Journal of Behavioral and Experimental Economics 119 (2025) 102471

Contents lists available at ScienceDirect

Journal of Behavioral and Experimental Economics

journal homepage: www.elsevier.com/locate/jbee

Consumers' privacy concerns and price setting in a multichannel monopoly

M. Sánchez [a], A. Urbano [b,c,*]

[a] Department of Quantitative Methods for Economy and Business, University of Murcia, Spain
[b] University of Valencia, Edificio Departamental Oriental, Campus del Tarongers, 46022 Valencia, Spain
[c] ERI-CES, University of Valencia, c/Serpis 29, 46022 Valencia, Spain

ARTICLE INFO

JEL classification:
D89
C72
D42
L12

Keywords:
Instrumental privacy concerns
Learning
Market channels
Fully revealing perfect bayesian equilibrium
Pooling equilibrium

ABSTRACT

This paper develops a dynamic model of multichannel price setting under market signaling and learning. A monopolist sells a good through two channels: brick-and-mortar and online. We introduce two novel features: (i) instrumental and idiosyncratic privacy concerns for consumers purchasing online, and (ii) the idea that firms possess better information about the market's average privacy concerns than individual consumers, and signal this information through prices. In any fully revealing equilibrium, price signaling can distort prices upward compared to a setting in which the monopolist's private information is common knowledge. Moreover, social welfare may be higher under a non-informative (pooling) equilibrium than under a fully revealing equilibrium if the realized market signal is sufficiently high. These results offer new insights into the interplay between privacy concerns, information asymmetry, and dynamic pricing in the digital economy, with implications for policy design.

1. Introduction

Consumer privacy has become a central concern in today's digital markets. The rapid expansion of online commerce has transformed industries worldwide.[1] Complementary technologies — such as smart devices, online platforms, big data analytics, and artificial intelligence — have facilitated the extensive collection and diverse use of consumers' digital footprints. As a result, many traditional brick-and-mortar firms have extended their operations online, evolving into multichannel or "brick-and-click" firms. In this new landscape, consumer data has emerged as a highly valuable asset, enabling firms to predict behavior and deliver personalized services, thereby creating novel forms of business value.

However, the benefits of such data-driven strategies are increasingly being undermined by rising consumer privacy concerns.[2] In the United States, evidence from the TRUST e-Survey found that nearly 92% of consumers have uninstalled retail apps over data concerns, 74% have reduced their online activity, and 45% report increased privacy anxiety compared to previous years (Hakobyan, 2019; Li et al., 2020). Similar patterns are observed globally—for instance, 69% of Australians report greater concern about online privacy compared to past years (Australian Government, 2017).

Acquisti et al. (2020) document enduring privacy concerns across decades of surveys, as well as experimental and observational evidence of privacy-sensitive behavior. Research in behavioral economics and psychology further suggests that when individuals are uncertain

* Correspondence to: Economic Analysis Department, Edificio departamental oriental, Avenida de los Naranjos s/n, 46022 Valencia, Spain.
E-mail addresses: mariola.sanchez@um.es (M. Sánchez), amparo.urbano@uv.es (A. Urbano).

[1] The beginning of online sales can be traced back to the late nineties. Despite the dot-com collapse in 2000, brick-and-mortar firms realized the benefits of online sales and started adopting it. Since then, there has been a gradual and continuous increase in the adoption of online sales. The number of online shoppers has increased in 2020 by 9. 5% to 3.4 billion. On-line sales revenue has increased by 25% to reach $2.43 trillion in 2020. Revenues and users of online markets are expected to grow to reach $3.4 trillion and 4.9 billion users, respectively by 2025. In the United States, the online market showed growth of 14. 9%, while total retail sales increased only 3. 4% during the fourth quarter of 2019 compared to the same period in 2018 (U.S, Department-of-Commerce, U., 2020). The growth in 2020 has almost tripled to reach 44%, the largest increase in the last two decades, largely due to the lockdown conditions of the COVID-19 pandemic. In addition, the online sales business represented 14. 3% of total sales in 2020 which was only 6.4% in 2015 (U.S, Department-of-Commerce, U., 2016).

[2] Privacy concerns have been recognized as a continual impediment to online retail growth. Also, privacy issues have been found to have a profound effect on consumer confidence when dealing with online transaction (Gal-Or et al., 2018). Understandably consumers have to disclose certain amount of their personal information in retailer's online platform in order to purchase certain products or services. Finally, there have been calls for revisiting and addressing privacy issues in the contemporary setting. As highlighted by Smith et al. (2011), "to date, many important threads of information privacy research have developed, but these threads have not been woven together into a cohesive fabric".

https://doi.org/10.1016/j.socec.2025.102471
Received 6 March 2025; Received in revised form 6 September 2025; Accepted 23 October 2025
Available online 30 October 2025

about their own preferences, they rely on contextual cues—a phenomenon known as "constructed preferences".

Lin (2022) distinguishes between two types of privacy preferences: intrinsic, where privacy is valued as a right,[3] and instrumental, where privacy is valued for its role in preventing harmful outcomes. For example, consider Insurance Telematics Programs. Auto insurers like Progressive or Allstate offer discounts if drivers use a tracking device (e.g., Snapshot) to monitor habits. Safe drivers may accept and enjoy lower premiums. Riskier drivers or privacy-conscious users reject it, even if premiums are higher. Insurers set prices based on beliefs about average risk inferred through voluntary participation. Thus, consumers show instrumental privacy concerns: avoiding data-sharing to prevent negative classification, the payoff of preventing their private "type" from being revealed through data (Posner 1981, Stigler 1980).[4] In other words, instrumental privacy concerns are concerns about privacy that stem from the anticipated negative outcomes associated with inappropriately accessed, used, or shared personal data. These concerns are instrumental in the sense that privacy is valued not necessarily for its own sake but because violating it can lead to harms such as identity theft, discrimination, surveillance, reputational damage, financial loss, or manipulation.[5]

In economic theory, privacy preferences are often treated as instrumentally motivated—dependent on how firms use personal data. Unlike intrinsic preferences, which are typically fixed, instrumental concerns are endogenous and context-dependent. For example, consumers may opt for privacy-preserving alternatives like DuckDuckGo over data-intensive services like Google, not because they dislike personalized ads per se, but to avoid potential harms from data misuse.[6] Recognizing this endogeneity helps reconcile findings that privacy preferences are sensitive to context.

As firms move online, they increasingly invest in market research to understand the population-level (average) instrumental privacy concerns of consumers. These insights are then reflected in pricing strategies. Intuitively, if privacy concerns are high, firms may lower prices to encourage purchases; if concerns are low, prices may be higher. At the same time, consumers learn about the relevance of their privacy concerns through market experiences and pricing cues.

We develop a two-period signaling model in which a monopolist sells a good through both physical and online channels. Consumers purchasing online have uncertain, idiosyncratic instrumental privacy concerns, which they begin to understand only after the initial purchasing experience. Meanwhile, the firm receives a noisy signal about the market population-average privacy concern and strategically sets prices to signal this information in the second period. Thus, as a novelty, our paper belongs to the few studies that assume instrumental privacy concerns, and thus, decouple privacy concerns from valuation of the good, as Choe et al. (2023) do. In this context, we wish to study how consumers' idiosyncratic privacy concerns affect price setting by this dynamic multichannel monopolist.

Our model is a two-period signaling game where a monopolist and consumers use market signals to learn about consumers' instrumental privacy concerns. Consumers who purchase in the online market have instrumental preferences for privacy concerns that are unknown to them when they make their purchases in the first period. These concerns are decomposed into the population-average concerns and the individual-idiosyncratic concerns. In addition, at the beginning of the second period, the monopolist receives a noisy signal about the population-average privacy concerns, which allows it to adjust the price in both sale channels in the second period. Consumers are aware that their beliefs about privacy depend on both their own experience and the population-average concerns, which they try to infer from the firm's information. Thus, consumers construct their instrumental preferences for privacy through the context. For example, consider Apple vs. Meta (Facebook). Apple made privacy a central part of its brand (e.g., App Tracking Transparency on iOS). Meta's business model depends heavily on personal data for targeted advertising. By highlighting strong privacy protections, Apple signaled low potential data misuse, building consumer trust. Apple charged premium prices for devices and services. However, after Apple's changes, Meta saw a drop in advertising efficiency. Firms reliant on Meta ads paid higher prices for worse targeting. Thus, Apple used "price and platform behavior" as a signal of average privacy sensitivity. Meta's reduced ability to extract data reflects high average privacy concerns in the market.

Another novelty is the use of the idea that the firm has better information about the market average privacy concerns than consumers and signals this information to consumers via prices. This signal can stem from market research, past data, or other analytics. The key innovation in our model is that the firm has better aggregate information about privacy concerns than individual consumers do at the time of purchase. By setting prices strategically in the second period, the firm can signal their private information. Consumers then update their beliefs about their own instrumental privacy preferences based on the observed price and their own past experiences. Thus, the prices designed by the monopolist in the second period serve as a signal to consumers on the monopolist's private information. Consumers' inference from prices, together with their first period purchasing experience, will determine their second period demand. Crucially, for the price to signal the average market privacy to consumers, the monopolist's private information must not be a sufficient statistic. In fact, the firm may use the price mechanism to endogenously change consumers' beliefs about their privacy concerns. Therefore, some idiosyncratic information learned by consumers is important because consumers' private information limits the extent to which the monopolist can deceive them. We study price setting by the monopolist under the two standard patterns: uniform pricing and price differentiation.

This study provides several key findings. First, in any second-period fully revealing equilibrium, price signaling can distort prices upward compared to a scenario in which the monopolist's private information is common knowledge to both consumers and the monopolist. This distortion arises because, as consumers update their information, an increase in price leads them to infer lower levels of privacy concerns, resulting in higher second-period demand. If consumers place significant weight on market prices when forming beliefs about the firm's signal, and if they also rely heavily on the firm's signal when estimating their own privacy concerns, then the second-period inverse demand curve becomes steeper. Thus, price signaling causes demand to become less sensitive to price increases. However, as previously noted, consumers' private information constrains the extent to which the monopolist can mislead them. Our two-period model is a signaling game, and we find that, in second-period separating equilibria, the firm's prices are a linear function of its private information. The crucial feature here is the active learning behavior of consumers. To our knowledge, this study is

[3] For instance, most people resist contact-tracing apps during the pandemic, citing concerns about the "big brother" overreach and violation of personal rights.

[4] For example, risky drivers may avoid installing devices that allow an insurance firm to monitor their driving habits (Jin and Vasserman 2021, Soleymanian et al. 2019).

[5] For example, concern about behavioral data being used to manipulate purchases or political opinions, discrimination via data profiling: Worry that employers or insurers might use data to offer worse terms or deny services, surveillance, and chilling effects: Changes in online behavior due to fear of being monitored or flagged by authorities, reputation damage: Risk of personal information being leaked and causing personal/professional harm.

[6] Consider DuckDuckGo vs. Google. DuckDuckGo markets itself as a private search engine, avoiding user tracking. Google offers more personalized services but collects extensive user data. DuckDuckGo cannot monetize as well via ads, so it must charge advertisers less or target privacy-conscious users. Google offers services for "free" but monetizes heavily via user data. DuckDuckGo appeals to consumers with high instrumental privacy concerns, while Google may "pool" users into a market where privacy is less of a concern.

the first to assess how price signaling influences price adjustment in a dynamic model of consumer signal extraction and learning, where privacy concerns are both instrumental and idiosyncratic, and where the monopolist has more information than consumers.

Second, when the signal regarding average population privacy concerns is sufficiently high, consumers fare better in the second period under a non-informative equilibrium (pooling) than under a fully revealing equilibrium. Similarly, the monopolist's profits in the second period may be higher with a pooling equilibrium than with a fully revealing one when the signal of market privacy concerns is sufficiently high. This result shows that signaling can be quite costly. Consequently, second-period social welfare may also be higher with the non-informative equilibrium.

Third, we study both uniform pricing and third-degree price discrimination. More personalized pricing—where prices are tailored based on individual data such as browsing history, location, or purchasing behavior—can raise concerns under the GDPR (General Data Protection Regulation enforced in the EU; see Zuiderveen Borgesius and Poort (2017)). The costs of ensuring GDPR compliance—including system modifications for transparency, managing data subject requests, and the risk of substantial fines for non-compliance—can be considerable. Faced with these challenges, some firms may prefer uniform pricing strategies, which are simpler to implement from a compliance perspective as they require less extensive data processing and disclosure. While personalized pricing may potentially improve profitability, concerns about consumer backlash, regulatory requirements, and complexity and costs of implementation often lead firms to favor uniform pricing. As a result, both pricing strategies remain relevant in the design of sellers' pricing models.

We perform a factorial experiment that shows that the main qualitative results of the model (e.g., how privacy concerns shape firm behavior and welfare outcomes) are stable across a wide range of parameter values. The experiment also highlights that interactions matter: Some parameters do not matter much in isolation, but their effects become important when combined with others (for instance, when both consumer privacy concerns and the precision of the market signal are high.).

Finally, this paper provides valuable insights into how privacy concerns, information asymmetry, and dynamic pricing interact in the digital economy. By tailoring policy, we can create a fairer, more transparent market where consumer privacy is respected, and businesses thrive on trust, not manipulation.

We assume fixed proportions of consumers across channels to preserve tractability. While this simplifies the model, it retains the core insights about signaling, learning, and price-setting under privacy concerns.

Related literature

This paper contributes to several strands of literature, most notably in privacy economics, signaling theory, and markets for information. First, we build on the literature on privacy preferences, especially where privacy is viewed as an economic good. Acquisti et al. (2016) provide a comprehensive survey of privacy economics, emphasizing the subjective and idiosyncratic nature of privacy sensitivities. Our work focuses specifically on instrumental privacy concerns, as in Lin (2022), and Choe et al. (2023), and decouples them from product valuation. This approach contrasts with earlier studies that model privacy as a fixed utility loss or cost. Papers such as Acquisti and Varian (2005) and Conitzer et al. (2012), and Belleflamme and Vergote (2016) examine how access to or concealment of private data affects pricing, consumer surplus, and firm profits. We differ by modeling consumers' privacy concerns as uncertain and learned over time, rather than known in advance. Villas-Boas (2014) and Chen and Zhang (2009) explore dynamic pricing strategies under customer learning, but do not focus on privacy concerns.

Second, our study contributes to signaling theory, particularly in dynamic games with asymmetric information. Signaling models are used to understand how agents with private information communicate indirectly through observable actions. In our model, the monopolist's second-period price plays a signaling role, akin to the literature on noisy learning in monopoly settings (Matthews & Mirman, 1983; Judd & Riordan, 1994; Mirman et al., 2015). Consumers, in turn, interpret price changes as signals about average privacy concerns and adjust their demand accordingly. Jeitschko and Normann (2012) show that firms in a duopoly may under-invest in private information acquisition, whereas we study a monopoly that uses its private signal to influence consumers' beliefs through pricing. Unlike Bonatti (2011), who focuses on publicly observable sales as information signals, we emphasize private learning by consumers based on their own purchasing experience and price signals.

Third, we relate to the broader literature on markets for information (Bergemann & Bonatti, 2019; Bergemann et al., 2022), where firms collect, analyze, and use consumer data to improve market outcomes. In contrast to intermediaries or platforms that sell or aggregate data, our model focuses on how a single firm uses internal information to adjust its pricing dynamically in a context where consumers have both idiosyncratic and common uncertainty.

Lastly, our paper contributes to the literature on data-driven pricing strategies and their implications under regulatory constraints. GDPR compliance challenges have made uniform pricing attractive to many firms (Zuiderveen Borgesius & Poort, 2017), and our model reflects this dual pricing environment by examining both personalized and uniform pricing settings.

Our key contribution lies in combining the literature on privacy, learning, and signaling to model the strategic pricing behavior of a brick-and-click monopolist when consumers have uncertain and instrumental privacy concerns. We show how prices serve as both economic and informational tools, shaping consumer learning and market outcomes.

The paper is organized as follows. Section 2 presents the general model, and Section 3 describes the belief updating. Sections 4 and 5 analyze the monopolist's two main strategies to set prices and derive the fully revealing equilibria. Sections 6 and 7 derive the pooling and hybrid equilibria of the model, respectively. Social Welfare is analyzed in Section 8, and Section 9 offers a parameter sensitivity analysis through a factorial experiment. Section 10 presents some policy implications, and Section 11 concludes the paper.

2. The model

2.1. Product market

Consider a two-period monopoly market and a mass of consumers. The monopolist has an overall demand consisting of consumers who purchase via two channels: the traditional channel (the brick-and-mortar channel) and the Internet (the online channel). The company may misuse consumer data, for example, selling consumers' data to third parties, but we do not model the company misbehavior because it does not add anything relevant to the analysis.

It is assumed (as in Bergemann and Bonatti 2024) that the proportion of consumers buying in each market is fixed to obtain closed-form solutions.[7] Let parameter λ represent the proportion of consumers buying from the brick-and-mortar channel and $(1-\lambda)$ that of consumers purchasing from the online channel. Thus, λ is exogenous and the total mass of consumers is normalized to 1. All consumers know their willingness to pay—the product is well established, and they are familiar

[7] The model of endogenous proportions is intractable, even with a model with a continuum of consumers, each buying a unit of product.

with its quality or taste. We take a linear-quadratic approach to make the analysis tractable and provide closed-form solutions.

Let $U_{it} = \theta_i q_{it} - \frac{q_{it}^2}{2}$ be the utility function of the consumer i in the brick-and-mortar channel in period t, and let $q_{it} = \theta_i - p_t$ be the consumer demand in period t, where p_t is the price in period t and θ_i is the known intercept of demand.

However, if an individual i purchases through the online channel, privacy concerns may diminish their utility. This is so because consumers do not know how the firm will use their data. We assume that consumers have an *instrumental* need for privacy because revealing their private information can lead to hurtful usage by the monopolist in the market. By instrumental preferences for privacy, we refer to preferences that arise not from an intrinsic valuation of privacy (such as valuing privacy as a right or a source of autonomy) but from the economic consequences of disclosing personal data. That is, consumers care about privacy because sharing their data might result in negative outcomes, such as price discrimination, identity theft, or misuse of their personal information. This aligns with the approach taken by Lin (2022) and Choe et al. (2023), and others, where privacy is seen as a strategic concern: consumers anticipate possible future harm and therefore develop preferences accordingly.

