Published for SISSA by Springer

Received: *November 27, 2023*
Revised: *December 22, 2023*
Accepted: *December 23, 2023*
Published: *January 16, 2024*

The majoron coupling to charged leptons

Antonio Herrero-Brocal[a] **and Avelino Vicente**[a,b]

[a] *Instituto de Física Corpuscular — CSIC-Universitat de València, Parc Científic de Paterna, C/ Catedrático José Beltrán, 2 E-46980 Paterna (Valencia), Spain*
[b] *Departament de Física Teòrica, Universitat de València, Dr. Moliner 50, E-46100 Burjassot (València), Spain*

E-mail: antonio.herrero@ific.uv.es, avelino.vicente@ific.uv.es

Abstract: The particle spectrum of all Majorana neutrino mass models with spontaneous violation of global lepton number include a Goldstone boson, the so-called majoron. The presence of this massless pseudoscalar changes the phenomenology dramatically. In this work we derive general analytical expressions for the 1-loop coupling of the majoron to charged leptons. These can be applied to any model featuring a majoron that have a clear hierarchy of energy scales, required for an expansion in powers of the low-energy scale to be valid. We show how to use our general results by applying them to some example models, finding full agreement with previous results in several popular scenarios and deriving novel ones in other setups.

Keywords: Axions and ALPs, Baryon/Lepton Number Violation, Lepton Flavour Violation (charged)

ArXiv ePrint: 2311.10145

https://doi.org/10.1007/JHEP01(2024)078

Contents

1 Introduction

Lepton number is an accidental symmetry in the Standard Model (SM) of particle physics. Many extensions of this minimal model actually include sources of lepton number violation. This is the case of all Majorana neutrino mass models, in which the $\mathrm{U}(1)_L$ lepton number symmetry is broken. When this symmetry is global and its breaking is spontaneous, a massless Goldstone boson arises in the spectrum, the majoron (J) [1–5]. This massless pseudoscalar has dramatic phenomenological implications and can be probed by many experiments, including the search for rare low-energy processes [6] and invisible Higgs decays at high-energy colliders [7], as well as due to its impact on many cosmological observables [8–11].

The flavor structure of the majoron couplings to charged leptons is crucial for the phenomenology [12, 13]. As explained below, flavor diagonal couplings are strongly constrained by astrophysical observations, while the flavor off-diagonal ones may induce exotic processes like $\ell_\alpha \to \ell_\beta J$, with $\alpha \neq \beta$. Different models also induce majoron couplings to charged leptons at different loop orders. Tree-level diagonal majoron couplings to charged leptons are actually induced in many scenarios beyond the SM. For instance, in models that generate them via mixing between the Higgs and the singlet that breaks lepton number spontaneously. However, in this case the couplings turn out to be diagonal in the charged lepton mass basis. An alternative mechanism must be introduced in order to generate tree-level off-diagonal couplings. For instance, these are induced if the majoron couples directly to the charged leptons or to other fermions that mix with them [14].

We study the 1-loop coupling of the majoron to a pair of charged leptons. In contrast to previous works that focus on specific models, typically the type-I seesaw with spontaneous

JHEP01(2024)078

lepton number violation [2, 15–17], we derive general expressions valid for (virtually) any model. We consider all 1-loop diagrams leading to a majoron coupling to charged leptons and expand the resulting analytical expressions in powers of the light neutrino masses (or other related low-energy scale). The main result of our work is a set of formulae that can be readily applied to any majoron model of interest to obtain the couplings to charged leptons. In order to demonstrate their use, we illustrate the application of our analytical results to several example models. This allows us to recover well-known results in some popular scenarios, which constitutes a non-trivial cross-check of our calculation. We also obtain novel results in other less studied models.

The rest of the manuscript is organized as follows. We present our setup, discuss the current bounds on the majoron couplings to charged leptons and derive general expressions for them in section 2. These general results are applied to specific models in section 3. This allows us to show how to use our analytical results. We summarize our work and discuss further directions in section 4. Finally, appendices A and B contain additional details about our results.

2 The majoron coupling to charged leptons

The Lagrangian describing the interaction of a majoron with a pair of charged leptons can be generally written as [6],

$$\mathcal{L}_{\ell\ell J} = J\,\bar{\ell}_\beta \left(S_L^{\beta\alpha}\, P_L + S_R^{\beta\alpha}\, P_R \right) \ell_\alpha + \text{h.c.} = J\,\bar{\ell}_\beta \left[S^{\beta\alpha}\, P_L + \left(S^{\alpha\beta} \right)^*\, P_R \right] \ell_\alpha\,. \tag{2.1}$$

Here $P_{L,R} = \frac{1}{2}\left(1 \mp \gamma_5\right)$ are the usual chiral projectors while $\ell_{\alpha,\beta}$ are the charged leptons, with $\alpha, \beta = 1, 2, 3$ two flavor indices, and

$$S^{\beta\alpha} = S_L^{\beta\alpha} + \left(S_R^{\alpha\beta} \right)^*\,. \tag{2.2}$$

All flavor combinations are considered: $\beta\alpha = \{ee, \mu\mu, \tau\tau, e\mu, e\tau, \mu\tau\}$. We note that majorons are pseudoscalar states. This implies that the diagonal $S^{\beta\beta} = S_L^{\beta\beta} + \left(S_R^{\beta\beta} \right)^*$ couplings are purely imaginary at all orders in perturbation theory [18]. We are interested in models that induce flavor off-diagonal couplings at the 1-loop level. Therefore, while flavor diagonal couplings will be allowed at tree-level, the off-diagonal ones will be absent and only generated at 1-loop. In models that generate off-diagonal couplings at tree-level, the 1-loop contributions considered in our work can be regarded as just corrections, hence expected to be subdominant. For similar reasons, we will consider only diagonal tree-level couplings with quarks.

There are stringent constraints on both diagonal and off-diagonal majoron couplings to charged leptons [6]. The former are constrained by astrophysical observations. If produced, majorons would escape astrophysical scenarios without interacting with the medium, thus leading to a very efficient energy loss mechanism. This has been studied in several recent works [19–24], which have derived the limits

$$|\text{Im}\, S^{ee}| < 2.1 \times 10^{-13} \tag{2.3}$$

and

$$|\text{Im}\, S^{\mu\mu}| < 3.1 \times 10^{-9}\,. \tag{2.4}$$

The off-diagonal majoron couplings to charged leptons also receive strong constraints, in this case from searches of the flavor violating decays $\ell_\alpha \to \ell_\beta \, J$. In order to specify the bound, it proves convenient to define the combination

$$\left|\widetilde{S}^{\beta\alpha}\right| = \left(\left|S_L^{\beta\alpha}\right|^2 + \left|S_R^{\beta\alpha}\right|^2\right)^{1/2} , \tag{2.5}$$

that enters the $\ell_\alpha \to \ell_\beta \, J$ decay width as

$$\Gamma(\ell_\alpha \to \ell_\beta \, J) = \frac{m_\alpha}{32\,\pi} \left|\widetilde{S}^{\beta\alpha}\right|^2 . \tag{2.6}$$

The current limit on the branching ratio of $\mu^+ \to e^+ \, J$ was obtained at TRIUMF [25]. Taking into account all possible chiral structures for the majoron coupling, one can estimate the limit $\text{BR}\,(\mu \to e\,J) \lesssim 10^{-5}$ [26], which in turn implies

$$\left|\widetilde{S}^{e\mu}\right| < 5.3 \times 10^{-11} . \tag{2.7}$$

Finally, the currently best experimental limits on τ decays including majorons were set by the Belle II collaboration [27]. They can be used to derive the bounds

$$\begin{aligned} \left|\widetilde{S}^{e\tau}\right| &< 3.5 \times 10^{-7} , \\ \left|\widetilde{S}^{\mu\tau}\right| &< 2.7 \times 10^{-7} . \end{aligned} \tag{2.8}$$

2.1 General setup

Our goal is to obtain generic expressions for the majoron coupling to a pair of charged leptons. We adopt a general setup with 3 light charged leptons, $\ell_\alpha = \{e, \mu, \tau\}$, N neutral leptons, $n_i = \{n_1, \ldots, n_N\}$, out of which at least 3 must be light, and 6 quarks, $q_i = \{u, c, t, d, s, b\}$.[1] Note that only the usual SM charged leptons and quarks are considered and that all quarks are generically denoted by q_i, without any distinction between up- and down-type quarks. We also assume the neutral leptons to be of Majorana nature, which implies that their mass matrix in the gauge basis is symmetric. We also introduce a massive scalar ρ, a massive pseudoscalar σ, a singly charged scalar $\eta^\pm$, a scalar leptoquark X and a vector leptoquark Y_μ. All these fields are mass eigenstates. In case more than one massive scalar, pseudoscalar, charged scalar or leptoquark (of any type) is present in the model, our results can be easily adapted by summing over all mass eigenstates. In addition, we note that at least one pseudoscalar will always be part of the spectrum, namely the massless majoron. We will consider the following general interaction Lagrangian:

$$\mathcal{L} = \mathcal{L}_J + \mathcal{L}_\rho + \mathcal{L}_\sigma + \mathcal{L}_\eta + \mathcal{L}_{\text{LQ}} + \mathcal{L}_Z + \mathcal{L}_W + \mathcal{L}_S . \tag{2.9}$$

The first term describes the interaction of the majoron with a pair of neutrinos, a pair of charged leptons or a pair of quarks, and is given by[2]

$$\mathcal{L}_J = \bar{n}_i \left(A_{ij} P_R + A_{ji}^* P_L\right) n_j \, J + \bar{\ell}_\alpha \, K_{\alpha\alpha} \gamma_5 \, \ell_\alpha \, J + \bar{q}_i \, I_{ii} \gamma_5 \, q_i \, J . \tag{2.10}$$

[1] In the following, we denote the charged lepton flavor index with a Greek character, $\alpha = 1, 2, 3$, while the neutral fermion and quark flavor indices will be denoted with a Latin character, $i = 1, \ldots, N$ and $i = 1, \ldots, 6$, respectively.

[2] It may seem *exotic* to consider a majoron coupling to quarks, but this interaction is indeed induced at tree-level in models in which the scalar singlet responsible for lepton number violation gets a non-zero mixing with the Higgs doublet after the electroweak and lepton number symmetries are spontaneously broken.

As already explained, we have considered only flavor-diagonal couplings between the majoron and charged leptons and between the majoron and quarks (here, K and I are purely imaginary diagonal matrices). The Yukawa interactions of the scalar ρ and the pseudoscalar σ can be written as

$$\mathcal{L}_\rho = \bar{\ell}_\alpha \left(G_{\alpha\beta} P_L + G^*_{\beta\alpha} P_R \right) \ell_\beta \, \rho \,, \tag{2.11}$$

$$\mathcal{L}_\sigma = \bar{n}_i \left(B_{ij} P_R + B^*_{ji} P_L \right) n_j \, \sigma + \bar{\ell}_\alpha \left(C_{\alpha\beta} P_L + C^*_{\beta\alpha} P_R \right) \ell_\beta \, \sigma \,. \tag{2.12}$$

In principle, the scalar ρ can also couple to a pair of neutrinos, but we have decided to ignore these couplings, since they would not contribute (at 1-loop) to the majoron coupling to charged leptons. We also note that eq. (2.12) reduces to $\bar{\ell}_\alpha \, C^*_{\alpha\alpha} \gamma_5 \, \ell_\alpha \, \sigma$, with $C_{\alpha\alpha}$ purely imaginary, when one considers the interaction term between the pseudoscalar σ and a pair of charged leptons with the same flavor. Similarly, eq. (2.11) reduces to $\bar{\ell}_\alpha \, G_{\alpha\alpha} \, \ell_\alpha \, \sigma$, with $G_{\alpha\alpha}$ purely real, in the flavor diagonal case. The charged scalar η can also have a Yukawa coupling to a charged lepton and a neutrino, given by

$$\mathcal{L}_\eta = \bar{\ell}_\alpha \left(D_L^{\alpha i} P_L + D_R^{\alpha i} P_R \right) n_i \, \eta + \text{h.c.} \,. \tag{2.13}$$

The X and Y leptoquarks couple to a quark and a charged lepton in the general form

$$\mathcal{L}_{\text{LQ}} = \bar{\ell}_\alpha \left(T_L^{\alpha i} P_L + T_R^{\alpha i} P_R \right) q_i \, X + \bar{\ell}_\alpha Y_\mu \, \gamma^\mu \left(H_L^{\alpha i} \, P_L + H_R^{\alpha i} \, P_R \right) q_i \; + \text{h.c.} \,. \tag{2.14}$$

Since we denote up- and down-quarks as q_i indiscriminately, the electric charges of X and Y are not fixed, but left as free parameters. However, these electric charges do not affect any of the results that follow and hence there is no need to distinguish between quark types. $\mathcal{L}_Z$ and $\mathcal{L}_W$ contain the usual gauge bosons interactions with neutrinos and charged leptons, which can be written as [15–17, 28, 29]

$$\mathcal{L}_Z = \frac{g}{4 \, \cos\theta_W} \bar{n}_i \, \not{Z} \left(E_{ij} \, P_L - E_{ji} \, P_R \right) n_j + \frac{g}{\cos\theta_W} \bar{\ell}_\alpha \, \not{Z} \left(v_\ell - a_\ell \gamma_5 \right) \ell_\alpha \,, \tag{2.15}$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \left(\bar{\ell}_\alpha F_{\alpha i} \not{W}^- P_L \, n_i + \text{h.c.} \right) \,, \tag{2.16}$$

where E is an Hermitian matrix and v_ℓ and a_ℓ are the usual Z-boson couplings to charged leptons. Finally, $\mathcal{L}_S$ describes a cubic scalar interaction of the majoron with one scalar and one pseudoscalar,

$$\mathcal{L}_S = \omega \, J \, \sigma \, \rho \,, \tag{2.17}$$

where ω is a real parameter with dimensions of mass. Sums over charged lepton and neutrino flavor indices are implicitly assumed in eqs. (2.10)–(2.16). The generic couplings A, B, C, $D_{L,R}$, E, F and $G_{L,R}$ have model-dependent expressions in terms of the parameters of the specific model under consideration. Nevertheless, they can generally be written as

$$A_{ij} = U_{ki}\,\bar{A}_{kr}\,U_{rj}\,, \tag{2.18}$$

$$B_{ij} = U_{ki}\,\bar{B}_{kr}\,U_{rj}\,, \tag{2.19}$$

$$D_L^{\alpha i} = \bar{D}_L^{\alpha k}\,U_{ki}^*\,, \tag{2.20}$$

$$D_R^{\alpha i} = \bar{D}_R^{\alpha k}\,U_{ki}\,, \tag{2.21}$$

$$E_{ij} = U_{ki}\,\bar{E}_{kr}\,U_{rj}^* = \sum_{k=1}^{3} U_{ki}\,U_{kj}^*\,, \tag{2.22}$$

$$F_{\alpha i} = \bar{F}_{\alpha k}\,U_{ki}^* = U_{\alpha i}^*\,, \tag{2.23}$$

where U is the unitary matrix implicitly defined by

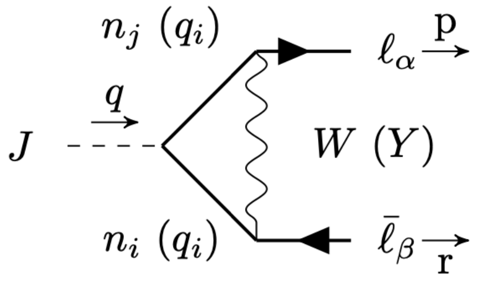
$$M_{ij} = U_{ki}\,\bar{M}_{kr}\,U_{rj}\,, \tag{2.24}$$

with $M \equiv \mathrm{diag}\,(m_{n_1}, \ldots, m_{n_N})$ and $\bar{M}$ the neutral lepton mass matrices in the mass and gauge bases, respectively. We have assumed that we work in the charged lepton mass basis. Therefore, $\bar{A}$, $\bar{B}$, $\bar{D}_{L,R}$, $\bar{E}$ and $\bar{F}$ are the couplings in the gauge basis, while $C = \bar{C}$ and $G = \bar{G}$. Eqs. (2.22) and (2.23) are completely general, since they just correspond to the transformation of the usual neutral and charged currents, and the index $i = 1, 2, 3$ in these equations tags a gauge non-singlet. In the case of quark couplings we prefer to keep them in the mass basis, just for simplicity, since we are not interested in the gauge basis parameters.

