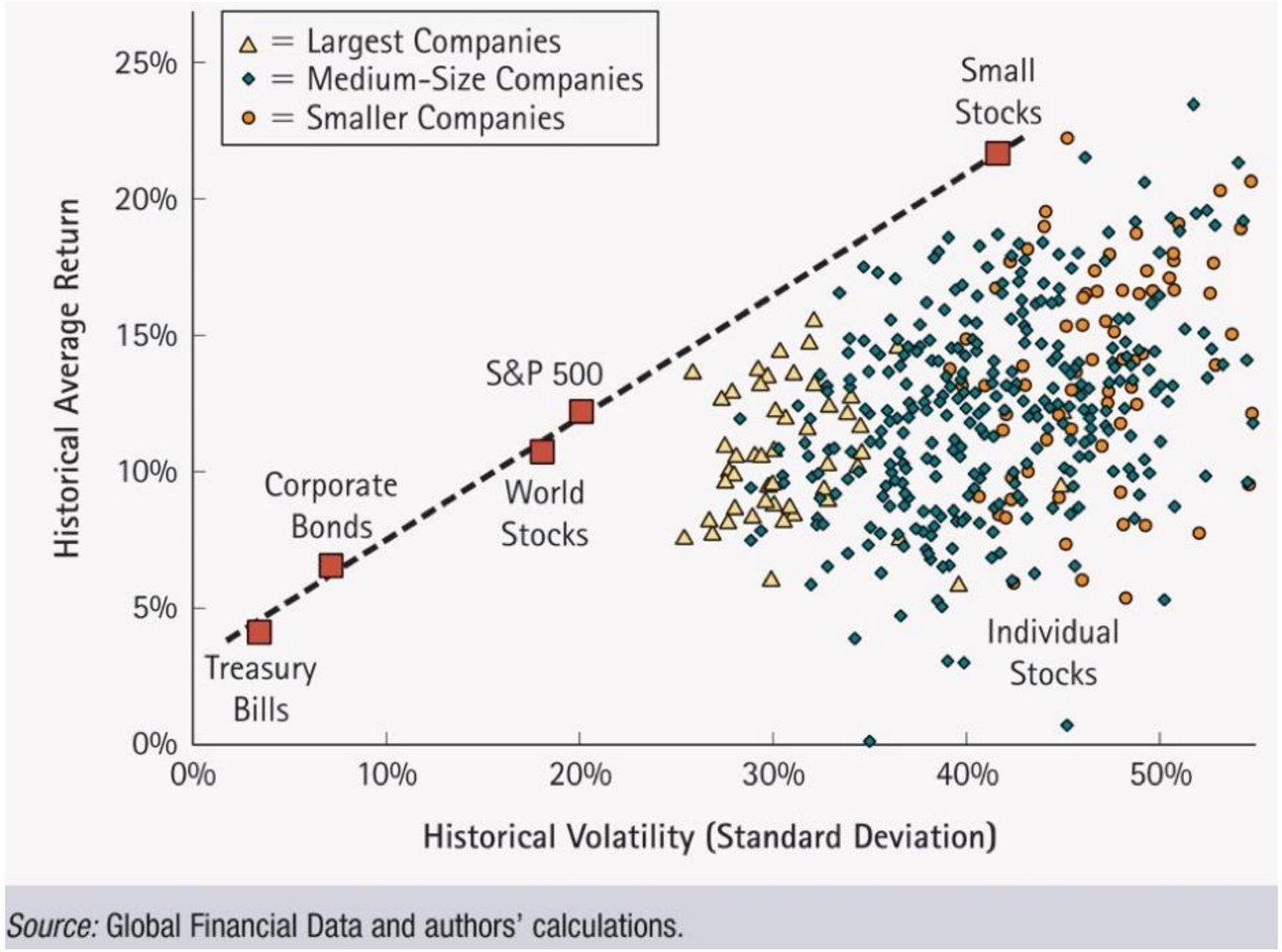
- **The returns of large portfolios:**

- Investments with higher volatility, as measured by standard deviation, tend to have higher average returns.