Thus, consumers do not know their *potential losses from online shopping*, represented by the random variable α_{it}, at the time of the purchase. As privacy concerns may diminish the consumers' utility, we assume that consumers i's utility function in the internet channel in period t is $U_{it} = \left(\theta_i - \alpha_{it}\right) q_{it} - \frac{q_{it}^2}{2}$. If individual i knew α_{it}, she would demand $q_{it} = \theta_i - \alpha_{it} - p_t$ units of the good in period t if price were p_t. However, since she does not know α_{it} at the time of purchase, the expected demand of the consumer i' for the online channel in period t is

$$E\left\{\theta_i - \alpha_{it} - p_t | \Omega_{it}\right\}, \tag{1}$$

where Ω_{it} is consumer i's information at time t. We assume that the unit production cost in each period is linear, common knowledge, and normalized to zero. In what follows we will use the shorter term *private concerns* to refer to the expression "potential losses from online shopping".

2.2. Information environment

2.2.1. Consumers

Consumers do not know how the firm is going to use their data when purchasing from the online channel, and fear monopolist misuse. Their instrumental preferences for privacy are modeled as endogenous and uncertain in our framework. That is, when a consumer first purchases online, she does not know exactly how much she should fear a privacy loss—because she does not know how the firm might use her data. Over time, through direct experience and through inference from market signals (especially prices), the consumer gradually constructs her privacy concerns. We assume that these concerns are private or individual to each consumer. We borrow from Acquisti et al. (2015) the idea that when people are uncertain about their preferences and they often look for cues in their environment to provide guidance.[8] One of the signals that influence perceptions of privacy is the behavior of other people. For example, observing other people reveal information increases the likelihood that one will reveal it oneself. Also, data externalities might explain why people are more willing to share data online if many other people already shared data. In our context, the behavior of other people is represented by the market population-average privacy concerns (market-average potential losses from online shopping). In addition, given the consumer lack of precision in their privacy preferences, we assume that consumers are building them over time. Thus, in our model, consumers try to learn their privacy concerns from the available information at each time.

Hence, consumers' privacy concerns are decomposed into population-average concerns and individual-idiosyncratic concerns, and represent the privacy concerns of consumer i buying through the online channel in period $(t = 1, 2)$ by an index α_{it} equal to,

$$\alpha_{it} = \tilde{x} + \widetilde{\omega_i} + \widetilde{v_{it}}. \tag{2}$$

Random variables $\tilde{x}$, $\widetilde{\omega_i}$, and $\widetilde{v_{it}}$ represent the population-average privacy in that specific product market, individual i's persistent deviation from that population-average privacy, and individual i's specific-time deviation, respectively.

We assume that the random variables are all normally and independently distributed of each other,[9] $\widetilde{\omega_i} \sim N\left(0, \sigma_\omega^2\right)$, $\widetilde{v_{it}} \sim N\left(0, \sigma_v^2\right)$, and $\tilde{x} \sim N\left(\bar{x}, \sigma_x^2\right)$. Variable $\tilde{\omega_i}$ captures differences between consumers. Some consumers do not care about privacy, whereas others may consider privacy of vital importance. In particular, if a consumer detects that some private information is used in a harmful way, she will increase the value of α_{it}, hence lowering the utility of this channel. Variable $\tilde{v_{it}}$ is an external shock. For example, there may be a new security system that allows consumers to prevent being tracked by cookies and which depends on the device to access the Internet of each consumer.

We study a context where the consumer demand for products is nonnegative. Thus, we assume that $\bar{x}$ (the mean of the population-average privacy) is smaller than θ_i. In addition, we will provide bounds on the realization of the market signal z, ensuring nonnegative equilibrium prices and quantities. The fact that the willingness to pay is equal and known between channels allows us to focus on the concerns of consumers about potential losses from online shopping, that is, the uncertainty of the demand, as a distorting element in the analysis of the equilibrium of the market.

After first period purchases and knowing θ_i, p_1 and q_{i1}, consumer i observes $\hat{\alpha_{i1}} = \theta_i - p_1 - q_{i1}$ and gets a realization of the random variable α_{i1}. We also assume that each consumer action (quantity choice) and payoffs (experience with privacy concerns) is privately observed, so that information is not publicly observable in the market. Hence, the other buyers and the firm do not observe it. In fact, the firm only observes the aggregated expected demand of the online channel, contrary to Bonatti (2011). We examine the scenario where a firm uses the price mechanism to signal its information and try to endogenously change consumers' beliefs about their privacy concerns.

2.2.2. The firm

The firm receives a private signal about consumers' average privacy concerns at the beginning of period 2. This signal can stem from market research, past data, or other analytics. The key innovation in our model is that firms have better aggregate information about privacy concerns than individual consumers do at the time of purchase. By setting prices strategically in the second period, firms can signal their private information. Consumers then update their beliefs about their own instrumental privacy preferences based on the observed price and their own past experiences. The signal received by the monopolist is,

$$z = \tilde{x} + \tilde{\varphi}, \tag{3}$$

where $\tilde{x}$ represents the random variable, showing, as before, the average privacy in the market, and $\tilde{\varphi}$ is an external shock, which is distributed normally $\tilde{\varphi} \sim N\left(0, \sigma_\varphi^2\right)$.

[8] Context-dependence of privacy preferences. In fact, assuming instrumental preference as endogenous can also help us reconcile previous findings that privacy preferences are context-specific.

[9] Normality has the disadvantage of unbounded support, allowing for negative demand and prices. However, normality has the highly desirable feature of implying linear updating rules for consumers, which simplifies our analysis considerably. Negative prices will be considered off equilibrium signals.

Information about privacy concerns is important to the monopolist because it is required to set prices in the online channel. Therefore, the signal z represents important information for the monopolist's second-period action, and it will be observed after first-period sales. This can be interpreted to mean that the firm undertakes some market research following initial sales, to have an idea of consumers' privacy concerns. This timing assumption on the firm's private information simplifies the exposition in places. However, we could also have assumed that the monopolist observes this information before period-one sales, for example, the market research could have be done before first period sales. It turns out that such first-period information is neither valuable to the firm nor in equilibrium can be signaled until consumers have acquired some corroborating information of their own. Therefore, private information of the firm only matters in period two, and we can adopt either a market research after first period sales or before them. This signal is common knowledge but only observed by the firm. The important point to keep in mind is that the signal that the monopoly observes does not depend on the number of buyers of the online channel.

Let γ_z be the relative precision of signal z (i.e., $\gamma_z = \frac{\sigma_x^2}{\sigma_z^2}$). To focus on signaling issues and discard manipulative behavior by the firm, we take γ_z to be exogenous and common knowledge among consumers and the monopolist. This basic case is enough to offer the main signaling results.

To complete the description of our model, we next discuss the order in which actions are taken and information revealed. First, after period-one sales, γ_z is determined by the firm (or by nature) and is directly observed by consumers.

2.2.3. Timing of the game

Period 1

In period 1, the market for the product opens. The monopolist must decide on its price strategy (i.e., whether to practice price discrimination) for both channels and must announce the first-period price or prices.

In this first period, no information is generated by any player, so there is neither private information for the monopolist nor learning by customers. Therefore, their information sets consist of simple expectations.

Thus, at the beginning of the first period, consumers of the online channel are uncertain about their precise privacy. Therefore, they observe the market price and decide how much of the product to purchase given their expectations regarding privacy concerns. The remaining consumers observe the market price and buy the product through the traditional (brick-and-mortar) channel. The firm set price (s) according to its perceived expected demands.

Period 2

The firm

The firm observes at the beginning of period 2 a private signal about the population-average privacy concern on the online channel, $z = \tilde{x} + \tilde{\varphi}$. Let us denote this observation as $\hat{z}$. This observation constitutes its information set in $t = 2$. Then, based on this information, the firm sets and announces its period 2 pricing. Note that since we have assumed that each consumer's information is not publicly observable in the market, observation of first-period online purchases does not give any consumers' private information to the firm.

Online consumers

As already said, after purchasing in the first period, the realization of α_{i1} at the end of this period, $\hat{\alpha_{i1}} = \theta_i - p_1 - q_{i1}$, will give online consumers some information to update their information about α_{i1} at the beginning of period 2.

Thus, online consumers are aware that their privacy beliefs in the first period have to be updated with the realization of α_{i1} at the end of period 1, and the realization of a signal on population-average concerns that the monopoly has privately observed. Consumers will infer the monopoly's signal realization from the monopoly's second-period price once this price is announced. As common in the signaling literature, we assume that consumers use a linear inference rule such as $z = a + bp_2$. Finally, consumers make a purchasing decision. The following figure displays the timing of signals and actions (see Fig. 1).

This two-period game with incomplete information is a dynamic Bayesian game. Therefore, the corresponding equilibrium concept is perfect Bayesian equilibrium. The perfect Bayesian equilibrium prescribes equilibrium strategies for the firm and consumers that are sequentially rational (for each of their information sets) to the other players' equilibrium strategies, and beliefs (coming from Bayesian updating) consistent with the equilibrium strategies. We focus first on the unique separating equilibrium when consumer inferences about $\hat{z}$ are a linear function of p_2. In this fully revealing equilibrium the firm's second-period prices are a linear function of its private information. This situation arises because if consumers use linear inference rules to compute mean expectations about $\hat{z}$, then linear decision rules by the firm will be optimal. Afterwards, we calculate the pooling and semi-separating equilibrium of this game.

3. Bayesian updating of beliefs

To construct their demand in period 2, consumers must update their privacy beliefs, from the realization of α_{i1} at the end of period 1, and from the inference of the signal on population-average concerns received from the monopolist through the price in period 2. Likewise, Firms must update their information regarding the consumers' privacy concerns to construct its perceived demand. Therefore, we first calculate several Bayesian updates for future reference. Because all random variables are normally distributed, Bayesian updates are just regression equations. Let δ_α be the relative precision of $\hat{\alpha_{i1}}$ (the first period of online sales observation), δ_z be the relative precision of the signal z in period 2, and δ_x be the relative precision of the prior distribution of α_{i2}.[10] In the Appendix, we prove the following Proposition.

Proposition 1. *In period 2, the updating of α_{i1} after the observation of z and first-period sales via the online channel is*

$$E\left\{\alpha_{i2}|\alpha_{i1}, z\right\} = \overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z. \tag{4}$$

with

$$\delta_z = \gamma_z\left(1 - \delta_\alpha\right), \tag{5}$$

and

$$\delta_x = \left(1 - \gamma_z\right)\left(1 - \delta_\alpha\right) = \gamma_x + (1 - \delta_\alpha) \tag{6}$$

As expected, the posterior average beliefs of consumers' privacy concerns in the second period is a linear combination of three relevant variables: the realization of the private signal of the monopolist, $\hat{z}$, the observation of the first period of online sales, $\hat{\alpha_{i1}}$, and the prior about the population population-average privacy, $\overline{x}$, weighted by their relative precision, δ_z, δ_α, and δ_x, respectively. Furthermore, Eqs. (5) and (6) imply that updated beliefs only depend on two key parameters, namely δ_α and γ_z. The parameter δ_α can be interpreted as the degree of importance consumers place on their experience from the first-period purchases on the Internet. The parameter γ_z is the relative precision of the private information of the monopolist (that is, the relative precision of z). Note that an improvement in the precision of γ_z means a decrease in the precision of γ_x (i.e., the first period relative precision of the true average privacy concern).

[10] When using the reference prior, the posterior mean coincides with the ordinary least square (OLS) linear regression models. Thus, the updating expression may be interpreted as a weighted average of the prior estimator, and the maximum likelihood estimator, with weights proportional to their precisions.

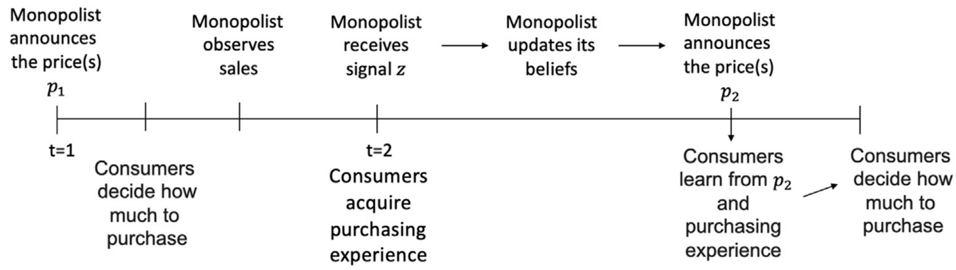


Fig. 1. Timing of signals and actions.

Before presenting the analysis under the different scenarios of equilibrium prices, it is worth thinking about the nature of signaling equilibrium prices in period 2. At the beginning of period 2, both consumers and the firm have new information. Each consumer i remembers her first period experience, which yields an observation $\hat{\alpha_{it}}$ of α_{it}, and the firm has an observation $\hat{z}$ of z. A high value of $\hat{z}$ indicates a high value of the population-average privacy concerns, which in turn indicates that consumer $i's$ expectation of α_{it} is likely to be high. Thus, the firm concludes from a high $\hat{z}$ that second-period demand is likely to be low (the difference between the willingness-to-pay for the product and privacy concerns), which points to a low maximizing price in period 2. Therefore, the firm has an incentive to learn the average privacy concerns in the online channel because its expected price and profits in period 2 depend on it. Consumers understand the firm's incentives and therefore infer from the price some information about the firm's observation of z. Information about $\hat{z}$ is useful to consumers because it provides an independent signal of the true value of the concerns of the population average about privacy, a component of their random concerns about privacy. However, because each consumer's utility experience with the good is idiosyncratic, each consumer will continue to use her personal information (the observation $\hat{\alpha}_{it}$ of the first period purchases) when making privacy concerns inferences. We later conclude that the equilibrium does in fact possess these features. However, the presence of idiosyncratic signals to the consumer is crucial.

Two main scenarios or strategies are now analyzed. In these scenarios, the monopolist can choose either to practice third degree price discrimination between the brick-and-mortar and online channels or to set a uniform price for both channels. We first analyze the uniform price strategy and then third degree price discrimination between channels, accounting for the fact that the proportion of channels, λ, is exogenous.

4. The benchmark model: Uniform pricing

In this section, we analyze how the monopolist sets a uniform price in periods $t = 1, 2$ given the available information in each period. Note that the firm could sell the data from the online shoppers to third parties and receive a payment for it. If we assume that the payment is a fixed amount, it would have no impact on the price the monopolist sets, so we do not consider it, to shorten the algebra and the profit calculations.

Let Ω_{it} and Ω_{mt} the consumer i's information and the monopoly's information, respectively, at time t, $t = 1, 2$. In period 1, consumers have expected demand given their information set in $t = 1$. As specified earlier, no additional information for either the monopolist or consumers has been generated, and both consumers' expected demand and the firm's expected profits consist of simple expectations. Let p_1^u be the uniform price expected by consumers in period 1, and q_{iB1} and q_{iC1} the demand in period 1 for the brick-and-mortar and the online channels, respectively. Then, as usual, the demand for the brick-and-mortar channel is $q_{iB1}^u = \theta_i - p_1^u$, and the expected demand of consumer i for the online channel, conditional on her information set is, $q_{iC1}^u = \theta_i - \bar{x} - p_1^u$. Letting Π_1^u be the profits for the two channels profits in period 1 means that the firm's expected profits, conditional on its information set, are,

$$E\left[\Pi_1^u|\Omega_{m1}\left(\alpha_1\right)\right] = \lambda\left(\theta_i - p_1^u\right)p_1^u + (1-\lambda)\left(\theta_i - \bar{x} - p_1^u\right)p_1^u. \tag{7}$$

Similarly, let q_{iB2}^u and q_{iC2}^u be the expected demand in period 2 for the brick-and-mortar and the online channels respectively, and let p_2^u be the expected uniform price for consumers in period 2. Then, consumer i's demand for the brick-and-mortar channel in period 2 is $q_{iB2}^u = \theta_i - p_2^u$, and the expected demand for the online channel, conditional on the consumer's information set in period 2 (first-period sales and second-period price) is $q_{iC2}^u = \theta_i - E\left\{E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}|\alpha_{i1}, p_2^u\right\} - p_2^u$, which from the updated beliefs of $E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}$ (see Eq. (4)) translates to,

$$q_{iC2}^u = \theta_i - E\left\{\bar{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z|\alpha_{i1}, p_2^u\right\} - p_2^u. \tag{8}$$

The perceived demand curve of the monopolist in period 2 is again the sum of the expected demands for the two channels. After period 1, the firm observes first-period sales and gets a signal about the population-average privacy concerns in the market. The firm then uses this information to set the second-period price. As in period 1, λ is the proportion of consumers buying through the brick-and-mortar channel. Let $E[q_2^u]$ be the expected demand faced by the firm in period 2. Then, $E\left[q_2^u|\Omega_{m2}\left(z, p_2(z)\right)\right] = \lambda q_{i2B}^u + (1-\lambda)E\left[q_{i2C}^u|\alpha_{i1}, z\right]$.

