

Article

Reactive Elements Control in LC Series Resonant Inverters by Current-Controlled Variable-Transformer and Magnetic Energy Recovery Switch for Induction Heating

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Abstract: This work consists of the analysis and design of a LC series resonant inverter with reactive element control for induction heating hardening applications. This novel method uses a current-controlled variable transformer (VT) to control the reflected inductance of the inductor in the resonant, and a magnetic energy recovery switch (MERS) to vary the influence of the capacitor as a reactive power compensation element. This converter topology allows quality factor (Q) or operating frequency (f_{sw}) to be adjusted, making it possible to harden workpieces of different geometries and materials with a single converter. In the article, the design of both elements will be studied and tested. Experimental results were carried out with a 10 kW induction heating inverter prototype, with a frequency range of 60 kHz to 100 kHz and a quality factor of 6 to 10, measuring efficiencies above 95%.

Keywords: LC resonant inverters; induction heating; current-controlled variable-transformer; magnetic energy recovery switch; PS power control; silicon carbide (SiC) power MOSFETs

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1. Introduction

The influence of induction heating at industrial level has increased due to the transition towards less polluting systems that do not rely on non-renewable energies such as gas, which was one of the main energy sources for heat treatment of metal pieces. Within induction heating applications, scan hardening is one of the most challenging, as this surface heat treatment of workpieces has to harden only the surface part without affecting the internal part of the workpiece, in order to increase the abrasion and fatigue resistance without affecting the hardness and flexibility of the internal part of the workpiece [1]. Surface hardening consists of subjecting the workpiece to be heated to a magnetic field generated by a coil called heating inductor, powered by a high current and high-frequency power supply. Foucault or Eddy currents induced inside the workpiece cause heat due to Joule and hysteresis losses. The depth of the heated zone depends on the frequency of the current due to the skin effect as well as the heat conduction and magnetic property.

The advent of the electric vehicle has led to the development of new hardening scanning systems due to the appearance of parts with complex geometries that must withstand the torque of electric motors, such as cogwheel, gears and splined hubs, with the added difficulty that they are combined into a single shaft shaped part that must be hardened at different frequencies, see Figure 1. The most basic heating systems base their operation on adjusting the resonant tank by changing transformer taps or by making combinations of series and parallel capacitors to reach the optimum operating point. The problem is that these manual changes cannot be made during the heating process, so they are only optimal if the working load does not physically change or modify its electrical resistive properties during heating, which is not true in a real induction heating process. Until now,

the sequential use of dual frequencies has been considered [2], a medium frequency (MF) to heat the quasi-cylindrical zone close to the surface of the workpiece and a high frequency (HF) to heat the surfaces closer to the inductor, such as the tip and flanges of the teeth. But this process requires two resonant tanks with two different inductors and two power supplies, which means having four reactive elements set at two working frequencies, as well as needing a fast mechanical transfer system of the workpiece between the two inductors (Figure 2a).

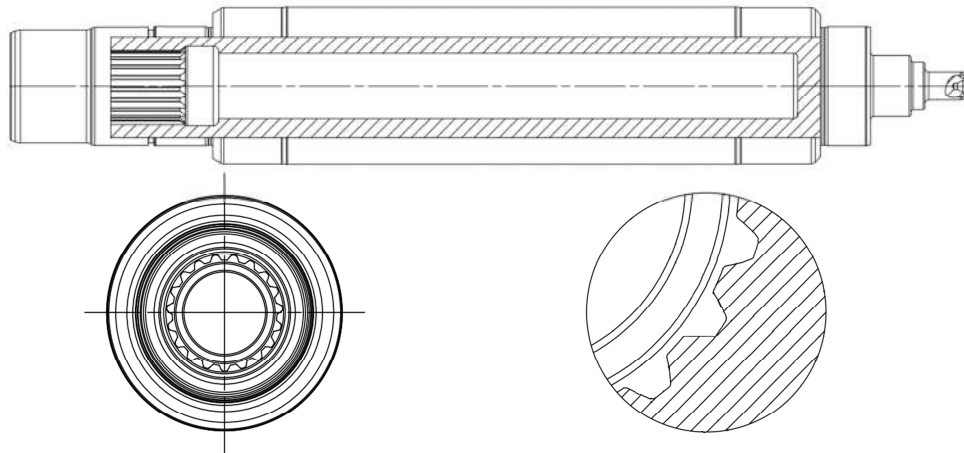


Figure 1. Drive Spline Shaft scheme for Electric Vehicle.

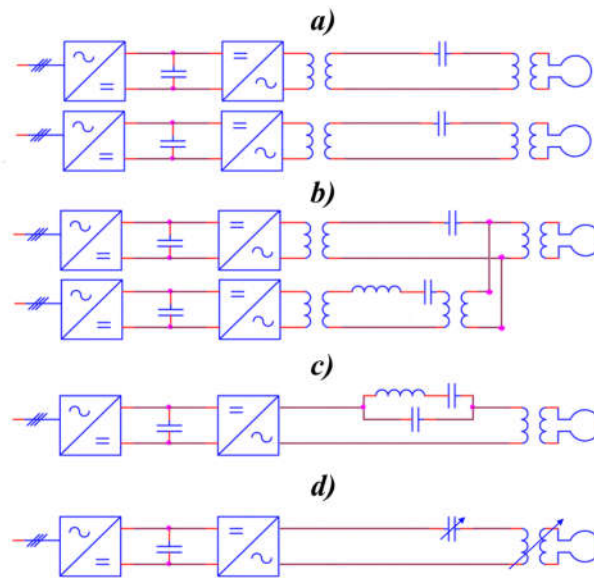


Figure 2. Multiple frequency converter topologies. (a) Dual frequency independent, (b) Dual frequency with single inductor, (c) Dual frequency with single converter and (d) Variable frequency with reactive element control.

As an alternative to this topology, several authors have proposed different combinations of resonant tanks and power supplies, in [3] they propose a common inductor for both resonant tanks, but it is still necessary to have four reactive elements and two power supplies (Figure 2b). Another improvement was introduced in [4], where using the same inductor and a sinusoidal pulse width modulation (SPWM), one of the power supplies can be dispensed with, at the cost of having to keep the four reactive elements and losing

the zero-voltage soft switching (ZVS) (Figure 2c). These proposals are currently used in the industry, but they have drawbacks such as being limited by design to only two hardening frequencies, which does not make them valid for parts with variable geometries like the one in Figure 1. Furthermore, all of them use variable frequency modulation (FM) [5,6] or combinations of variable frequency with phase shift (FM-PS) [7,8]. As demonstrated, the heat generated by the induced currents in the workpiece is concentrated in a surface layer of thickness δ , which is the penetration depth and is defined by

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}, \quad (1)$$

where μ is the magnetic permeability ρ is the electrical resistivity of the part and f is the applied frequency, which means that if during heating the frequency varies due to modulation, the depth of penetration varies, which can invalidate the workpiece. Figure 3 shows the penetration depth of 1040 carbon steel as a function of frequency. To address this problem, several authors have focused their studies on fixed frequency induction systems. In [9] the output power of the inverter is regulated only by the phase shift (PS), with the drawback that the system loses ZVS soft switching if the applied power decreases or load changes are performed. In [10] a narrow frequency range is used, which combines PS with small frequency variations, thus improving the efficiency of the inverter and further optimising the resonant tank. In [11], a topology of three reactive elements LLC is used, with the series inductance being a current-controlled variable inductor, which together with the PS allows the output power to be regulated without varying the frequency, increasing the efficiency of the converter over the entire operating range and maintaining the switching in ZVS even with load changes. However, this topology has the disadvantage of having to use three reactive elements for a single frequency. Finally, in [12] another solution is proposed to keep the frequency fixed and improve efficiency by using a current-controlled variable transformer, which allows the impedance of the load to be adapted at any moment. However, despite the availability of solutions to regulate the power at a fixed frequency, these solutions only allow the part to be heated at one frequency, which does not make them suitable for hardening complex geometries or parts with different materials.