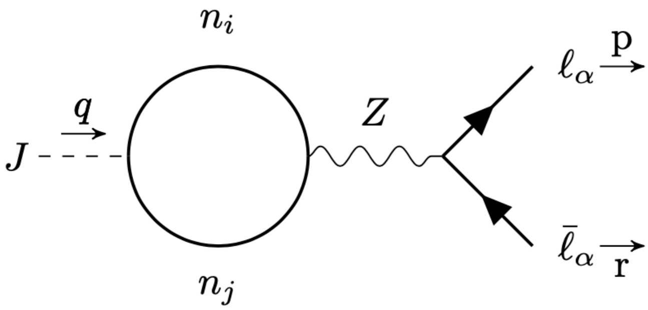
2.2 1-loop coupling

Let us now move on to the computation of the 1-loop coupling of the majoron to a pair of charged leptons. First of all, we consider all possible 1-loop diagrams leading to this coupling. They are shown in figure 1. Other 1-loop diagrams vanish due to the pseudoscalar nature of the majoron. For instance, this would be the case for the self-energy diagram in figure 1(d) if we replaced the pseudoscalar σ by the scalar ρ. It is important to notice a key aspect. In the scalar diagrams, the fermion in the loop can either be a neutral or a charged lepton. We shall consider both scenarios. We could in principle include charged leptons in the Z boson diagram. However, given that this diagram is always flavor diagonal, such an inclusion would constitute a correction to the tree-level majoron coupling, thus rendering it negligible. For this reason we will not consider this option. If σ couples to quarks, an additional σq contribution, analogous to $\sigma\ell$, would exist. The resulting $\mathcal{M}_{\sigma q}$ amplitude can be obtained by applying obvious $\ell \to q$ replacements to the $\mathcal{M}_{\sigma\ell}$ amplitude and we decided to omit it. Similarly, an additional Z boson contribution with a quark closed loop can be added, again with a trivial $n \to q$ change. Finally, let us note that non-trivial electrically charged particles have been added to the spectrum. In such cases, we would have infinitely many different possibilities (due to the charge combinations). However, we emphasize once again that these electric charges do not affect our results.

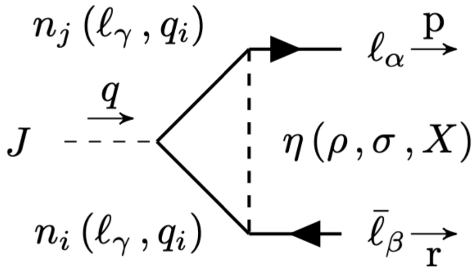
The amplitudes of the Feynman diagrams in figure 1 will be denoted by $\mathcal{M}_A$, with $A = \{W, Z, \eta, \rho, \sigma, X, Y, \sigma n, \sigma\ell, S\}$, and each amplitude corresponds to a different diagram. For simplicity, we will work in the unitarity gauge, where the diagrams including SM Goldstone



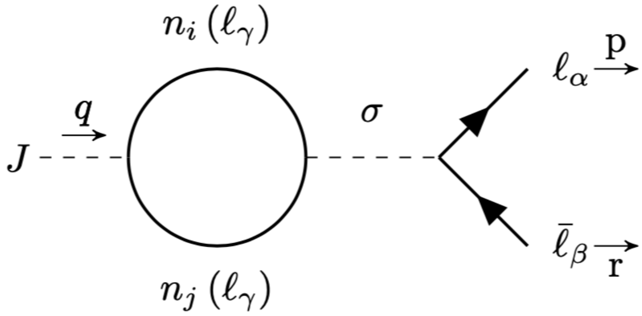
(a) $\boldsymbol{W}$ boson and $\boldsymbol{Y}$ contributions: $\mathcal{M}_W + \mathcal{M}_Y$.



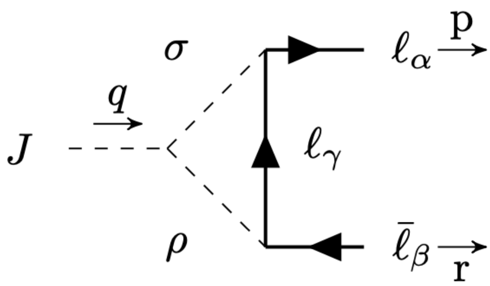
(b) $\boldsymbol{Z}$ boson contribution: $\mathcal{M}_Z$.



(c) $\boldsymbol{\eta}$, $\boldsymbol{\rho}$, $\boldsymbol{\sigma}$ and $\boldsymbol{X}$ contributions: $\mathcal{M}_\eta + \mathcal{M}_\rho + \mathcal{M}_\sigma + \mathcal{M}_X$.



(d) $\boldsymbol{\sigma n}$ and $\boldsymbol{\sigma \ell}$ contributions: $\mathcal{M}_{\sigma n} + \mathcal{M}_{\sigma\ell}$.



(e) $\boldsymbol{S}$ contribution: $\mathcal{M}_S$.

Figure 1. Feynman diagrams leading to the 1-loop coupling of the majoron to a pair of charged leptons.

bosons are absent. Thus, the amplitudes are given by

$$
\begin{aligned}
\mathcal{M}_W = 2\,\bar{v}_{\ell_\beta} \sum_{i,j} \int \frac{d^4k}{(2\pi)^4} \frac{g}{\sqrt{2}}\, F_{\beta j}\, \gamma_\mu P_L \frac{1}{k^2 - M_W^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{M_W^2} \right) \frac{\not{k} - \not{r} + m_j}{(k-r)^2 - m_j^2} \\
\times \left(A_{ij} P_R + A^*_{ji} P_L \right) \frac{\not{k} + \not{p} + m_i}{(k+p)^2 - m_i^2} \frac{g}{\sqrt{2}}\, \gamma_\nu P_L\, F^*_{\alpha i}\, u_{\ell_\alpha}\,,
\end{aligned}
\tag{2.25}
$$

$$
\mathcal{M}_Z = -2\frac{g}{\cos\theta_W}\bar{v}_{\ell_\beta}\gamma_\mu\left(v_\ell - a_\ell\gamma_5\right)u_{\ell_\alpha}\frac{1}{-M_Z^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{M_Z^2}\right) \tag{2.26}
$$

$$
\times\sum_{i,j}\int\frac{d^4k}{(2\pi)^4}\frac{g}{4\cos\theta_W}\,\mathrm{Tr}\left[\gamma_\nu\left(E_{ij}P_L - E_{ji}P_R\right)\frac{\not{k}+\not{q}+m_j}{(k+q)^2-m_j^2}\left(A_{ij}P_R + A_{ji}^*P_L\right)\frac{\not{k}+m_i}{k^2-m_i^2}\right],
$$

$$
\mathcal{M}_\eta = 2\sum_{i,j}\bar{v}_{\ell_\beta}\int\frac{d^4k}{(2\pi)^4}\left(D_L^{\beta j}P_L + D_R^{\beta j}P_R\right)\frac{\not{k}-\not{r}+m_j}{(k-r)^2-m_j^2}\left(A_{ij}P_R + A_{ji}^*P_L\right)
$$

$$
\times\frac{\not{k}+\not{p}+m_i}{(k+p)^2-m_i^2}\left[\left(D_L^{\alpha i}\right)^*P_R + \left(D_R^{\alpha i}\right)^*P_L\right]\frac{1}{k^2-m_\eta^2}u_{\ell_\alpha}, \tag{2.27}
$$

$$
\mathcal{M}_\rho = \sum_\gamma\bar{v}_{\ell_\beta}\int\frac{d^4k}{(2\pi)^4}\left(G_{\beta\gamma}P_L + G_{\gamma\beta}^*P_R\right)\frac{\not{k}-\not{r}+m_{\ell_\gamma}}{(k-r)^2-m_{\ell_\gamma}^2}K_{\gamma\gamma}\gamma_5\frac{\not{k}+\not{p}+m_{\ell_\gamma}}{(k+p)^2-m_{\ell_\gamma}^2}
$$

$$
\times\left(G_{\gamma\alpha}P_L + G_{\alpha\gamma}^*P_R\right)\frac{1}{k^2-m_\rho^2}u_{\ell_\alpha}, \tag{2.28}
$$

$$
\mathcal{M}_\sigma = \mathcal{M}_\rho\left(G\leftrightarrow C\right), \tag{2.29}
$$

$$
\mathcal{M}_X = \bar{v}_{\ell_\beta}\sum_j\int\frac{d^4k}{(2\pi)^4}\left(T_L^{\beta j}P_L + T_R^{\beta j}P_R\right)\frac{\not{k}-\not{r}+m_{q_j}}{(k-r)^2-m_{q_j}^2}I_{jj}\gamma_5\frac{\not{k}+\not{p}+m_{q_j}}{(k+p)^2-m_{q_j}^2}
$$

$$
\times\left[\left(T_L^{\alpha j}\right)^*P_R + \left(T_R^{\alpha j}\right)^*P_L\right]\frac{1}{k^2-M_X^2}u_{\ell_\alpha}, \tag{2.30}
$$

$$
\mathcal{M}_Y = \bar{v}_{\ell_\beta}\sum_j\int\frac{d^4k}{(2\pi)^4}\gamma_\mu\left(H_L^{\beta j}P_L + H_R^{\beta j}P_R\right)\frac{1}{k^2-M_Y^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{M_Y^2}\right)\frac{\not{k}-\not{r}+m_{q_j}}{(k-r)^2-m_{q_j}^2}
$$

$$
\times I_{jj}\gamma_5\frac{\not{k}+\not{p}+m_{q_j}}{(k+p)^2-m_{q_j}^2}\gamma_\nu\left[\left(H_L^{\alpha j}\right)^*P_L + \left(H_R^{\alpha j}\right)^*P_R\right]u_{\ell_\alpha}, \tag{2.31}
$$

$$
\mathcal{M}_{\sigma n} = -2\sum_{i,j}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\left(B_{ij}P_R + B_{ji}^*P_L\right)\frac{\not{k}+\not{q}+m_j}{(k+q)^2-m_j^2}\left(A_{ij}P_R + A_{ji}^*P_L\right)\frac{\not{k}+m_i}{k^2-m_i^2}\right]
$$

$$
\times\bar{v}_{\ell_\beta}\left(C_{\beta\alpha}P_L + C_{\alpha\beta}^*P_R\right)\frac{1}{-m_\sigma^2}u_{\ell_\alpha}, \tag{2.32}
$$

$$
\mathcal{M}_{\sigma\ell} = -\sum_\gamma\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[K_{\gamma\gamma}\gamma_5\frac{\not{k}+\not{q}+m_{\ell_\gamma}}{(k+q)^2-m_{\ell_\gamma}^2}C_{\gamma\gamma}^*\gamma_5\frac{\not{k}+m_{\ell_\gamma}}{k^2-m_{\ell_\gamma}^2}\right]
$$

$$
\times\bar{v}_{\ell_\beta}\left(C_{\beta\alpha}P_L + C_{\alpha\beta}^*P_R\right)\frac{1}{-m_\sigma^2}u_{\ell_\alpha}, \tag{2.33}
$$

$$
\mathcal{M}_S = \sum_\gamma\bar{v}_{\ell_\beta}\int\frac{d^4k}{(2\pi)^4}\left(G_{\beta\gamma}P_L + G_{\gamma\beta}^*P_R\right)\frac{\not{k}+m_{\ell_\gamma}}{k^2-m_{\ell_\gamma}^2}\left(C_{\gamma\alpha}P_L + C_{\alpha\gamma}^*P_R\right)
$$

$$
\times\frac{1}{(k-r)^2-m_\rho^2}\frac{1}{(k+p)^2-m_\sigma^2}\,\omega\,u_{\ell_\alpha}. \tag{2.34}
$$

The amplitudes in the previous equations have been computed with the help of `Package X` [30]. Since the resulting exact analytical expressions are very involved, we have derived approximate expressions, valid in most scenarios of interest. We now explain the main ingredients in our calculation:

- A hierarchy of scales in the neutrino sector has been assumed, namely

$$
m_{\text{light}} \ll m_{\text{EW}} \ll M_H\,. \tag{2.35}
$$

Here m_{light} is a low-energy scale, to be identified with the scale of neutrino masses or with other related low mass scales, $m_{\text{EW}} \sim m_W$ is the usual electroweak scale and M_H is a high-energy scale where the heavy mediators responsible for neutrino masses lie. This hierarchy of scales allows us to expand our results in powers of the small mass ratios $m_{\text{light}}/m_{\text{EW}}$ and m_{EW}/M_H and obtain approximate (but also simpler) analytical expressions.

- As usual, all states at a given energy scale will be assumed to be degenerate at order zero in m_{light}. For instance, all the heavy mediators will be assumed to have a mass M_H, with possible corrections of order m_{light}. In practice, this means that $m_{\text{light}}/m_{\text{EW}}$ and m_{EW}/M_H will be single expansion parameters and our approximations may be invalid if large mass splittings exist among the states that lie at a given energy scale.

- In order to properly apply the mass hierarchy in eq. (2.35), we split the sum over all neutral fermion mass eigenstates as

$$\sum_{i,j} = \sum_{i,j\sim l} + \sum_{i,j\sim h} + \sum_{i\sim l}\sum_{j\sim h} + \sum_{j\sim l}\sum_{i\sim h} , \tag{2.36}$$

where $\sim l$ and $\sim h$ refer to the sum over light and heavy neutral fermions, respectively.

- The couplings in the interaction vertices may include the unitary matrix U, see eqs. (2.18)–(2.23). This must be taken into account to properly expand our results, since the entries of the U matrix involve the mass ratios that we use as expansion parameters. In fact, we will make use of the identity

$$\sum_{j\sim h} U_{rj}\, m_j\, U_{kj} = \bar{M}^\dagger_{rk} - \sum_{j\sim l} U_{rj}\, m_j\, U_{kj} , \tag{2.37}$$

which can be readily derived from the definition of U. This expression allows us to identify up to which order in m_{light} we must expand our analytical expressions in order to be fully consistent. The first term, $\bar{M}^\dagger_{rk}$, is much larger than the neutrino mass scale, while $\sum_{j\sim l} U_{rj}\, m_j\, U_{kj} \sim m_\nu$ is of order one in m_{light}. Therefore, if the model under consideration predicts $\bar{M}_{kr} \neq 0$, then the part proportional to the light neutrino masses does not contribute at leading order. In contrast, if $\bar{M}_{kr} = 0$, the dominant term will be proportional to m_{light}, and we must expand up to this order. In general, when we expand the integrals using eq. (2.36), we find, mostly, that we must compare expressions of the form $\sum_{j\sim h} U_{rj}\, m_j\, U_{kj}$ with $\sum_{j\sim l} U_{rj}\, m_j\, U_{kj}$. A priori, one can think that the first one is dominant because m_j is much larger if $j \sim h$, rather than when $j \sim l$. However, we do not know the shape of the U matrix. In some models, the suppression introduced by U in the heavy block compensates for the suppression of the light mass. Ultimately, we cannot compare these expressions. For this reason, we use eq. (2.37) intensively to transform these expressions into comparable ones.[3]

[3]The exception to this rule takes place when the heavy mass appears in the denominator, $\sum_{j\sim h} U_{rj}\, m_j^{-1}\, U_{kj}$. In this case, it does not make sense to perform this transformation, as we would artificially introduce the dominant term into the total sum, and this will cancel out with the sum over the light states. Therefore, we will include it in the zeroth order, as it is not explicitly proportional to any power of the light mass. The actual order will depend on the model under consideration. Then, given a model, one can certainly derive the form of the light neutrino mass matrix, and therefore, upon substitution, determine the order of this term.