However, note that the firm's observation of first-period sales is not useful because it does not contain any consumers' private information. Therefore, the online demand curve perceived by the monopolist is the expectation of the representative consumer's demand conditional on its own information and the second-period price. That is, $E\left[q_2^u|\Omega_{m2}\left(z, p_2(z)\right)\right] = \lambda q_{i2B}^u + (1-\lambda)E\left[q_{i2C}^u|z, p_2^u\right]$, which, by the process of Bayesian updating described earlier, is specified as,

$$E\left[q_2^u|\Omega_{m2}\left(z, p_2(z)\right)\right] = \lambda\left(\theta_i - p_2^u\right) + (1-\lambda)\left(\theta_i - E\left\{\bar{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z|z, p_2^u\right\} - p_2^u\right). \tag{9}$$

Letting Π_2^u be the two-channels profits in period 2, then,

$$E\left[\Pi_2^u|\Omega_{m2}\left(z, p_2(z)\right)\right] = \lambda\left(\theta_i - p_2^u\right)p_2^u + (1-\lambda) \times \left(\theta_i - E\left\{\bar{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z|z, p_2^u\right\} - p_2^u\right)p_2^u. \tag{10}$$

After specifying the representative consumer's expected demand and the monopolist's expected profits in $t = 1, 2$, we look for the market equilibrium in fully revealing strategies. As explained earlier, the equilibrium concept is that perfect Bayesian equilibrium and consists of the following: the monopolist's pricing strategy in each period, given its information sets Ω_{mt} at $t = 1, 2$; consumers' demands from consumers' utility maximization, given their information sets, Ω_{it}, at $t = 1, 2$; and beliefs of both consumers and the monopolist, derived from Bayesian updating and consistent with the equilibrium strategies of the firm and consumers.

The first step to compute the equilibrium consists of specifying exactly what consumer i believes when she decides to purchase the product through the online channel with information set Ω_{i2}. A consumer who purchases the product via the brick-and-mortar channel knows exactly what her utility is and thus has no privacy concerns.

However, the information set at the beginning of period 2 for consumers buying via the online channel consists of consumers' own experience (i.e., the observation of α_{i1}) plus the commonly observed p_2^u, which potentially indicates the firm's observation of its private signal, z.

Suppose that consumers make inferences on z from p_2^u according to the linear (regression) rule $z = a + bp_2^u$. Note that a is the expected value of p_2^u when z is zero (the intercept) and b is the slope (the regression coefficient), i.e., the expected change in p_2^u when z changes.

Then, consumer i's second-period expected demand in the online channel is,

$$q_{iC2}^u = \theta_i - (\overline{x}\delta_x + \hat{\alpha}_{i1}\delta_\alpha + \left(a + bp_2^u\right)\delta_z) - p_2^u. \tag{11}$$

In period 2, the demand curve perceived by the monopolist is the expectation over the representative consumer demand, conditional on its own information, Ω_{m2}, consisting of the observation $\hat{z}$ of z, the inference rule, and second-period price. We derive in the Appendix such online channel expected demand function, yielding $E\left[q_2^u|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right] = \theta_i - \left(\overline{x}\gamma_x + \delta_z\left(a + bp_2^u\right) + \hat{z}\delta_\alpha\gamma_z\right) - p_2^u$, and thus, the two channels expected demand function perceived by the monopolist in period 2 is $E\left[q_2^u|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right] = \lambda\left(\theta_i - p_2^u\right) + \left((1-\lambda)\left(\theta_i - \left(\overline{x}\gamma_x + \delta_z\left(a + bp_2^u\right) + \hat{z}\delta_\alpha\gamma_z\right) - p_2^u\right)\right)$, with second-period expected profits for the monopolist,

$$E\left[\Pi_2^u|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right] = \lambda\left(\theta_i - p_2^u\right)p_2^u + \left((1-\lambda)\left(\theta_i - \left(\overline{x}\gamma_x + \delta_z\left(a + bp_2^u\right) + \hat{z}\delta_\alpha\gamma_z\right) - p_2^u\right)\right)p_2^u. \tag{12}$$

Reordering the terms of the two channel expected demand function in period 2, it can be easily seen that an increase in price in period 2 has two distinct effects on expected demand,

$$E\left[q_2^u|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right] = \theta_i - (1-\lambda)\left(\overline{x}\gamma_x + \delta_z a + \hat{z}\delta_\alpha\gamma_z\right) - p_2^u\left(1 + b(1-\lambda)\delta_z\right).$$

In the above expression, the demand slope is $-\left(1 + b(1-\lambda)\delta_z\right)$. The first term (-1) represents the (usual) direct effect of price on expected linear demand in period 2. The indirect effect, which is the term $(-b(1-\lambda)\delta_z)$, depends on the sign of $\varepsilon b\varepsilon$.[11] We will see later that $\varepsilon b\varepsilon$ is negative (see (17) below) and δ_z positive, so this term is positive. Signaling makes the slope of expected demand less than 1 and hence increases the slope of the inverse demand function, which ultimately acts to dampen how prices respond to changes in privacy concerns. The inverse demand function becomes steeper because, under price signaling, raising the price may signal to the consumer that the market has a low population-average privacy concerns. Thus, expected demand does not necessarily monotonically fall with prices. The dynamic effect arises because a forward-looking firm recognizes that if it changes the price of a product today, it affects consumer beliefs about the relationship between price and privacy in the future. If the value of $(-b(1-\lambda)\delta_z)$ is high (i.e., if each consumer places great importance on p_2^u in drawing inferences about z, and places great importance on z when estimating α_{i2} -see (7)), then the expected inverse demand will become quite steep.

Crucially, an increase in prices will not reduce expected demand by as much as it would in the absence of signaling. In other words, the signaling problem makes consumers less price sensitive.[12]

Let the tuple (p_1^{u*}, p_2^{u*}) be the equilibrium prices. That is, given $E\left[\Pi_1^u|\Omega_{m1}\left(\alpha_1\right)\right]$ in $t=1$ and $E\left[\Pi_2^u|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right]$ in $t=2$, then,

$$p_1^{u*} = \arg\max_{p_1^u} \left\{\lambda\left(\theta_i - p_1^u\right)p_1^u + (1-\lambda)\left(\theta_i - \overline{x} - p_1^u\right)p_1^u\right\}, \tag{13}$$

$$p_2^{u*} = \arg\max_{p_2^u}\left\{\lambda\left(\theta_i - p_2^u\right)p_2^u + (1-\lambda) \times \left(\theta_i - \left(\overline{x}\gamma_x + \delta_z\left(a + bp_2^u\right) + \hat{z}\delta_\alpha\gamma_z\right) - p_2^u\right)p_2^u\right\}. \tag{14}$$

The first-order conditions give the optimal price in period 2, yielding,

$$p_2^{u*} = \frac{\theta_i - (1-\lambda)(\overline{x}\gamma_x + \hat{z}\delta_\alpha\gamma_z + a\delta_z)}{2\left(1 + b(1-\lambda)\delta_z\right)}. \tag{15}$$

We are searching for an equilibrium in which the representative consumer's inference rule is correct. Consumers are correct in believing that $\hat{z}$, the observation of z by the firm, is equal to $\hat{z} = a + bp_2^u$. Then, the consumers' second-period expected price p_2^u must satisfy the condition that $p_2^u = (\hat{z} - a)/b$. At equilibrium, both consumers and the firm share the same linear rule for $\hat{z}$. Hence, prices must be the same, which implies that the price set by the monopolist is also equal to $p_2^{u*} = (\hat{z} - a)/b$. Thus, equating (17) to $p_2^{u*} = (\hat{z} - a)/b$, isolating $\hat{z}$ in both of them, matching the terms in a and those in b in both equations, and solving for a and b gives the result that, in a linear separating equilibrium,

$$a = \frac{\theta_i - \overline{x}\gamma_x(1-\lambda)}{(1-\lambda)\gamma_z}, \tag{16}$$

and

$$b = -\frac{2}{(1-\lambda)\left(2-\delta_\alpha\right)\gamma_z}. \tag{17}$$

Substituting a and b in (15) gives the expected price and therefore demand and profits in period 2. Proposition 2 characterizes the separating perfect Bayesian equilibrium.[13]

Proposition 2. *Suppose that the proportion of consumers shared between channels is given by $\lambda \in (0,1)$ and that the consumers inference rule is linear, then there exists a separating perfect Bayesian equilibrium with uniform pricing. In equilibrium:*

1. *The firm sets price and quantity in $t = 1$,*

$$p_1^{u*} = \frac{1}{2}(\theta_i - (1-\lambda)\overline{x}), \qquad q_1^{u*} = \frac{1}{2}(\theta_i - (1-\lambda)\overline{x}).$$

2. *In period 2, the equilibrium price and quantity are,*

$$p_2^{u*} = \frac{(2-\delta_\alpha)}{2}(\theta_i - (1-\lambda)(\overline{x}\gamma_x + \hat{z}\gamma_z)), \qquad q_2^{u*} = \frac{\delta_\alpha}{2}(\theta_i - (1-\lambda)(\overline{x}\gamma_x + \hat{z}\gamma_z)). \tag{18}$$

3. *This equilibrium is the unique separating equilibrium when consumer inferences about z are a differentiable and invertible function of p_2. Also, any consumers' inference rule has to be linear.*

The proof of point 3 is in the Appendix.

Note that given that $\gamma_x = (1-\gamma_z)$, the separating equilibrium price and demand, p_2^{u*} and q_2^{u*}, can be expressed as,

$$p_2^{u*} = 2(\underbrace{\frac{1}{2}(\theta_i - (1-\lambda)\overline{x})}_{\text{Monopolist with no signaling}} + \underbrace{\frac{(1-\lambda)}{2}\gamma_z(\overline{x} - \hat{z}))}_{\text{market signal effect}} - \underbrace{\frac{\delta_\alpha}{2}(\theta_i - (1-\lambda)(\overline{x}\gamma_x + \hat{z}\gamma_z))}_{\text{Consumers signal effect}}. \tag{19}$$

[11] It is required that $\delta_z b(1-\lambda) < 1$, otherwise no finite monopoly price exists, since the slope of the demand will be positive. In equilibrium this condition is satisfied, the monopoly price always exists, and the second-period pricing problem is well defined. To see this use (17) below, substitute δ_z by (5), simplify, and the result follows.

[12] Notably, our model imposes uncertainty in the intercept of the demand and not on its slope. Note that some uncertainty in the slope would give consumers incentives to increase their purchases to increase their information (learning by experimentation). We leave experimentation issues aside.

[13] The second-order condition holds in each period. It is trivial in period 1. In period 2, the second-order condition is $-2(1-\lambda)\left(1 + b\delta_z\right)$. Plugging in "b" as specified in (17) gives $-2\delta_\alpha/(2-\delta_\alpha) < 0$.

$$q_2^{u*} = \delta_\alpha(\underbrace{\frac{1}{2}(\theta_i - (1-\lambda)\overline{x})}_{\text{Demand with no signaling}} + \underbrace{\frac{(1-\lambda)}{2}\gamma_z(\overline{x} - \hat{z})}_{\text{market signal effect}}) \tag{20}$$

The first term in p_2^{u*} is the price of the monopolist without signaling, the second is the difference between the mean of the population-average privacy concerns and the realization of z, and the last term refers to the consumers experience. The second term reflects the cost of signaling for the monopolist, related to its own signal, since the realization of z is passed on to the prices. Obviously, if the realization of z is positive and higher than $\overline{x}$, this term will be subtracted from the first one, but for negative realizations of z's the price could be quite high. When the realization of z is equal to its mean, $\bar{x}$, the market signal effect disappears because it does not carry additional information. The third term is related to the parameter of consumers' experience from first period purchases. The higher this parameter the smaller the monopolist' price and vice versa.

Similarly, the equilibrium quantity will depend on the consumer's experience and the signal received from the monopolist through prices. The parameter of consumers' experience reduces demand reflecting the consumers' uncertainty about their privacy concerns. The market signal effect will increase or decrease the quantity demanded. For negative realizations of z's, indicating a lower population-average privacy concerns, demand will increase, and will decrease for very positive realizations of z. As above if $\hat{z} = \bar{x}$, the signal from the monopolist is non-informative.

The separating perfect Bayesian equilibrium with uniform pricing is based on equilibrium market signals. Thus, note that for positive realizations of z, $p_2^{u*} = 0$ when $\hat{z} = \frac{1}{(1-\lambda)\gamma_z}(\theta_i - (1-\lambda)\overline{x}\gamma_x) > 0$, and so is q_2^{u*} by Eq. (18). Denote by z_0^u such realization. Then, any $\hat{z} > z_0^u$ will result in negative prices. Negative prices are off-equilibrium signals and the consumer does not purchase in the market. Therefore, at equilibrium $\hat{z}$ is bounded by above by z_0^u, i.e., $\hat{z} < z_0^u$.

The second-period price is indeed a linear function of $\hat{z}$. Furthermore, signaling might distort prices upward in comparison to the scenario in which z is common knowledge to both consumers and the monopolist and there is no signaling. To study this distortion, let p_2^z be the price in equilibrium in period 2 in which z is common knowledge (i.e., both consumers and the monopolist receive information from signal z). Then, for any observation $\hat{z}$ of z, the equilibrium price is,

$$p_2^z = \frac{1}{2}(\theta_i - (1-\lambda)(\overline{x}\gamma_x + \hat{z}\gamma_z)).$$

Because $p_2^{u*} - p_2^z > 0$, signaling always distorts prices upward compared to the scenario where z is common knowledge. By increasing price with respect to the no signaling scenario, the monopolist increases its profits. On the one hand, it manipulates consumers' beliefs regarding privacy concerns, which increases demand. On the other, given that the demand under signaling is less sensitive, the increase in price does not translate, in general, to the same quantity reduction. The lower elasticity of demand in equilibrium has important consequences for the costs of the monopolist' signaling. Consider the realizations of z as the monopolist's types. The lower demand sensitivity to prices benefits the high types of the monopolist, i.e., those for which signaling implies a price increase (when the $\hat{z}$'s are negatives), because higher prices reduce demand but less than proportionally. However, for the bad types of the monopolist, those who have to reduce the price (positive $\hat{z}$'s) signaling represents a higher cost, because price reductions are not accompanied by proportional increases in consumers demand.[14]

[14] Note however, that this is independent of the fact that these prices are still higher than those under z common knowledge.

4.1. Consumers' learning: Higher consumers' experience or more precise market signal?

Recall that the consumer learning process depends on two key parameters, δ_α and γ_z, representing how much importance consumers place on their experience regarding privacy concerns from the online channel and the relative precision of the monopolist's private information, respectively. We can analyze some interesting limiting cases.

If the precision of the consumers' experience, δ_α, tends to zero, the second period demand will tend to zero (see (18)). The second-period expected price will depend on $\hat{z}$, in particular, it could be higher than θ_i if $\hat{z}$ is negative enough.[15] Note that the precision of consumers' experience could tend to zero if $\gamma_z = 1$ and hence $\gamma_x = 0$. This limiting case corresponds to the case where the private information of the monopolist is a sufficient statistic for the consumers' estimates of their privacy concerns. In this situation, the monopolist has an incentive to set a high price to signal to consumers that the average market privacy is low enough. Anticipating the monopolist's incentives, consumers will not trust the monopolist. Hence, the only separating equilibrium demand for the online channel will be zero. Therefore, unless the price can signal the population-average market privacy to consumers, the monopolist's private information must not be a sufficient statistic in a separating equilibrium. It is important for consumers to learn some idiosyncratic information.

On the other hand, as consumers' private information becomes a sufficient statistic for market privacy concerns (i.e., δ_α tends to 1), the second-period price and quantity tend to the equilibrium monopoly price and quantity without signaling by the firm.[16] In this limiting case, the monopolist has nothing to signal to consumers because its information is useless for them.

For intermediate cases, as consumers place more importance on their first-period purchasing experience, the lower the second-period expected price and the higher the consumer's purchases.[17] These effects will be higher the lower the relative signal precision γ_z. Any attempt by the monopolist to increase the relative precision γ_z of z, will only translate to a price increase if $\overline{x} \geq \hat{z}$ (i.e., if the mean of the population-average privacy in the market is higher than the monopolist's private observation).[18] The economic intuition is that, after observing $\hat{z}$, the higher the precision of z is, the more convinced the monopolist will be that the observed $\hat{z}$ is higher (in absolute terms) than the true population-average privacy. Hence, it will set a higher price. On the contrary, if $\hat{z} \geq \overline{x}$, then any increase in the precision of z will reaffirm the monopolist's beliefs that the observation of $\hat{z}$ is smaller than the true population-average privacy. Hence, it will set a lower price.

5. Third degree price discrimination

Our benchmark is to study the simplest strategy that the monopolist adopts regarding its second-period price. However, the firm may choose to practice third-degree price discrimination between the two channels to ensure the maximum willingness to pay.

Let superscript "d" denote price discrimination. Then, let p_{1B}^d and p_{2B}^d be the prices of the brick-and-mortar channel in periods 1 and

[15] When $\delta_\alpha = 0$, $\gamma_z = 1$, and $\gamma_x = 0$, then $P_2^{u*} = (\theta_i - (1-\lambda)\hat{z})$.

[16] When $\delta_\alpha = 1$, and since $\gamma_x = (1-\gamma_z)$, then $P_2^{u*} = \frac{1}{2}\theta_i - (1-\lambda)(\overline{x} + (\hat{z} - \overline{x})\gamma_z)$ and $q_2^{u*} = \frac{1}{2}\theta_i - (1-\lambda)(\overline{x} + (\hat{z} - \overline{x})\gamma_z)$, which, for a sufficiently small value of γ_z, tend to the P_1^{u*} and q_1^{u*}, the first period equilibrium price and quantity, respectively, when consumers do not take into account any signaling by the monopolist.

[17] See (19), or check that he partial derivative of the expected price in period 2 is $\frac{\partial p_2^u}{\partial \delta_\alpha} = -((\theta_i - (1-\lambda)(\overline{x}\gamma_x + \hat{z}\gamma_z))/2) < 0$.

[18] From (19) and since, $\gamma_x = (1-\gamma_z)$, then $(\partial p_2^u)/(\partial \gamma_z) = (2-\delta_\alpha)(1-\lambda)(\bar{x} - \hat{z})/2 \gtreqless 0$ as $\overline{x} \gtreqless \hat{z}$.

2, and let p^d_{1C} and p^d_{2C} be the prices of the online channel in periods 1 and 2, respectively. In period 1, as in uniform pricing, consumers have expected demands given their information set Ω_1 and the channel chosen for purchasing. As described earlier, the demand for the brick-and-mortar channel is $q_{iB1} = \theta_i - p^d_{1B}$, and the expected demand for the online channel is $E\left[q_{iC1}|\Omega_{i1}\left(\alpha_{i1}\right)\right] = \theta_i - \overline{x} - p^d_{1C}$. Similarly, let $q^d_{iB2} = \theta_i - p^d_{2B}$ the demand for the brick-and-mortar channel in period 2. As consumers' information set changes to Ω_{i2}, then consumers update their beliefs about their privacy concerns, as long as they purchased through the online channel, yielding the expected demand $q^d_{iC2} = \theta_i - E\left\{E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}|\alpha_{i1}, p^d_{2C}\right\} - p^d_{2C}$.