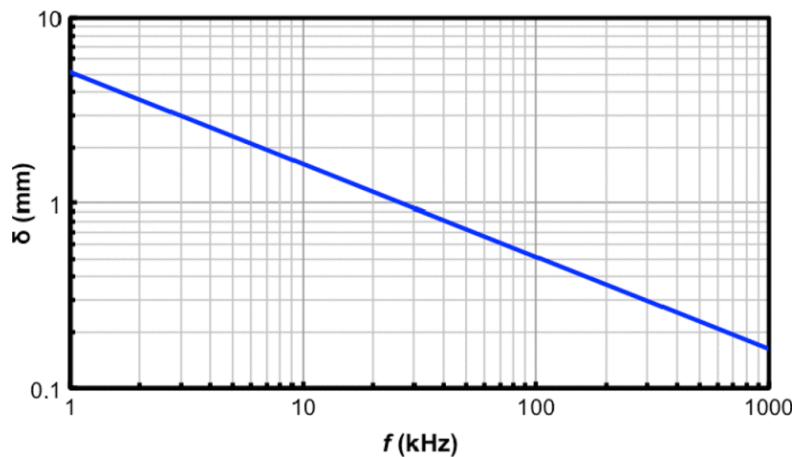


Figure 3. Penetration depth in carbon steel 1040 in front of frequency.

In contrast of the topologies presented previously, Figure 2a–c, in this article the control of the reactive elements of the resonant tank is proposed, for which a current controlled variable transformer (VT) is used to vary the ratio of the transformer to be coupled to the heating inductor, in order to control the reflected inductance of the inductor in the resonant tank and a magnetic energy recovery switch (MERS) to vary the influence of the capacitor as a reactive energy compensation element, Figure 2d. As studied below, the

combination of these elements together with the PS regulation to vary the output power of the inverter, allows switching to the optimum frequency required by the hardening for the depth of penetration taking into account the position of the workpiece and the inductor in the scanning process.

The article is structured as follows. Section 2 analyses the series resonant LC circuit where the influence of the reactive elements in the induction system is analysed. In Section 3, the current controlled variable transformer is analysed and designed, while in Section 4, the MERS is analysed. In Section 5, the resonant tank and the inverter are designed for the specifications of an application, the working principle of the control system is discussed and the theoretical efficiency of the inverter is calculated. In Section 6, experimental tests are performed to validate the design and, finally, conclusions are presented in Section 7.

2. LC Series Resonant Converter Topology

The series LC circuit, common to all the topologies considered previously, is shown in Figure 4. It consists of a heating inductor (L_s) in series with a capacitor (C_p) that compensates the reactive power of the circuit. As can be seen, a transformer is also used to adapt the inductor to the resonant tank by amplifying the inductance. Together with them, a series resistor (R_s) is represented, which represents the value of the equivalent resistance of the piece to be heated.

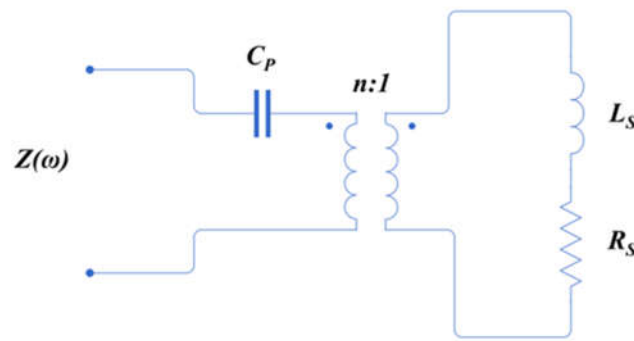


Figure 4. LC series resonant circuit.

The behaviour of the transformer consists in adapting the inductance to the desired capacitor and operating frequency, so that the inductance reflected in the primary of the transformer is expressed as

$$L_p = L_s n^2, \quad (2)$$

where the secondary series inductance (L_s) of the transformer is reflected in the primary multiplied by the square of the transformer turns ratio (n). This effect also occurs with the equivalent resistance reflected from the primary (R_p),

$$R_p = R_s n^2. \quad (3)$$

As can be seen from the following equation, the resonant frequency is affected by the transformation ratio,

$$\omega_o = \omega_p = \frac{1}{\sqrt{L_p C_p}} = \frac{1}{\sqrt{n^2 L_s C_p}}, \quad (4)$$

and also affects the quality factor of the resonant tank for a fix active power

$$Q = \frac{L_p \omega_o I_o^2}{P_o} = \frac{n^2 L_s \omega_o I_o^2}{P_o}. \quad (5)$$

The quality factor is a decisive parameter in induction heating, it measures the amount of reactive power in the inductor in relation to the active power that can be applied. The choice of the quality factor is determined by aspects such as the material of the workpiece to be heated, the geometry, the type of heat treatment and the coupling between the workpiece and the inductor [13]. In induction heating systems, the inductor has physical dimensions that cannot be varied during heating. Therefore, in the case of parts with irregular dimensions or different material combinations, it is important to choose a quality factor value that allows the part to be correctly coupled throughout the heating process to deliver the desired power. This parameter is important because as is known the reflected impedance of the work piece in the inductor varies during heating, either because of the variable geometry of the part, because it is composed of different materials, because of temperature-dependent changes in the permeability, penetration depth and electrical resistance of the piece, or because of the quality factor of the circuit.

Figure 5a, shows how the shape of the quality factor of a series LC resonant varies as a function of frequency, where the two reactive elements are kept constant. The curve shows at its peak the value of Q that defines its shape. As can be seen, the maximum power deliver occurs at the resonant frequency. On the other hand, Figure 5b shows the effect of varying only the primary reflected inductance of the heating inductor by varying the transformation ratio. Due to the relationship between inductance and frequency in the quality factor, the quality factor increases with increasing inductance. It has to be taken into account that the equivalent series resistor (R_p) is set to a value that does not exceed the maximum active power of the equipment, in order to achieve this, it is considered that the resistance of the workpiece to be heated is not fixed, as it varies during the heating process to maintain a constant (R_p). The opposite case is represented in Figure 5c, where the increase in capacitor capacitance causes the resonant frequency to decrease, without being compensated by an increase in inductance, which causes the quality factor to decrease. Therefore, as shown in Figure 5d, the effect of being able to control the two reactive elements of the system ensures that the quality factor is maintained over an entire frequency range, or decreased or increased by varying only one of the two elements and the working frequency.

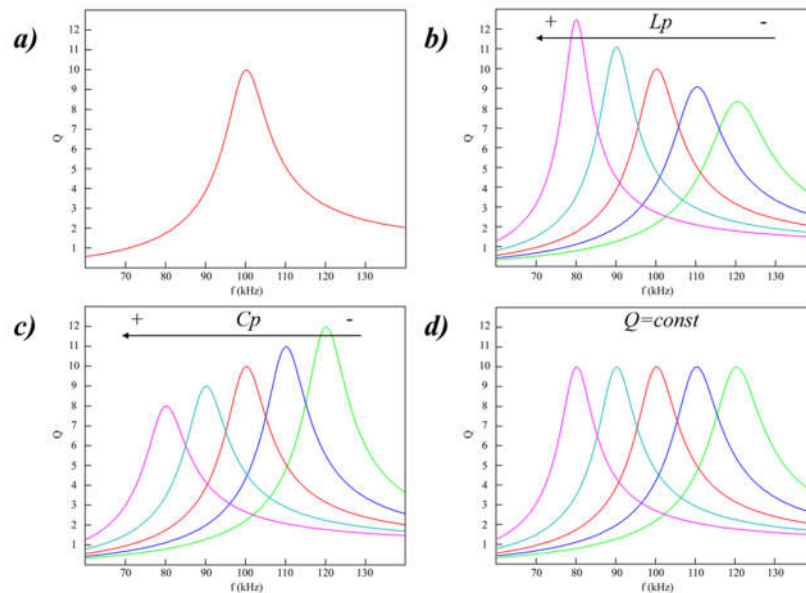


Figure 5. Quality factor behavior for a constant R_p . (a) Two elements resonant, (b) Resonant with variable transformer, (c) Resonant with variable capacitor and (d) Resonant with control of the two reactive elements. The peak of the curve corresponds to the quality factor value of the curve.