JHEP01(2024)078

- We have simplified the results and written them using matrix notation. In this process, sums over repeated indices have been identified as matrix products whenever possible.
- The new scalar and vector states will be assumed to be much heavier than m_{light} and all charged leptons. In the case of leptoquarks, we impose that their masses are significantly above the electroweak scale.

Before we present our results it proves convenient to introduce some matrices that will allow us to write them in a more compact way:

$$\Gamma_{spj}^{n,m,t} \equiv \sum_{k,r} \bar{A}_{kr} \left[\left(\bar{M}^\dagger \right)^n \left(\bar{M}\bar{M}^\dagger \right)^m \right]_{pk} U_{rj} m_j^t U_{sj}^* \,, \tag{2.38}$$

$$\tilde{\Gamma}_{spj}^{n,m,t} \equiv \sum_{k,r} \bar{A}_{kr} \left[\left(\bar{M}^\dagger \right)^n \left(\bar{M}\bar{M}^\dagger \right)^m \right]_{sr} U_{kj} m_j^t U_{pj}^* \,, \tag{2.39}$$

$$\Delta_{spj}^{n,m,t} \equiv \sum_{k,r} \bar{A}_{kr} \left[\left(\bar{M}^\dagger \right)^n \left(\bar{M}\bar{M}^\dagger \right)^m \right]_{pk} U_{rj} m_j^t U_{sj} \,, \tag{2.40}$$

$$\tilde{\Delta}_{spj}^{n,m,t} \equiv \sum_{k,r} \bar{A}_{kr} \left[\left(\bar{M}^\dagger \right)^n \left(\bar{M}\bar{M}^\dagger \right)^m \right]_{sr} U_{kj} m_j^t U_{pj} \,, \tag{2.41}$$

$$\gamma_{sp} \equiv \sum_{k,r} \sum_{i,j\sim l} \bar{A}_{kr}^* U_{kj}^* U_{sj} U_{ri}^* U_{pi} \,. \tag{2.42}$$

We are now in position to present our results. We will do so explicitly splitting them into different orders of m_{light}. The leading order contributions, as well as the divergent pieces (when present), will be provided here. However, one should keep in mind that this splitting in orders of m_{light} is performed based on the explicit appearance of m_{light} in the resulting analytical expressions. As explained above, a term that is a priori of order zero in m_{light} might actually be of order one if the specific shape of the U matrix introduces one power of the low scale. Therefore, the leading order contributions given below might be *polluted* with higher order terms, which can in general be safely neglected. This is no longer true if the order zero contributions vanish or turn out to be of order one due to cancellations. In this case, a consistent calculation requires the consideration of all order one contributions. For this reason, the expressions for higher order terms are given in appendix A.

First of all, the Z boson contribution can be written as

$$\mathcal{M}_Z = -\frac{1}{8\pi^2} \, \bar{v}_{\ell_\alpha} \gamma_5 \, \Gamma_Z^\alpha \, u_{\ell_\alpha} \,, \tag{2.43}$$

where

$$\Gamma_Z^\alpha = -i \, \frac{m_{\ell_\alpha}}{v^2} \, \text{Im}\Big\{ (\Gamma_Z)^{(\text{div})} + (\Gamma_Z)^{(0)} + \mathcal{O}(m_{\text{light}}) \Big\} \,, \tag{2.44}$$

with $\langle H^0 \rangle = \frac{v}{\sqrt{2}}$ the usual Higgs vacuum expectation value (VEV) that breaks the electroweak symmetry and

$$(\Gamma_Z)^{(\text{div})} = \sum_{s=1}^{3} \sum_{k,r} \delta_{ks} \, \bar{A}_{kr} \left[\bar{M}_{rk}^\dagger \left(\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{M_H^2} \right) - \frac{5}{3} \right) - M_H \sum_{j\sim h} U_{kj} U_{rj} \right] , \tag{2.45}$$

$$(\Gamma_Z)^{(0)} = \sum_{s=1}^{3} \left(\sum_{j\sim l} \frac{\tilde{\Gamma}_{ssj}^{1,0,0} + \Gamma_{ssj}^{1,0,0}}{6} - \sum_{j\sim h} \frac{\tilde{\Delta}_{ssj}^{0,1,-1} + \Delta_{ssj}^{0,1,-1}}{3} \right) . \tag{2.46}$$

The term $(\Gamma_Z)^{(\mathrm{div})}$ contains the dimensional regularization divergence $\frac{1}{\epsilon}$. It will only be relevant in models leading to $\delta_{ks}\,\bar{A}_{kr} \neq 0$. Since the index $s = 1, 2, 3$ runs over gauge non-singlet states, this will happen in models in which the majoron couples to non-singlet fermion representations and in this case one expects a majoron coupling to charged leptons already at tree-level. We have also included a finite piece in $(\Gamma_Z)^{(\mathrm{div})}$, the last term in eq. (2.45), simply because it is also proportional to $\delta_{ks}\,\bar{A}_{kr}$ and vanishes whenever the divergent piece does. Furthermore, the rest of the contributions can be generally written as

$$\mathcal{M}_B = \frac{1}{8\pi^2}\,\bar{v}_{\ell_\beta}\left(L_B^{\beta\alpha}\,P_L + R_B^{\beta\alpha}\,P_R\right)u_{\ell_\alpha}\,, \tag{2.47}$$

with $B = \{W, \eta, \rho, \sigma, X, Y, \sigma n, \sigma\ell, S\}$. The W-boson contributions are given by

$$L_W^{\beta\alpha} = \frac{2\,m_{\ell_\beta}}{v^2}\left\{\left(L_W^{\beta\alpha}\right)^{(0)} + \mathcal{O}(m_{\mathrm{light}})\right\}\,, \tag{2.48}$$

with

$$\left(L_W^{\beta\alpha}\right)^{(0)} = \sum_{j\sim l}\left(\frac{\Gamma^{1,0,0\,*}_{\alpha\beta j}}{12} + \frac{2}{3}\Gamma^{1,0,0}_{\beta\alpha j}\right) - \sum_{j\sim h}\left(\frac{\tilde{\Delta}^{0,1,-1\,*}_{\alpha\beta j}}{6} + \frac{7}{12}\tilde{\Delta}^{0,1,-1}_{\beta\alpha j}\right)\,. \tag{2.49}$$

The charged scalar contribution is given by

$$\begin{aligned} L_\eta^{\beta\alpha} = m_{\ell_\alpha}&\left[\bar{D}_L^{\beta p}\left(\bar{D}_L^{\alpha s}\right)^*\left(L_\eta^{LL}\right)^*_{sp} - \left(\bar{D}_L^{\alpha p}\right)^*\bar{D}_L^{\beta s}\left(\widetilde{L}_\eta^{LL}\right)_{sp}\right] \\ &+ m_{\ell_\beta}\left[\bar{D}_R^{\beta p}\left(\bar{D}_R^{\alpha s}\right)^*\left(L_\eta^{RR}\right)^*_{sp} - \left(\bar{D}_R^{\alpha p}\right)^*\bar{D}_R^{\beta s}\left(\widetilde{L}_\eta^{RR}\right)_{sp}\right] \\ &+ \bar{D}_L^{\beta p}\left(\bar{D}_R^{\alpha s}\right)^*\left(L_\eta^{RL}\right)^*_{sp}\,, \end{aligned} \tag{2.50}$$

where

$$L_\eta^{LL} = \left(L_\eta^{LL}\right)^{(0)} + \mathcal{O}(m_{\mathrm{light}})\,, \tag{2.51}$$

$$L_\eta^{RR} = \left(L_\eta^{RR}\right)^{(0)} + \mathcal{O}(m_{\mathrm{light}})\,, \tag{2.52}$$

$$L_\eta^{RL} = \left(L_\eta^{RL}\right)^{(0)} + \mathcal{O}(m_{\mathrm{light}})\,, \tag{2.53}$$

$$\widetilde{L}_\eta^{LL} = L_\eta^{LL}\left(f_{(1,2,5,6,7,8)} \leftrightarrow f_{(3,4,9,10,15,16)},\ f_{13} \leftrightarrow F_{1,-3},\ f_{14} \leftrightarrow F_{2,-4}\right)\,, \tag{2.54}$$

$$\widetilde{L}_\eta^{RR} = L_\eta^{RR}\left(f_{(1,2,5,6,7,8)} \leftrightarrow f_{(3,4,9,10,15,16)},\ f_{13} \leftrightarrow F_{1,-3},\ f_{14} \leftrightarrow F_{2,-4}\right)\,. \tag{2.55}$$

Here we have introduced the f and F loop functions. Their explicit expressions can be found in appendix B. We note that in the Z and W bosons contributions given above, no loop functions appeared. This is because $m_Z \sim m_W \sim m_{\mathrm{EW}}$ and we already expanded our results in powers of m_{EW}/M_H. However, in contributions involving η, ρ and σ a new mass scale appears, namely the mass of the new heavy scalar. Then, a complete expansion cannot be made without introducing further assumptions on their mass scale. As already explained above, our calculation assumes $m_\eta, m_\rho, m_\sigma \gg m_{\mathrm{light}}$ but, for the sake of generality, we prefer to stay agnostic about how these scalar masses compare to M_H. Furthermore, the notation $f_{(i_1,\ldots,i_n)} \leftrightarrow f_{(j_1,\ldots,j_n)}$ means that the replacement $f_{i_1} \leftrightarrow f_{j_1}, \ldots, f_{i_n} \leftrightarrow f_{j_n}$ is to be applied. We found

$$\left(L_\eta^{LL}\right)^{(0)} = \left(L_\eta^{RR}\right)^{(0)} \left(\Gamma \leftrightarrow \tilde{\Gamma}, \Delta \leftrightarrow \tilde{\Delta}\right) , \tag{2.56}$$

$$\begin{aligned}\left(L_\eta^{RR}\right)^{(0)}_{sp} &= f_7 \sum_{j\sim h} \Delta^{0,1,-1}_{spj} + \sum_j \left\{ f_8\, \Delta^{0,1,1}_{spj} - F_{5,7}\, \tilde{\Gamma}^{1,0,0}_{spj} - F_{6,8}\, \tilde{\Gamma}^{1,1,0}_{spj} \right\} \\ &\quad + \sum_{j\sim l} \left\{ F_{5,7,-1}\, \tilde{\Gamma}^{1,0,0}_{spj} + F_{6,8,-2}\, \tilde{\Gamma}^{1,1,0}_{spj} \right\} ,\end{aligned} \tag{2.57}$$

$$\begin{aligned}\left(L_\eta^{RL}\right)^{(0)}_{sp} &= \gamma^*_{sp} \frac{m^2_{\ell_\beta} + m^2_{\ell_\alpha}}{2m_\eta^2} - \gamma_{sp} \frac{m_{\ell_\beta} m_{\ell_\alpha}}{2m_\eta^2} + f_{11} \sum_{j\sim h} \Delta^{1,1,-1}_{spj} \\ &\quad + \sum_j \left[-f_{10} \left(\tilde{\Gamma}^{0,2,0}_{psj}\right)^* - f_9 \left(\tilde{\Gamma}^{0,1,0}_{psj}\right)^* - F_{3,11}\, \tilde{\Delta}^{1,0,1}_{spj} + f_{12}\, \Delta^{1,1,1}_{spj} - F_{4,12}\, \tilde{\Delta}^{1,1,1}_{spj} \right] \\ &\quad + \sum_{j\sim l} \left[F_{9,-1} \left(\tilde{\Gamma}^{0,1,0}_{psj}\right)^* + F_{10,-2} \left(\tilde{\Gamma}^{0,2,0}_{psj}\right)^* + F_{3,-9} \left(\Gamma^{0,1,0}_{psj}\right)^* + F_{4,-10} \left(\Gamma^{0,2,0}_{psj}\right)^* \right] .\end{aligned} \tag{2.58}$$

Let us note that we have included a term proportional not to the mass of neutrinos but to the square of the mass of charged leptons in the zeroth order expressions. Although this term is not explicitly proportional to m_{light}, we expect it to be subdominant, simply because $m_\ell \ll M_H$. However, in the event that both remaining terms of order zero and order one were to cancel out, then this term would dominate over a possible order two, as $m_{\text{light}} \ll m_\ell$.