Since the procedure and all the calculations to find the equilibrium prices are similar to those of the uniform pricing, we relegate them to the Appendix, and offer here the equilibrium result. Proposition 3 characterizes the linear equilibrium.[19]

Proposition 3. *Suppose that market-channel proportions are given by $\lambda \in (0,1)$ and that the inference rule is linear. Then, there exists a separating perfect Bayesian equilibrium with third degree price discrimination. In equilibrium,*

1. *In period 1, price and quantity of the brick-and-mortar channel are,*

$$p^{d*}_{1B} = \frac{\theta_i}{2}, \qquad q^{d*}_{1B} = \lambda\frac{\theta_i}{2},$$

and those of the online channel,

$$p^{d*}_{1C} = \frac{1}{2}\left(\theta_i - \overline{x}\right), \qquad q^{d*}_{1C} = \frac{1}{2}(1-\lambda)(\theta_i - \overline{x}).$$

2. *In period 2, price and quantity of the brick-and-mortar channel are,*

$$p^{d*}_{2B} = \frac{\theta_i}{2}, \qquad q^{d*}_{2B} = \lambda\frac{\theta_i}{2}, \tag{21}$$

and those the online channel,

$$p^{d*}_{2C} = \frac{1}{2}\left(2-\delta_\alpha\right)\left(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)\right), \qquad q^{d*}_{2C} = \frac{1}{2}(1-\lambda)\delta_\alpha\left(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)\right). \tag{22}$$

3. *This equilibrium is the unique separating equilibrium under third degree price discrimination when consumer inferences about z are a differentiable and invertible function of p_2. Also, any consumers' inference rule has to be linear.*

As in the uniform scenario, p^{d*}_{2C} and q^{d*}_{2C} can be expressed as,

$$p^{d*}_{2C} = 2(\underbrace{\frac{1}{2}(\theta_i - \overline{x})}_{\text{Monopolist with no signaling}} + \underbrace{\frac{1}{2}\gamma_z(\overline{x} - \hat{z})}_{\text{Market signal effect}}) - \underbrace{\frac{1}{2}\delta_\alpha\left(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)\right)}_{\text{Consumers signal effect}} \tag{23}$$

$$q^{d*}_{2C} = \delta_\alpha(1-\lambda)(\underbrace{\frac{1}{2}(\theta_i - \overline{x})}_{\text{Demand with no signaling}} + \underbrace{\frac{1}{2}\gamma_z(\overline{x} - \hat{z})}_{\text{market signal effect}}) \tag{24}$$

showing the effects of the monopolist's information and the consumer's information on both the monopolist's price and the consumer's demand.

Additionally, $p^{d*}_2 = 0$ when positive realizations $\hat{z} = z^d_0 = \frac{1}{\gamma_z}(\theta_i - \overline{x}\gamma_x) > 0$, where z^d_0 is such realization. Then, any $\hat{z} > z^d_0$ will result is negative prices. Therefore, as in uniform pricing, negative prices are off-equilibrium signals and the consumer does not purchase in the internet channel. Therefore, at equilibrium $\hat{z}$ is bounded by above by z^d_0. As in uniform pricing, the second-period equilibrium price in the online channel p^{d*}_{2C} is the unique separating equilibrium where consumers' inferences on $\hat{z}$ are a differentiable and invertible function of p^{d*}_{2C}.[20]

The distortion on the second-period online channel price because of market signaling with respect to the scenario of common knowledge of z follows the same pattern as with uniform pricing. As expected, the second-period price in the online channel is a linear function of $\hat{z}$, and, under price discrimination, the learning process only affects to the second-period price. Again, the monopolist will distort second-period prices upward in the online channel in comparison to the scenario in which z is common knowledge to both consumers and the monopolist. To study this distortion, let p^z_{2C} be the equilibrium price in period 2 for the online channel in which z is common knowledge. At equilibrium, $p^z_{2C} = \frac{1}{2}\left(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)\right)$, and trivially, $p^d_{2C} - p^z_{2C} = \frac{1}{2}(1-\delta_\alpha)\left(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)\right) > 0$.

Similar analysis and intuition as for the uniform case can be translated to the limiting case where the private information of the monopolist is a sufficient statistic for the consumer's estimates of their private concerns about the online channel (i.e., when δ_α tends to zero). In an attempt to manipulate consumers' privacy concerns, the monopolist will set a high second-period price on the online market.[21] However, consumers will not be deceived, and the monopolist will only sell through the brick-and-mortar channel (i.e., $q^{d*}_{2c} = 0$). On the contrary, as consumers' private information becomes a sufficient statistic for the market privacy concerns (i.e., when δ_α tends to 1), then the online market second-period price and quantity will tend to the price and quantity without market signaling by the monopolist, respectively (i.e., $P^{d*}_{2C} = \frac{1}{2}(\theta_i - \overline{x})$ and $q^{d*}_{2C} = \frac{1}{2}(1-\lambda)(\theta_i - \overline{x})$). Finally, for intermediate cases, the second-period expected price in the online channel is lower when consumers place greater importance on their first-period purchasing experience.

6. Pooling equilibrium

We investigate now the pooling equilibrium of the second stage of the game. At a pooling equilibrium the monopolist ignores its private information and signals nothing. The consumer cannot make inferences about z, when observing the second period price, but can update α_{i1}, after observing its first period purchasing market experience, $\hat{\alpha}_{i1}$ then,

$$E\left\{\alpha_{i2}|\alpha_{i1}\right\} = E\left\{\alpha_{i2}\right\} + \rho\frac{\sigma_{\alpha_1}}{\sigma_{\alpha_2}}\left(\alpha_1 - E\left\{\alpha_1\right\}\right) = \bar{x}\left(1 - \frac{\sigma_1^2}{\sigma_{\alpha_1}\sigma_{\alpha_2}}\right) + \frac{\sigma_1^2}{\sigma_{\alpha_1}\sigma_{\alpha_2}}\alpha_{i1},$$

since $Var\left(\alpha_{i1}\right) = \sigma^2_x + \sigma^2_\omega$, $Var\left(\alpha_{i2}\right) = \sigma^2_x + \sigma^2_\omega + \sigma^2_\upsilon$, and $\rho(\alpha_{i1}, \alpha_{i2}) = \frac{Cov(\alpha_{i1}, \alpha_{i2})}{\sqrt{Var(\alpha_{i1})Var(\alpha_{i2})}} = \frac{\sigma_1^2}{\sigma_{\alpha_1}\sigma_{\alpha_2}}$.

The monopolist ignores the realization of z and takes expectations of $E\left\{\alpha_{i2}|\alpha_{i1}\right\}$ consequently,

$$E(E\left\{\alpha_{i2}|\alpha_{i1}\right\}) = \bar{x}\left(1 - \frac{\sigma_1^2}{\sigma_{\alpha_1}\sigma_{\alpha_2}}\right) + \frac{\sigma_1^2}{\sigma_{\alpha_1}\sigma_{\alpha_2}}E(\alpha_{i1}) = \bar{x}\left(1 - \frac{\sigma_1^2}{\sigma_{\alpha_1}\sigma_{\alpha_2}}\right) + \frac{\sigma_1^2}{\sigma_{\alpha_1}\sigma_{\alpha_2}}\bar{x} = \bar{x}.$$

Let p^{up}_2 be the second period price under pooling equilibrium and uniform pricing in both markets, i.e.,

$$p^{up}_2 = arg \max_{p^{up}_2}\left\{\lambda\left(\theta_i - p^{up}_2\right)p^{up}_2 + (1-\lambda)\left(\theta_i - \overline{x} - p^{up}_2\right)p^{up}_2\right\},$$

then, $p^{up}_2 = \frac{1}{2}(\theta_i - (1-\lambda)\overline{x})$, and letting q^{up}_2 and $E\Pi^{up}_2$ be the corresponding quantity and expected profits under pooling uniform pricing, $q^{up}_2 = \frac{1}{2}(\theta_i - (1-\lambda)\overline{x})$, and $E\Pi^{up}_2 = \frac{1}{4}(\theta_i - (1-\lambda)\overline{x})^2$.

Similarly let p^{dp}_{2C} be the second period in the online channel under pooling pricing and third degree price discrimination between markets,

[19] The second-order conditions are again trivially satisfied. For the brick-and-mortar channel in periods 1 and 2, the second-order condition is $-2\lambda < 0$. For the online channel, the second-order condition is $-2(1-\lambda) < 0$ in period 1 and $2(1-\lambda)(-1 + b\delta_z) < 0$ in period 2. Plugging in "b", as in (17), in the second-order conditions for the online channel results in a negative sign (i.e., $-(2(1-\lambda)\delta_\alpha)/(2-\delta_\alpha) < 0$.

[20] The proof is similar to that of the uniform case, so we do not repeat it here.

[21] $p^{d*}_{2C} = \left(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)\right)$.

then $p_{2C}^{dp} = \frac{1}{2}(\theta_i - \overline{x})$. Again, letting q_{2C}^{dp} and $E\Pi_{2C}^{dp}$ be the corresponding quantity and expected profits under price discrimination between markets, $q_{2C}^{dp} = \frac{1}{2}(1-\lambda)(\theta_i - \overline{x})$, and $E\Pi_{2C}^{dp} = \frac{1}{4}((1-\lambda)(\theta_i - \overline{x})^2)$. Summing up,

Proposition 4. *A second period uniform pooling perfect Bayesian equilibrium is given by* $p_2^{up} = \frac{1}{2}(\theta_i - (1-\lambda)\overline{x})$, *and* $q_2^{up} = \frac{1}{2}(\theta_i - (1-\lambda)\overline{x})$.

A second period third degree price discrimination pooling perfect Bayesian equilibrium in the online channel is given by $p_{2C}^{dp} = \frac{1}{2}(\theta_i - \overline{x})$, *and* $q_{2C}^{dp} = \frac{1}{2}(1-\lambda)(\theta_i - \overline{x})$.

Comparing the pooling prices p_2^{up} and p_{2C}^{dp} to those of the separating equilibrium p_2^u and p_{2C}^d in (18) and (22), respectively, it is not difficult to see that the former will be higher or lower than the later depending on the value of $\hat{z}$. In the next section we will discuss on that issue more carefully. As already mentioned, consider that the realization of z defines the monopolist's type, thus each $\hat{z}$ defines a type. Then, the important question about the pooling equilibrium is whether the monopolist prefers to conceal its information or to signal it. More precisely, for which types the monopolist prefers to hide its information and for which other types it prefers to reveal it. This is the aim of the next section.

7. Hybrid equilibrium

In a hybrid equilibrium some monopolist's types will hide their information while others will instead reveal it. Obviously, the bad types of the monopolist (those with positive high realizations of z) will prefer to conceal their information while the good types (those with low realizations of z) would like to separate and signal it.

To elaborate in this question, we compare in the Appendix the corresponding profits. Recall that $E\Pi_2^{u*}$ are the monopolist's second-period expected profits under uniform pricing, $E\Pi_2^{u*}$ and $E\Pi_2^{up}$ those under pooling. Equating both profits and solving for z, we find the realization of z for which profits under either signaling or concealing are the same. Let $\hat{z}_\pi^{u*}$ be such a z. Then, for any realization of z higher than z_π^{u*} profits under fully revealing uniform prices will be smaller than under pooling pricing.

Repeating the analysis under third degree price discrimination, we find that $\hat{z}_\pi^{d*}$ is the realization of z for which the second-period expected profits in the online channel under price discrimination are the same as those under pooling in the online market. Then, as above, for any realization of z higher than z_π^{d*} profits under discrimination in the online channel will be smaller than under pooling.

Given the above thresholds, the monopolist could, after the realization of z, choose either to conceal or to signal its information. Then, for any $\hat{z} \geq z_\pi^{u*}$, the monopolist will play an uninformative pooling price, while for $\hat{z} < z_\pi^{u*}$, will price uniformly. The same rule will apply for price discrimination with the threshold being now z_π^{d*}. Obviously, when either $\hat{z} = z_\pi^{u*}$, or $\hat{z} = z_\pi^{d*}$, profits are the same under both regimes.

The consumer chooses a quantity to purchase after seeing the second period price. She knows that the monopolist is maximizing profits and then, under not signaling at all, the pooling price is either p_2^{up} or p_{2C}^{dp}, as in Proposition 4, depending on the uniform pricing or price discrimination scenarios. With this information the consumer will demand the pooling quantity whenever she sees the pooling price and will follow her inference rule for any other price, p_2^{u*}, or p_{2C}^{d*} (as in Propositions 2 and 3, respectively). Recall that q_2^{up} and q_{2C}^{dp}, are consumer's demand under both pooling scenarios, respectively, and q_{2C}^{u*}, and q_{2C}^{d*}, those under the inference rule. Also, in the Appendix it is shown that $z_\pi^{u*} < z_{\pi=0}^{u*} = z_0^u$, where $z_{\pi=0}^{u*}$ is the threshold of z for which profits under uniform pricing are zero, and similarly, $z_\pi^{d*} < z_{\pi=0}^{d*} < z_0^d$, where $z_{\pi=0}^{d*}$ is the threshold of z for which profits under discrimination are zero.

Proposition 5. *A hybrid perfect Bayesian equilibrium for the second stage of the game is given by,*

Under uniform pricing, for any $\hat{z} \in [z_\pi^{u*}, z_{\pi=0}^{u*}]$ *the monopolist will conceal and set* p_2^{up} *and the consumer will demand* q_2^{up} *for* $p_2 = p_2^{up}$. *For any* $\theta_i < \hat{z} < z_\pi^{u*}$, *the monopolist will signal and set the corresponding price* $p_2 \neq p_2^{up}$, *with* $p_2 > 0$, *and the consumer will use her inference rule, giving rise to the equilibrium prices and quantities depending on* $\hat{z}$, p_2^{u*} *and* q_2^{u*}.

Under third degree price discrimination, for any $\hat{z} \in [z_\pi^{d*}, z_{\pi=0}^{d*}]$ *the monopolist will conceal in the online market and set* p_2^{dp}, *and the consumer will demand* q_{2C}^{dp} *for any signal* $p_2 = p_2^{dp}$. *For any* $\hat{z} < z_\pi^{d*}$, *the monopolist will separate in this market and set the corresponding price* $p_2 \neq p_2^{dp}$, $p_2 > 0$, *the consumer will use her inference rule, giving rise to the equilibrium prices and quantities depending on* $\hat{z}$, p_{2C}^{d*} *and* q_{2C}^{d*}.

At this point it would be interesting to analyze the optimal amount of information to include in the signal. In other words, whether this setting admits equilibrium with intervals. However, this goes beyond the scope of the paper, and, in addition, as we will see in the next Section, both the monopolist and consumers are worse off as $\hat{z}$ increases, so that both economic agents are better off under the pooling equilibrium when $\hat{z}$ is sufficiently positive.

8. Welfare

Next, we derive the impact of consumers' uncertainty and market signaling on the second period consumer surplus, monopolist profits and social welfare.

Expected consumer surplus is, $CS_{it} = U_{it} - p_{it}q_{it} = (\theta_i - \alpha_{it})\, q_{it} - \frac{q_{it}^2}{2} - p_{it}q_{it}$, for $t = 1, 2$. Therefore, social welfare is specified as $SW_t = CS_{it} + \Pi_t$, where Π_t are the monopolist's expected profits in period t, respectively.[22] We first pay attention to second period consumers surplus.

Because signals play a crucial role in the second period, we start by computing CS_{i2} in period $t = 2$ under uniform and third degree price discrimination strategies under separating equilibrium, and under uniform and third degree price discrimination strategies under pooling equilibrium. Firstly, under separating equilibrium, let CS^u and CS^d be expected consumers surplus under uniform and discrimination pricing, respectively. Simple calculations yield $CS^u = \frac{1}{8}\delta_\alpha^2(\theta_i - (1-\lambda)(\overline{x}\gamma_x + \gamma_z\hat{z}))^2$ and $CS^d = \frac{1}{8}\left(\theta_i^2\lambda + \delta_\alpha^2(1-\lambda)(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z))^2\right)$.[23]

To gain informational insights, let $CS^{uz} = \frac{1}{8}(\theta_i - (1-\lambda)(\overline{x}\gamma_x + \gamma_z\hat{z}))^2$ and $CS^{dz} = \frac{1}{8}(\theta_i^2\lambda + (1-\lambda)(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z))^2)$ be expected consumer surplus under uniform and third degree price discrimination strategies, respectively, when signal z is common knowledge, that is, consumers observe the public signal and there is no need to infer information from the second period price. Furthermore, let $CS^{up} = \frac{1}{8}(\theta_i - (1-\lambda)\overline{x})^2$ and $CS^{dp} = \frac{1}{8}(\lambda\theta_i^2 + (1-\lambda)(\theta_i - \overline{x})^2)$ be expected consumer surplus in the pooling equilibrium under uniform and price discrimination strategies, respectively. Note that CS^{up} and CS^{dp} does not depend on the realization of the monopolist's signal z. The following lemma states the main relations between CS's.

Lemma 1. *In the second period of the game,*

- *(1) Consumers are always better off under a third degree price discrimination policy than under uniform pricing, that is,* $CS^d > CS^u$.
- *(2) Consumers' surplus with z common knowledge is higher than consumers' surplus under either uniform pricing or third degree price discrimination separating equilibrium.*

[22] Note that the existence of signals, and different sets of information, result in an expected consumer surplus that will depend on the signals and precision's realization (Jeitschko et al., 2018).

[23] Under third degree price discrimination, the consumer surplus of the brick channel is $CS_b^d = \frac{\theta_i^2\lambda}{8}$, and that of the online channel $CS_c^d = \frac{1}{8}\delta_\alpha^2(1-\lambda)(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z))^2$.