3. Analysis and Design of Current-Controlled Variable Transformer

The analysis of the current controlled variable transformer has its origin in previous articles [14,15]. Its principle of operation is based on distributing the magnetic flux generated by the primary to the secondary by varying the effective permeability of the core through a path that decreases its reluctance when connected to a direct current source [16], thus increasing the influence of the reluctance of the secondary path, which includes an air gap. In transformer design, core geometry, windings and air gap position have been discussed in numerous articles [17,18]. But for simplicity of design by using a commercial core and because it can be easily modelled mathematically, the one proposed in [12] is used for this work, with the difference that now this transformer will not be used to match the impedance of the resonant, but to match the influence of the inductor inductance in the resonant tank.

The structure used is shown in Figure 6 and uses two «E» shaped commercial cores with a central air gap distributed over the two cores. The primary winding is wound between the two outer legs of both cores, which are linked together. The secondary winding is split between the centre legs of the cores, and their windings are connected in parallel. The two side arms form the saturation control inductor under DC bias, which is wound in series with opposite polarity to decouple the primary and secondary from DC bias, neutralising the influence of magnetic flux and avoiding induced voltages. Therefore, when the core is not subjected to DC bias, the reluctance of the secondary will be lower than that of the primary, as it has an air gap, which causes the magnetic flux to be closed to a lesser extent by the secondary. However, when the core is DC biased, the effective permeability of the core decreases by saturating it and, therefore, its reluctance, so the reluctance of the secondary is higher in relation to that of the core and, therefore, the magnetic flux is closed by this path.

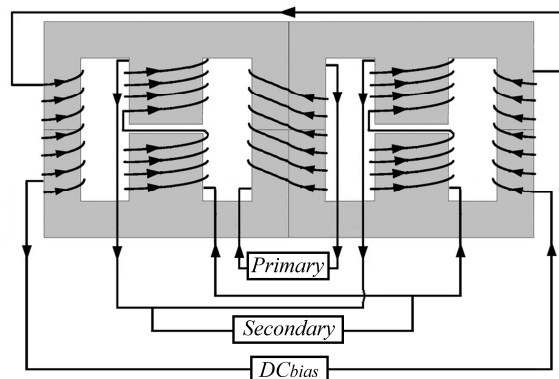


Figure 6. Structure of the Current-Controlled Variable-Transformer.

The mathematical modelling of the transformer is based on the magnetic circuit due to the relationships that balance the ampere-turns and magnetic fluxes. As shown in Figure 7, to maintain symmetry, the primary is analysed as a single core and the secondary as two in parallel.

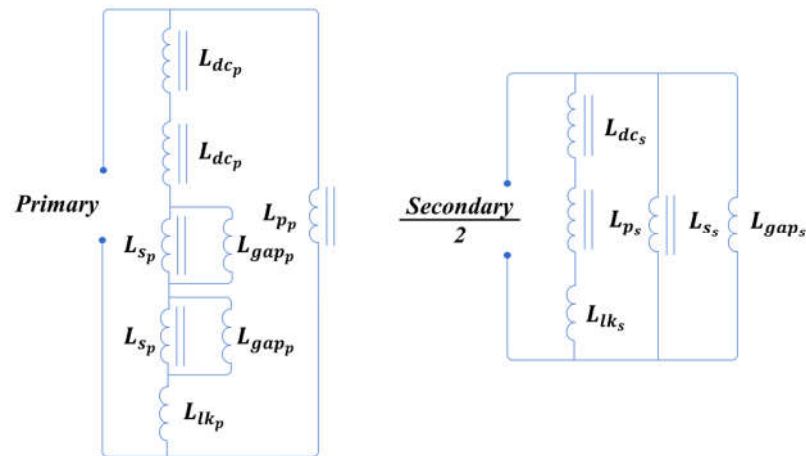


Figure 7. Equivalent inductance circuit of primary at the left and secondary at the right.

As for the primary inductance, it is reflected in primary and secondary by the following expressions,

$$L_{pp} = \frac{\mu_0 \mu_i A_p (n_p)^2}{l_{pp}} \text{ and } L_{ps} = \frac{\mu_0 \mu_i A_p (n_s)^2}{l_{ps}}, \quad (6)$$

where μ_0 is the air permeability, the initial permeability of the core is μ_i , the effective area of the primary branch corresponds to A_p , n_p and n_s are the number of turns of the primary and secondary and the magnetic path length is l_{pp} y l_{ps} for primary to primary and primary arm to the secondary.

The expressions of the inductances of the secondary branches reflected from the primary and the secondary are as follows

$$L_{sp} = \frac{\mu_0 \mu_i A_s (n_p)^2}{l_{sp}} \text{ and } L_{ss} = \frac{\mu_0 \mu_i A_s (n_s)^2}{l_{ss}}, \quad (7)$$

where the effective area of the secondary branch is A_s , the magnetic path length of the secondary arm corresponds to l_{ss} , and the magnetic path length from the secondary to the primary arm belongs to l_{sp} .

The expression of the leakage inductance is also obtained, this inductance belongs to the inductance that closes through the air instead of through the ferrite core and has been obtained experimentally from [16]

$$L_{lk_p} = \frac{\mu_0 16 A_p (n_p)^2}{l_{pp}} \text{ and } L_{lk_s} = \frac{\mu_0 16 A_s (n_s)^2}{l_{ss}}. \quad (8)$$

As for the inductance of the saturation control branches it is expressed for primary and secondary as

$$L_{dc_p} = \frac{\mu_0 \tilde{\mu} A_{dc} (n_p)^2}{l_{dc_p}} \text{ and } L_{dc_s} = \frac{\mu_0 \tilde{\mu} A_{dc} (n_s)^2}{l_{dc_s}}, \quad (9)$$

where $\tilde{\mu}$ is the relative permeability under DC bias, the effective area of the saturation control branch is A_{dc} and magnetic path length is l_{dc_p} .

Finally, for both the primary and secondary branches, the inductances due to the air gap located in the secondary branches are expressed as follows

$$L_{gap_p} = \frac{\mu_0 A_{gap} (n_p)^2}{l_{gap}} \text{ and } L_{gap_s} = \frac{\mu_0 \tilde{\mu} A_{dc} (n_s)^2}{l_{dc_s}}, \quad (10)$$

where A_{gap} is the effective area and l_{gap} correspond to the and magnetic path length of the air gap.

The distribution of the magnetic path lengths is shown in Figure 8.

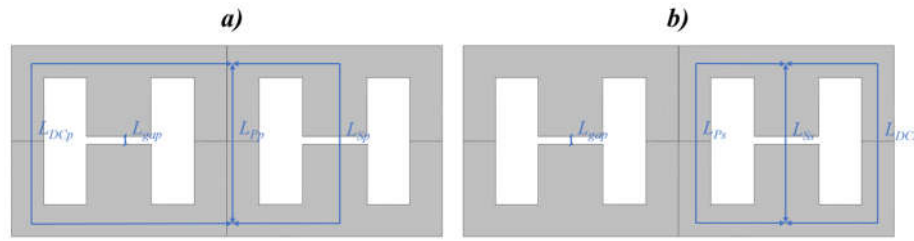


Figure 8. Magnetic path lengths distribution. (a) primary side and (b) secondary side.