We move on to the contribution induced by the neutral scalar ρ, which can be written as

$$L_\rho^{\beta\alpha} = \frac{1}{2m_\rho^2} \left[\left(L_\rho^{\beta\alpha}\right)^{(\text{div})} + \left(L_\rho^{\beta\alpha}\right)^{(0)} \right] , \tag{2.59}$$

with

$$\left(L_\rho^{\beta\alpha}\right)^{(\text{div})} = -\frac{1}{2} \left[\left(m^2_{\ell_\beta} + m^2_{\ell_\alpha}\right) + 2m_\rho^2 \left(1 + \frac{1}{\epsilon} + \log \frac{\mu^2}{m_\rho^2}\right) \right] (GKG)_{\beta\alpha} , \tag{2.60}$$

$$\begin{aligned}\left(L_\rho^{\beta\alpha}\right)^{(0)} &= m_{\ell_\beta} \left[\left(G^\dagger \left(\mathbb{I}_3 + \log \frac{M_\ell^2}{m_\rho^2} \right) M_\ell K G \right)_{\beta\alpha} - \text{h.c.} \right] - \left(G \log \frac{M_\ell^2}{m_\rho^2} M_\ell^2 K G \right)_{\beta\alpha} \\ &\quad + \frac{m_{\ell_\alpha} m_{\ell_\beta}}{2} \left(G^\dagger K G^\dagger\right)_{\beta\alpha} .\end{aligned} \tag{2.61}$$

where $\mathbb{I}_n$ is the $n \times n$ identity matrix. Again, we have included a finite piece in $\left(L_\rho^{\beta\alpha}\right)^{(\text{div})}$ because it vanishes whenever the divergent piece does. A similar triangle diagram contribution is induced by the pseudoscalar σ, with

$$L_\sigma^{\beta\alpha} = L_\rho^{\beta\alpha} \left(G \leftrightarrow C, m_\rho \leftrightarrow m_\sigma\right) . \tag{2.62}$$

The scalar leptoquark contribution is given by

$$L_X^{\beta\alpha} = \frac{1}{2M_X^2} \left[\left(L_X^{\beta\alpha}\right)^{(\text{div})} + \left(L_X^{\beta\alpha}\right)^{(0)} \right] , \tag{2.63}$$

with

$$\left(L_X^{\beta\alpha}\right)^{(\mathrm{div})} = -\frac{1}{2}\left[\left(m_{\ell_\beta}^2 + m_{\ell_\alpha}^2\right) + 2M_X^2\left(1+\frac{1}{\epsilon}+\log\frac{\mu^2}{M_X^2}\right)\right]\left(T_L I T_R^\dagger\right)_{\beta\alpha}, \tag{2.64}$$

$$\begin{aligned}\left(L_X^{\beta\alpha}\right)^{(0)} &= m_{\ell_\beta}\left[\left(T_R\left(\mathbb{I}_3+\log\frac{M_\ell^2}{M_X^2}\right)M_\ell I T_R^\dagger\right)_{\beta\alpha} - \mathrm{h.c.}\right] - \left(T_L\log\frac{M_\ell^2}{M_X^2}M_\ell^2 I T_R^\dagger\right)_{\beta\alpha}\\ &+ \frac{m_{\ell_\alpha}m_{\ell_\beta}}{2}\left(T_R I T_L^\dagger\right)_{\beta\alpha}.\end{aligned} \tag{2.65}$$

In the same way, we find for the vector leptoquark contribution

$$L_Y^{\beta\alpha} = \frac{1}{M_Y^2}\left[\left(L_Y^{\beta\alpha}\right)^{(\mathrm{div})} + \left(L_Y^{\beta\alpha}\right)^{(0)}\right], \tag{2.66}$$

where

$$\begin{aligned}\left(L_Y^{\beta\alpha}\right)^{(\mathrm{div})} &= \frac{1}{2}\Big[M_\ell\, H_L\, M_Q I\, H_L^\dagger + H_R\, M_Q I\, H_R^\dagger\, M_\ell + 2M_\ell H_L I H_R^\dagger M_\ell\\ &+ 2H_R\left(M_Q^2 - 3M_Y\mathbb{I}_3\right)IH_L^\dagger\Big]_{\beta\alpha}\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{M_Y^2}\right)\end{aligned} \tag{2.67}$$

and

$$\begin{aligned}\left(L_Y^{\beta\alpha}\right)^{(0)} &= \frac{1}{4}M_\ell H_L M_Q\left(8\log\frac{M_Y^2}{M_Q^2} - 7\,\mathbb{I}_3\right)IH_L^\dagger + \frac{1}{4}H_R M_Q\left(8\log\frac{M_Y^2}{M_Q^2} - 7\,\mathbb{I}_3\right)IH_R^\dagger M_\ell\\ &+ M_\ell H_L I H_R^\dagger M_\ell + H_R\left(M_Q^2 + 4M_Q^2\log\frac{M_Y^2}{M_Q^2} - M_Y^2\mathbb{I}_3\right)IH_L^\dagger\end{aligned} \tag{2.68}$$

Note that the expressions in eqs. (2.59), (2.62) , (2.63) and (2.66) are exactly of order zero (no expansion has been made). This is easy to understand since there are no neutrinos in the associated loops. Lastly, the σn and $\sigma\ell$ contributions are given by

$$L_{\sigma n}^{\beta\alpha} = \frac{1}{m_\sigma^2}C_{\beta\alpha}\left[(L_{\sigma n})^{(\mathrm{div})} + (L_{\sigma n})^{(0)} + \mathcal{O}(m_{\mathrm{light}})\right], \tag{2.69}$$

with

$$(L_{\sigma n})^{(\mathrm{div})} = \mathrm{Re}\left\{\sum_j\left[\bar{B}_{sp}\Delta_{psj}^{1,0,1} + \bar{B}_{sp}^*\left(\Gamma_{spj}^{0,1,0} + \tilde{\Gamma}_{spj}^{0,1,0}\right)\right]\left(3+2\,\frac{1}{\epsilon}+2\,\log\frac{\mu^2}{M_H^2}\right)\right\}, \tag{2.70}$$

$$\begin{aligned}(L_{\sigma n})^{(0)} = \mathrm{Re}\Bigg\{&\sum_j 2\bar{B}_{sp}^*\left[\tilde{\Gamma}_{spj}^{0,1,0} - \Gamma_{spj}^{0,1,0} - \frac{1}{M_H^2}\left(\tilde{\Gamma}_{spj}^{0,2,0} + \tilde{\Gamma}_{spj}^{0,1,2}\right)\right] - \frac{2}{M_H^2}\bar{B}_{sp}\tilde{\Delta}_{psj}^{1,1,1}\\ &- \sum_{j\sim l}[\bar{B}_{sp}^*\left(\frac{2}{M_H^2}\Gamma_{spj}^{0,2,0} - 3\Gamma_{spj}^{0,1,0} + \tilde{\Gamma}_{spj}^{0,1,0}\right) - \sum_{j\sim h}\bar{B}_{sp}\Delta_{psj}^{1,1,-1}\Bigg\},\end{aligned} \tag{2.71}$$

and

$$L_{\sigma\ell}^{\beta\alpha} = -\frac{1}{m_\sigma^2}C_{\beta\alpha}\,\mathrm{Tr}\left[(L_{\sigma\ell})^{(\mathrm{div})} + (L_{\sigma\ell})^{(0)}\right], \tag{2.72}$$

with

$$(L_{\sigma\ell})^{(\mathrm{div})} = K M_\ell^2 C^\dagger \left(I + \frac{1}{\epsilon} \right) \tag{2.73}$$

$$(L_{\sigma\ell})^{(0)} = K M_\ell^2 \log \frac{\mu^2}{M_\ell^2} C^\dagger \,. \tag{2.74}$$

The cubic scalar interaction contribution leads to

$$L_S^{\beta\alpha} = \frac{\omega}{4\left(m_\rho^2 - m_\sigma^2\right)^2} \left(L_{\sigma\rho}^{\beta\alpha} \right)^{(0)} \,, \tag{2.75}$$

with

$$\begin{aligned}\left(L_S^{\beta\alpha}\right)^{(0)} = &\left(G C^\dagger M_\ell\right)_{\beta\alpha} \left(-m_\rho^2 + m_\sigma^2 + m_\rho^2 \log \frac{m_\rho^2}{m_\sigma^2} \right) - \left(M_\ell G^\dagger C\right)_{\beta\alpha} \left(-m_\rho^2 + m_\sigma^2 + m_\sigma^2 \log \frac{m_\rho^2}{m_\sigma^2} \right) \\ &-2 (G M_\ell C) \left(m_\rho^2 - m_\sigma^2\right) \log \frac{m_\rho^2}{m_\sigma^2} \,. \end{aligned} \tag{2.76}$$

Again, the last three expressions are exactly of order zero in m_{light} since there are no neutrinos in the loop. We also note that only the real parts contribute in eqs. (2.70) and (2.71). To conclude, the right chirality couplings are given by

$$R_B^{\beta\alpha} = L_B^{\alpha\beta\,*} \,. \tag{2.77}$$

Finally, the majoron coupling to a pair of charged leptons, defined in eq. (2.1), can be written in terms of the results presented in this section as

$$S^{\beta\alpha} = \frac{1}{8\pi^2} \left(\delta^{\beta\alpha} \, \Gamma_Z^\alpha + \sum_B L_B^{\beta\alpha} \right) \,. \tag{2.78}$$

Some comments are in order. First of all, we emphasize that these expressions are very long because they are meant to be completely general. In specific models, most of the terms will simply vanish or take very simple forms, as we will show explicitly in section 3. Secondly, while most contributions are perfectly finite, some include divergent pieces. As already explained, these only appear in contributions that would necessarily lead to the existence of a majoron coupling to a pair of charged leptons already at tree-level. In this case they are expected to induce just small (finite) corrections, but we included them for the sake of completeness.

3 Application to specific models

In this section we show several example models and illustrate how our general results for the majoron coupling to charged leptons apply to these specific cases. We consider several neutrino mass models, some of them very well known. Instead of breaking lepton number explicitly, as usually done, we promote it to a global conserved $\mathrm{U}(1)_L$ symmetry that gets spontaneously broken by the vacuum expectation value (VEV) of a scalar χ. This implies the presence of a majoron in the particle spectrum of the theory.

field	spin	generations	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_L$
N	$\frac{1}{2}$	3	**1**	**1**	0	1
χ	0	1	**1**	**1**	0	-2

Table 1. New particles in the type-I seesaw with spontaneous lepton number violation.

3.1 Type-I seesaw

The type-I seesaw with spontaneous lepton number violation [2] extends the SM particle content with 3 singlet fermions N and a scalar singlet χ, all charged under the global $\mathrm{U}(1)_L$ as shown in table 1. The Yukawa terms relevant for our discussion are

$$-\mathcal{L} = y\,\bar{L}\tilde{H}N + \frac{1}{2}\lambda\,\chi\,\bar{N}^c N + \text{h.c.}\,. \tag{3.1}$$

Here H is the SM Higgs doublet, with $\tilde{H} = i\sigma_2 H^*$, y a general 3×3 matrix and λ a symmetric 3×3 matrix. We will work in the basis in which λ is a diagonal matrix with real entries. The χ singlet can be decomposed as

$$\chi = \frac{1}{\sqrt{2}}\left(v_\chi + \rho + iJ\right)\,, \tag{3.2}$$

where $\langle\chi\rangle = \frac{v_\chi}{\sqrt{2}}$ is the χ VEV responsible for the breaking of $\mathrm{U}(1)_L$, ρ is a massive CP-even scalar and J is the massless majoron, the Goldstone boson associated to the spontaneous violation of lepton number. After symmetry breaking, a Dirac mass term, M_D, that mixes the left-handed neutrinos in the lepton doublets with the singlet neutrinos, as well as Majorana mass term for the singlet neutrinos, M_R, are induced. They are given by

$$M_D = y\,\frac{v}{\sqrt{2}}\,,\qquad M_R = \lambda\,\frac{v_\chi}{\sqrt{2}}\,. \tag{3.3}$$

In the basis $\{\nu_L^c, N\}$, the resulting 6×6 Majorana mass matrix for the neutral fermions can be written as

$$\bar{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}\,. \tag{3.4}$$

If one assumes the hierarchy $M_D \ll M_R$ (equivalent to $m_{\mathrm{EW}} \ll M_H$ in eq. (2.35)), the light neutrinos mass matrix is given by $m_\nu = -M_D\,M_R^{-1}\,M_D^T$, hence leading to naturally suppressed masses. This is the usual type-I seesaw mechanism [31–35].

Since ρ does not couple to the charged leptons at tree-level, the only diagrams that induce a majoron coupling to charged leptons are those with gauge bosons, shown in figure 1(a) and 1(b). Therefore, we only need to adapt the general results for Γ_Z and L_W to the type-I seesaw. We first identify the general couplings of eqs. (2.18)–(2.23) that are involved in these contributions. In this model, the majoron coupling to a pair a neutral fermions is given in the gauge basis by

$$\bar{A}_{kr} = \begin{cases} \dfrac{i}{2v_\chi}\,\bar{M}_{kr}\,, & \text{if } k,r = 4,5,6 \\[2ex] 0\,, & \text{otherwise} \end{cases} \tag{3.5}$$

The A, E and F couplings in the mass basis are then given by eqs. (2.18), (2.22) and (2.23), although they are not necessary for our computation. First, we note that $(\Gamma_Z)^{(\text{div})}$ vanishes exactly since $\delta_{ks}\bar{A}_{kr} = 0$ for $s = 1, 2, 3$. Then, we just need to compute $\sum_{j\sim l}\tilde{\Gamma}^{1,0,0}_{\beta\alpha j}$, $\sum_{j\sim l}\Gamma^{1,0,0}_{\beta\alpha j}$, $\sum_{j\sim h}\Delta^{0,1,-1}_{\beta\alpha j}$ and $\sum_{j\sim h}\tilde{\Delta}^{0,1,-1}_{\beta\alpha j}$. In order to do that we need to know the form of the U matrix. In the type-I seesaw this unitary matrix can be written as [36]

$$U = \begin{pmatrix} U_l & 0 \\ 0 & U_h \end{pmatrix} \begin{pmatrix} \sqrt{\mathbb{I}_3 - PP^\dagger} & P \\ -P^\dagger & \sqrt{\mathbb{I}_3 - P^\dagger P} \end{pmatrix} \equiv U_2\, U_1\,. \tag{3.6}$$

Here U_1 brings the neutral fermions mass matrix into a block-diagonal form, while U_2 finally diagonalizes, independently, the light and heavy sectors of the matrix. U_l, U_h and P are 3×3 matrices. The matrix P depends on the matrices M_D and M_R in a non-trivial way. However, one can expand P in inverse powers of the large M_R scale as

$$P = \sum_{i=1}^{\infty} P_i\,, \tag{3.7}$$

with $P_i \propto M_R^{-i}$. At leading order in M_R^{-1}, one finds

$$\sqrt{\mathbb{I}_3 - PP^\dagger} = \sqrt{\mathbb{I}_3 - P^\dagger P} = \mathbb{I}_3 + \mathcal{O}(M_R^{-2})\,, \tag{3.8}$$

and

$$P = P_1 + \mathcal{O}(M_R^{-3}) = M_D^*\, M_R^{-1} + \mathcal{O}(M_R^{-3})\,. \tag{3.9}$$

With these expressions at hand it is straightforward to expand the sums in eqs. (2.38)–(2.41) and find

$$\begin{aligned}
\sum_{j\sim l}\Gamma^{1,0,0}_{\beta\alpha j} &= \frac{i}{2v_\chi}\sum_{k,r=4}^{6}\sum_{j=1}^{3}\bar{M}_{kr}\bar{M}^\dagger_{\alpha k}U_{rj}U^*_{\beta j} \\
&\simeq \frac{i}{2v_\chi}\left(-\mathbb{I}_3\, M_D M_R^{-1} M_R M_D^\dagger\right)_{\beta\alpha} = -\frac{i}{2v_\chi}\left(M_D M_D^\dagger\right)_{\beta\alpha}\,, && (3.10)\\
\sum_{j\sim l}\tilde{\Gamma}^{1,0,0}_{\beta\alpha j} &= \frac{i}{2v_\chi}\sum_{k,r=4}^{6}\sum_{j=1}^{3}\bar{M}_{kr}\bar{M}^\dagger_{\beta r}U_{kj}U^*_{\alpha j} \\
&\simeq \frac{i}{2v_\chi}\left(-M_D^* M_R M_R^{-1} M_D^T\, \mathbb{I}_3\right)_{\beta\alpha} = -\frac{i}{2v_\chi}\left(M_D M_D^\dagger\right)_{\alpha\beta}\,, && (3.11)\\
\sum_{j\sim h}\Delta^{0,1,-1}_{\beta\alpha j} &= \frac{i}{2v_\chi}\sum_{k,r=4}^{6}\sum_{j=4}^{6}\bar{M}_{kr}\left(\bar{M}\bar{M}^\dagger\right)_{\alpha k}U_{rj}m_j^{-1}U_{\beta j} \\
&\simeq \frac{i}{2v_\chi}\left(M_D^* M_R^{-1} M_R^{-1} M_R M_R M_D^T\right)_{\beta\alpha} = \frac{i}{2v_\chi}\left(M_D M_D^\dagger\right)_{\alpha\beta}\,, && (3.12)\\
\sum_{j\sim h}\tilde{\Delta}^{0,1,-1}_{\beta\alpha j} &= \frac{i}{2v_\chi}\sum_{k,r=4}^{6}\sum_{j=4}^{6}\bar{M}_{kr}\left(\bar{M}\bar{M}^\dagger\right)_{\beta r}U_{kj}m_j^{-1}U_{\alpha j} \\
&\simeq \frac{i}{2v_\chi}\left(M_D M_R M_R M_R^{-1} M_R^{-1} M_D^\dagger\right)_{\beta\alpha} = \frac{i}{2v_\chi}\left(M_D M_D^\dagger\right)_{\beta\alpha}\,. && (3.13)
\end{aligned}$$

field	spin	generations	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_L$
N	$\frac{1}{2}$	3	**1**	**1**	0	-1
S	$\frac{1}{2}$	3	**1**	**1**	0	1
χ	0	1	**1**	**1**	0	-2

Table 2. New particles in the inverse seesaw with spontaneous lepton number violation.