- *(3) Consumers are always better off under a separating uniform pricing than under a uniform pooling equilibrium if signal $\hat{z}$ is sufficiently small, i.e.,* $\hat{z}_{c*} \leq \frac{-(1-\delta_\alpha)\theta_i+(1-\lambda)\overline{x}}{(1-\lambda)\delta_\alpha\gamma_z} - \frac{\overline{x}\gamma_x}{\gamma_z}$.
- *(4) Consumers are always better off under a third degree price discrimination separating equilibrium than under a third degree price discrimination pooling equilibrium if signal $\hat{z}$ is sufficiently small, i.e.* $\hat{z}_{c**} \leq \frac{-(1-\delta_\alpha)\theta_i+\overline{x}}{\delta_\alpha\gamma_z} - \frac{\overline{x}\gamma_x}{\gamma_z}$.
- *(5) For $\hat{z} > \bar{x}$ consumers are always better off under either a uniform pooling equilibrium or a third degree price discrimination pooling equilibrium than under the corresponding equilibria when z is common knowledge, that is, for $\hat{z} > \bar{x}$, $CS^{up} > CS^{uz}$ and $CS^{dp} > CS^{dz}$.*

The first point in the lemma indicates that in a separating equilibrium, consumers achieve a higher surplus when the monopolist follows a third price discrimination strategy instead of uniform pricing. The rationale behind this is that third degree price discrimination allows the monopolist to tailor prices more closely to the individual privacy concerns and willingness to pay of consumers in the different channels (online and offline). By doing so, the monopolist can better match prices to the consumers' perceived value, leading to an increase of consumer surplus. This finding underscores the efficiency of third degree price discrimination in addressing the heterogeneity in consumer privacy concerns and maximizing consumer welfare in a multichannel market.

The second point in the lemma says that in a separating equilibrium, consumer surplus is higher when the signal z is common knowledge. The condition $\delta_\alpha < 1$ implies that the precision of consumers' private information about their privacy concerns is not perfect. When z is common knowledge, consumers can make more informed decisions based on the average privacy concerns in the market, leading to a higher consumer surplus. This highlights the importance of transparency and information sharing in enhancing consumer welfare, as it allows consumers to better understand and respond to their privacy concerns.

The monopolist's ability to signal information about population-average privacy concerns through prices plays a crucial role. Accurate signaling helps consumers make better-informed decisions, enhancing their surplus. However, points 3 and 4 in the lemma stress the role of consumers' fear of potential losses from online shopping on their welfare. The threshold conditions in these result provide a clear criterion for when signaling and separating equilibrium are more beneficial for consumers, than no information at all. Specifically, when the market signal realization, $\hat{z}$, is very low, even negative, suggesting low population-average privacy concerns, consumers benefit more from separating equilibrium than from pooling (not signaling). However, as $\hat{z}$ increases, the CSs of separating equilibria decrease. Namely, if the signal realization is positive and high, that means that the expected population-average privacy concerns are high, and the monopolist lowers its price. Because of the existence of noisy signals and learning, the demand is more inelastic; the change in the expected demand is less than proportional to the price change. Consumers "understand" that if the monopolist lowers his price, its intention is to increase its demand of the online channel, but the demand does not increases proportionally to the price reduction. As a result, the consumers surplus under any of the separating equilibrium is decreasing in the signal $\hat{z}$. The precise change depends on the precision of signals, γ_z (public one) and δ_α (private one). In theses cases, CS under pooling equilibrium can be higher than CS under separating equilibrium. Abounding in the consumers' fear of potential losses from online shopping, point 5 in the lemma, states that for signal realizations higher than the mean of z, consumers are better off without information than when z is common knowledge.

Partial derivatives show that expected consumer surplus is increasing in the value of the private signal δ_α, that is, $\frac{\partial CS^u}{\partial \delta_\alpha} > 0$, $\frac{\partial CS^d}{\partial \delta_\alpha} > 0$, $\frac{\partial CS^{uz}}{\partial \delta_\alpha} > 0$ and, $\frac{\partial CS^{dz}}{\partial \delta_\alpha} > 0$. However, when it comes to the signal's precision γ_z, the sign of partial derivatives depend on the relationship between the mean $\overline{x}$ of z and the realization of z, $\hat{z}$, i.e., $\overline{x} \lessgtr \hat{z}$.

Picture 2 illustrates for positives and negatives values of $\hat{z}$, how consumers surplus behave under the different price scenarios. We plot CS's under different monopolist's behavior, with $\theta_i = 10$, $\overline{x} = 5$, $\lambda = 0.5$, and $\gamma_z = \gamma_x = \delta_\alpha = 0.5$. Under these fixed values, $\hat{z}_{c*} = -25$ and $\hat{z}_{c**} = -5$. The picture reproduces the lemma results.

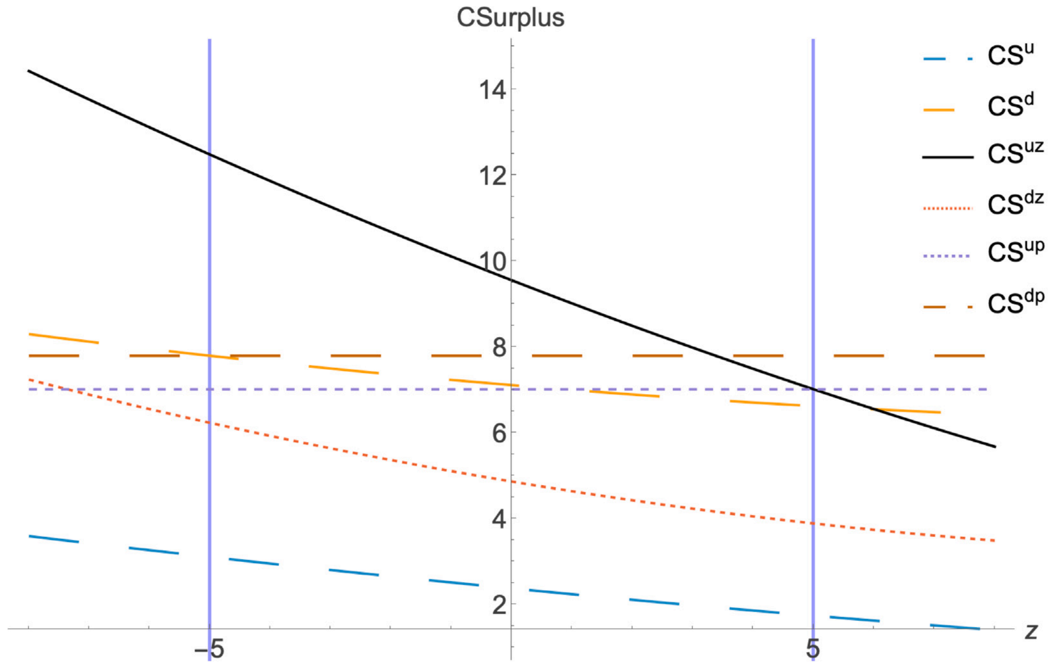
Turning now to the monopolist's profits, it is not difficult to show that the separating second period expected profits under third degree price discrimination, $E\Pi^{d*}_{2C}$, are higher than the monopolist's second-period expected profits under uniform pricing, $E\Pi^{u*}_2$. However, it is easy to show that, because the two channels are segmented markets, third degree price discrimination does not change the mean and precision of the updating of consumers' beliefs about privacy concerns as compared with uniform pricing updating. Therefore, there are no informational gains from discrimination. The reason for price discrimination comes from the monopolist's gains in profits from setting different prices in the certain (brick-and-mortar) and the uncertain (online) channels, and the resulting prices will depend on the degree of uncertainty of the market, as measured by the population-average privacy concerns.

We already saw that profits from separating equilibria either $E\Pi^{u*}_2$ or $E\Pi^{d*}_{2C}$ can be smaller than the corresponding pooling equilibrium if $\hat{z}$ is positive and sufficiently high, that is, $E\Pi^{u*}_2 < E\Pi^{up}_2$ and $E\Pi^{d*}_{2C} < E\Pi^{up}_2$, whenever $\hat{z} > z^{u*}_\pi$ and $\hat{z} > z^{d*}_\pi$ respectively, indicating the monopolist's bad types cost of signaling. As $\hat{z}$ increases, the price reduction by the monopolist is not accompanied by sales increases in the same proportion and profits decrease. Therefore, from thresholds z^{u*}_π and z^{d*}_π on the monopolist gains more by not signaling at all. The cost of signaling is also reflected in greater profits when z is common knowledge than under full disclosure of information. However, for sufficiently large realizations of z, not conveying any information is best.

Picture 3 plots the relationship among the several equilibrium profits under the same parameters than Picture 2. For example, note that $E\Pi^{u*}_2 > E\Pi^{up}_2$, for $\hat{z} < 0.4$, $E\Pi^{d*}_{2C} > E\Pi^{dp}_2$, for $\hat{z} < 3.7$, and that the profits under price discrimination with z common knowledge are higher than under the corresponding pooling equilibrium up to some $\hat{z}$. All the separating equilibrium profits decrease with $\hat{z}$, showing the increase in the monopolist cost of signaling as $\hat{z}$ increases.

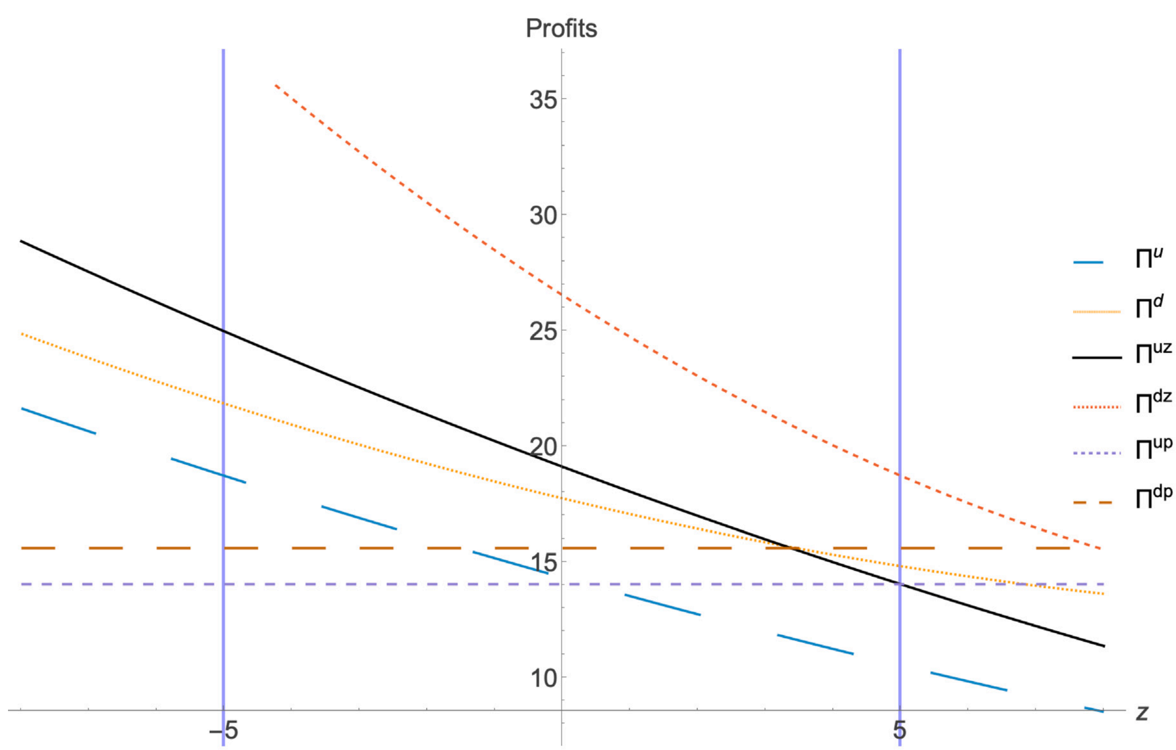
Finally, picture 4 exhibits social welfare under each scenario described above. Regarding to SW, we find that the implications of signals precision are in the same line as the impacts on CS. Partial derivatives show that social welfare (SW) is increasing in the value of the private signal δ_α, that is, $\frac{\partial SW^u}{\partial \delta\alpha} > 0$, $\frac{\partial SW^d}{\partial \delta\alpha} > 0$, $\frac{\partial SW^{uz}}{\partial \delta\alpha} > 0$, and $\frac{\partial SW^{dz}}{\partial \delta\alpha} > 0$. However, as in the case of CS, when it comes to the signal's precision γ_z, the sign of the partial derivatives depends on the relationship between the mean $\overline{x}$ and the realization of z, $\hat{z}$, i.e., $\overline{x} \lessgtr \hat{z}$.

The impact of $\hat{z}$ on the level on social welfare (SW) is significantly influenced by consumer perception of potential losses when buying online. High signals $\hat{z}$, indicating high expected losses, lead to reduced consumer participation and lower social welfare. Conversely, low signals $\hat{z}$, indicating low expected losses, lead to increased consumer participation and higher social welfare. Understanding these dynamics is crucial for predicting the exact impact on social welfare, as can be seen in Picture 4. Indeed, as $\hat{z}$ gets higher, the combination of reduced consumer participation and limited increase in quantity sold results in a decrease in social welfare. The market becomes less efficient, and fewer transactions occur, leading to lower overall economic well-being. For high enough $\hat{z}$, pooling equilibria become a better social scenario.



$\theta_i = 10,\ \overline{x} = 5,\ \lambda = 0.5,$ and $\gamma_z = \gamma_x = \delta_\alpha = 0.5.$

Fig. 2. Consumer Surplus as a function of $\hat{z}$. $\theta_i = 10$, $\overline{x} = 5$, $\lambda = 0.5$, and $\gamma_z = \gamma_x = \delta_\alpha = 0.5$..



$\theta_i = 10,\ \overline{x} = 5,\ \lambda = 0.5,$ and $\gamma_z = \gamma_x = \delta_\alpha = 0.5.$

Fig. 3. Profits as a function of $\hat{z}$. $\theta_i = 10$, $\overline{x} = 5$, $\lambda = 0.5$, and $\gamma_z = \gamma_x = \delta_\alpha = 0.5$..

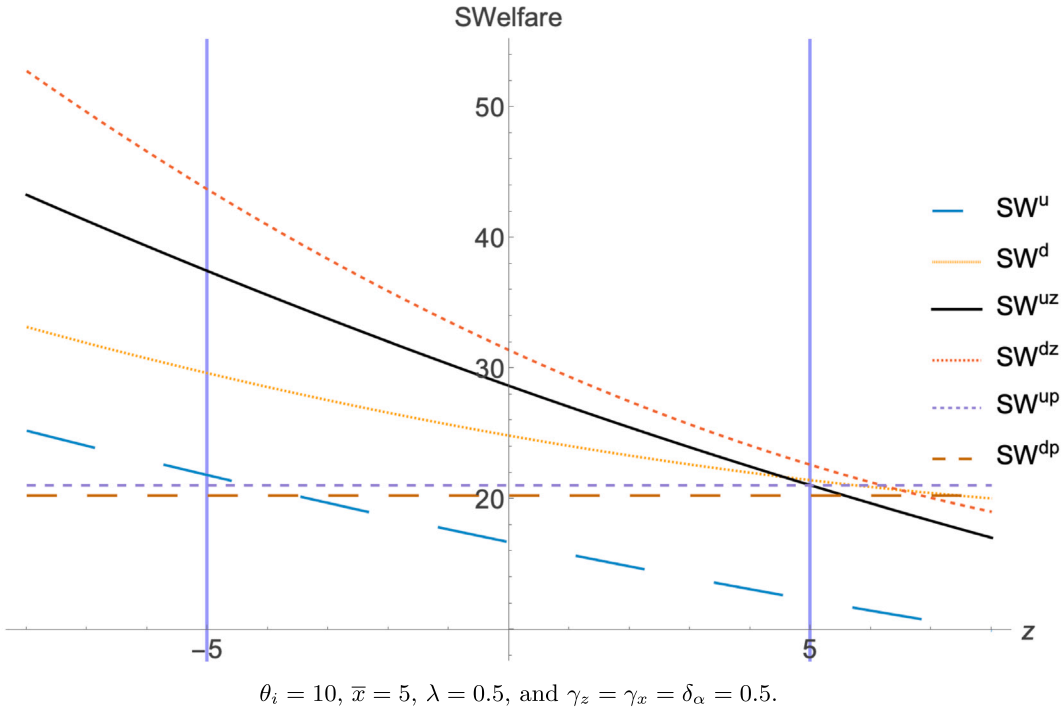


Fig. 4. Social Welfare as a function of $\hat{z}$. $\theta_i = 10$, $\overline{x} = 5$, $\lambda = 0.5$, and $\gamma_z = \gamma_x = \delta_\alpha = 0.5$..

9. Parameter sensitivity analysis: Factorial experiment

In this Section, we conduct a factorial experiment to test how sensitive their model's outcomes are to different parameter values. Economically, this matters because the model relies on assumptions about key parameters δ_α and γ_z. If small changes in these assumptions lead to very different results, then the conclusions would be fragile. If not, the conclusions are robust. A *factorial experiment* is a systematic approach in which two or more factors are varied simultaneously at predefined levels, and all possible combinations of these levels are considered. In this way, incorporating this sensitivity analysis into the study offers a valuable opportunity to dive deeper into the complex interactions between our key variables.[24] In our case, the factors are the degree of valuation of consumers' previous experience in terms of privacy, δ_α, the relative precision of the public signal γ_z, and the realization of the signal $\hat{z}$. With three levels of γ_z, three levels of δ_α, and two regimes for $\hat{z}$ (greater or smaller than true level of the average population's privacy concern $\bar{x}$), the design results in $3 \times 3 \times 2 = 18$ parameter combinations with values $\{0.2, 0.5, 0.8\}$ for both parameters γ_z and δ_α. This experimental setup allows us to identify not only the isolated effect of each parameter but also their interactions, thereby providing a comprehensive mapping of how the capacity of learning from the private and public signals shape equilibrium outcomes. The rest of parameters are fixed at $\theta_i = 10$, $\overline{x} = 3$, and $\lambda = 0.4$.