2. Estimating Risk and Return based on Historical Data

- **The returns of individual stocks:**
  - Larger stocks have lower volatility overall.
  - Even the largest stocks are typically more volatile than a portfolio of large stocks.
  - The standard deviation of an individual security does not explain the size of its average return.
  - In the chart in the next slide, all individual stocks have lower returns and/or higher risk than the portfolios.

2. Estimating Risk and Return based on Historical Data



Source: Global Financial Data and authors' calculations.

- The historical tradeoff between risk and return in large portfolios, 1926-2015

3. Calculating Asset Portfolio Return

- Portfolio weights
  - The fraction of the total portfolio held in each investment in the portfolio:

$$w_i = \frac{\text{Value of investment } i}{\text{Total value of portfolio}}$$

  - Portfolio weights add up to 100% (i.e. $w_1 + w_2 + \ldots + w_n = 100\%$).

3. Calculating Asset Portfolio Return

- The portfolio weights for a portfolio of 200 Apple shares at $200 per share and 1,000 Coca-Cola shares at $60 per share will be:

$$w_{Apple} = \frac{200 \times \$200}{100,000} = 40\% \qquad w_{Coca-Cola} = \frac{1000 \times \$60}{100,000} = 60\%$$

3. Calculating Asset Portfolio Return

- The return on a portfolio, Rp
  - The weighted average of the returns on the investments in the portfolio:

$$R_p = w_1 R_1 + w_2 R_2 + \ \ldots \ + w_n R_n$$

3. Calculating Asset Portfolio Return

PROBLEM

- Suppose you invest $100,000 to buy 200 shares of Apple at $200 per share ($40,000) and 1,000 shares of Coca-Cola at $60 per share ($60,000).
- If Apple’s stock goes up to $240 per share and Coca-Cola’s stock falls to $57 per share and neither paid dividends, what is the new value of the portfolio?
- What return did the portfolio earn?
- If you do not buy or sell any shares after the price changes, what are the new weights of the assets in the portfolio?

3. Calculating Asset Portfolio Return

SOLUTION

- Portfolio:
  - 200 shares of Apple: $200 ➡ $240 ($40 capital gain per share)
  - 1,000 shares of Coca-Cola: $60 ➡ $57 ($3 capital loss per share)
  - To determine your portfolio's return, calculate its value with the new prices and compare the result to your initial investment of $100,000.

3. Calculating Asset Portfolio Return

CALCULATION

- The initial portfolio weights were $40,000 / $100,000 = 40% for Apple and $60,000 / $100,000 = 60% for Coca-Cola; we can therefore calculate the portfolio's return as we have seen:

$$R_P = w_{Apple} R_{Apple} + w_{Coke} R_{Coke} = 0.40 \times 20\% + 0.60(-5\%) = 5\%$$

- After the price changes, the new portfolio weights are equal to the value of your investment in each stock divided by the new portfolio value:

$$w_{Apple} = \frac{200 \times \$240}{105,000} = 45.71\% \qquad w_{Coca-Cola} = \frac{1000 \times \$57}{105,000} = 54.29\%$$

3. Calculating Asset Portfolio Return

CALCULATION

- The new value of your Apple stock is 200 × $240 = $48,000, and the new value of your Coca-Cola stock is 1,000 × $57 = $57,000. So, the new value of the portfolio is $48,000 + $57,000 = $105,000, giving a gain of $5,000 or a 5% return on your initial investment of $100,000.

- Since neither stock paid any dividends, we calculate their returns simply as the capital gain or loss divided by the purchase price. The return on Apple stock was $40 / $200 = 20%, and the return on Coca-Cola stock was -$3 / $60 = -5%.