At this point we just need to combine all these results into a final expression for the majoron coupling to charged leptons. For the Z boson contribution we make use of eq. (2.46) to obtain

$$(\Gamma_Z)^{(0)} = -\frac{i}{2v_\chi}\,\mathrm{Tr}(M_D M_D^\dagger)\,, \tag{3.14}$$

whereas for the W boson contribution, using eq. (2.49), we find

$$\left(L_W^{\beta\alpha}\right)^{(0)} = -\frac{i}{2v_\chi}\left(M_D M_D^\dagger\right)_{\beta\alpha}\,. \tag{3.15}$$

Finally, replacing these contributions into eqs. (2.44) and (2.48), and using the generic Lagrangian in eqs. (2.1) and the expression for the coupling S in eq. (2.78), we recover the known formula for the majoron coupling to charged leptons in the type-I seesaw [15–17],

$$\mathcal{L}_{\ell\ell J} = \frac{iJ}{16\pi^2 v^2 v_\chi}\bar{\ell}\left[M_\ell\,\mathrm{Tr}(M_D M_D^\dagger)\gamma_5 + 2M_\ell M_D M_D^\dagger P_L - 2M_D M_D^\dagger M_\ell P_R\right]\ell\,. \tag{3.16}$$

The *natural* size of these couplings can be readily estimated from this expression. The coupling S in eq. (2.78) is estimated as

$$S_{\text{Type-I}} \sim \frac{M_\ell\, M_D^2}{16\pi^2\, v^2\, v_\chi} \sim \frac{M_\ell\, m_\nu}{16\pi^2\, v^2}\,, \tag{3.17}$$

where we have used that $m_\nu \sim M_D^2/M_R$ and $M_R \sim v_\chi$ in this model. The resulting coupling is strongly suppressed, not only by the loop factor, but also by charged lepton and neutrino masses. For instance, with $M_\ell \sim m_\mu$ and $m_\nu \sim 0.1\,\mathrm{eV}$, one finds $S_{\text{Type-I}} \sim 10^{-18}$. This generic result clearly respects the bounds in section 2 (eqs. (2.3), (2.4), (2.7) and (2.8)). In fact, using eq. (2.6) one finds branching ratios as low as $\mathrm{BR}(\mu \to e\,J) \sim 10^{-20}$, well below any foreseen experimental search.

In summary, in the type-I seesaw one generally expects tiny majoron couplings to the charged leptons. The only way out of this conclusion is to take advantage of the fact that neutrino masses and the majoron coupling to charged leptons actually depend on a different product of matrices ($M_D M_R^{-1} M_D^T$ versus $M_D M_D^\dagger/v_\chi$). Therefore, one can in principle evade the generic expectation of eq. (3.17) by adopting specific textures for the y Yukawa matrix.

3.2 Inverse seesaw

The inverse seesaw with spontaneous lepton number violation [37] introduces the new fields in table 2. In this case, in addition to the usual 3 N fermion singlets, 3 S fermion singlets are included too, with opposite lepton number. The relevant Yukawa Lagrangian terms are given by

$$-\mathcal{L} = y\,\bar{L}\tilde{H}N + M_R\,\bar{N}^c S + \frac{1}{2}\,\lambda'\,\chi^*\bar{N}^c N + \frac{1}{2}\,\lambda\,\chi\bar{S}^c S + \text{h.c.}\,. \tag{3.18}$$

We note the presence of two Yukawa terms involving χ, with couplings λ' and λ, two symmetric 3×3 matrices. Moreover, M_R is a general 3×3 matrix with dimensions of mass. However, we will work in the basis in which M_R is a diagonal matrix with real entries. This choice is particularly convenient for our computation, since M_R will be taken below as expansion parameter. Again, the singlet χ can be decomposed as in eq. (3.2), including a massless majoron, J. Furthermore, symmetry breaking leads to a Dirac mass term, M_D, as in the type-I seesaw, as well as to Majorana mass terms for the singlet neutrinoss, μ' and μ, given by

$$M_D = y\,\frac{v}{\sqrt{2}}\,, \quad \mu' = \lambda'\,\frac{v_\chi}{\sqrt{2}}\,, \quad \mu = \lambda\,\frac{v_\chi}{\sqrt{2}}\,. \tag{3.19}$$

In the basis $\{\nu_L^c, N, S^c\}$, the 9×9 Majorana mass matrix for the neutral fermions can be written as

$$\bar{M} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & \mu' & M_R \\ 0 & M_R & \mu \end{pmatrix} \equiv \begin{pmatrix} 0 & m_D \\ m_D^T & Q \end{pmatrix}\,, \tag{3.20}$$

with

$$Q = \begin{pmatrix} \mu' & M_R \\ M_R & \mu \end{pmatrix}\,, \tag{3.21}$$

and

$$m_D = \begin{pmatrix} M_D & 0 \end{pmatrix}\,. \tag{3.22}$$

If one assumes the hierarchy $\mu, \mu' \ll M_D \ll M_R$, the light neutrinos mass matrix is given by $m_\nu = M_D M_R^{-1} \mu M_R^{-1} M_D^T$, hence leading to naturally small neutrino masses. We note that the μ' parameter does not contribute to neutrino masses at leading order in the expansion.

In this model, just as in the type-I seesaw, the only diagrams contributing to the majoron coupling to charged leptons are those with gauge bosons, shown in figure 1(a) and 1(b). Then, we only need to compute Γ_Z and L_W at leading order. In order to do that we must identify the general couplings of eqs. (2.18)–(2.23) that participate in these contributions. In the gauge basis, the majoron coupling to a pair a neutral fermions is given by

$$\bar{A}_{kr} = \begin{cases} -\dfrac{i}{2v_\chi}\,\bar{M}_{kr}\,, & \text{if } k,r = 4,5,6 \\[2ex] \dfrac{i}{2v_\chi}\,\bar{M}_{kr}\,, & \text{if } k,r = 7,8,9 \\[2ex] 0\,, & \text{otherwise} \end{cases} \tag{3.23}$$

The A, E and F couplings in the mass basis can be readily obtained with the help of eqs. (2.18), (2.22) and (2.23). Again, the divergent piece $(\Gamma_Z)^{(\mathrm{div})}$ vanishes exactly due to $\delta_{ks}\bar{A}_{kr}=0$ for $s=1,2,3$. Therefore, we must compute $\sum_{j\sim l}\tilde{\Gamma}^{1,0,0}_{\beta\alpha j}$, $\sum_{j\sim l}\Gamma^{1,0,0}_{\beta\alpha j}$, $\sum_{j\sim h}\Delta^{0,1,-1}_{\beta\alpha j}$ and $\sum_{j\sim h}\tilde{\Delta}^{0,1,-1}_{\beta\alpha j}$. In order to do that we need the form of the U matrix, which in this model is a 9×9 unitary matrix. When the mass matrix $\bar{M}$ in eq. (3.20) is written in terms of the matrices Q and m_D in eqs. (3.21) and (3.22), one obtains an expression that is formally equivalent to that in the type-I seesaw, see eq. (3.4). Therefore, U takes the same form, namely

$$U=\begin{pmatrix} U_l & 0 \\ 0 & U_h \end{pmatrix}\begin{pmatrix} \sqrt{\mathbb{I}_3-PP^\dagger} & P \\ -P^\dagger & \sqrt{\mathbb{I}_6-P^\dagger P} \end{pmatrix}\equiv U_2\,U_1\,, \tag{3.24}$$

with U_l and U_h two 3×3 and 6×6 matrices, respectively. P is a 3×6 matrix that can be expanded in inverse powers of the large M_R scale as in eq. (3.21). At leading order one finds

$$\sqrt{\mathbb{I}_3-PP^\dagger}=\mathbb{I}_3+\mathcal{O}(M_R^{-2})\,, \tag{3.25}$$

$$\sqrt{\mathbb{I}_6-P^\dagger P}=\mathbb{I}_6+\mathcal{O}(M_R^{-2})\,, \tag{3.26}$$

and

$$P=P_1+\mathcal{O}(M_R^{-3})=m_D^*\left(Q^{-1}\right)^\dagger+\mathcal{O}(M_R^{-3})\,. \tag{3.27}$$

With these expressions, one can readily find

$$\begin{aligned}\sum_{j\sim l}\Gamma^{1,0,0}_{\beta\alpha j}&=\frac{i}{2v_\chi}\sum_{j=1}^{3}\left(\sum_{k,r=7}^{9}-\sum_{k,r=4}^{6}\right)\bar{M}_{kr}\left(\bar{M}^\dagger\right)_{\alpha k}U_{rj}U^*_{\beta j}\\ &\simeq-\frac{i}{2v_\chi}\left(M_DM_R^{-1}\mu M_R^{-1}\mu' M_D^\dagger\right)_{\beta\alpha},\end{aligned} \tag{3.28}$$

$$\begin{aligned}\sum_{j\sim l}\tilde{\Gamma}^{1,0,0}_{\beta\alpha j}&=\frac{i}{2v_\chi}\sum_{j=1}^{3}\left(\sum_{k,r=7}^{9}-\sum_{k,r=4}^{6}\right)\bar{M}_{kr}\left(\bar{M}^\dagger\right)_{\beta r}U_{kj}U^*_{\alpha j}\\ &\simeq-\frac{i}{2v_\chi}\left(M_D^*\mu' M_R^{-1}\mu M_R^{-1}M_D^T\right)_{\beta\alpha},\end{aligned} \tag{3.29}$$

$$\begin{aligned}\sum_{j\sim h}\Delta^{0,1,-1}_{\beta\alpha j}&=\frac{i}{2v_\chi}\sum_{j=4}^{9}\left(\sum_{k,r=7}^{9}-\sum_{k,r=4}^{6}\right)\left(\bar{M}\bar{M}^\dagger\right)_{\alpha k}U_{rj}m_j^{-1}U_{\beta j}\\ &\simeq-\frac{i}{2v_\chi}\left(M_D^*M_R^{-1}\mu^\dagger M_R^{-2}\mu M_RM_D^T+M_D^*M_R^{-2}\mu'\mu'^\dagger M_D^T+M_D^*M_R^{-2}\mu' M_R^{-1}\mu M_D^T\right)_{\beta\alpha},\end{aligned} \tag{3.30}$$

$$\begin{aligned}\sum_{j\sim h}\tilde{\Delta}^{0,1,-1}_{\beta\alpha j}&=\frac{i}{2v_\chi}\sum_{j=4}^{9}\left(\sum_{k,r=7}^{9}-\sum_{k,r=4}^{6}\right)\bar{M}_{kr}\left(\bar{M}\bar{M}^\dagger\right)_{\beta r}U_{kj}m_j^{-1}U_{\alpha j}\\ &\simeq\frac{-i}{2v_\chi}\left(M_DM_R\mu M_R^{-2}\mu^\dagger M_R^{-1}M_D^\dagger+M_D\mu'^\dagger\mu' M_R^{-2}M_D^\dagger+M_DM_R\mu M_R^{-1}\mu' M_R^{-2}M_D^\dagger\right)_{\beta\alpha}.\end{aligned} \tag{3.31}$$

Now we just need to introduce these results into the expressions for the leading order Z and W boson contributions. For the Z boson contribution we use eq. (2.46) to obtain

$$(\Gamma_Z)^{(0)}=\frac{i}{6v_\chi}\,\mathrm{Tr}\left(M_D\left[\mu\mu'+2\mu\mu^\dagger+2\mu'^\dagger\mu'\right]M_R^{-2}M_D^\dagger\right)\,, \tag{3.32}$$

JHEP01(2024)078

whereas for the W boson contribution, using eq. (2.49), we find

$$\left(L_W^{\beta\alpha}\right)^{(0)} = \frac{i}{24 v_\chi} \left(M_D \left[-\mu\mu' - \mu'^\dagger \mu^\dagger + 5\mu\mu^\dagger + 5\mu'\mu'^\dagger \right] M_R^{-2} M_D^\dagger \right)_{\beta\alpha} . \tag{3.33}$$

Finally, replacing these contributions into eqs. (2.44) and (2.48), and using the generic expressions in eqs. (2.1) and (2.78) we can write the coupling of the majoron to a pair of charged leptons in the inverse seesaw as

$$\begin{aligned} \mathcal{L}_{J\ell\ell} = -\frac{iJ}{96\pi^2 v^2 v_\chi} \bar{\ell} \Big\{ & 2M_\ell \operatorname{Tr}\left(M_D \left[\mu\mu' + 2\mu\mu^\dagger + 2\mu'^\dagger \mu' \right] M_R^{-2} M_D^\dagger \right) \gamma_5 \\ & + M_\ell\, M_D \left(5\mu\mu^\dagger + 5\mu'\mu'^\dagger - \mu\mu' - \mu'^\dagger \mu^\dagger \right) M_R^{-2} M_D^\dagger\, P_L \\ & - M_D \left(5\mu\mu^\dagger + 5\mu'\mu'^\dagger - \mu'^\dagger\mu^\dagger - \mu\mu' \right) M_R^{-2} M_D^\dagger\, M_\ell\, P_R \Big\} \ell . \end{aligned} \tag{3.34}$$

In the case of $\mu' = 0$ (or, equivalently, $\lambda' = 0$), an assumption that is often made in the literature, this expression reduces to

$$\begin{aligned} \mathcal{L}_{J\ell\ell} = -\frac{iJ}{96\pi^2 v^2 v_\chi} \bar{\ell} \Big\{ & 4M_\ell \operatorname{Tr}\left(M_D\, \mu\mu^\dagger M_R^{-2} M_D^\dagger \right) \gamma_5 \\ & + 5M_\ell\, M_D\, \mu\mu^\dagger M_R^{-2} M_D^\dagger\, P_L - 5M_D\, \mu\mu^\dagger M_R^{-2} M_D^\dagger\, M_\ell\, P_R \Big\} \ell . \end{aligned} \tag{3.35}$$

To the best of our knowledge, this is the first time that this coupling is computed in the context of the inverse seesaw. We can now proceed in the same way as in the type-I seesaw and estimate its size in the inverse seesaw. In this case, and assuming $\mu' = 0$ for simplicity, the coupling S can be estimated as

$$S_{\text{ISS}} \sim \frac{M_\ell\, M_D^2\, \mu^2}{96\pi^2\, v^2\, v_\chi\, M_R^2} \sim \frac{M_\ell\, \mu\, m_\nu}{96\pi^2\, v^2\, v_\chi} \sim \frac{M_\ell\, m_\nu}{96\pi^2\, v^2} , \tag{3.36}$$

where we have used that $m_\nu \sim M_D^2 \mu / M_R^2$ and $\mu \sim v_\chi$ in this model. The resulting coupling is again very strongly suppressed. In fact, one finds the same suppression factors as in the type-I seesaw (see eq. (3.17)) and thus expects $S_{\text{ISS}} \sim S_{\text{Type-I}}$. One could in principle increase S_{ISS} by introducing a large non-vanishing μ' (or λ'), since this coupling does not affect neutrino masses at leading order. However, it does enter subleadingly, see for instance [38]. Therefore, μ' cannot be increased indefinitely.