Tables A.1, A.2, and A.3 in the Appendix offer the results for prices, profits, consumer surplus and social welfare for each combination of factors that are different experiments, e_i, with $i = 1, \ldots, 18$. Table A.1 offers the results under uniform and price discrimination scenarios. Table A.2 shows the figures for pooling equilibria and Table A.3 shows the results under the Hybrid equilibrium. Furthermore, the column *Regime* in the tables refers to the relative realization of the public signal $\hat{z}$ with respect to the true population's average privacy concerns $\bar{x}$. The notation "lt" indicates the case $\hat{z} < \bar{x}$, while "gt" corresponds to $\hat{z} > \bar{x}$. This distinction is important because of the distortion caused by the existence of noisy signals in the market and its implications in the learning process. Consequently, this fact directly influences the demand response, the profitability of pricing schemes, and the welfare implications. Finally, the results are plotted in Fig. 5 to illustrate the resulting prices, profits, consumer surplus, and social welfare under various scenarios: Uniform(U), third degree price discrimination(D), hybrid, and pooling equilibria.

In all panels, the horizontal axis corresponds to δ_α, while the three values of γ_z are represented by different colors: orange ($\gamma_z = 0.2$), green ($\gamma_z = 0.5$), and blue ($\gamma_z = 0.8$). Solid lines denote the case where the signal $\hat{z}$ lies above the mean $\bar{x}$ (high type), whereas dashed lines correspond to the case $\hat{z} < \bar{x}$ (low type). Note that Pooling equilibria do not depend on variations of the parameters. Therefore, we get a unique flat line for prices, profits, CS, and social welfare. Furthermore, there are cases where we get a flat line for the price in the brick channel that do not depend on δ_α and γ_z: third-degree price discrimination, pooling discrimination, and hybrid discrimination.

The comparative statics in the panel provide a clear account of how consumer learning and information precision shape equilibrium outcomes. As δ_α increases, demand becomes more state-dependent, and consumers exhibit a form of loyalty or inertia. Consequently, prices tend to decrease, but profits tend to increase, especially under hybrid or third-degree price discrimination regimes, where firms can target locked-in consumers. As the panels translate the data from Tables A.1–A.3, you can check the precise values in them. For example, consider Table A.1 under uniform pricing, fix $\gamma_z = 0.2$, the regime "lt" ($\hat{z} < \bar{x}$), and consider the three possible values of δ_α. As δ_α increases to 0.5 (from row 1 to row 7), the uniform price decreases, and it does so when $\delta_\alpha = 0.8$ (row 13). However, profits, CS^u, and SW^u keep on increasing and for fixed $\gamma_z = 0.2$ they are highest at $\delta_\alpha = 0,8$. The same logic applies to price discrimination. These results validate the model's results.

An analogous logic applies to γ_z, the precision of the market signal, which reflects the market signal effect. As γ_z increases, beliefs about

[24] Recently, theoretical studies have incorporated factorial experiments to not only validate the model's results, but also provide nuanced insights into the policy implications. See, for example, Nerja and Sánchez (2025b) or Nerja and Sánchez (2025a).

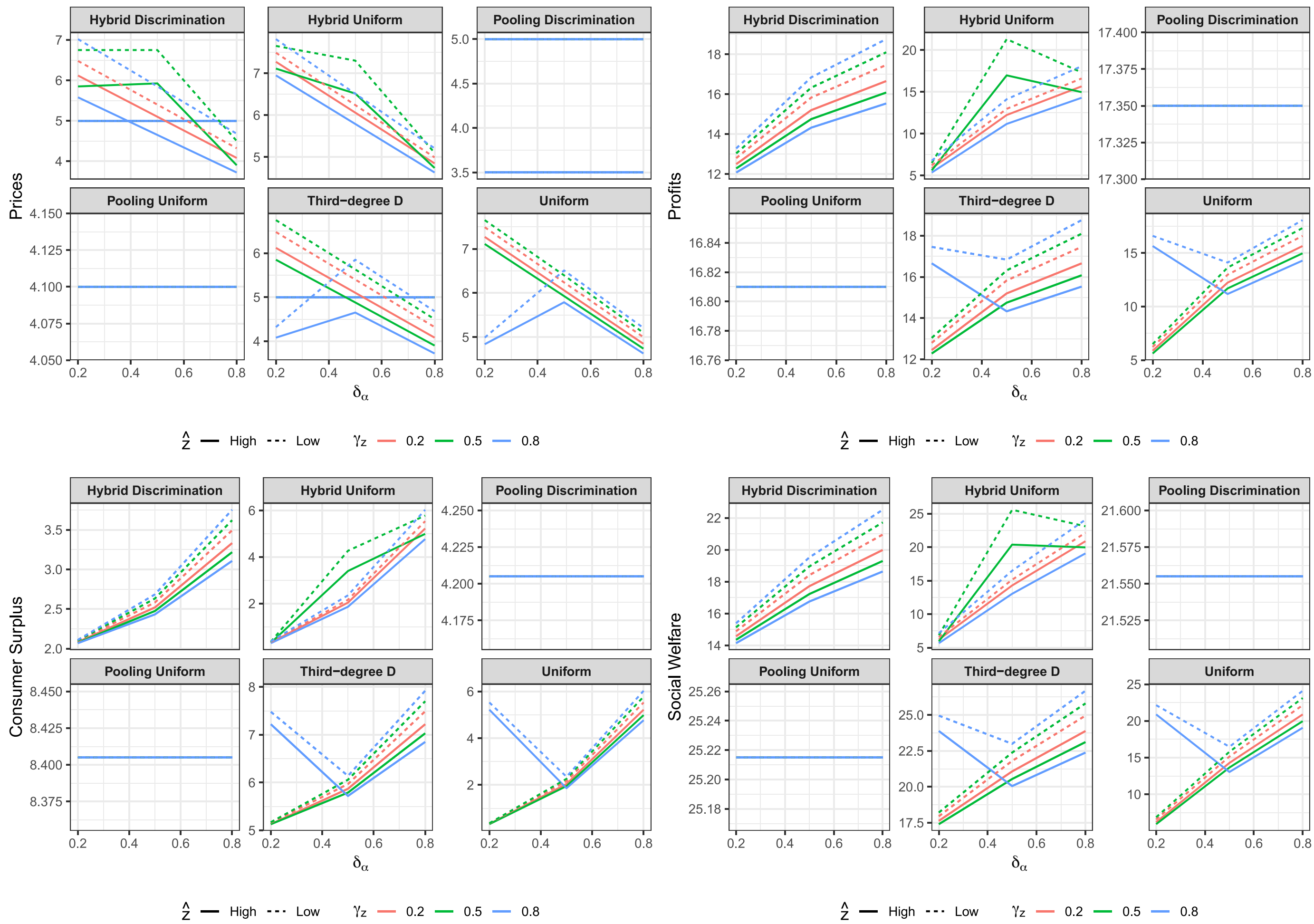


Fig. 5. Simulations depending on δ_α and γ_z.

market privacy become less dispersed and firms face lower risks of adverse selection, but the impact of regimes is crucial. With a downwardly biased signal ($\hat{z}$ = Low), greater precision prevents overpricing and consumers benefit, even if firms leave some rents unexploited. Prices and profits increase, but CS and SW also increase (see, for example, rows 7, 9, and 11 in Table A.1, for a fixed $\delta_\alpha = 0.5$). This strengthens the profitability of separation strategies: the more accurately firms can infer consumers' privacy, the more profitable it becomes to depart from pooling and engage in hybrid or fully discriminatory pricing. Accordingly, profits increase monotonically in γ_z under this regime, while prices also rise as firms exploit better information to tailor markups. With an upwardly biased signal ($\hat{z}$ = High), more precision implies lower prices, lower CS, and lower SW. Profits are also decreasing, showing the cost of signaling for the monopolist (see, for example, rows 8, 10, and 12 in Table A.1, for $\delta_\alpha = 0.5$), as predicted for the theoretical model. As we have seen, CS reacts asymmetrically depending on the regime. This asymmetry underscores the double-edged nature of information: the signal precision amplifies whichever bias prevails in the signal.

The complex interaction between parameters is illustrated by the fact that the maximum profits, CS^u and SW^u, are when $\delta_\alpha = 0.8$, $\gamma_z = 0.8$, and $\hat{z} < \bar{x}$ (for example, see row 17 in Table A.1, under uniform and price discrimination). Thus, as both parameters increase, prices decrease, and hence CS and profits generally rise, reflecting stronger market power and more effective price discrimination. Consumer surplus shows the opposite tendency, declining when firms capture a larger share of surplus, especially under upwardly biased signals. Social welfare, in turn, increases with greater signal precision due to improved allocation, although the gains are reduced when the signal is systematically biased. Overall, the figures clearly illustrate that higher values of δ_α and γ_z shift the distribution of gains toward producers while shaping the efficiency of the market outcome.

As mentioned above, the impact of regimes is also crucial. Under pooling, neither δ_α nor γ_z substantially alters outcomes because prices cannot be tailored to types, leaving profits relatively flat. Hybrid regimes show intermediate sensitivity, with profits and prices moderately increasing as both parameters rise. Third-degree discrimination displays the strongest effects: higher δ_α intensifies loyalty-driven markups, while higher γ_z enables sharper segmentation. This combination pushes both prices and profits upward most strongly in discriminatory settings, at the cost of consumer surplus whenever $\hat{z} > \bar{x}$. These regime-dependent patterns reveal the central role of market design in shaping how information translates into rents and welfare.

From the perspective of welfare, the patterns reflect the interplay of allocative efficiency and rent redistribution. Higher γ_z improves the match between consumer's privacy and consumption, which raises total welfare. Yet bias in the signal distorts this channel: when $\hat{z}$ is High, consumers under-consume, while when $\hat{z}$ is Low, they over-consume. In both cases, misallocation reduces welfare relative to an unbiased benchmark, even if one side of the market appears to gain in surplus, highlighting that transfers and allocative distortions must be considered jointly.

In economic terms, the results emphasize that consumer learning and information precision are not unambiguously beneficial. Greater reliance on experience (δ_α) reduces elasticity and facilitates firm market power, while greater precision (γ_z) strengthens the profitability of discrimination. Whether consumers benefit depends on the direction of bias in the market signal. Social welfare is more robust to these asymmetries, as improved matching typically dominates, but welfare is also dragged down when bias persists. Overall, the panel demonstrates how the interaction of market information precision, consumer purchasing experience, and the market signal bias jointly determines the distribution of gains between firms and consumers' surplus.

10. Policy implications

This paper offers valuable insights into the interplay between privacy concerns, information asymmetry, and dynamic pricing in the digital economy. By adopting well-targeted policy measures, regulators can foster a fairer and more transparent marketplace—one that respects consumer privacy and enables businesses to build trust rather than exploit it.

The study shows that consumers form privacy preferences both from personal experience and from inferences drawn from pricing. When firms use prices to signal information, these signals can distort market outcomes. To counter such effects, government agencies must safeguard privacy, promote transparency, and ensure economic efficiency. In particular, regulators should establish nationwide standards for privacy disclosures on online platforms and encourage the creation of public "privacy ratings" for firms—similar to nutritional labels.

The risks of privacy-sensitive price discrimination are illustrated by Amazon's early experiments with personalized pricing based on browsing history or location. Consumer backlash led to the abandonment of the practice, as many perceived it as exploitative. This example shows how third-degree price discrimination using sensitive data can backfire, reducing demand and trust.

As a notable empirical case, Choi et al. (2025) investigate how privacy protections and clear communication about privacy benefits influence adoption of Central Bank Digital Currencies (CBDCs)—a digital alternative to cash or cards. Unlike cash, which ensures high anonymity, CBDCs may offer only limited anonymity due to their electronic nature, raising concerns about how personal and transaction data are stored and managed. Using a nationally representative sample of over 3,500 participants, their randomized experiment finds that privacy-preserving features — such as storing personal and transaction data separately — increase willingness to adopt CBDCs by 7.4% for offline and 5% for online purchases. Providing information about privacy benefits boosts willingness by an additional 5.8% for offline purchases. When privacy-preserving design is combined with assurances that personal data will not be used commercially, willingness rises by 14% offline and 15% online—gains of 64% and 44% over the baseline, respectively. These findings provide practical guidance for CBDC design and underscore the importance of communicating privacy protections.

In the same line, Tsai et al. (2011) designed an experiment in which a shopping search engine interface clearly and compactly displays privacy policy information. When such information is made available, consumers tend to purchase from online retailers who better protect their privacy. In fact, their study indicates that when privacy information is made more salient and accessible, some consumers are willing to pay a premium to purchase from privacy protective websites.

Thus, studies where privacy information is made salient — and affects willingness to pay — connect directly with the idea that prices and presentation (signals) shape beliefs about privacy risk. Both experiments highlight the central role of privacy in consumer decision-making and how design and information can influence behavior, supporting and reinforcing the theoretical model developed in the manuscript.

Price signaling can sometimes raise prices above the full-information benchmark, harming consumers and reducing welfare. The analysis shows that pooling equilibria (non-informative prices) can, under certain conditions, yield higher welfare than fully revealing equilibria. Fair policy in digital markets with data asymmetry should therefore aim to assess the effects of price signaling on efficiency and consumer welfare. Regulators should consider intervening when price signaling causes excessive distortions or undermines consumer welfare.

Real-world examples, such as region-based pricing on streaming platforms like Netflix or Spotify, highlight these dynamics. Many consumers use VPNs to bypass location-based price differences, revealing both price sensitivity and privacy concerns. In line with regulations like the GDPR, firms often limit personalized pricing to avoid backlash and compliance risks. Bó et al. (2024) conducted an experiment to further demonstrate that when consumers know prices are based on their data, they strategically alter the information they disclose—supporting the model's assumption that privacy preferences are heterogeneous and evolve over time. This highlights strategic consumer behavior in response to data-driven pricing. Their findings highlight the importance of data privacy policies in the age of big data, in which behavior in apparently unrelated contexts might affect prices. Manipulation of data disclosure or varying willingness to pay reflect our model's assumption of consumers with different, uncertain privacy concerns who learn over time. Thus, regulation should limit or monitor personalized pricing practices, especially when they exploit uncertainty around privacy.

In conclusion, regulatory bodies such as the FTC or GDPR enforcement agencies should prevent exploitation, ensure compliance, and uphold market fairness. This may include auditing firms that practice third-degree price discrimination to determine whether pricing exploits inferred privacy vulnerabilities, imposing restrictions or disclosure requirements on algorithmic pricing, and strengthening rules on privacy-related price signals. In particularly sensitive sectors such as healthcare or finance, uniform pricing could be mandated to eliminate risks arising from asymmetric knowledge about consumer privacy.

11. Conclusions and final remarks

This paper examines a two-period monopolist operating through both brick-and-mortar and online channels, setting prices to maximize expected profits. Although demand is identical across the two channels, online demand is influenced by instrumental consumers' privacy concerns—concerns that are not directly observable. These are broken down into population-average and individual-specific concerns. At the start of the second period, the monopolist receives a private, noisy signal about population-average concerns and uses it to adjust online prices. Consumers, in turn, understand that their beliefs about privacy are shaped both by personal experience and inferred market-level concerns—deduced from observed prices. As a result, second-period prices act as signals of privacy conditions. These signals, combined with first-period experiences, influence second-period demand. In this dynamic setting, both firms and consumers learn from market signals. Price signaling reduces price sensitivity of expected demand and may lead to higher prices compared to scenarios where the average concern level (z) is publicly known. The monopolist benefits from higher expected profits through channel-based third-degree price discrimination.

Finally, the paper shows that if the realized population-average privacy concern is sufficiently high, (1) the monopolist earns less profit in a fully revealing equilibrium than in a pooling equilibrium—highlighting the costs of signaling, and (2) consumer surplus, firm profits, and overall social welfare are maximized under the non-informative (pooling) equilibrium.

The factorial experiment suggests that the main qualitative results of the model (e.g., how privacy concerns shape firm behavior and welfare outcomes) are stable across a wide range of parameter values. However, interactions matter: Some parameters do not matter much in isolation, but their effects become important when combined with others (for instance, when both consumer privacy concerns and the precision of the market signal are high). Thus, the factorial sensitivity analysis shows that the model's predictions are not driven by a narrow set of assumptions, but hold across plausible economic environments. This strengthens the policy relevance: regulators and firms can expect similar dynamics of consumers' privacy concerns, market signals, and welfare, even if the exact parameter values differ in reality.

Therefore, the theoretical analysis offers important insights into how privacy concerns, information asymmetry, and dynamic pricing interact in the digital economy. The interaction of these features may create market failures, and the need for regulatory policy to create a fairer and transparent market where consumer privacy is respected and businesses thrive on trust, not manipulation. In particular, transparency and privacy protection are essential in digital markets, as shown by the empirical application of Choi et al. (2025) and others.

We plan to conduct a laboratory experiment to empirically test the behavioral predictions of our model in a two-period simulated market with two purchasing channels. The key treatments vary along two dimensions. (1) Price–Signal Treatments: Treatment A (No signal): Price is random/no info, and Treatment B (Price reveals privacy risk): Higher prices imply lower privacy risk (as per model logic). (2) Feedback Precision Treatments: The information precision treatments will vary with how much personal experience consumers gain from the first-period purchase (low vs. high feedback privacy concerns). We would like to test several behavioral hypotheses. H1: Price Signaling Works (participants interpret higher prices in the online channel as a signal of lower privacy risks), H2: Learning Shape Beliefs (consumers who received stronger private signals in period 1 rely more on their own experience and less on prices), and H3: Demand is less elastic in the Signaling Treatment (when prices signal privacy concerns, participants become less price sensitive).