3. Calculating Asset Portfolio Return

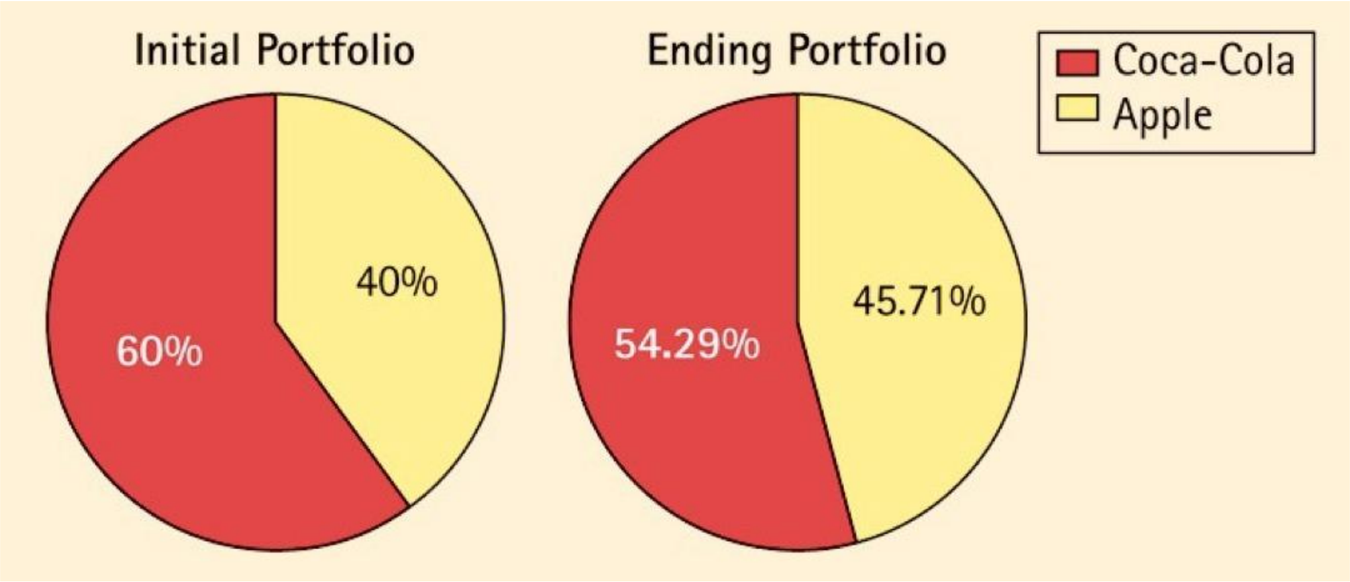
INTERPRETATION

- The loss of $3,000 on your investment in Coca-Cola was offset by the gain of $8,000 on your investment in Apple, giving a total gain of $5,000 or a return of 5%.
- The same result is obtained by giving the 20% return on Apple a weight of 40% and the -5% loss on Coca-Cola a weight of 60%.
- Your net return is 5%.
- After a year, Apple's weight in the portfolio has increased and Coca-Cola's weight has decreased.
- Note that the stock whose return was greater than the overall portfolio return has increased its weight in the portfolio.

3. Calculating Asset Portfolio Return

INTERPRETATION

- The charts below show the initial and end weights of Apple (yellow) and Coca-Cola (red).



3. Calculating Asset Portfolio Return

PROBLEM

- Suppose you invest $158,000 to buy 2,000 shares of Microsoft at $25 per share ($50,000) and 3,000 shares of Pepsi at $36 per share ($108,000). If Microsoft shares rise to $33 per share and Pepsi shares fall to $32 per share and neither pays dividends, what is the new value of the portfolio?
- What return did the portfolio earn?
- If you do not buy or sell any shares after the price changes, what are the new portfolio weights?

3. Calculating Asset Portfolio Return

SOLUTION

- Portfolio:
  - 2,000 MSFT shares: $25  $33 ($8 capital gain)
  - 3,000 PEP shares: $36  $32 ($4 capital loss)

3. Calculating Asset Portfolio Return

APPROACH

- To determine your portfolio's return, calculate its value with the new prices and compare the result to your original investment of $158,000.The new value of your Microsoft shares is 2,000 × $33 = $66,000, and the new value of your Pepsi shares is 3,000 X $32 = $96,000.
- The new value of your portfolio is $66,000 + $96,000 = $162,000, giving a gain of $4,000 or a 2.5% return on your initial investment of $158,000.