We conclude that in the version of the inverse seesaw with spontaneous lepton number violation considered here one also expects tiny majoron couplings to the charged leptons. Again, as in the type-I seesaw, one can evade this conclusion by properly tuning the Yukawa matrices. This may be used to break the generic expectation in eq. (3.36). Furthermore, this conclusion may not hold in alternative versions of the inverse seesaw with spontaneous lepton number violation [39, 40].

3.3 Scotogenic model

In the Scotogenic model [41, 42] with spontaneous lepton number violation, the particle spectrum is extended with 3 fermion singlets, N, a scalar doublet, η, and a scalar singlet,

field	spin	generations	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_L$	$\mathbb{Z}_2$
N	$\frac{1}{2}$	3	**1**	**1**	0	1	-
η	0	1	**1**	**2**	$\frac{1}{2}$	0	-
χ	0	1	**1**	**1**	0	-2	+

Table 3. New particles in the Scotogenic model with spontaneous lepton number violation.

χ. A new $\mathbb{Z}_2$ parity is introduced as well, under which N and η are odd while the rest of the fields in the model are even. The fields and their charges under the symmetries of the model are given in table 3. The terms of the Yukawa Lagrangian that are relevant for our discussion are given by

$$-\mathcal{L} = y\,\bar{L}\tilde{\eta}N + \frac{1}{2}\,\lambda\,\chi\bar{N}^c N + \text{h.c.}\,. \tag{3.37}$$

with $\tilde{\eta} = i\sigma_2\eta^*$. The doublet η is assumed to have a vanishing VEV, $\langle\eta\rangle = 0$ in order to preserve the $\mathbb{Z}_2$ symmetry. This ensures that the lightest $\mathbb{Z}_2$-odd state is completely stable. The singlet χ can be decomposed as in eq. (3.2), giving rise to a massless majoron, J. The breaking of $\mathrm{U}(1)_L$ induces a Majorana mass term for the singlet fermions, M_N, defined as

$$M_N = \lambda\,\frac{v_\chi}{\sqrt{2}}\,. \tag{3.38}$$

Note, however, that the $\mathbb{Z}_2$ symmetry precludes any Dirac mass term mixing the SM neutrinos with the fermion singlets. This in turn implies that neutrinos remain massless at tree-level.[4] In fact, in the basis $\{\nu_L^c, N\}$, the tree-level 6×6 Majorana mass matrix for the neutral fermions can be written as

$$\bar{M} = \begin{pmatrix} 0 & 0 \\ 0 & M_N \end{pmatrix}\,, \tag{3.39}$$

which trivially leads to $U = \mathbb{I}_6$.

In the Scotogenic model, the majoron interacts at tree-level only with the N fermion singlets, which are mass eigenstates since the $\nu_L - N$ mixing is exactly zero. For this reason, the gauge bosons contributions to the majoron coupling to charged leptons are absent in this model. The only 1-loop contribution is given by the η charged scalar, as shown in figure 1(c). We just need to compute L_η, given in eq. (2.50). The relevant couplings in the gauge basis are

$$\bar{A}_{kr} = \begin{cases} i\dfrac{M_N}{2v_\chi}\,\delta_{kr}\,, & \text{if } k,r = 4,5,6 \\[2ex] 0\,, & \text{otherwise} \end{cases} \tag{3.40}$$

[4] In the Scotogenic model, neutrinos get non-zero masses at the 1-loop level. This is due to the breaking of lepton number in two units by the χ VEV. In this work we are interested in the 1-loop coupling of the majoron to charged leptons. Any effect on this coupling associated to the 1-loop neutrino mass would be of higher order in pertubation theory, hence not relevant for our discussion.

and

$$D_L^{\alpha i} = 0\,, \tag{3.41}$$

$$D_R^{\alpha i} = -\sum_{k=4}^{6} y^{\alpha k}\,\delta_{ki}\,. \tag{3.42}$$

The couplings in the mass basis can be computed using eqs. (2.18), (2.20) and (2.21). Since $D_L^{\alpha i} = 0$, we only need to compute $\left(L_\eta^{RR}\right)^{(0)}$. This contribution involves the sums over light states $\sum_{j\sim l}\tilde{\Gamma}_{spj}^{1,0,0}$ and $\sum_{j\sim l}\tilde{\Gamma}_{spj}^{1,1,0}$, but these are trivially zero, since they both involve the product $\bar{A}_{kr}U_{kj} = \bar{A}_{kr}\delta_{kj} = 0$ for $j = 1,2,3$. Therefore, we are left with $\sum_{j\sim h}\Delta_{spj}^{0,1,-1}$, $\sum_j \Delta_{spj}^{0,1,1}$, $\sum_j \tilde{\Gamma}_{spj}^{1,0,0}$ and $\sum_j \tilde{\Gamma}_{spj}^{1,1,0}$. They are found to be

$$\sum_{j\sim h}\Delta_{spj}^{0,1,-1} = i\frac{M_N}{2v_\chi}\sum_{k,r=4}^{6}\delta_{kr}\left(\bar{M}\bar{M}^\dagger\right)_{pk}\delta_{rj}\,m_j^{-1}\,\delta_{sj} = i\frac{M_N^2}{2v_\chi}\,\delta_{sp}\,, \tag{3.43}$$

$$\sum_{j}\Delta_{spj}^{0,1,1} = i\frac{M_N}{2v_\chi}\sum_{k,r=4}^{6}\delta_{kr}\left(\bar{M}\bar{M}^\dagger\right)_{pk}\delta_{rj}\,m_j^{1}\,\delta_{sj} = i\frac{M_N^4}{2v_\chi}\,\delta_{sp}\,, \tag{3.44}$$

$$\sum_{j}\tilde{\Gamma}_{spj}^{1,0,0} = i\frac{M_N}{2v_\chi}\sum_{k,r=4}^{6}\left(\bar{M}^\dagger\right)_{sr}\delta_{kj}\,\delta_{pj} = i\frac{M_N^2}{2v_\chi}\,\delta_{sp}\,, \tag{3.45}$$

$$\sum_{j}\tilde{\Gamma}_{spj}^{1,1,0} = i\frac{M_N}{2v_\chi}\sum_{k,r=4}^{6}\left(\bar{M}^\dagger\bar{M}\bar{M}^\dagger\right)_{sr}\delta_{kj}\,\delta_{pj} = i\frac{M_N^4}{2v_\chi}\,\delta_{sp}\,. \tag{3.46}$$

With these results one can directly obtain from (2.57) and (2.55)

$$\left(L_\eta^{RR}\right)^{(0)}_{sp} = -\frac{i}{2v_\chi}\left(f_5\,M_N^2 + f_6\,M_N^4\right)\delta_{sp}\,,\qquad \left(\tilde{L}_\eta^{RR}\right)^{(0)}_{sp} = -\frac{i}{2v_\chi}\left(f_9\,M_N^2 + f_{10}\,M_N^4\right)\delta_{sp}\,. \tag{3.47}$$

Using the f_i and $F_{i,j,\dots,n}$ loop functions defined in appendix B, we find

$$(L_\eta)^{\beta\alpha} = i\,M_\ell\,y\,\frac{M_N^2}{2v_\chi\left(M_N^2 - m_\eta^2\right)^2}\left(M_N^2 - m_\eta^2 + m_\eta^2\log\frac{m_\eta^2}{M_N^2}\right)y^\dagger \tag{3.48}$$

and we recover the known result of [43, 44][5]

$$-\mathcal{L}_{J\ell\ell} = \frac{iJ}{16\pi^2 v_\chi}\bar{\ell}\left(M_\ell\,y\,\Theta\,y^\dagger\,P_L - y\,\Theta\,y^\dagger\,M_\ell\,P_R\right)\ell\,, \tag{3.49}$$

with

$$\Theta_{sp} \equiv \frac{M_N^2}{\left(M_N^2 - m_\eta^2\right)^2}\left(M_N^2 - m_\eta^2 + m_\eta^2\log\frac{m_\eta^2}{M_N^2}\right)\delta_{sp}\,. \tag{3.50}$$

It is important to emphasize here that calculating this coupling in this general form in this model is an overkill. In this model, as explained above, the majoron only interacts with the fermion singlets and the coupling can be taken to be diagonal without loss of generality. This

[5]The definition of the y Yukawa matrix in [44] differs from ours by a complex conjugation.

field	spin	generations	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_L$
N	$\frac{1}{2}$	3	$\mathbf{1}$	$\mathbf{1}$	0	1
χ	0	1	$\mathbf{1}$	$\mathbf{1}$	0	-2
H_2	0	1	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	0

Table 4. New particles in the type-I seesaw with two Higgs doublets and spontaneous lepton number violation.

fact implies that the resulting loop integral is straightforward, since the light neutrinos do not enter the loop. Therefore, if we were to perform the explicit calculation, we would obtain a simple integral that would directly yield the reproduced result. In other words, we would not need to carry out any expansion. Hence, the fact that our calculation, after considering the most general possible model and carrying out all expansions while accounting for all possibilities, returns the correct result is a very strong validation of our outcome.

Finally, one can estimate the size of the majoron couplings to charged leptons in the Scotogenic model. In this model, the coupling S can be estimated as

$$S_{\text{Scot}} \sim \frac{M_\ell \, y^2}{16\pi^2 \, v_\chi} \sim \frac{2 \, M_\ell \, m_\nu}{\lambda_5 \, v^2} \sim \frac{32\pi^2}{\lambda_5} \, S_{\text{Type-I}} \,, \tag{3.51}$$

where we have used that $m_\nu \sim \lambda_5 \, v^2 y^2/(32\pi^2 M_N)$, with λ_5 the coupling of the quartic scalar term $\left(H^\dagger \eta\right)^2$, and $M_N \sim v_\chi$ in this model. We have also assumed that all loop functions are of $\mathcal{O}(1)$ and have compared to the result for the type-I seesaw in eq. (3.17). We find a large enhancement compared to the tiny value obtained in the type-I seesaw. In fact, a relatively small λ_5 coupling would naturally lead to the violation of the bounds in section 2. For instance, the current limit on $\text{BR}(\mu \to e \, J) \sim 10^{-17}/\lambda_5^2$ would be violated for $\lambda_5 \lesssim 10^{-6}$. This result confirms previous findings in the literature, which have already shown a very rich majoron phenomenology in the Scotogenic model. We refer to [44] for a detailed exploration of the phenomenology of the $\ell_\alpha \to \ell_\beta \, J$ decays in this model.

3.4 Type-I seesaw with two Higgs doublets

In order to better illustrate our results, we now consider a non-minimal model: the type-I seesaw with two Higgs doublets and spontaneous lepton number violation. This model extends the Two Higgs Doublet Model (2HDM) particle content with 3 singlet fermions N and a scalar singlet χ, all charged under the global $\mathrm{U}(1)_L$ as shown in table 4, just as the type-I seesaw discussed in section 3.1. In addition, this model adds a second Higgs doublet, H_2. In the following we assume that we are working in the Higgs basis, in which H_2 does not acquire a VEV. The Yukawa terms relevant for our discussion are

$$-\mathcal{L} = Y \, \bar{L} H e_R + Y_2 \, \bar{L} H_2 e_R + y \, \bar{L} \tilde{H} N + y_2 \, \bar{L} \tilde{H}_2 N + \frac{1}{2} \, \lambda \, \chi \, \bar{N}^c N + \text{h.c.} \,. \tag{3.52}$$

Since we work in the Higgs basis, M_D and M_R are given by the same expressions as in eq. (3.3). Similarly, the charged leptons mass matrix is

$$M_\ell = Y \, \frac{v}{\sqrt{2}} \,. \tag{3.53}$$

Furthermore, Y_2 and y_2 are two general 3×3 matrices. Neutrino mass generation takes place in exactly the same way as in the minimal type-I seesaw and the same holds true for the majoron. Consequently, the gauge diagrams remain identical to those we computed previously in section 3.1. However, as in any 2HDM, the presence of a second Higgs doublet implies the existence of physical charged and CP-odd scalars. Following the same notation as in section 2, they will be denoted as η and σ. These states induce the Feynman diagrams with amplitudes $\mathcal{M}_\eta$ and $\mathcal{M}_{\sigma n}$ shown in figures 1(c) and 1(d). In order to compute them, we first identify the new couplings, not present in section 3.1, defined in eqs. (2.18)–(2.23). These are

$$\bar{B}_{kr} = \begin{cases} i\,(y_2)_{kr}\ , \text{ if } k = 1,2,3 \text{ and } r = 4,5,6 \\ \\ 0\,, \qquad \text{otherwise} \end{cases} \tag{3.54}$$

$$C^{\alpha\beta} = -\,(Y_2)^*_{\beta\alpha}\ , \tag{3.55}$$

$$\bar{D}_L^{\alpha k} = \begin{cases} (Y_2)_{\alpha k}\ , \text{ if } k = 1,2,3 \\ \\ 0\,, \qquad \text{otherwise} \end{cases} \tag{3.56}$$

$$\bar{D}_R^{\alpha k} = \begin{cases} (y_2)_{\alpha k}\ , \text{ if } k = 4,5,6 \\ \\ 0\,, \qquad \text{otherwise} \end{cases} \tag{3.57}$$

In our computation we will assume that $m_\eta, m_\sigma \ll M_H$. Let us start with the computation of the charged scalar contribution. We have to obtain expressions for L_η^{RR}, $\tilde{L}_\eta^{RR}$, L_η^{LL}, $\tilde{L}_\eta^{LL}$ and L_η^{RL}. We will start by computing the first order of L_η^{RR} and $\tilde{L}_\eta^{RR}$. One can easily find

$$\sum_j \Delta^{0,1,1}_{spj} = \sum_j \tilde{\Gamma}^{1,1,0}_{spj} = \frac{i}{2v_\chi} M_R^4\,\delta_{sp} + \mathcal{O}(M_R^2)\,, \tag{3.58}$$

$$\sum_{j\sim h} \Delta^{1,1,-1}_{spj} = \sum_j \tilde{\Gamma}^{1,0,0}_{spj} = \frac{i}{2v_\chi} M_R^2\,\delta_{sp} + \mathcal{O}(M_R^0)\,, \tag{3.59}$$

while other terms are of lower order in M_R and hence subdominant. With these results one can directly obtain

$$\left(L_\eta^{RR}\right)^{(0)}_{sp} = -\frac{i}{2v_\chi}\left(f_5\,M_R^2 + f_6\,M_R^4\right)\,\delta_{sp}\,, \quad \left(\tilde{L}_\eta^{RR}\right)^{(0)}_{sp} = -\frac{i}{2v_\chi}\left(f_9\,M_R^2 + f_{10}\,M_R^4\right)\,\delta_{sp}\,. \tag{3.60}$$