The maximum primary inductance obtained from the equivalent circuit in Figure 4, can be expressed as

$$\frac{1}{L_{p_{max}}} = \left(\frac{1}{L_{p_p}} + \frac{1}{L_{lk_p} + 2L_{dc_p} + \frac{2}{L_{s_p} + L_{gap_p}}} \right), \quad (11)$$

while the maximum secondary inductance is given by

$$\frac{1}{2L_{s_{max}}} = \left(\frac{1}{L_{p_s} + L_{dc_s} + L_{lk_s}} + \frac{1}{L_{s_s}} + \frac{1}{L_{gap_s}} \right). \quad (12)$$

To obtain the ratio between the maximum inductance, with the core unsaturated, and the minimum L_{min} , with the core saturated, it must be considered that it is due to the minimum $\tilde{\mu}$ resulting from the maximum exposure to H_{bias} , because $\tilde{\mu}$ decreases when exposed to a continuous magnetic field strength. Therefore, the expression for primary is obtained as

$$\frac{L_{p_{max}}}{L_{p_{min}}} = \left(\frac{\frac{1}{2\mu_i} + \frac{6l_{gap}}{2l_{gap}(74 + 6\mu_i) + 3l_{p_s}}}{\frac{1}{2\mu_i} + \frac{24(l_{gap} + l_{p_s})}{2\mu_e l_{gap} l_{p_s}(74 + 4\mu_i + \tilde{\mu}) + 1}} \right), \quad (13)$$

and for the secondary as

$$\frac{L_{s_{max}}}{L_{s_{min}}} = \left(\frac{\frac{2\mu_e l_{s_p}}{\tilde{\mu} + 32} + \frac{\mu_e l_{s_s}}{\mu_i} + 1}{\frac{2\mu_e l_{s_p}}{\mu_i + 32} + \frac{\mu_e l_{s_s}}{\mu_i} + 1} \right), \quad (14)$$

where μ_e is obtained from the elementary equation of an inductor and corresponds to the effective relative permeability of the core with a gap,

$$L = \frac{\mu_0 \mu_e A_e n^2}{l}. \quad (15)$$

4. Analysis of Magnetic Energy Recovery Switch

The electronic control of the capacitor of a resonant tank has already been studied in previous works. In [19], a MERS was used in a parallel LC converter to vary the working frequency of the resonant, but using a parallel topology presented the problem of having to use a controlled rectifier with a series inductance to act as a current source for the parallel capacitor. To implement a MERS, four insulated gate bipolar transistors (IGBTs) are used, limiting the maximum frequency to 30 kHz, which is not high enough for small penetration depths. On the other hand, increasing the duty cycle of the capacitor increases its influence on the resonant, decreasing the frequency, which implies a loss of the quality factor, i.e., the ability to deliver power to the part to be heated. In [20] the topology is simplified using a series LC converter, but in this case four IGBTs are used to form the full

bridge and another four for the MERS, and the frequency is still limited to 30 kHz and also the quality factor is lost as the frequency decreases. The controllable capacitor topology has also been published in other papers as Gate-Controlled Series Capacitors (GCSC) [21,22], where it is applied to the subsynchronous resonance phenomenon of a steam turbine-generator, but the application frequencies are low, allowing the use of thyristors to control the capacitor. Therefore, and because the desired frequencies for hardening applications are usually in the range of 50 kHz to 100 kHz [1], this work presents a new MERS design that uses only two SiC Metal-Oxide Semiconductor Field-Effect Transistors (MOSFETs) to control the influence of the capacitor. The proposed topology can be seen in Figure 9.

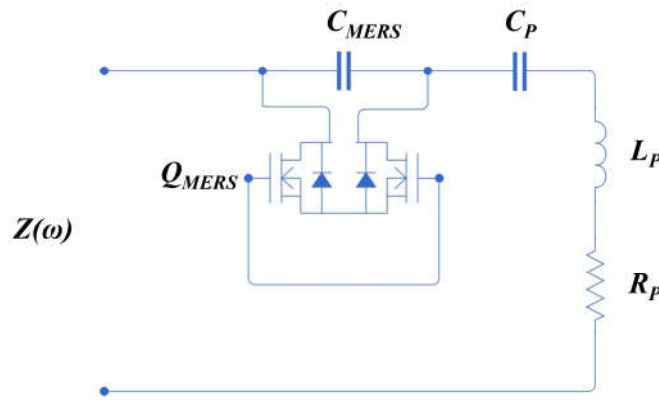


Figure 9. LC series resonant MERS topology.

The operating principle consists of short-circuiting the capacitor during a certain phase to reduce the reactive energy compensation of the capacitor, so that during that instant the voltage provided by the MERS capacitor to the inductor will be zero, due to the fact that no current is flowing through it. To avoid stretches in which the compensated energy is zero, the topology has another capacitor in series (C_s) that maintains the minimum value as current is continuously flowing through it. This effect allows the power applied to the load to be varied without varying the input frequency of the series resonant tank or decrease the resonance frequency of the LC tank by varying the effective capacity of the capacitor.

The conduction angle of the capacitor (γ) is between 0 and π radians, and the reactance of the MERS capacitor can be expressed by the following equation,

$$X_{MERS}(\gamma) = \frac{V_{eff}(\gamma)}{I_{eff}(\gamma)} = X_C \sqrt{\frac{4\pi - 4\gamma + 3\sin(2\gamma) - 2\gamma\cos(2\gamma) + 2\pi\cos(2\gamma)}{2\pi - 2\gamma + \sin(2\gamma)}}, \quad (16)$$

where the effective current of the capacitor is defined as

$$I_{eff}(\gamma) = \frac{I}{\sqrt{\pi}} \sqrt{\int_{\gamma}^{\pi} \sin(\phi)^2 d\phi}, \quad (17)$$

and the effective voltage as

$$V_{eff}(\gamma) = \frac{IX}{\sqrt{\pi}} \sqrt{\int_{\gamma}^{\pi} (-\cos(\phi) + \cos(\gamma))^2 d\phi}. \quad (18)$$

where (X_C) is the reactance of the capacitor used to form the MERS, (I) is the peak current of the capacitor and (ϕ) is the angle between 0 and 2π radians.

Figure 10 shows the typical waveforms of the circuit when it is excited with a square voltage wave and the MERS intervenes to reduce the influence of the capacitor. As can be

seen during the MERS conduction period, the voltage of this capacitor cancels out, thus observing a change in the trend of the output current, which is only influenced by the series capacitor.

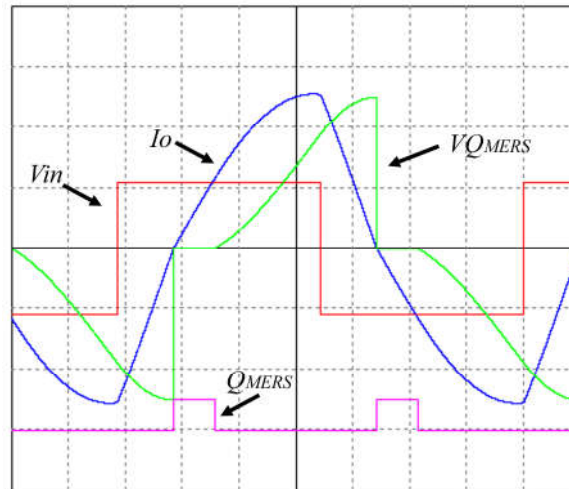


Figure 10. Example of waveforms of the influence of MERS on the serial LC resonant. Blue represents the output current, red shows the input voltage, green shows the capacitor voltage and magenta displays the MERS gate signal.

To avoid losses in MERS, the turn ON of the MOSFETs conduction is synchronised with the zero crossing of the series resonant current, thus making the ON conduction of the transistor without current, which allows ON losses to be neglected. In Figure 11, this effect can be seen in the behaviour of the MERS. As can be seen in the figure, the conduction of the MERS MOSFETs occurs when the current goes through zero. It can also be seen how the current through the capacitor compensation circuit corresponds to the output current, which is the current flowing through the resonant tank.