3. Calculating Asset Portfolio Return

SOLUTION

- The new value of your portfolio is $66,000 + $96,000 = $162,000, giving a gain of $4,000 or a 2.5% return on your initial investment of $158,000.

$$RP = w\,Microsoft * R\,Microsoft + w\,Pepsi * R\,Pepsi = 0.316 \times 32\% + 0.684 \times -11.1\% = 2.5\%$$

3. Calculating Asset Portfolio Return

SOLUTION

- After the price changes, the new portfolio weights are equal to the value of your investment in each stock divided by the new portfolio value:

$$w_{Microsoft} = \frac{2{,}000 \times \$33}{\$162{,}000} = 40.7\% \quad w_{Pepsi} = \frac{3{,}000 \times \$32}{\$162{,}000} = 59.3\%$$

3. Calculating Asset Portfolio Return

INTERPRETATION

- The loss of $12,000 on your investment in Pepsi was offset by the gain of $16,000 on your investment in Microsoft, resulting in a total gain of $4,000 or a return of 2.5%. The same result is obtained by giving the 32% return on Microsoft a weight of 31.6% and the loss of -11.1% on Pepsi a weight of 68.4%.

- Your total net return is 2.5%.

3. Calculating Asset Portfolio Return

- The expected return of a portfolio

-The weighted average of the expected returns of the investments (shares) that make up the portfolio, using the portfolio weights:

$$E[RP] = w1*E[R1] + w2*E[R2] + \ldots + wn*E[Rn]$$

3. Calculating Asset Portfolio Return

Term	Concept	Equation
Portfolio weight	The relative investment in your portfolio	$w_i = \frac{\text{Value of Investment } i}{\text{Total Value of Portfolio}}$
Portfolio return	The total return earned on your portfolio, accounting for the returns of all of the securities in the portfolio and their weights	$R_P = w_1R_1 + w_2R_2 + \cdots + w_nR_n$
Portfolio expected return	The return you can expect to earn on your portfolio, given the expected returns of the securities in that portfolio and the relative amount you have invested in each	$E[R_P] = w_1E[R_1] + w_2E[R_2] + \cdots + w_nE[R_n]$

3. Calculating Asset Portfolio Return

PROBLEM

•Suppose you invest $10,000 in Boeing (BA) stock and $30,000 in Merck (MRK) stock. There is a return of 10% for Boeing and 16% for Merck.

What is the expected return for your portfolio?

What is the expected return for your portfolio?

3. Calculating Asset Portfolio Return

SOLUTION
APPROACH

- The total investment is $40,000:
  - $10,000 / $40,000 = 25% in Boeing: Y [RBA] = 10%
  - $30,000 / $40,000 = 75% in Merck: Y [RMERCK] = 16%

- We calculate the portfolio's expected return from the weighted average of the expected returns of the assets that make up the portfolio, using the portfolio weights.

3. Calculating Asset Portfolio Return

CALCULATION

- The expected return for your portfolio is:

$$E[RP] = wBA^{*}E[R_{BA}] + wMRK^{*}E\,[R_{MERCK}]$$

$$E[RP] = 0.25^{*}10\% + 0.75^{*}16\% = 14.5\%$$

4. Systematic and Unsystematic Risk

- Types of risk
  - Common risk: outcomes are linked (e.g. an earthquake).
  - Independent risk: outcomes are NOT linked (e.g. theft).
  - Diversification: investment strategy that combines assets with different risk profiles.