It is now useful to notice that $f_i \sim M_H^{-2}$ for i odd and $f_i \sim M_H^{-4}$ for i even. Therefore, the two terms in the previous expressions are of the same order in M_R. In fact, making use of the expressions for the f_i and $F_{i,j,\ldots,n}$ loop functions in appendix B, and using $m_\eta \ll M_H$, we find

$$\left(L_\eta^{RR}\right)^{(0)}_{sp} = \frac{-3i}{8v_\chi}\,\delta_{sp}\,, \quad \left(\tilde{L}_\eta^{RR}\right)^{(0)}_{sp} = \frac{-i}{8v_\chi}\,\delta_{sp}\,, \tag{3.61}$$

where we have used that $\frac{(M_R)_{sp}}{M_H} = \delta_{sp}$ at first order. Our result is easy to understand: since both the majoron and the charged scalar couple directly to the singlets N, which are essentially the heavy neutrinos, there is no mixing in the loop (no mass suppression) at leading order. For this reason, one expects L_η^{LL} and $\tilde{L}_\eta^{LL}$ to be subdominant, as two mixings are required in the loop. This fact is reflected in our results and an explicit calculation confirms this reasoning. Moreover, one could be tempted to make a similar argument for L_η^{RL}, which requires one mixing. However, we must be careful with this term because it is not proportional to the charged lepton masses in eq. (2.51). In fact, the introduction of M_D is equivalent to the charged lepton mass, since $M_D \sim M_\ell$. Let us see it explicitly. One can easily find

$$-\sum_j \left(\tilde{\Gamma}^{0,2,0}_{psj}\right)^* = \sum_j \tilde{\Delta}^{1,1,1}_{spj} = \sum_j \Delta^{1,1,1}_{spj} = \sum_{j\sim l} \left(\Gamma^{0,2,0}_{psj}\right)^* = \frac{i}{2v_\chi}\left(M_R^4 M_D^\dagger\right)_{s-3,p} + \mathcal{O}(M_R^2), \tag{3.62}$$

$$\sum_{j\sim h} \Delta^{0,1,-1}_{spj} = \sum_j \tilde{\Delta}^{1,0,1}_{spj} = -\sum_j \left(\tilde{\Gamma}^{0,1,0}_{psj}\right)^* = \sum_{j\sim l} \left(\Gamma^{0,1,0}_{psj}\right)^* = \frac{i}{2v_\chi}\left(M_R^2 M_D^\dagger\right)_{s-3,p} + \mathcal{O}(M_R^0), \tag{3.63}$$

while other terms are of lower order in M_R and can be neglected. With these results,

$$\left(L_\eta^{RL}\right)^{(0)}_{sp} = -2\frac{i}{2v_\chi}\left[\left(f_3\, M_R^2 + f_4\, M_R^4\right) M_D^\dagger\right]_{s-3,\,p}, \tag{3.64}$$

and, using the expressions for f_i and $F_{i,j,\ldots,n}$, we obtain

$$\left(L_\eta^{RL}\right)^{(0)}_{sp} = -\frac{i}{2v_\chi}\left(M_D^\dagger\right)_{s-3,\,p}. \tag{3.65}$$

And, with both results, finally,

$$(L_\eta)^{\beta\alpha} = \frac{i}{2v_\chi}\left(M_\ell\, y_2\, y_2^\dagger + Y_2\, M_D\, y_2^\dagger\right)_{\beta\alpha}. \tag{3.66}$$

Let us continue with the CP-odd scalar contribution, $\mathcal{M}_{\sigma n}$. First, we note that the divergent piece vanishes, as expected. This is because only the real part contributes, see eq. (2.70), and $(L_{\sigma n})^{(\mathrm{div})}$ turns out to be purely imaginary. To show it we compute $\sum_j \Delta^{1,0,1}_{psj}$, $\sum_j \Gamma^{0,1,0}_{spj}$ and $\sum_j \tilde{\Gamma}^{0,1,0}_{spj}$,

$$\sum_j \Delta^{1,0,1}_{psj} = \frac{i}{2v_\chi}\left(M_R^2 M_D^\dagger\right)_{p-3,\,s}, \tag{3.67}$$

$$\sum_j \tilde{\Gamma}^{0,1,0}_{spj} = \frac{i}{2v_\chi}\left(M_D M_R^2\right)_{p-3,\,s}, \tag{3.68}$$

$$\sum_j \Gamma^{0,1,0}_{spj} = 0. \tag{3.69}$$

Therefore,

$$(L_{\sigma n})^{(\mathrm{div})} = \mathrm{Re}\left\{-\frac{1}{2v_\chi}\left[\mathrm{Tr}\left(y_2 M_R^2 M_D^\dagger\right) - \mathrm{Tr}\left(y_2^* M_R^2 M_D^T\right)\right]\left(3 + 2\,\frac{1}{\epsilon} + 2\,\log\frac{\mu^2}{M_H^2}\right)\right\} = 0. \tag{3.70}$$

We then consider the finite piece. We have already computed several terms required for the evaluation of L_η^{RL}. With these results (and a few more), one can show that $L_{\sigma n}^{(0)}$ vanishes for the same reason as for the divergent piece. This means that the leading order contribution from the σ scalar is, at most, of order one. This includes the explicit order one contribution given in appendix A as well as the *hidden* order one contribution in $L_{\sigma n}^{(0)}$. However, one can show that this order also cancels out, so $L_{\sigma n} = \mathcal{O}(M_R^{-2})$, and in this way, we can safely neglect this contribution. In summary, the majoron coupling to a pair charged leptons in the type-I seesaw with two Higgs doublets is given by

$$\mathcal{L}_{\ell\ell J} = \frac{iJ}{16\pi^2 v_\chi}\bar{\ell}\left\{ M_\ell \operatorname{Tr}\left(\frac{M_D M_D^\dagger}{v^2}\right)\gamma_5 + 2\left(M_\ell \frac{M_D M_D^\dagger}{v^2} - M_\ell\, y_2\, y_2^\dagger - Y_2\, M_D\, y_2^\dagger\right) P_L \right.$$
$$\left. - 2\left(\frac{M_D M_D^\dagger}{v^2} M_\ell - y_2\, y_2^\dagger\, M_\ell - y_2\, M_D^\dagger\, Y_2^\dagger\right) P_R \right\}\ell\,. \tag{3.71}$$

Let us now study the size of the coupling S in this model. It can be split as

$$S_{\rm Type-I+2HDM} = S_{\rm Type-I} + S_{\rm 2HDM}\,, \tag{3.72}$$

where $S_{\rm Type-I}$ is given in eq. (3.17) and $S_{\rm 2HDM}$ contains the new terms associated to our 2HDM version of the model, not present in the type-I seesaw (with one Higgs doublet). The latter can be estimated as

$$S_{\rm 2HDM} \sim \frac{1}{16\pi^2\, v_\chi}\left(M_\ell\, y_2^2 + y_2\, Y_2\, M_D\right) \sim \frac{1}{16\pi^2\, v^2\, v_\chi}\left(M_\ell\, M_D'^2 + M_\ell'\, M_D\, M_D'\right)\,, \tag{3.73}$$

where we have defined M_ℓ' and M_D' analogously to M_ℓ and M_D, simply replacing Y and y by Y_2 and y_2. This allows us to obtain the ratio

$$\frac{S_{\rm 2HDM}}{S_{\rm Type-I}} = \left(\frac{M_D'}{M_D}\right)^2 + \frac{M_\ell'\, M_D'}{M_\ell\, M_D}\,. \tag{3.74}$$

This result hints at an enhancement of the majoron coupling to charged leptons compared to the type-I seesaw if $M_D' \gg M_D$ and/or $M_\ell' \gg M_\ell$. Let us suppose $M_\ell' = 0$ (or, equivalently, $Y_2 = 0$) and consider $M_D' \gg M_D$ (or, equivalently, $y_2 \gg y$). One should note that y_2 does not participate in the type-I seesaw contribution to neutrino masses, but it does induce a Scotogenic-like 1-loop contribution, $m_\nu^{\rm 1-loop} \sim \lambda_5\, v^2 y_2^2/(32\pi^2 M_R)$.[6] Imposing $m_\nu^{\rm 1-loop} \lesssim m_\nu \equiv m_\nu^{\rm tree}$ then leads to

$$\frac{\lambda_5}{32\pi^2}\left(\frac{y_2}{y}\right)^2 \lesssim 1\,. \tag{3.75}$$

Assuming now $y \sim 10^{-6}$, the typical size of the Yukawa couplings for a type-I seesaw at the TeV scale, eq. (3.75) implies $\lambda_5\, y_2^2 \lesssim 10^{-10}$. This is fulfilled for $y_2 \sim 1$ if $\lambda_5 \lesssim 10^{-10}$. In this case, $M_D'/M_D \sim 10^6$ and one expects an enhancement of the majoron couplings of about 12 orders of magnitude with respect to the type-I seesaw. Such a huge increase would indeed lead to conflict with the bounds in eqs. (2.3), (2.4), (2.7) and (2.8). However, less pronounced

[6] In the absence of mixing between the CP-even scalars, λ_5 is the coupling of the quartic scalar term $\left(H^\dagger H_2\right)^2$. Otherwise, if the CP-even components of H^0 and H_2^0 mix, λ_5 is a combination of scalar potential parameters associated to this mixing.

JHEP01(2024)078

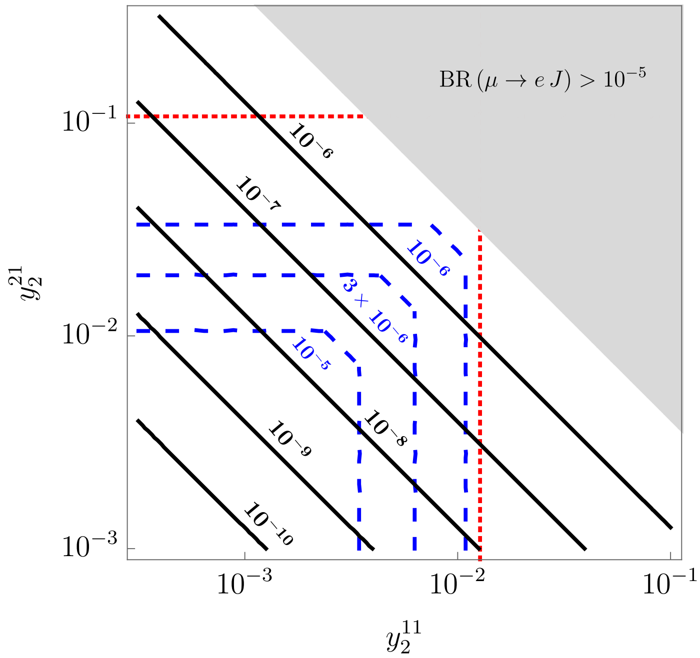


Figure 2. Contours of BR$(\mu \to e\, J)$ (black) and $\lambda_5^{\max}$ (blue, dashed) in the y_2^{11}-y_2^{21} plane. The rest of y_2 Yukawa couplings are set to zero for simplicity, while $\lambda = 1$ and $v_\chi = 1\,$TeV. Here $\lambda_5^{\max}$ is the maximal value of λ_5 compatible with all the entries in $m_\nu^{1-\text{loop}}$ being smaller than those in m_ν^{tree}. The shaded region is excluded by $\mu \to e\, J$ searches while the vertical and horizontal red dotted lines determine the upper limits on y_2^{11} and y_2^{21}, respectively, derived from eqs. (2.3) and (2.4).

hierarchies lead to observable effects, for instance in $\ell_\alpha \to \ell_\beta\, J$ decays, in agreement with the current constraints. This is shown in figure 2, which displays contours of BR$(\mu \to e\, J)$ and $\lambda_5^{\max}$ in the y_2^{11}-y_2^{21} plane. Here $\lambda_5^{\max}$ is the maximal value of λ_5 compatible with all the entries in $m_\nu^{1-\text{loop}}$ being smaller than those in m_ν^{tree}. The rest of y_2 Yukawa couplings are set to zero for simplicity, while $\lambda = 1$ and $v_\chi = 5\,$TeV. This plot clearly shows that the model can achieve large $\mu \to e\, J$ braching ratios, almost as large as the current bound, in the region of parameter space that has been considered. In conclusion, the extension of the type-I seesaw with a second Higgs doublet induces novel contributions to the majoron couplings to charged leptons that may increase them notably and lead to observable consequences.

4 Final discussion

The majoron is present in any model that breaks a global lepton number symmetry spontaneously. In the absence of explicit sources of $\mathrm{U}(1)_L$ breaking, its mass is exactly zero. This has a strong impact on the phenomenology of the model, which changes dramatically with respect to the variant with explicit breaking. We have derived general analytical expressions for the 1-loop coupling of the majoron to charged leptons. These couplings are known to be crucial for the phenomenology of the model and, in fact, they often lead to the most stringent constraints. Our analytical results can be used in virtually all models featuring a majoron. We have provided several example models that illustrate how to use them. Due to our lack of imagination, we could not find examples for all possible contributions, but they can be

JHEP01(2024)078

present in some (less popular) models. In any case, we found perfect agreement with the results obtained in all models for which the majoron coupling to charged leptons was known. This is a clearly non-trivial cross-check of our analytical expressions.

The main limitation of our approach is the assumption of a hierarchy of mass scales, which is nevertheless common to most Majorana neutrino mass models, such as those based on the seesaw mechanism. We have also focused on scenarios that actually *require* the computation of the 1-loop majoron couplings to charged leptons. For instance, since we did not consider additional fermions in the particle spectrum, one may wonder about the applicability of our results in models with heavy leptons. While our analytical expressions cannot be directly applied in this case, we note that (i) the analogous expressions with heavy neutral leptons can be easily adapted, and, more importantly, (ii) models with heavy charged leptons induce majoron couplings with the light charged leptons already at tree-level, thus making our 1-loop results mere corrections. In conclusion, our analytical expressions are applicable to any model in which the 1-loop majoron couplings to charged leptons *must* be computed.

Our results go beyond the majoron. One can use them to obtain the 1-loop couplings of any massless (or very light) pseudoscalar to a pair of charged leptons. In fact, the majoron can be regarded as a particular and theoretically well-motivated type of axion-like particle (ALP). Whenever the assumptions of our approach are fulfilled, our results can also be applied to compute the 1-loop couplings of an ALP to a pair of charged leptons. Therefore, they may be of interest in scenarios not necessarily related to neutrino mass generation and lepton number violation.

The study of majorons and lepton number violation represents a fascinating frontier in particle physics, pushing the boundaries of our understanding of the fundamental building blocks of the Universe. Our contribution constitutes just another step in the ongoing quest to detect and characterize majorons. As we delve deeper into the mechanisms behind neutrino masses and lepton number violation, we not only expand our knowledge of the SM but also pave the way for potential breakthroughs that could revolutionize our understanding of particle interactions and the origins of mass.

Acknowledgments

The authors are grateful to Salvador Centelles Chuliá for discussions. Work supported by the Spanish grants PID2020-113775GB-I00 (AEI/10.13039/501100011033) and CIPROM/2021/054 (Generalitat Valenciana). The work of AHB is supported by the grant No. CIACIF/2021/100 (also funded by Generalitat Valenciana). AV acknowledges financial support from MINECO through the Ramón y Cajal contract RYC2018-025795-I.