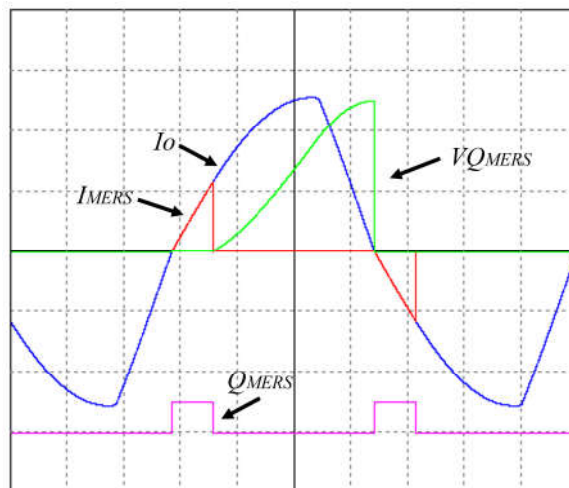


Figure 11. Example of waveforms of MERS operation in the LC series resonant. Blue represents the output current, red shows the MERS current, green shows the MERS voltage on one transistor and magenta displays the MERS gate signal.

5. Reactive Elements Control Inverter Design, Control and Efficiency

5.1. Design Procedure

In practical hardening applications, single coil inductors closely coupled to the workpiece are used. As stated above, the purpose of this design is to use a single heating station with a single inductor, therefore, the inductor geometry, which will be set by a diameter larger than the largest diameter of the part to be heated, will provide the inductance value of the inductor [23]. The diameter of the workpiece will also set the quality factor, as this must increase as the workpiece is decoupled from the inductor to continue applying the desired power to the workpiece [1]. Another characteristic set by the part to be heated is the depth of penetration, which is used to determine the frequency at which the inverter should switch. Table 1 shows the example parameters for the scan hardening of a variable geometry workpiece made of 1040 carbon steel, using a one turn inductor with a diameter of 40 mm.

Table 1. Initial Requirements.

Scan Step	Workpiece Diameter (mm)	Penetration Depth (mm)	Frequency (kHz)	Quality Factor	Maximum Power (kW)
1	23.5	0.7	67	10	6
2	30.2	0.6	77.5	8	9
3	25.5	0.55	89	9	10
4	35	0.5	100	6	10
5	32	0.65	72	7	10

From the initial requirements, the design parameters of the individual heating stages can be obtained. For this purpose, the resonance frequency of the LC tank is obtained by the following equation

$$\omega_o = \frac{2Q\omega_{sw}}{\tan \alpha + \sqrt{\tan^2 \alpha + 4Q^2}}, \quad (19)$$

where ω_{sw} , is the switching frequency expressed in angular form and α is the phase angle between the output voltage and current of the inverter.

With the power requirement and quality factor, the current that must flow through the inductor can be obtained using the following expression

$$I_L = \sqrt{\frac{P_o Q}{\omega_{sw} L}}, \quad (20)$$

as well as the output current of the inverter,

$$I_o = \frac{P_o}{\frac{2}{\pi} \sqrt{2} V_d \cos \alpha}, \quad (21)$$

where the DC input voltage of the inverter corresponds to V_d . Obtaining from these two the transformer ratio required for each of the heating cycles

$$n = \frac{I_L}{I_o}, \quad (22)$$

and therefore the primary inductance of the transformer is expressed as

$$L' = n^2 L. \quad (23)$$

Therefore, the capacitance value of the combination between the series capacitor C_s and the MERS capacitor C_{MERS} , is expressed as

$$C_{Eq} = \frac{\left(\frac{1}{\omega_o}\right)^2}{L'} \quad (24)$$

By means of the value of the voltage to be distributed between the two capacitors,

$$V_{C_{Eq}} = \frac{I_o}{\omega_{sw} C_{Eq}}, \quad (25)$$

the value of the two capacitors composing the resonant is selected, so that the equivalent value of the MERS can be obtained by the following expression

$$C_{MERS} = \frac{1}{\frac{1}{C_{Eq}} - \frac{1}{C_s}} \quad (26)$$

In Table 2, the values resulting from the design can be found, where the capacitance of both capacitors is 0.66 μF .

Table 2. Results of the Design.

Scan Step	Workpiece Diameter (mm)	Inductor Current (Arms)	Transformer Ratio	Equivalent Capacitor (μF)	MERS Capacitance (μF)
1	23.5	567	4.7	0.65	3.58
2	30.2	608	4.77	0.48	1.73
3	25.5	634	4.98	0.33	0.66
4	35	488	3.84	0.45	1.4
5	32	622	4.88	0.53	2.73

Regarding the design of the variable transformer, the selection of the core with the number of turns of the windings is the most important part, as the inductance ratio must be respected. Therefore, the product area, which is defined by the effective area A_e and the winding area A_w , is applied by the following expression

$$A_p = A_e A_w = \frac{V_{C_{Eq}} I_o}{2 f_{sw} B_{max} J K}, \quad (27)$$

where:

B_{max} corresponds to the maximum flux density (T).

J corresponds to the maximum current density (A/m^2).

K is a constant defined between 0.5 and 0.9, indicating the capacity of the winding.

The next step consists of iterating to obtain a less restrictive value than the one used in the area product. To do this, a core of larger dimensions than those calculated in the previous step is chosen, and using the core loss curves at the lowest operating frequency, it is sought that the value obtained for the flux density ($B = \mu\text{H}$) is less than the maximum, the permeability being that of maximum variation obtained from the curves of variation of the relative permeability under DC bias polarization. Therefore, with the characteristics of the core, the minimum number of turns in the primary is obtained with the following equation:

$$n_p = \frac{V_{C_{Eq}}}{4 f_{sw} B_{max} A_e}. \quad (28)$$

Using equations (11) and (13), the value of the minimum primary inductance is obtained using the above number of turns. This inductance will correspond to the maximum DC bias inductance [24], which is the minimum open circuit primary inductance $L_{p_{min_{oc}}}$, where 20% can be assumed to be the typical value of the minimum short circuit inductance $L_{p_{min_{cc}}}$ resulting in a minimum primary coupling coefficient (K_p) of 0.8.

$$K_p = 1 - \frac{L_{pmincc}}{L_{pminoc}}. \quad (29)$$

Then, the mutual inductance (L_M) is obtained from the following expression

$$L_M = \frac{L_{pminoc} K_p}{1 - K_p}. \quad (30)$$

Maximum secondary inductance, reflected from the primary of the transformer, is expressed as

$$L_{secmax} = \frac{(L_M/K_p)^2}{2L_{pmin}}. \quad (31)$$

Maximum transformer ratio is obtained from the inductance of the secondary when the primary is short-circuited.

$$L_{smincc} = \frac{L_{secmax}}{TR_{max}^2}. \quad (32)$$

The open circuit test value corresponds to the value of the minimum secondary inductance. To obtain it, it is considered that 0.8 is the typical secondary coupling factor (K_s), which allows us to obtain its value with the following expression

$$L_{sminoc} = \frac{L_{smincc}}{1 - K_s}. \quad (33)$$

Therefore using (12) and (14) the number of turns can be obtained.

The value of the minimum transformation ratio is obtained from the minimum primary and secondary coupling coefficients. Because when the core is not exposed to DC bias, the magnetic flux is mostly closed through the outer branches of the core. Resulting in a coupling coefficient (K) of 0.05. This will be experimentally confirmed later when the design is validated.

$$K = \sqrt{K_p K_s}. \quad (34)$$

Given the ratio of the maximum primary and secondary inductances obtained from (11), (13) and (12), (14) gives the minimum transformer ratio

$$TR_{min} = \frac{1}{K} \sqrt{L_{smax}/L_{pmax}}. \quad (35)$$

The design will be successful if the minimum ratio obtained is less than or equal to the desired ratio, leading to the last procedure. This consists of reaching the minimum inductance values by means of the value of $\tilde{\mu}$ used in equations (13) and (14) in order to determine the necessary H_{bias} .

For this purpose, using the following expression it can be optimize the relationship between the bias current (I_{bias}) and the number of turns of the saturation winding (n_{dc})

$$I_{bias} 2n_{dc} = H_{bias} 2l_{dc}, \quad (36)$$

where:

H_{bias} is the continuous magnetic field intensity.

l_{dc} is the length of the dc control branch from center to center of the core (m).