4. Systematic and Unsystematic Risk

Type of Risk	Definition	Example	Risk Diversified in Large Portfolio?
Common Risk	Linked across outcomes	Risk of earthquake	No
Independent Risk	Risks that bear no relation to each other	Risk of theft	Yes

4. Systematic and Unsystematic Risk

PROBLEM

- You are playing a very simple gambling game with your friend: a $1 bet based on a coin flip. In other words, you each bet $1 and flip a coin: heads means you win your friend's $1, while tails means you lose and your friend takes your $1. How is your risk different if you play this game 100 times as opposed to just betting $100 (instead of $1) on a single coin flip?

4. Systematic and Unsystematic Risk

SOLUTION
APPROACH

- The risk of losing one coin flip is independent of the risk of losing the next: you have a 50% chance of losing each time, and one flip does not affect any other. We can calculate the expected outcome of each flip as a weighted average, where wins (+$1) are weighted by 50% and losses (-$1) are weighted by 50%. We can then calculate the probability of losing $100 under either scenario.

4. Systematic and Unsystematic Risk

CALCULATION

- If you play the game 100 times, you should lose 50 times. So, the expected outcome is $50 (+$1) + $50 (-$1) = $0: you should break even. However, even if you do not win exactly half of the time, the probability of losing all 100 coin flips (and, thus, losing $100) is extremely low (in fact, it is 0.50^100, which is much less than even 0.0001%). If it happened, you should check the coin!

4. Systematic and Unsystematic Risk

CALCULATION

- If, on the other hand, you were to bet $100 on the outcome of a single coin flip, you would have a 50% chance of winning $100 and a 50% chance of losing $100, so the expected outcome would be the same: breaking even. However, as there is a 50% chance of losing $100, your risk would be much higher than with 100 $1 bets.

4. Systematic and Unsystematic Risk

INTERPRETATION

- In each case, you risk losing $100. By spreading your risk across 100 different bets, however, you will have reduced it compared to placing a single $100 bet.

4. Systematic and Unsystematic Risk

Something similar happens with stocks:

- Investing in a single stock is riskier than investing in multiple stocks.
- However, the maximum potential gain from a single stock will always be greater than that of a portfolio of several stocks (diversification).
- If you knew in advance which stock would be the most profitable, you would be better off investing in just that one stock. But since this is not possible, investors resort to diversification to reduce risks.