CRediT authorship contribution statement

M. Sánchez: Writing – original draft, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **A. Urbano:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Funding

This paper is an extension of a chapter of the thesis of M. Sánchez, which was awarded with the Enrique Fuentes Quintana of Funcas Award - Social Sciences Category - Call 2019–2020. This author thanks the Spanish Ministry of Economy, Industry and Competitiveness for providing financial support BES-2014-069278. The authors also thanks the support of project PID2022-136547NB-I00, founded by MICIU/AEI/10.13039/501100011033 and by FEDER, UE (M. Sánchez), and PID2019-110790RB-I00 financed by MCIN/ AEI /10.13039/501100011033 (A. Urbano), and the Generalitat Valenciana under the Excellence Programs Prometeo 2019/095 and 2022/029 (A. Urbano).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors thank Simon Anderson, Nathan Larson, Yair Tauman, participants of several International Conferences in Game Theory at Stony Brook, Simposios of the Spanish Economic Association and Jornadas de Economía Industrial for their comments and suggestions. We also thank two anonymous referees who suggested the parameter sensitivity analysis and the policy implications of our model. The usual disclaimer applies.

Appendix A

Proof of Proposition 1. Bayesian updating To express the updated beliefs in a convenient form, we first compute the Bayesian update of the random variable α_{i1} once the private signal z has been observed. By the specification of α_{i1} and z in (2) and (3), respectively, the expected values of α_{i1} and z are $E\left\{\alpha_{i1}\right\} = E\left\{z\right\} = \overline{x}$, and the variances of α_{i1} and z are $Var\left(\alpha_{i1}\right) = \sigma_x^2 + \sigma_\omega^2$ and $Var(z) = \sigma_x^2 + \sigma_\varphi^2$. To simplify, we label $Var\left(\alpha_{i1}\right) = \sigma_{\alpha_1}^2 = \sigma_\alpha^2$ and $Var(z) = \sigma_z^2$.[25]

Then, the Bayesian update is[26]

$$E\left\{\alpha_{i1}|z\right\} = \overline{x}\left(1 - \frac{\sigma_x^2}{\sigma_z^2}\right) + z\left(\frac{\sigma_x^2}{\sigma_z^2}\right). \tag{A.1}$$

Recalling that γ_z is the relative precision[27] of signal z (i.e., $\gamma_z = \frac{\sigma_x^2}{\sigma_z^2}$) and letting γ_x be the relative precision of the prior distribution of α_{i1} (i.e, $\gamma_x = 1 - \gamma_z = \left(1 - \frac{\sigma_x^2}{\sigma_z^2}\right)$), then,

$$E\left\{\alpha_{i1}|z\right\} = \gamma_z z + \gamma_x \overline{x}. \tag{A.2}$$

Clearly, the Bayesian updates of α_{i1} conditional on z are a linear combination of z and $\overline{x}$, weighted by their respective relative precision (γ_x and γ_z). Next, we calculate the updating of α_{i1} after the observation of z and first-period sales via the online channel in period 2.

At the beginning of period 2, new information enters the market. Therefore, the conditional expectation of α_{i2} will take place after the observation of z and the first-period sales, q_1. That is, there are three correlated and normally distributed random variables, where the variance–covariance matrix is as follows:

$$\begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ z \end{pmatrix} \sim N\left(\begin{pmatrix} \overline{x} \\ \overline{x} \\ \overline{x} \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 & \sigma_x^2 + \sigma_\omega^2 & \sigma_x^2 \\ \sigma_x^2 + \sigma_\omega^2 & \sigma_\alpha^2 & \sigma_x^2 \\ \sigma_x^2 & \sigma_x^2 & \sigma_z^2 \end{pmatrix}\right). \tag{A.3}$$

Then, the updating of α_{i1} after the observation of z and first-period sales via the online channel in period 2 is,

$$\begin{aligned} E\left\{\alpha_{i2}|\alpha_{i1}, z\right\} = \; & E\left\{\alpha_{i2}\right\} + \begin{pmatrix} Cov\left(\alpha_{i2}, \alpha_{i1}\right) & Cov\left(\alpha_{i2}, z\right) \end{pmatrix} \\ & \times \begin{pmatrix} Var\left(\alpha_{i1}\right) & Cov\left(\alpha_{i2}, \alpha_{i1}\right) \\ Cov\left(\alpha_{i2}, \alpha_{i1}\right) & Var(z) \end{pmatrix}^{-1} \begin{pmatrix} \alpha_{i1} - E\left\{\alpha 1\right\} \\ z - E\left\{z\right\} \end{pmatrix}. \end{aligned}$$

that given the expected values, variances, and the variance–covariance matrix and substituting the corresponding terms into the above equation gives the following expression,

$$E\left\{\alpha_{i2}|\alpha_{i1}, z\right\} = \overline{x}\left(1 - \frac{\sigma_x^2\sigma_\varphi^2 + \sigma_\omega^2\sigma_z^2}{|\Sigma|} - \frac{\sigma_x^2\sigma_\upsilon^2}{|\Sigma|}\right) + \alpha_{i1}\left(\frac{\sigma_x^2\sigma_\varphi^2 + \sigma_\omega^2\sigma_z^2}{|\Sigma|}\right) + z\left(\frac{\sigma_x^2\sigma_\upsilon^2}{|\Sigma|}\right),$$

where $|\Sigma| = \sigma_\alpha^2\sigma_z^2 - \sigma_x^4$ is the determinant of the variance–covariance matrix specified earlier.

Let $\delta_\alpha = \frac{\sigma_x^2\sigma_\varphi^2 + \sigma_\omega^2\sigma_z^2}{\sigma_\alpha^2\sigma_z^2 - \sigma_x^4}$; $\delta_z = \frac{\sigma_x^2\sigma_\upsilon^2}{\sigma_\alpha^2\sigma_z^2 - \sigma_x^4}$, and $\delta_x = 1 - \delta_\alpha - \delta_z$.

[25] The correlation between the two variables is specified by the index ρ. Calculations show that $\rho\left(\alpha_{it}, z\right) = \frac{Cov(\alpha_{it}, z)}{\sqrt{Var(\alpha_{i1})Var(z)}} = \frac{\sigma_x^2}{\sigma_\alpha\sigma_z}$.

[26] Following DeGroot (2005), the Bayesian updating of the mean μ of a normal random variable after n observation of a random sample is $\mu' = (\tau\mu + ns\overline{x})/(s + ns)$, where τ is the prior precision, and $\overline{x}$ and s are the mean and precision of the random sample.

[27] Recall that we have assumed that γ_z is common knowledge among consumers and the monopolist.

Table A.1
Factorial experiment: prices (p^u, p^d_B, p^d_C), profits (π), consumer surplus (CS), and social welfare (SW) under Uniform (U), Third-Degree (D). Fixed parameters: $\theta = 10$, $x = 3$, $\lambda = 0.4$.

e_i	δ_α	γ_z	$\hat{z}$	Reg.	Uniform (U)				Third-Degree (D)				
					p	π	CS	SW	p_b	p_c	π	CS	SW
1	0.2	0.2	2	lt	7.49	6.23	0.35	6.57	5.00	6.48	12.80	5.16	17.95
2	0.2	0.2	4	gt	7.27	5.88	0.33	6.20	5.00	6.12	12.45	5.13	17.63
3	0.2	0.5	2	lt	7.65	6.50	0.36	6.87	5.00	6.75	13.04	5.17	18.21
4	0.2	0.5	4	gt	7.11	5.62	0.32	5.93	5.00	5.85	12.28	5.13	17.40
5	0.2	0.8	2	lt	4.99	16.61	5.53	22.15	5.00	4.32	17.46	7.48	24.95
6	0.2	0.8	4	gt	4.84	15.66	5.22	20.89	5.00	4.08	16.66	7.22	23.88
7	0.5	0.2	2	lt	6.24	12.98	2.16	15.14	5.00	5.40	15.83	5.97	21.80
8	0.5	0.2	4	gt	6.06	12.24	2.04	14.28	5.00	5.10	15.20	5.87	21.07
9	0.5	0.5	2	lt	6.40	13.65	2.26	15.80	5.00	5.63	16.32	6.06	22.38
10	0.5	0.5	4	gt	5.93	11.70	1.95	13.65	5.00	4.88	14.75	5.79	20.54
11	0.5	0.8	2	lt	6.51	14.12	2.35	16.48	5.00	5.85	16.84	6.14	22.99
12	0.5	0.8	4	gt	5.79	11.18	1.86	13.04	5.00	4.65	14.33	5.72	20.05
13	0.8	0.2	2	lt	5.00	16.61	5.53	22.15	5.00	4.32	17.46	7.49	24.95
14	0.8	0.2	4	gt	4.85	15.66	5.22	20.89	5.00	4.08	16.66	7.22	23.88
15	0.8	0.5	2	lt	5.10	17.34	5.78	23.12	5.00	4.50	18.10	7.70	25.80
16	0.8	0.5	4	gt	4.74	14.98	4.99	19.97	5.00	3.90	16.08	7.03	23.11
17	0.8	0.8	2	lt	5.21	18.08	6.03	24.11	5.00	4.68	18.76	7.92	26.68
18	0.8	0.8	4	gt	4.63	14.30	4.77	19.07	5.00	3.72	15.53	6.85	22.38

Then, a new substitution in $E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}$ gives the firm's Bayesian update of consumers' privacy concerns in period 2, conditional on z and α_{i1},

$$E\left\{\alpha_{i2}|\alpha_{i1}, z\right\} = \overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z. \tag{A.4}$$

which is Eq. (4) in Proposition 1. Also, by above note that $\sigma^2_\alpha = \sigma^2_x + \sigma^2_\omega + \sigma^2_\vartheta$ and $\sigma^2_z = \sigma^2_x + \sigma^2_\varphi$, and then we can rewrite, $\delta_z = \gamma_z\left(1-\delta_\alpha\right)$, and $\delta_x = \left(1-\gamma_z\right)\left(1-\delta_\alpha\right) = \gamma_x + (1-\delta_\alpha)\delta_x$, which are Eqs. (5) and (6) in Proposition 1.

Derivation of the second period perceive demand by the monopolist in the online channel: Section 4. The benchmark model: Uniform pricing

In period 2, the demand curve perceived by the monopolist in the online channel is the expectation over the representative consumer demand, conditional on its information set, Ω_{m2}, consisting of the observation $\hat{z}$ of z, the second-period price, and the Bayesian updating of α_{i1}.

$$E\left[q^u_2|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right] = \theta_i - E\left\{\left(\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + \left(a + bp^u_2\right)\delta_z\right)\right\} - p^u_2.$$

By Eqs. (A.1) and (A.2), $E\left\{\alpha_{i1}|z\right\} = \overline{x}\left(1-\frac{\sigma^2_x}{\sigma^2_z}\right) + z\left(\frac{\sigma^2_x}{\sigma^2_z}\right) = \gamma_z z + \gamma_x\overline{x}$, then

$$E\left[q^u_2|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right] = \theta_i - (\overline{x}\delta_x + (\gamma_z z + \gamma_x\overline{x})\delta_\alpha + \left(a + bp^u_2\right)\delta_z) - p^u_2.$$

By Eq. (6), $\delta_x = \gamma_x(1-\delta_\alpha)$, then $\overline{x}(\delta_x + \gamma_x\delta_\alpha) = \overline{x}(\gamma_x(1-\delta_\alpha) + \gamma_x\delta_\alpha) = \overline{x}\gamma_x$.

Thus,

$$E\left[q^u_2|\Omega_{m2}\left(z, p_2\left(\hat{z}\right)\right)\right] = \theta_i - \left(\overline{x}\gamma_x + \delta_z\left(a + bp^u_2\right) + \hat{z}\delta_\alpha\gamma_z\right) - p^u_2,$$

which is the expression for the monopolist's update of the second period demand in the online channel.

Proof of point 3 of Proposition 2. Section 4. The benchmark model: Uniform pricing

We prove here that the uniform equilibrium specified in the main text is the unique separating equilibrium where consumers' inferences about z are a differentiable and invertible function of p^u_2. First, we prove that the equilibrium is linear, second that the inference rule is unique, and third that any equilibrium inference rule has to be linear.

To see that this is the case, note that the calculations show that this a linear equilibrium. Assume that consumers infer $\hat{z} = z(p^u_2)$ if the second-period price is p^u_2, and that z is differentiable. The demand curve faced by the firm in period 2 is,

$$\lambda\left(\theta_i - p^u_2\right) + (1-\lambda)\left(\theta_i - \left(\overline{x}\gamma_x + \delta_z z(p^u_2) + z\delta_\alpha\gamma_z\right) - p^u_2\right).$$

The profit maximization price satisfies the first-order condition:

$$\theta_i - 2p^u_2 - (1-\lambda)(\bar{x}\gamma_x + \delta_z z(p^u_2) + \delta_z z'(p^u_2)p^u_2 + z\delta_\alpha\gamma_z) = 0,$$

which implicitly defines the correct rule, $z(p^u_2)$. In a Bayes-Nash equilibrium, consumers use the correct inference rule, hence, $z\left(p^u_2\right)$ must solve the ordinary differential equation:

$$(1-\lambda)((\delta_z + \delta_\alpha\gamma_z)z(p^u_2) + \delta_z z'(p^u_2)p^u_2) = \theta_i - 2p^u_2 - (1-\lambda)\bar{x}\gamma_x$$

We proceed by ordering and simplifying the terms in the previous differential equation to look for general/particular solutions.

To that end, divide first the ordinary differential equation by $(1-\lambda)\delta_z p^u_2$,

$$\frac{(\delta_z + \delta_\alpha\gamma_z)z(p^u_2)}{\delta_z p^u_2} + z'(p^u_2) = \frac{2}{\delta_z}\left(\frac{1}{(1-\lambda)}\right) - \frac{\theta_i}{p^u_2\delta_z}\left(\frac{1}{(1-\lambda)}\right) + \frac{\overline{x}\gamma_x}{\delta_z p^u_2}$$

Letting $s = \frac{2}{\delta_z}\left(\frac{1}{(1-\lambda)}\right)$, $m = \frac{\theta_i}{\delta_z}\left(\frac{1}{(1-\lambda)}\right)$, $t = \frac{\overline{x}\gamma_x}{\delta_z}$, and $r = \frac{(\delta_z + \delta_\alpha\gamma_z)}{\delta_z}$, and let us rename p^u_2 as p_2 to ease notation, and reordering the terms yields,

$$z'\left(p_2\right) + z\left(p_2\right)p_2^{-1}r = mp_2^{-1} - p_2^{-1}t - s.$$

Second, multiply the above expression by p^r_2 (the integrating factor), which gives,

$$\frac{d}{dp_2}p_2{}^r z(p_2) = p_2{}^r\left(z'\left(p_2\right) + z\left(p_2\right)p_2{}^{-1}r\right) = p_2{}^r\left(mp_2^{-1} - tp_2^{-1} - s\right),$$

Table A.2
Factorial experiment: prices (p^{up}, p_B^{dp}, p_C^{dp}), profits (π), consumer surplus (CS), and social welfare (SW) under Pooling Scenario, both Uniform (UP) and discrimination (DP). Fixed parameters: $\theta = 10$, $x = 3$, $\lambda = 0.4$.

e_i	δ_α	γ_z	$\hat{z}$	Reg.	Pooling-U (PU)				Pooling-Third D (PD)				
					p^{up}	Π^{up}	CS^{up}	SW^{up}	p_b^{dp}	p_c^{dp}	π^{dp}	CS^{dp}	SW^{dp}
1	0.2	0.2	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
2	0.2	0.2	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
3	0.2	0.5	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
4	0.2	0.5	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
5	0.2	0.8	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
6	0.2	0.8	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
7	0.5	0.2	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
8	0.5	0.2	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
9	0.5	0.5	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
10	0.5	0.5	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
11	0.5	0.8	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
12	0.5	0.8	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
13	0.8	0.2	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
14	0.8	0.2	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
15	0.8	0.5	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
16	0.8	0.5	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
17	0.8	0.8	2	lt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56
18	0.8	0.8	4	gt	4.10	16.81	8.41	25.22	5.00	3.5	17.35	4.21	21.56

Table A.3
Factorial experiment: prices (p^{uh}, p_B^{dh}, p_C^{dh}), profits (π), consumer surplus (CS) and social welfare (SW) under Hybrid Scenario, both Uniform (UP) and discrimination (DP). Fixed parameters: $\theta = 10$, $x = 3$, $\lambda = 0.4$.

e_i	δ_α	γ_z	$\hat{z}$	Reg.	Hybrid-U (UH)				Hybrid-Third D (PD)				
					p^{uh}	Π^{uh}	CS^{uh}	SW^{uh}	p_b^{dh}	p_c^{dh}	π^{dh}	CS^{dh}	SW^{dh}
1	0.2	0.2	2	lt	7.49	6.23	0.351	6.58	5.00	6.48	12.79	2.09	14.89
2	0.2	0.2	4	gt	7.27	5.87	0.32	6.20	5.00	6.12	12.49	2.08	14.58
3	0.2	0.5	2	lt	7.65	6.50	0.36	6.86	5.00	6.75	13.03	2.10	15.13
4	0.2	0.5	4	gt	7.11	5.61	0.31	5.92	5.00	5.85	12.28	2.07	14.35
5	0.2	0.8	2	lt	7.81	6.78	0.37	7.15	5.00	7.02	13.28	2.10	15.39
6	0.2	0.8	4	gt	6.94	5.36	0.29	5.66	5.00	5.58	12.07	2.06	14.14
7	0.5	0.2	2	lt	6.24	12.97	2.16	15.14	5.00	5.40	15.83	2.58	18.41
8	0.5	0.2	4	gt	6.06	12.24	2.04	14.28	5.00	5.10	15.20	2.52	17.72
9	0.5	0.5	2	lt	7.30	21.31	4.26	25.58	5.00	6.75	16.32	2.63	18.96
10	0.5	0.5	4	gt	6.51	16.98	3.39	20.38	5.00	5.92	14.75	2.47	17.22
11	0.5	0.8	2	lt	6.51	14.12	2.35	16.48	5.00	5.85	16.84	2.68	19.52
12	0.5	0.8	4	gt	5.79	11.17	1.86	13.03	5.00	4.65	14.32	2.43	16.75
13	0.8	0.2	2	lt	4.99	16.61	5.53	22.15	5.00	4.32	17.46	3.49	20.95
14	0.8	0.2	4	gt	4.84	15.66	5.22	20.89	5.00	4.08	16.65	3.33	19.99
15	0.8	0.5	2	lt	5.10	17.34	5.78	23.12	5.00	4.50	18.10	3.62	21.72
16	0.8	0.5	4	gt	4.74	14.97	4.99	19.97	5.00	3.90	16.08	3.21	19.30
17	0.8	0.8	2	lt	5.20	18.08	6.02	24.10	5.00	4.68	18.76	3.75	22.51
18	0.8	0.8	4	gt	4.63	14.30	4.76	19.07	5.00	3.72	15.53	3.10	18.64

and may be integrated to,

$$p_2{}^r\left(z\left(p_2\right)\right) = p_2{}^r\left(\frac{m-t}{r}\right) - \frac{p^{r+1}s}{1+r} + C.$$

for some constant C. This is a general solution. To determine C, we need the value of the function $z(p_2)$ at one point. For example, if $z(0)$ is finite (the initial condition), then evaluating the differential equation at $p_2 = 0$ implies that $C = 0$. Hence, $z(p_2)$ is linear in p_2.