To regulate the converter power once the resonant is matched with the variable transformer and MERS, phase shift (PS) control is used. This phase shift varies the voltage applied to the resonant between 0 and 180° and is expressed as (φ). The following expression defines the fundamental harmonic rms output voltage of the inverter

$$V_o(\varphi) = \frac{2\sqrt{2}}{\pi} V_d \cos \frac{\varphi}{2}. \quad (37)$$

And, therefore, the output current of the inverter can be expressed as a function of the phase shift, the transformation ratio and the equivalent capacity due to the MERS influence by the following equation

$$I_o(\varphi, TR, C_{Eq}) = \frac{2V_d Q \sqrt{2C_{Eq}L}}{\pi TR L} \cos \frac{\varphi}{2} \cos(\alpha). \quad (38)$$

In the same way that the output power of the converter is expressed by means of

$$P_o(\varphi, TR, C_{Eq}) = \frac{8V_d^2 Q}{\pi^2 TR} \sqrt{\frac{C_{Eq}}{L}} \cos^2 \frac{\varphi}{2} \cos^2(\alpha). \quad (39)$$

5.2. Control Operation Principle

The control of the reactive elements has to act on the MERS capacitor to vary the effective capacitance of the resonant, on the variable transformer ratio to increase or decrease the reflected inductance of the inductor in the primary, and on the phase angle to vary the voltage applied to the load. And these act on the control setpoints, i.e., the inverter power and switching frequency, which are set by the part to be heated at each point in the scanning process.

Figure 12 shows the simplified scheme of the proposed control. In the first stage, each input acts independently with its setpoint by means of a proportional integral and differential (PID) control where the first control loop has as input the inverter input power, with which the error is obtained by comparing it with the setpoint. The PID control obtains the phase shift percentage which in the final block will result in the transistors gate signals at the switching frequency. Its behaviour is expressed by the equation (40). The second control loop is the converter output current, which is compared to the optimum current, equation (39). This is because the inductor current can vary by the transformer ratio or by the inverter output current, being more optimal to obtain the current by the transformer ratio than by the inverter output, taking into account that the resonance frequency will vary with the transformer ratio. The last block will be responsible for controlling the bias current injected into the transformer. The third control loop is the optimal switching angle, i.e., the angle between the inverter output voltage and the output current, which is expressed by the following equation

$$\omega_{sw} = \omega_o \frac{\tan \frac{\varphi}{2} + \sqrt{\tan^2 \frac{\varphi}{2} + 4Q^2}}{2Q}, \quad (40)$$

where the switching frequency must be varied so that the angle α is approximately equal to $\varphi/2$, thus maintaining a constant optimum angle [25]. As shown in the equation and considering the fixed switching frequency as the heating parameter, the system must vary by the MERS the resonant frequency to obtain the optimal value, equations (4) and (27). Therefore, this loop regulates the MERS duty cycle, adapting to the switching frequency and the zero crossing of the current in the last block.

As has been observed, the system equations are all related to each other and, therefore, being a 3x3 system, with multiple inputs and multiple outputs (MIMO), a coupled non-linear system is used, where each regulation is decoupled with the diagonalisation network [26,27].

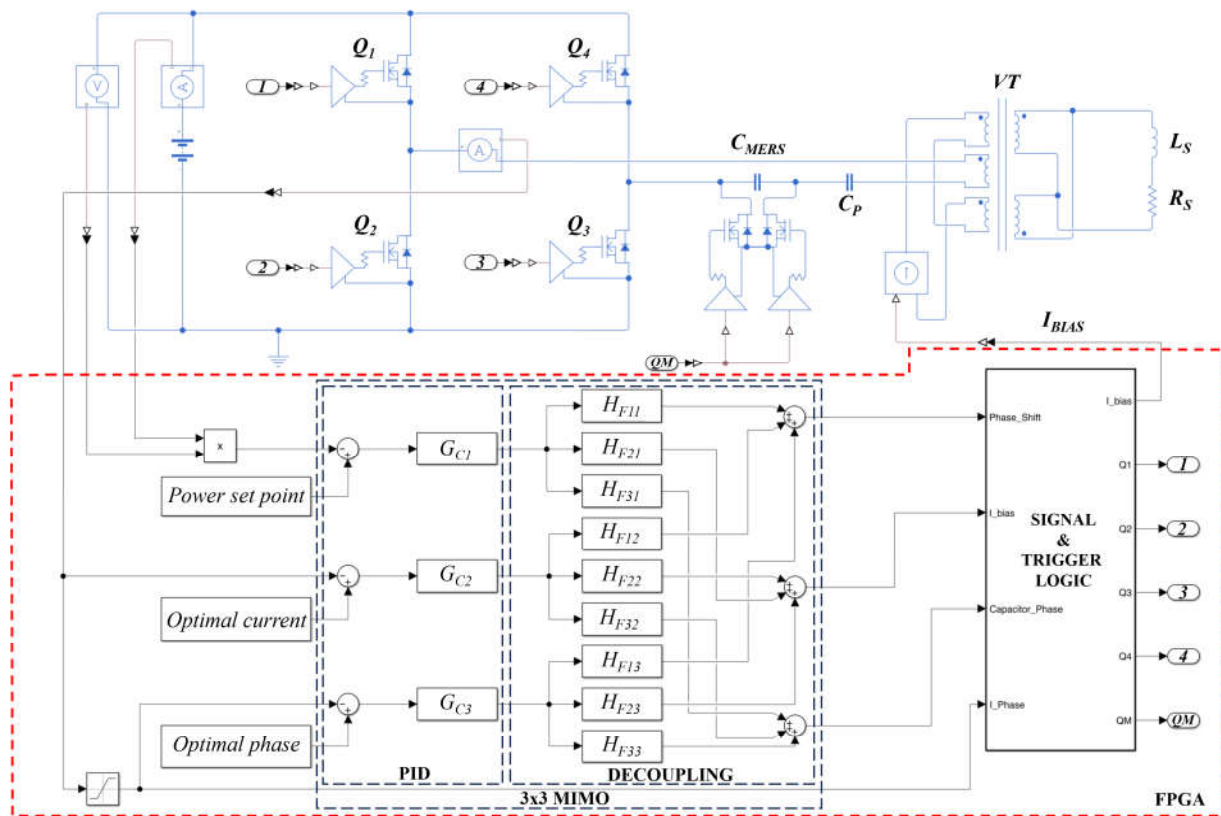


Figure 12. Schematic of the proposed system, detailing the inverter topology, the resonant load LC with the variable inductor, MERS and the control diagram.

5.3. Losses Analysis

This section analyzes the theoretical losses of the inverter, the MERS and the variable transformer.

As for the inverter losses, the losses due to the current conducted through the channel of the MOSFETs are given by the following expression

$$P_{cd}(\varphi, TR, C_{Eq}) = \left(\frac{I_o(\varphi, TR, C_{Eq})}{\sqrt{2}} \right)^2 R_{DS_{on}}, \quad (41)$$

where $R_{DS_{on}}$ is the ON state MOSFET resistance channel.