4. Systematic and Unsystematic Risk

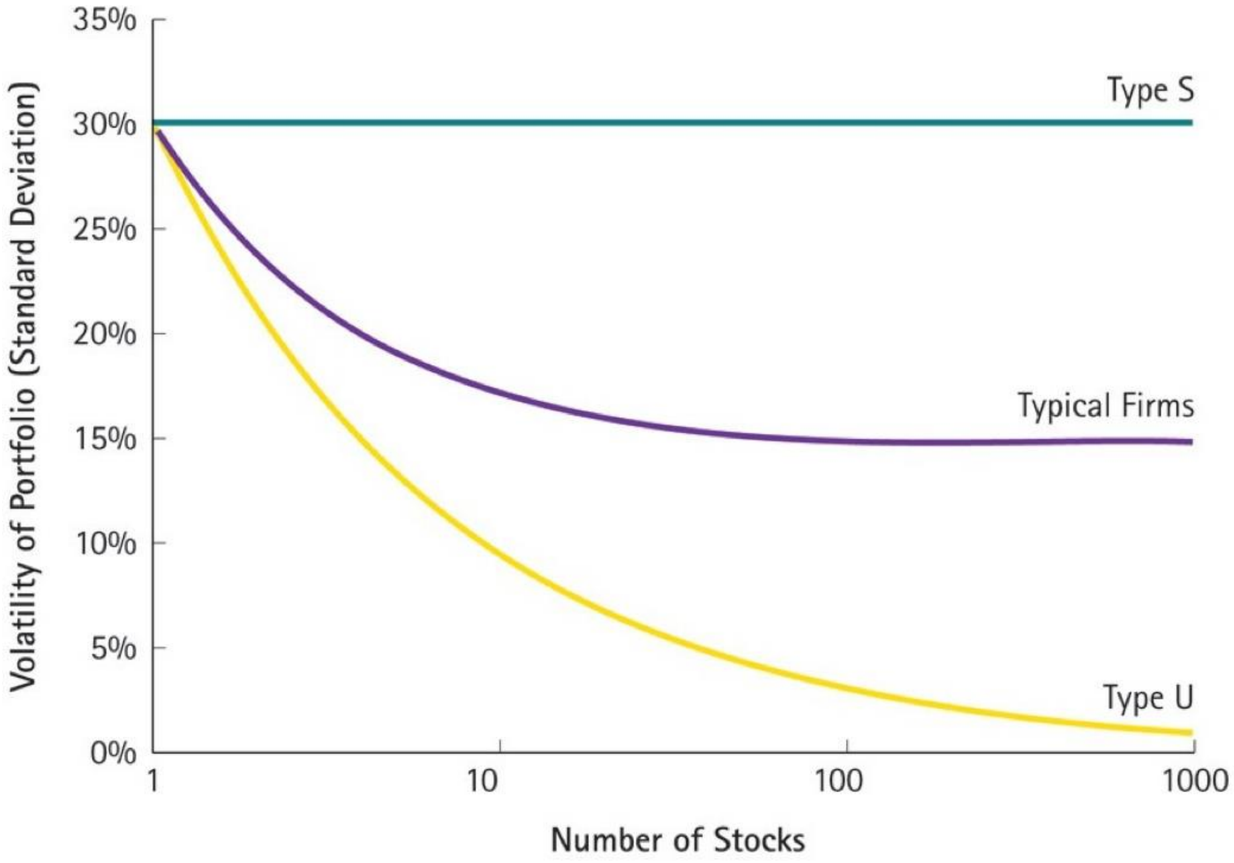
- Unsystematic vs. systematic risk

- Stock prices are affected by two types of news:
    1. News specific to the company's sector (unsystematic)
    2. Market-wide news (systematic)
    - Unsystematic risk (diversifiable) vs. systematic risk (non-diversifiable)

4. Systematic and Unsystematic Risk

- Unsystematic vs. systematic risk
  - Stock prices are affected by two types of news:
    1. News specific to the company's sector
    2. Market-wide news
  - Unsystematic risk (diversifiable): this type of risk only affects a single company or sector (a change in a company's management, a new tax on the banking sector, new European regulations on vehicle emissions, etc.).
  - Systematic risk (non-diversifiable): interest rate rises, energy price rises, pandemics, etc.