Next, we check that the linear rule is unique. The uniqueness property follows from the nature of the signaling differential equation. Consider an arbitrary linear rule: $z(p_2) = \beta + \alpha p_2$. $z(p_2)$ has to satisfy the first order conditions and solve the ordinary differential equation:

$$(1-\lambda)((\delta_z + \delta_\alpha\gamma_z)z(p_2^u) + \delta_z z'(p_2^u)p_2^u) = \theta_i - 2p_2^u - (1-\lambda)\bar{x}\gamma_x$$

which translates to:

$$(1-\lambda)(\delta_z + \delta_\alpha\gamma_z)\beta + (1-\lambda)(\delta_z + \delta_\alpha\gamma_z)\alpha p_2^u + \delta_z\alpha p_2^u = \theta_i - 2p_2^u - (1-\lambda)\bar{x}\gamma_x,$$

which simplifying gives,

$$(1-\lambda)(\delta_z + \delta_\alpha\gamma_z)\beta + (1-\lambda)\alpha p_2^u(2\delta_z + \delta_\alpha\gamma_z) = \theta_i - 2p_2^u - (1-\lambda)\bar{x}\gamma_x.$$

Then, parameters α and β solve:

$$\alpha = \frac{-2}{(1-\lambda)(2\delta_z + \delta_\alpha\gamma_z)} = \frac{-2}{(1-\lambda)(2-\delta_\alpha)\gamma_z},$$

since $\delta_z = \gamma_z(1-\delta_\alpha)$, and

$$\beta = \frac{\theta_i - (1-\lambda)\bar{x}\gamma_x}{(1-\lambda)(\delta_z + \delta_\alpha\gamma_z)} = \frac{\theta_i - (1-\lambda)\bar{x}\gamma_x}{(1-\lambda)\gamma_z},$$

since $\delta_z = \gamma_z(1-\delta_\alpha)$. These expressions are Eqs. (16) and (17) in the main text to the inference rule $z(p) = a + bp$. Hence, the linear rule is unique.

Finally, we show that there does not exist a non-linear rule, satisfying the differential equation. Renaming p_2^u as p_2 to ease notation, and totally differentiating the differential equation,

$$(1-\lambda)((\delta_z + \delta_\alpha\gamma_z)z(p_2) + \delta_z z'(p_2)p_2) = \theta_i - 2p_2 - (1-\lambda)\bar{x}\gamma_x,$$

yields,

$$(1-\lambda)((\delta_z + \delta_\alpha\gamma_z)z'(p_2) + \delta_z z''(p_2)p_2 + \delta_z z'(p_2)) = \theta_i - 2 - (1-\lambda)\bar{x}\gamma_x$$

At the initial value $z(0)$,

$$(1-\lambda)((\delta_z + \delta_\alpha\gamma_z)z'(p_2) + \delta_z z'(p_2)) = \theta_i - 2 - (1-\lambda)\bar{x}\gamma_x.$$

Then, since the left hand side term is $(1-\lambda)(2\delta_z + \delta_\alpha\gamma_z)z'(p_2)$

$$z'(p_2) = \frac{\theta_i - 2 - (1-\lambda)\bar{x}\gamma_x}{(1-\lambda)(2\delta_z + \delta_\alpha\gamma_z)} = \frac{\theta_i - 2 - (1-\lambda)\bar{x}\gamma_x}{(1-\lambda)(2-\delta_\alpha)\gamma_z}$$

and since $z''(p_2) = 0$, then $z(p_2)$ has to be linear. □

Proof of Proposition 3 Section 5. Third degree price discrimination

Here, we offer the calculations which prove Proposition 2. Recall that superscript "d" denotes price discrimination. Then, p_{1B}^d and p_{2B}^d are the prices of the brick-and-mortar channel in periods 1 and 2, and p_{1C}^d and p_{2C}^d those of the online channel in periods 1 and 2, respectively. Then, under price discrimination, the monopolist's expected profits in period 1 are,

$$E\left[\Pi_1^d|\Omega_{m1}\left(\alpha_1\right)\right] = \lambda\left(\theta_i - p_{1B}^d\right)p_{1B}^d + (1-\lambda)\left(\theta_i - \overline{x} - p_{1C}^d\right)p_{1C}^d.$$

In period 2, consumers' information set changes to Ω_{i2}, and consumers update their beliefs about their privacy concerns, as long as they purchased through the online channel. At equilibrium, the updating of consumers' beliefs implies that period 2 consumers' expected demand is $q_{iB2}^d = \theta_i - p_{2B}^d$ for the brick-and-mortar channel, and $q_{iC2}^d = \theta_i - E\left\{E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}|\alpha_{i1}, p_{2C}^d\right\} - p_{2C}^d$ for the online one, where, by Bayesian updating (see (7)), the second term on the right-hand side can be specified as,

$$q_{iC2}^d = \theta_i - E\left\{\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z|\alpha_{i1}, p_{2C}^d\right\} - p_{2C}^d,$$

and, by the linear inference rule, it translates to,

$$q_{iC2}^d = \theta_i - \left(\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + \left(a + bp_{2C}^d\right)\delta_z\right) - p_{2C}^d.$$

The demand curve perceived by the monopolist is the sum of the demand for the brick-and-mortar and online channels. After period 1, the monopolist observes online first-period sales, and in period 2, it gets some new information about consumers' average privacy concerns. As in the uniform price setting, the online demand curve perceived by the monopolist is the expectation of the representative consumer's demand conditional on its own information and the second-period price. That is,

$$\begin{aligned} E\left[q_{i2}^d|\Omega_{m2}\left(z, p_{2C}^d(z)\right)\right] &= \lambda q_{i2B}^d + (1-\lambda)E\left[q_{i2C}^d|\alpha_{i1}, z\right] \\ &= \lambda q_{i2B}^d + (1-\lambda)E\left[q_{i2C}^d|z, p_{2C}^d\right], \end{aligned}$$

which can be specified as,

$$\begin{aligned} E\left[q_{i2}^d|\Omega_{m2}\left(z, p_{2C}^d(z)\right)\right] &= \lambda\left(\theta_i - p_{2B}^d\right) + (1-\lambda) \\ &\quad\times\left(\theta_i - E\left\{\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z|z, p_{2C}^d\right\} - p_{2C}^d\right), \end{aligned}$$

and by the firm' s update rule of α_{i1}, given the realization of z

$$\begin{aligned} \left[q_{i2}^d|\Omega_{m2}\left(z, p_{2C}^d(\hat{z})\right)\right] &= \lambda\left(\theta_i - p_{2B}^d\right) + (1-\lambda) \\ &\quad\times\left(\theta_i - \overline{x}\gamma_x + \delta_z\left(a + bp_{2C}^d\right) + \hat{z}\delta_\alpha\gamma_z - p_{2C}^d\right). \end{aligned}$$

Thus, the monopolist's second-period expected profits are,

$$\begin{aligned} E\left[\Pi_2^d|\Omega_{m2}\left(z, p_2^d(\hat{z})\right)\right] &= \lambda(\theta_i - p_{2B}^d)p_{2B}^d \\ &\quad+(1-\lambda)(\theta_i - (\overline{x}\gamma_x + \delta_z\left(a + bp_{2C}^d\right) + \hat{z}\delta_\alpha\gamma_z)) - p_{2C}^d p_{2C}^d. \end{aligned} \tag{A.5}$$

Once the expected demands and the expected benefits in $t = 1, 2$ have been specified, let the tuple $(p_{1B}^{d*}, p_{1C}^{d*})$ be the equilibrium prices for period 1, and let $(p_{2B}^{d*}, p_{2C}^{d*})$ be those for period 2. That is,

$$p_1^{d*} = \arg\max_{p_{1B}^d, p_{1C}^d} \left\{\lambda\left(\theta_i - p_{1B}^d\right)p_{1B}^d + (1-\lambda)\left(\theta_i - \overline{x} - p_{1C}^d\right)p_{1C}^d\right\} \tag{A.6}$$

and

$$p_2^{d*} = \arg\max_{p_{2B}^d, p_{2C}^d} E\left[\Pi_2^d|\Omega_{m2}\left(z, p_2^d(\hat{z})\right)\right] \tag{A.7}$$

In particular, given the expected demand perceived by the monopolist, its optimal price in period 2 in the online channel is,

$$p_{2C}^{d*} = \frac{\theta_i - a\delta_z - (\overline{x}\gamma_x + \hat{z}\delta_\alpha\gamma_z)}{2(1 + b\delta_z)}. \tag{A.8}$$

Threshold calculations for Proposition 5

Compare first the corresponding profits. Recall that $E\Pi_2^{u*}$ are the monopolist's second-period expected profits under uniform pricing, $E\Pi_2^{u*} = \frac{1}{4}\left(2-\delta_\alpha\right)\delta_\alpha(\theta_i - (1-\lambda)\left(\overline{x}\gamma_x + \hat{z}\gamma_z\right))^2$, and that $E\Pi_2^{up} = \frac{1}{4}(\theta_i - (1-\lambda)\overline{x})^2$ are the monopolist's second period profits under pooling, then equating $E\Pi_2^{u*}$ with $E\Pi_2^{up}$, and solving for z, we find that profits under either signaling or concealing are the same whenever the realization of z is,

$$\hat{z}_\pi^{u*} = \frac{\theta_i(\sqrt{A} - 1) + (1-\lambda)\overline{x}}{\sqrt{A}(1-\lambda)\gamma_z} - \frac{\bar{x}\gamma_x}{\gamma_z} < z_{\pi=0}^{u*} \tag{A.9}$$

where $A = (2-\delta_\alpha)\delta_\alpha$, and $z_{\pi=0}^{u*} = \frac{\theta_i}{(1-\lambda)\gamma_z} - \frac{\bar{x}\gamma_x}{\gamma_z} = z_0^u$ is the threshold of z for which profits under uniform pricing are zero, and which corresponds with the threshold for which uniform prices are zero. Then, for any realization of z higher than z_π^{u*} profits under fully revealing uniform prices will be smaller than under pooling pricing.

Repeating the analysis under third degree price discrimination we have that second-period expected profits in the online channel under price discrimination are,

$$E\Pi_{2C}^{d*} = \frac{1}{4}\left(2-\delta_\alpha\right)\delta_\alpha(1-\lambda)(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z))^2,$$

and recalling that $E\Pi^{dp}_{2C} = \frac{1}{4}(1-\lambda)(\theta_i - \overline{x})^2$, are the second period expected profits under pooling in the online market, then as above the realization of z for which expected profits are the same is,

$$\hat{z}^{d*}_{\pi} = \frac{\theta_i(\sqrt{A}-1)+\overline{x}}{\sqrt{A}\gamma_z} - \frac{\bar{x}\gamma_x}{\gamma_z} < z^{d*}_{\pi=0} \tag{A.10}$$

where A is as above, and $z^{d*}_{\pi=0} = \frac{\theta_i}{\gamma_z} - \frac{\bar{x}\gamma_x}{\gamma_z} = z^d_0$ is the threshold of z for which profits under discrimination are zero, an which corresponds with the threshold for which prices are zero. Then, for any realization of z higher than z^{d*}_{π} profits under discrimination in the online channel will be smaller than under pooling.

Appendix B. Tables

See Tables A.1–A.3.

Data availability

No data was used for the research described in the article.

References

Acquisti, A., Brandimarte, L., & Loewenstein, G. (2015). Privacy and human behavior in the age of information. *Science*, *347*(6221), 509–514.

Acquisti, A., Brandimarte, L., & Loewenstein, G. (2020). Secrets and likes: The drive for privacy and the difficulty of achieving it in the digital age. *Journal of Consumer Psychology*, *30*(4), 736–758.

Acquisti, A., Taylor, C., & Wagman, L. (2016). The economics of privacy. *J. Econ. Lit.*, *54*(2), 442–492.

Acquisti, A., & Varian, H. R. (2005). Conditioning prices on purchase history. *Mark. Sci.*, *24*(3), 367–381.

Belleflamme, P., & Vergote, W. (2016). Monopoly price discrimination and privacy: The hidden cost of hiding. *Economics Letters*, *149*, 141–144.

Bergemann, D., & Bonatti, A. (2019). Markets for information: An introduction. *Annual Review of Economics*, *11*, 85–107.

Bergemann, D., & Bonatti, A. (2024). Data, competition, and digital platforms. *American Economic Review*, *114*(8), 2553–2595.

Bergemann, D., Bonatti, A., & Gan, T. (2022). The economics of social data. *Rand Journal of Economics*, *53*(2), 263–296.

Bó, I., Chen, L., & Hakimov, R. (2024). Strategic responses to personalized pricing and demand for privacy: An experiment. *Games and Economic Behavior*, *148*, 487–516.

Bonatti, A. (2011). Menu pricing and learning. *American Economic Journal: Microeconomics*, *3*(3), 124–163.

Chen, Y., & Zhang, Z. J. (2009). Dynamic targeted pricing with strategic consumers. *Int. J. Ind. Organ.*, *27*(1), 43–50.

Choe, C., Matsushima, N., & Shekhar, S. (2023). The bright side of the gdpr: Welfare-improving privacy management. Available At SSRN 4537982.

Choi, S., Kim, B., Kim, Y. S., & Kwon, O. (2025). Central bank digital currency and privacy: A randomized survey experiment. *International Economic Review*, *66*(2), 823–847.

Conitzer, V., Taylor, C. R., & Wagman, L. (2012). Hide and seek: Costly consumer privacy in a market with repeat purchases. *Marketing Science*, *31*(2), 277–292.

DeGroot, M. H. (2005). *Optimal statistical decisions*: *Vol. 82*, John Wiley & Sons.

Gal-Or, E., Gal-Or, R., & Penmetsa, N. (2018). The role of user privacy concerns in shaping competition among platforms. *Information Systems Research*, *29*(3), 698–722.

Hakobyan, M. (2019). The rise of online privacy concerns inhibits the customer online shopping experience. https://customerthink.com/the-rise-of-online-privacy-concerns-inhibits-the-customer-online-shopping-experience/. (Accessed 12 November 2019) from Customer Think site.

Jeitschko, T. D., Liu, T., & Wang, T. (2018). Information acquisition, signaling and learning in duopoly. *International Journal of Industrial Organization*, *61*, 155–191.

Jeitschko, T. D., & Normann, H.-T. (2012). Signaling in deterministic and stochastic settings. *Journal of Economic Behavior and Organization*, *82*(1), 39–55.

Jin, Y., & Vasserman, S. (2021). *Buying data from consumers: The impact of monitoring programs in US auto insurance*: *Technical Report*, National Bureau of Economic Research.

Judd, K. L., & Riordan, M. H. (1994). Price and quality in a new product monopoly. *Review of Economic Studies*, *61*(4), 773–789.

Li, Y., Liu, H., Lee, M., & Huang, Q. (2020). Information privacy concern and deception in online retailing: The moderating effect of online–offline information integration. *Internet Research*, *30*(2), 511–537.

Lin, T. (2022). Valuing intrinsic and instrumental preferences for privacy. *Marketing Science*, *41*(4), 663–681.

Matthews, S. A., & Mirman, L. J. (1983). Equilibrium limit pricing: The effects of private information and stochastic demand. *Econometrica: Journal of the Econometric Society*, 981–996.

Mirman, L. J., Salgueiro, E., & Santugini, M. (2015). Learning in a perfectly competitive market.

Nerja, A., & Sánchez, M. (2025a). The dynamics of HSR preference in the context of short-haul flight bans. *Transport Policy*, Article 103773.

Nerja, A., & Sánchez, M. (2025b). The impacts of airline corporate social responsibility in the air transport industry. *Economics of Transportation*, *43*, Article 100423.

Posner, R. A. (1981). The economics of privacy. *The American Economic Review*, *71*(2), 405–409.

Smith, H. J., Dinev, T., & Xu, H. (2011). Information privacy research: an interdisciplinary review. *MIS Quarterly*, 989–1015.

Soleymanian, M., Weinberg, C. B., & Zhu, T. (2019). Sensor data and behavioral tracking: Does usage-based auto insurance benefit drivers? *Marketing Science*, *38*(1), 21–43.

Stigler, G. J. (1980). An introduction to privacy in economics and politics. *The Journal of Legal Studies*, *9*(4), 623–644.

Tsai, J. Y., Egelman, S., Cranor, L., & Acquisti, A. (2011). The effect of online privacy information on purchasing behavior: An experimental study. *Information Systems Research*, *22*(2), 254–268.

U. S, Department-of-Commerce, U. (2016). U.s. commerce control list (CCL). Retrieved September 9, 2024 from Government site https://www.bis.gov/regulations.

U. S, Department-of-Commerce, U. (2020). Manufacturing and trade inventories and sales. Retrieved September 9, 2024 from Government site https://www.census.gov/programs-surveys/decennial-census/decade/2020/2020-census-main.html.

Villas-Boas, J. M. (2014). Price cycles in markets with customer recognition. *RAND J. Econ.*, *35*(3), 486–501.

Zuiderveen Borgesius, F., & Poort, J. (2017). Online price discrimination and EU data privacy law. *Journal of Consumer Policy*, *40*, 347–366.