The switching losses are due to the OFF losses of the MOSFETs, due to the ZVS operation of the converter, since it switches above the resonance frequency, which makes the ON switching losses negligible. For its analysis the energy is obtained from the polynomial equation of the manufacturer's OFF state switching loss curves for the drain to source voltage equal to V_d and the gate resistance used, resulting in

$$E_{off}(\varphi, TR, C_{Eq}) = aI_c(\varphi, TR, C_{Eq})^2 + bI_c(\varphi, TR, C_{Eq}) + c, \quad (42)$$

where I_c corresponds to the switching current of each transistor and is expressed by the following equation,

$$I_c(\varphi, TR, C_{Eq}) = \frac{2V_d Q \sqrt{2C_{Eq}L}}{\pi TR L} \cos \frac{\varphi}{2} \cos(\alpha) \sin(\alpha). \quad (43)$$

Therefore, the power losses for each transistor are obtained from

$$P_{sw}(\varphi, TR, C_{Eq}) = E_{off}(\varphi, TR) \frac{\omega_{sw}}{2\pi}. \quad (44)$$

In the same way that the conduction losses of the MOSFETs used in MERS are calculated using the following equation

$$P_{MERS_{cd}}(\varphi, TR, C_{Eq}) = (I_o(\varphi, TR, C_{Eq}) \sin(\gamma))^2 R_{DS_{on}}. \quad (45)$$

As well as only the OFF switching power losses of the transistor is considered

$$E_{MERS_{off}}(\varphi, TR, C_{Eq}) = aI_{MERS_c}(\varphi, TR, C_{Eq})^2 + bI_{MERS_c}(\varphi, TR, C_{Eq}) + c, \quad (46)$$

where the amplitude of the switching current is given by the following expression

$$I_{MERS_c}(\varphi, TR, C_{Eq}) = \frac{2V_d Q \sqrt{2C_{Eq}L}}{\pi TR L} \cos \frac{\varphi}{2} \cos(\alpha) \sin(\gamma). \quad (47)$$

This results in the following expression for the switching losses

$$P_{MERS_{sw}}(\varphi, TR, C_{Eq}) = E_{off}(\varphi, TR) \frac{\omega_{sw}}{2\pi}. \quad (48)$$

As for the transformer losses, the core losses due to magnetic flux density must be considered, as the influence of DC polarization being negligible [16,28]. Therefore, they can be obtained by means of the following equation

$$P_{TC} = P_{DT} V_T, \quad (49)$$

where P_{DT} is the transformer core loss density given by the manufacturer for the operating frequency and magnetic flux density, and V_T is its volume.

For the P_{TW} copper losses of the transformer, it is necessary to determine the effective resistance of the primary winding R_p and of the secondary winding R_s , taking into account the conductivity, the total length and the skin effect. The losses are obtained by

$$P_{TW} = I_o(\varphi, TR, C_{Eq})^2 (R_p + R_s). \quad (50)$$

Finally, the total losses are obtained from

$$P_{tot}(\varphi, TR, C_{Eq}) = 4P_{cd}(\varphi, TR, C_{Eq}) + 4P_{sw}(\varphi, TR, C_{Eq}) + 2P_{MERS_{cd}}(\varphi, TR, C_{Eq}) + 2P_{MERS_{sw}}(\varphi, TR, C_{Eq}) + P_{TC} + P_{TW}, \quad (51)$$

meanwhile, the efficiency is given by

$$\eta(\varphi, TR) = \frac{P_o(\varphi, TR, C_{Eq})}{P_o(\varphi, TR, C_{Eq}) + P_{tot}(\varphi, TR, C_{Eq})}. \quad (52)$$

Figure 13 shows the efficiency evolution in front of output power for the power ranges of each heating scan step. The components selected for the final design were SiC MOSFETs from Onsemi Scottsdale AZ with reference NTH4L022N120M3S where four were used for the inverter and two for the MERS. As for the transformer, two E-shaped cores from TDK Electronics Munich Germany with N27 material and 70/33/32 geometry were selected. As can be seen in the plotted results, the higher frequency required in steps 4, 3 and 2 makes the efficiency lower than in the rest, despite working with a resonant tank that adapts to each heating.

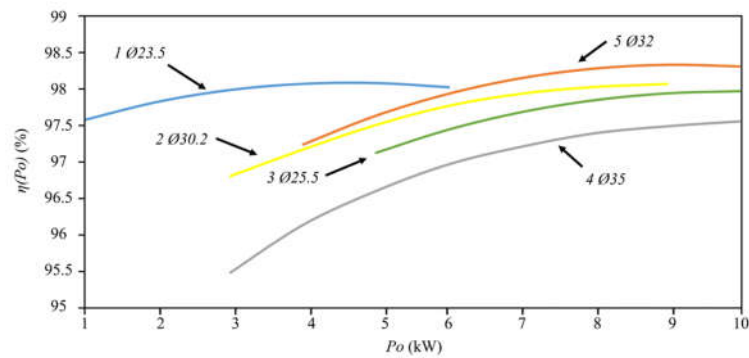


Figure 13. Calculated evolution of the efficiency of the inverter as a function of the output power for each scan step.

6. Experimental Results

For this section, the variable transformer and MERS designs were first verified and then used in the inverter prototype to verify the correct operation of the complete system.

In order to evaluate the design of the transformer, first step was to compare the theoretical values of the maximum and minimum inductance of the primary and secondary inductances with the experimental measurements, in order to check the veracity of the design and verify that the previous assumptions were correct. Table 3 shows these results for the maximum and minimum bias current. Despite observing a small dispersion in the values, the design is correct, since this dispersion is due to the parasitic elements not considered in the design and to the accuracy of the measurement.

Table 3. Theoretical and measured values of transformer inductances.

Control Current	Theoretical Inductance (μH)				Measured Inductance (μH)			
	Open Circuit		Short Circuit		Open Circuit		Short Circuit	
I_{dc} (A)	L_{poc}	L_{soc}	L_{psc}	L_{ssc}	L_{poc}	L_{soc}	L_{psc}	L_{ssc}
0	1103.1	79.37	1.22	0.793	1124.9	82.7	1.14	0.816
1	227.3	34.89	0.544	0.236	232.4	29.53	0.489	0.256

As for the measurements of the entire saturation range of the transformer designed, Table 4, the ratios and variations of its inductances were measured, resulting in a small dispersion that will not affect the operation of the converter and that is due to the same reasons as in the previous results.

Table 4. Theoretical and measured values of transformer ratios.

Control Current	Theoretical			Measured		
	L_p (μH)	L_s (μH)	TR	L_p (μH)	L_s (μH)	TR
0	1103.1	79.37	2	1124.9	82.7	1.82
0.25	478.06	60.45	3	472.89	58.16	3.1
0.5	333.12	42.79	4	327.93	41.68	4.34
0.75	258.32	35.43	5.5	275.5	35.01	5.36
1	227.3	34.89	7	232.4	29.53	6.58

To check the correct operation of the MERS, the voltage of the variable capacitor in operation was measured at the same time as the switching control signal of the capacitor. In Figure 14 below, two oscilloscope screenshots have been taken to show the behavior of the system at two different conditions. As can be seen in the image, the variable capacitor

voltage becomes zero when the MERS MOSFETs are in conduction, and it can also be seen that this signal coincides with the zero crossings of the inverter output current.

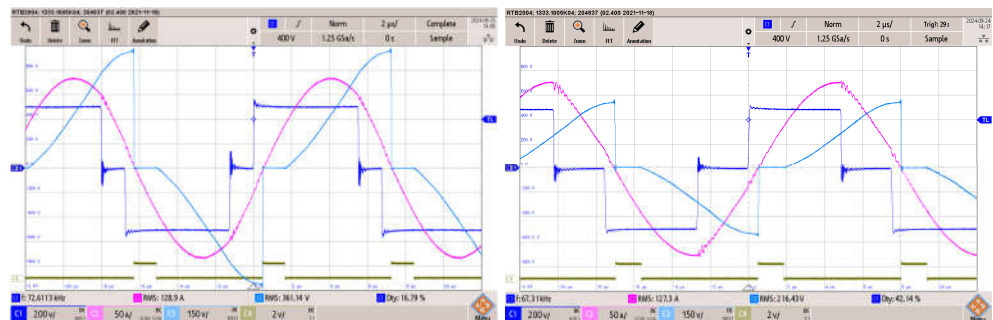


Figure 14. Oscillograms of experimental tests for MERS capacitor at two different conditions. C1 (dark blue) represents the output voltage (200 V/div), C2 (magenta) shows the output current (50 A/div), C3 (light blue) shows the voltage of the MERS capacitor (150 A/div) and C4 (brown) displays the MERS gate logic signal (2 V/div). The time base is 2 μs/div.