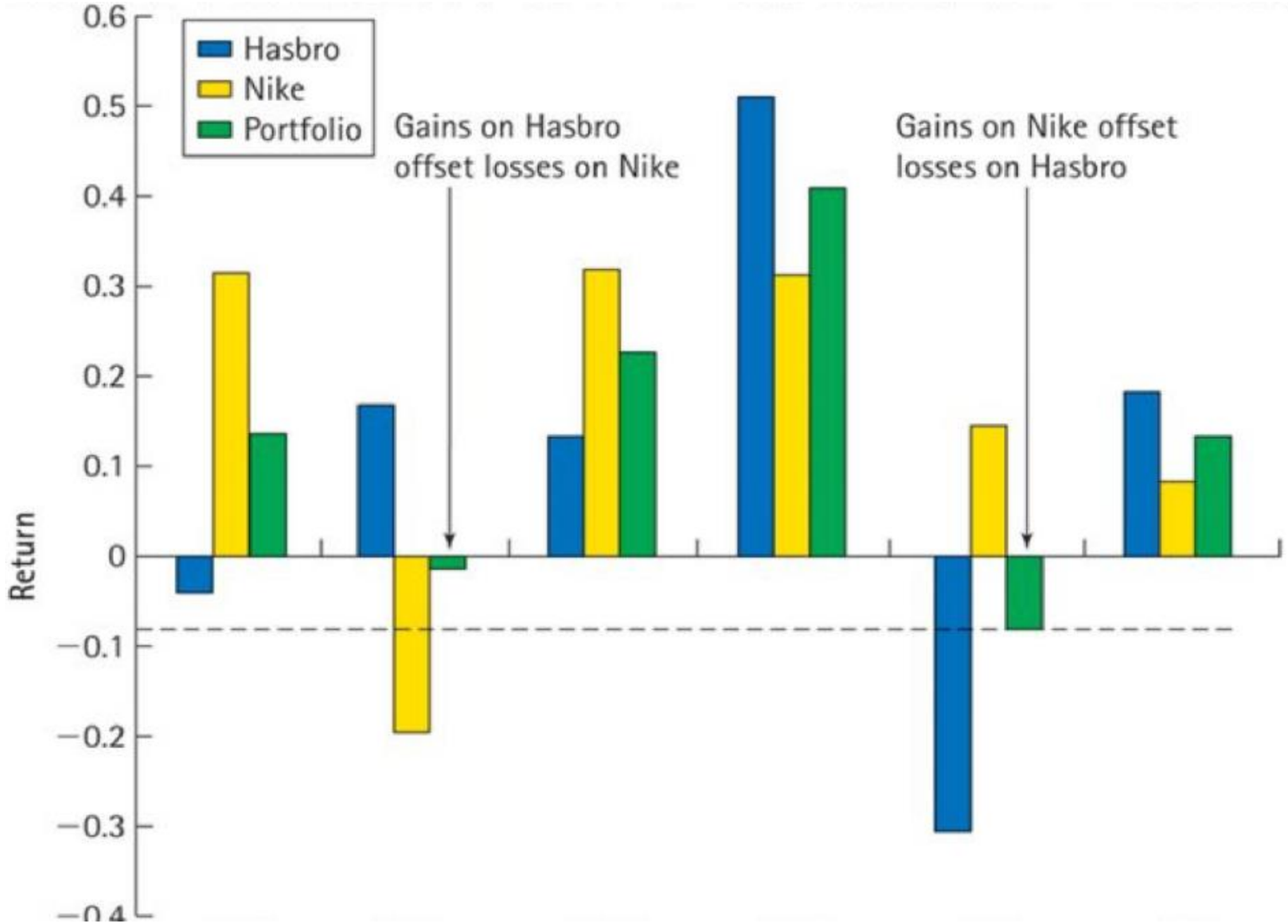
4. Systematic and Unsystematic Risk



As a rule, adding new stocks to a portfolio reduces risk until a certain point (approx. 20 stocks) is reached. Beyond that point, further risk reduction is not possible because, although all unsystematic risk has been removed, systematic risk cannot be eliminated.

Type S: portfolio with shares with systematic risk (non-diversifiable)
Type U: portfolio with shares with unsystematic risk (diversifiable)

4. Systematic and Unsystematic Risk



4. Systematic and Unsystematic Risk

- Diversifiable risk and the risk premium
  - The risk premium for diversifiable risk is zero.
    - Investors are not compensated for holding unsystematic risk.

4. Systematic and Unsystematic Risk

	Diversifiable?	Requires a Risk Premium?
Systematic Risk	No	Yes
Unsystematic Risk	Yes	No

	S Firm	U Firm
Volatility (standard deviation)	30%	30%
Risk-Free Rate	5%	5%
Risk Premium	5%	0%
Expected Return	10%	5%

FORMULAS

Concept	Formula
Realised returns	$\frac{D_1 + P_1 - P_o}{P_o}$
Compounding realised returns. When dividends are distributed more than once a year.	$(1 + R_{annual}) = (1 + R_1) * (1 + R_2) * \cdots .. * (1 + R_t)$
Average annual return	$\frac{R_1 + R_2 + \cdots + R_t}{t}$
Variance of returns	$\frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \cdots + (R_t - \bar{R})^2}{t - 1}$
Standard deviation	SD=$\sqrt{Var}$
95% prediction interval	$Average\ return \pm (2 \text{ x } \mathrm{SD(R)})$
Portfolio return	$(R_1 * w_1)+(R_2 * w_2)+(R_3 * w_3)+\ldots. (R_n * w_n)$