A 10 kW prototype inverter was used to test the whole system as a whole, which integrates the FPGA with the control, power supplies, firing controllers, sensors, MERS and the controllable current source of the variable transformer. The test bench used is shown in Figure 15, and each number corresponds to:

- (1) Induction heating inverter with six SiC NTH4L022N120M3S.
- (2) MERS capacitor.
- (3) Series capacitor (C_p).
- (4) Variable transformer.
- (5) Series inductor (L_s).
- (6) Set of workpieces.
- (7) 300 MHz bandwidth DSO.
- (8) Rogowski current probe.
- (9) Differential voltage probe.
- (10) Hall effect probe.

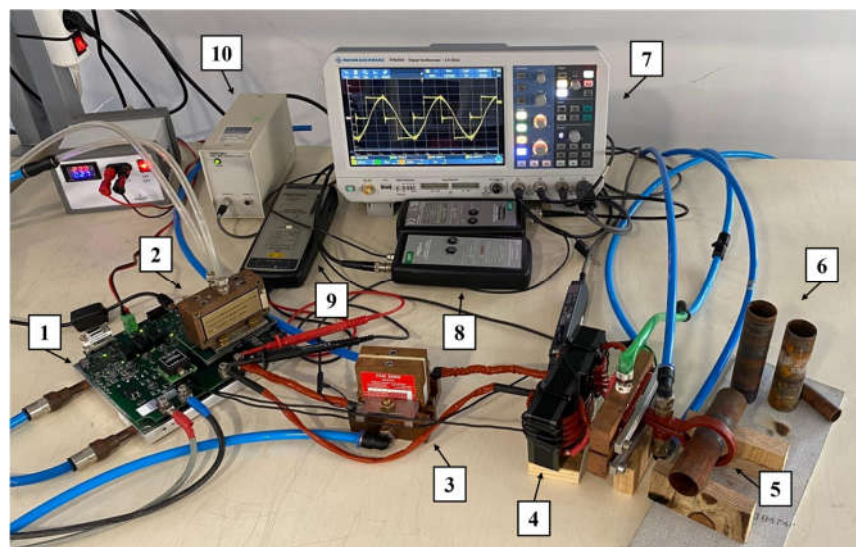


Figure 15. Inverter, MERS capacitor, variable transformer, and LC resonant load test bench. The numbered labels describe the components on the bottom.

In Table 5, the values of the main parameters obtained during the experimental tests of the converter operation have been recorded. In this table, the theoretical values of the design equations have also been added, in order to verify their veracity. Additionally, Figure 16 shows the screenshot of the oscilloscope for each of the measurements. In them it can be seen the converter operation for each step and setpoint of power. How it is observed, the system adapts to the frequency required in each of the steps.

Table 5. Comparison of theoretical and measured results at maximum power and 5 kW.

Scan Step	Power (kW)	Frequency (kHz)		Phase Shift (°)		Transformer Ratio		Equivalent Capacitor (μF)	
		Theor.	Meas.	Theor.	Meas.	Theor.	Meas.	Theor.	Meas.
1	5	67	66.98	79.3	80.46	4.7	4.87	0.66	0.6
	6		67.24	74.4	76.01		4.65		0.65
2	5	77.5	77.62	66.7	67.11	4.77	4.7	0.48	0.47
	9		77.48	38	38.84		4.76		0.47
3	5	89	89.43	60	59.58	4.98	5.11	0.33	0.3
	10		89.57	25.4	26.82		5.03		0.31
4	5	100	101.71	76.6	76.3	3.84	3.71	0.45	0.45
	10		102.3	54	53.89		3.82		0.42
5	5	72	72.17	61.7	63.18	4.88	4.95	0.53	0.5
	10		72.65	28	30.58		4.78		0.52

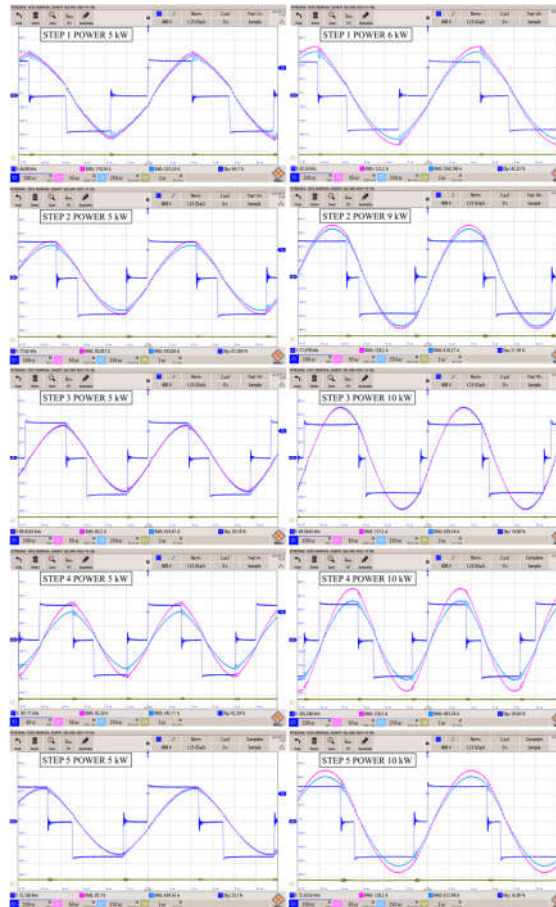


Figure 16. Oscillograms of experimental tests for the LC resonant inverter with rective elements control for different heating steps and power. The left column represents operation at 5 kW, and the

right column shows the operation at maximum power. C1 (dark blue) represents the output voltage (200 V/div), C2 (magenta) shows the output current (50 A/div), C3 (light blue) shows the output current at the secondary of the transformer (250 A/div) and C4 (brown) displays the DC bias current (2 A/div) of the variable transformer. The time base is 2 μ s/div.

Finally, the efficiency of the converter has been recorded for each heating step over the entire power range of each step. As can be seen in Figure 17, the recorded efficiency is similar to the theoretically calculated one, being this one between a percentage range of 98.5% and 95.9%, depending on the desired power and the required frequency in each heating step. As previously obtained, this efficiency has a high correlation with the switching frequency, since heating steps 3 and 4 are the ones that have registered the lowest efficiency, because they occur at frequencies of 89 kHz and 100 kHz respectively.

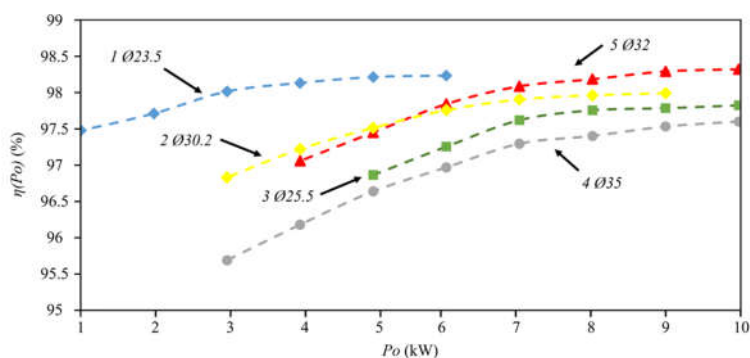


Figure 17. Results of the experimental measurements of the inverter efficiency in the power ranges of each step.

7. Conclusions

This paper presents a novel control method for series inverters in induction heating applications that require hardening of pieces with variable geometries, where the use of one or two frequencies is not sufficient to meet the requirements of the application. This system makes it possible to vary the working frequency maintaining the quality factor and allows the power and the working frequency to be varied independently. For this purpose, a current-controlled variable transformer has been used to adapt the inductor inductance to the resonant tank, a variable capacitor called MERS that varies the reactive power compensation to the inductor and the phase shift control to regulate the converter output power at fixed frequency. In the article, each of the stages has been analyzed and designed and mathematical expressions have been obtained to design and calculate the efficiency of the converter. All this has been experimentally tested with a prototype that has reached 10 kW with a maximum frequency of 100 kHz and a quality factor of 10. The measured efficiency of the system remained in the range of 98.5% and 95.9%.

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