A Higher order terms

We present here our results for the contributions of higher order in m_{light} not included in section 2, using exactly the same notation and conventions:

$$(\Gamma_Z)^{(1)} = -\sum_{s=1}^{3} \sum_{i,j\sim l} \bar{A}_{kr} U_{ki} U_{rj} \log\left(\frac{m_j^2}{M_H^2}\right) \left(U_{si} m_i U_{sj}^* + U_{sj} m_j U_{si}^*\right) , \tag{A.1}$$

$$\left(L_W^{\beta\alpha}\right)^{(1)} = \sum_{i,j\sim l} \bar{A}^*_{kr} U^*_{\beta i} m_i U^*_{ki} U_{\alpha j} U^*_{rj} \left[-\frac{5}{6} + 3\log\left(\frac{M_H}{M_W}\right) + 2\log\left(\frac{m_i}{M_H}\right)\right]$$
$$- \sum_{i,j\sim l} \bar{A}_{kr} U_{\alpha i} m_i U_{ki} U^*_{\beta j} U_{rj} \left[\frac{7}{6} + 2\log\left(\frac{m_i}{M_H}\right)\right], \tag{A.2}$$

$$\left(L_\eta^{LL}\right)^{(1)} = \left(L_\eta^{RR}\right)^{(1)} (m_i \leftrightarrow m_j, k \leftrightarrow r, s \leftrightarrow p), \tag{A.3}$$

$$\left(L_\eta^{RR}\right)^{(1)}_{sp} = \sum_{j\sim l} \bar{A}_{kr} U_{rj} m_j U_{sj} \Bigg[(F_{5,7,-1} - f_{13})\,\delta_{kp} - (F_{8,2} + f_{14})\left(\bar{M}^\dagger \bar{M}\right)_{kp}$$
$$+ \sum_{i\sim l} U_{ki} U^*_{pi} \frac{\log \frac{m_j}{m_\eta}}{m_\eta^2} \Bigg], \tag{A.4}$$

$$\left(L_\eta^{RL}\right)^{(1)}_{sp} = \sum_{j\sim l} \left[F_{3,11,-1}\left(\tilde{\Delta}^{1,0,1}_{sp} + \Delta^{1,0,1}_{sp}\right) + F_{4,12,-2}\,\tilde{\Delta}^{1,1,1}_{sp} + F_{12,-2}\,\Delta^{1,1,1}_{sp}\right], \tag{A.5}$$

$$(L_{\sigma n})^{(1)} = \mathrm{Re}\left\{\sum_{j\sim l}\left[\bar{B}_{sp}\left(-\frac{2}{M_H^2}\Delta^{1,1,1}_{ps} + \Delta^{1,0,1}_{ps} + \tilde{\Delta}^{1,0,1}_{ps}\right)\right]\right\}, \tag{A.6}$$

$$(L_{\sigma n})^{(2)} = \mathrm{Re}\left\{\sum_{i,j\sim l} \bar{A}_{kr} U_{rj} U_{ki}\left[\bar{B}_{sp} U_{pj} U_{si} m_j m_i + \bar{B}^*_{sp} U^*_{sj} U^*_{pi}\left(m_i^2 + m_j^2\right)\right]\log\frac{M_H^2}{m_j^2}\right\}. \tag{A.7}$$

B Loop functions

We define the following loop functions, which turn out to be relevant for the charged η scalar contributions:

$$f_1 = \frac{-M_H^2 + m_\eta^2 + (2M_H^2 - m_\eta^2)\log\left(\frac{M_H^2}{m_\eta^2}\right)}{2(M_H^2 - m_\eta^2)^2}, \tag{B.1}$$

$$f_2 = -\frac{-M_H^2 + m_\eta^2 + M_H^2\log\left(\frac{M_H^2}{m_\eta^2}\right)}{2M_H^2(M_H^2 - m_\eta^2)^2}, \tag{B.2}$$

$$f_3 = \frac{2M_H^2(M_H^2 - m_\eta^2) + (-3M_H^2 m_\eta^2 + m_\eta^4)\log\left(\frac{M_H^2}{m_\eta^2}\right)}{2(M_H^2 - m_\eta^2)^3}, \tag{B.3}$$

$$f_4 = \frac{-M_H^4 + m_\eta^4 + 2M_H^2 m_\eta^2 \log\left(\frac{M_H^2}{m_\eta^2}\right)}{2M_H^2(M_H^2 - m_\eta^2)^3}, \tag{B.4}$$

$$f_5 = \frac{6M_H^6 - 5M_H^4 m_\eta^2 - 2M_H^2 m_\eta^4 + m_\eta^6 - 2(6M_H^4 m_\eta^2 - 4M_H^2 m_\eta^4 + m_\eta^6)\log\left(\frac{M_H^2}{m_\eta^2}\right)}{4(M_H^2 - m_\eta^2)^4}, \tag{B.5}$$

$$f_6 = \frac{-3M_H^6 - 2M_H^4 m_\eta^2 + 7M_H^2 m_\eta^4 - 2m_\eta^6 + (8M_H^4 m_\eta^2 - 2M_H^2 m_\eta^4)\log\left(\frac{M_H^2}{m_\eta^2}\right)}{4M_H^2(M_H^2 - m_\eta^2)^4}, \tag{B.6}$$

$$f_7 = -\frac{-4M_H^8 - 19M_H^6 m_\eta^2 + 27M_H^4 m_\eta^4 - 5M_H^2 m_\eta^6 + m_\eta^8 + 6M_H^4 m_\eta^2(3M_H^2 + m_\eta^2)\log\left(\frac{M_H^2}{m_\eta^2}\right)}{6(M_H^2 - m_\eta^2)^5}, \tag{B.7}$$

$$f_8 = \frac{-M_H^6 - 9M_H^4 m_\eta^2 + 9M_H^2 m_\eta^4 + m_\eta^6 + 6M_H^2 m_\eta^2 (M_H^2 + m_\eta^2) \log\left(\frac{M_H^2}{m_\eta^2}\right)}{3(M_H^2 - m_\eta^2)^5}, \tag{B.8}$$

$$f_9 = \frac{2M_H^6 - 11M_H^4 m_\eta^2 + 10M_H^2 m_\eta^4 - m_\eta^6 + (8M_H^2 m_\eta^4 - 2m_\eta^6) \log\left(\frac{M_H^2}{m_\eta^2}\right)}{4(M_H^2 - m_\eta^2)^4}, \tag{B.9}$$

$$f_{10} = -\frac{M_H^6 - 6M_H^4 m_\eta^2 + 3M_H^2 m_\eta^4 + 2m_\eta^6 + 6M_H^2 m_\eta^4 \log\left(\frac{M_H^2}{m_\eta^2}\right)}{4M_H^2 (M_H^2 - m_\eta^2)^4}, \tag{B.10}$$

$$f_{11} = \frac{2M_H^6 + 3M_H^4 m_\eta^2 - 6M_H^2 m_\eta^4 + m_\eta^6 - 6M_H^4 m_\eta^2 \log\frac{M_H^2}{m_\eta^2}}{4\left(M_H^2 - m_\eta^2\right)^4}, \tag{B.11}$$

$$f_{12} = -\frac{M_H^4 + 4M_H^2 m_\eta^2 - 5m_\eta^4 - 4M_H^2 m_\eta^2 \log\frac{M_H^2}{m_\eta^2} - 2m_\eta^4 \log\frac{M_H^2}{m_\eta^2}}{4\left(M_H^2 - m_\eta^2\right)^4} \tag{B.12}$$

$$f_{13} = -\frac{3M_H^4 - 4M_H^2 m_\eta^2 + m_\eta^4 + 2M_H^4 \log\left(\frac{m_\eta^2}{M_H^2}\right)}{2(M_H^2 - m_\eta^2)^3}, \tag{B.13}$$

$$f_{14} = \frac{2(M_H^2 - m_\eta^2) + (M_H^2 + m_\eta^2) \log\left(\frac{m_\eta^2}{M_H^2}\right)}{2(M_H^2 - m_\eta^2)^3}, \tag{B.14}$$

$$f_{15} = \frac{M_H^8 - 8M_H^6 m_\eta^2 + 8M_H^2 m_\eta^6 - m_\eta^8 - 12M_H^4 m_\eta^4 \log\left(\frac{m_\eta^2}{M_H^2}\right)}{6\left(M_H^2 - m_\eta^2\right)^5}, \tag{B.15}$$

$$f_{16} = \frac{-M_H^6 + 9M_H^4 m_\eta^2 + 9M_H^2 m_\eta^4 - 17m_\eta^6 + 6(3M_H^2 m_\eta^4 + m_\eta^6) \log\left(\frac{m_\eta^2}{M_H^2}\right)}{12(M_H^2 - m_\eta^2)^5}. \tag{B.16}$$

In addition, we define

$$F_{i,j,k} \equiv \text{sign}(i) f_i + \text{sign}(j) f_j + \text{sign}(k) f_k \,. \tag{B.17}$$

The functions in this appendix take simplified expressions if $m_\eta \ll M_H$ and $m_\eta \gg M_H$. In the light η limit ($m_\eta \ll M_H$) they take the approximate expressions

$$f_1 = \frac{1}{2M_H^2} \left[-1 + 2 \log\left(\frac{M_H^2}{m_\eta^2}\right) \right], \tag{B.18}$$

$$f_2 = -\frac{1}{2M_H^4} \left[-1 + \log\left(\frac{M_H^2}{m_\eta^2}\right) \right], \tag{B.19}$$

$$f_{13} = -\frac{1}{2M_H^2} \left[3 - 2 \log\left(\frac{M_H^2}{m_\eta^2}\right) \right], \tag{B.20}$$

$$f_{14} = \frac{1}{2M_H^4} \left[2 - \log\left(\frac{M_H^2}{m_\eta^2}\right) \right], \tag{B.21}$$

$$f_3 = \frac{2}{3}f_5 = \frac{3}{2}f_7 = 2f_9 = 2f_{11} = 6f_{15} = \frac{1}{M_H^2}\,, \tag{B.22}$$

$$f_4 = \frac{2}{3}f_6 = \frac{3}{2}f_8 = 2f_{10} = 2f_{12} = 6f_{16} = -\frac{1}{2M_H^4}\,, \tag{B.23}$$

whereas in the heavy η limit ($m_\eta \gg M_H$) they can be approximately written as

$$f_1 = -\frac{1}{2m_\eta^2}\left[-1 + \log\left(\frac{M_H^2}{m_\eta^2}\right)\right], \tag{B.24}$$

$$f_3 = -\frac{1}{2m_\eta^2}\log\left(\frac{M_H^2}{m_\eta^2}\right), \tag{B.25}$$

$$f_5 = \frac{1}{4m_\eta^2}\left[1 - 2\log\left(\frac{M_H^2}{m_\eta^2}\right)\right], \tag{B.26}$$

$$f_9 = -\frac{1}{4m_\eta^2}\left[1 + 2\log\left(\frac{M_H^2}{m_\eta^2}\right)\right], \tag{B.27}$$

$$f_{12} = \frac{1}{4m_\eta^2 M_H^2}\left[5 + 2\log\left(\frac{M_H^2}{m_\eta^2}\right)\right], \tag{B.28}$$

$$f_{14} = -\frac{1}{2m_\eta^4}\left[-2 + \log\left(\frac{m_\eta^2}{M_H^2}\right)\right], \tag{B.29}$$

$$f_{16} = \frac{1}{12m_\eta^4}\left[17 - 6\log\left(\frac{m_\eta^2}{M_H^2}\right)\right], \tag{B.30}$$

$$f_2 = f_4 = f_6 = f_{10} = -\frac{1}{2m_\eta^2 M_H^2}\,, \tag{B.31}$$

$$3f_7 = 2f_{11} = f_{13} = 3f_{15} = \frac{1}{2m_\eta^2}\,, \tag{B.32}$$

$$f_8 = -\frac{1}{3m_\eta^4}\,. \tag{B.33}$$

References

[1] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, *Spontaneously broken lepton number and cosmological constraints on the neutrino mass spectrum*, *Phys. Rev. Lett.* **45** (1980) 1926 [INSPIRE].

[2] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, *Are there real Goldstone bosons associated with broken lepton number?*, *Phys. Lett. B* **98** (1981) 265 [INSPIRE].

[3] J. Schechter and J.W.F. Valle, *Neutrino decay and spontaneous violation of lepton number*, *Phys. Rev. D* **25** (1982) 774 [INSPIRE].

[4] G.B. Gelmini and M. Roncadelli, *Left-handed neutrino mass scale and spontaneously broken lepton number*, *Phys. Lett. B* **99** (1981) 411 [INSPIRE].

[5] C.S. Aulakh and R.N. Mohapatra, *Neutrino as the supersymmetric partner of the Majoron*, *Phys. Lett. B* **119** (1982) 136 [INSPIRE].

[6] P. Escribano and A. Vicente, *Ultralight scalars in leptonic observables*, *JHEP* **03** (2021) 240 [arXiv:2008.01099] [INSPIRE].

[7] A.S. Joshipura and J.W.F. Valle, *Invisible Higgs decays and neutrino physics*, *Nucl. Phys. B* **397** (1993) 105 [INSPIRE].

[8] D. Aristizabal Sierra, M. Tortola, J.W.F. Valle and A. Vicente, *Leptogenesis with a dynamical seesaw scale*, *JCAP* **07** (2014) 052 [arXiv:1405.4706] [INSPIRE].

[9] C. Bonilla et al., *Fermion dark matter and radiative neutrino masses from spontaneous lepton number breaking*, *New J. Phys.* **22** (2020) 033009 [arXiv:1908.04276] [INSPIRE].

[10] M. Escudero and S.J. Witte, *A CMB search for the neutrino mass mechanism and its relation to the Hubble tension*, *Eur. Phys. J. C* **80** (2020) 294 [arXiv:1909.04044] [INSPIRE].

[11] V. De Romeri, J. Nava, M. Puerta and A. Vicente, *Dark matter in the scotogenic model with spontaneous lepton number violation*, *Phys. Rev. D* **107** (2023) 095019 [arXiv:2210.07706] [INSPIRE].

[12] Y. Cheng, C.-W. Chiang, X.-G. He and J. Sun, *Flavor-changing Majoron interactions with leptons*, *Phys. Rev. D* **104** (2021) 013001 [arXiv:2012.15287] [INSPIRE].

[13] J. Sun, Y. Cheng and X.-G. He, *Structure of flavor changing Goldstone boson interactions*, *JHEP* **04** (2021) 141 [arXiv:2101.06055] [INSPIRE].

[14] P. Escribano, M. Hirsch, J. Nava and A. Vicente, *Observable flavor violation from spontaneous lepton number breaking*, *JHEP* **01** (2022) 098 [arXiv:2108.01101] [INSPIRE].

[15] A. Pilaftsis, *Astrophysical and terrestrial constraints on singlet Majoron models*, *Phys. Rev. D* **49** (1994) 2398 [hep-ph/9308258] [INSPIRE].

[16] C. Garcia-Cely and J. Heeck, *Neutrino lines from Majoron dark matter*, *JHEP* **05** (2017) 102 [arXiv:1701.07209] [INSPIRE].

[17] J. Heeck and H.H. Patel, *Majoron at two loops*, *Phys. Rev. D* **100** (2019) 095015 [arXiv:1909.02029] [INSPIRE].

[18] G. Ecker, *Vectorial versus axial Goldstone bosons*, *Z. Phys. C* **18** (1983) 173 [INSPIRE].

[19] L. Di Luzio, M. Giannotti, E. Nardi and L. Visinelli, *The landscape of QCD axion models*, *Phys. Rept.* **870** (2020) 1 [arXiv:2003.01100] [INSPIRE].

[20] R. Bollig, W. DeRocco, P.W. Graham and H.-T. Janka, *Muons in supernovae: implications for the axion-muon coupling*, *Phys. Rev. Lett.* **125** (2020) 051104 [*Erratum ibid.* **126** (2021) 189901] [arXiv:2005.07141] [INSPIRE].

[21] L. Calibbi, D. Redigolo, R. Ziegler and J. Zupan, *Looking forward to lepton-flavor-violating ALPs*, *JHEP* **09** (2021) 173 [arXiv:2006.04795] [INSPIRE].

[22] D. Croon, G. Elor, R.K. Leane and S.D. McDermott, *Supernova muons: new constraints on Z' bosons, axions and ALPs*, *JHEP* **01** (2021) 107 [arXiv:2006.13942] [INSPIRE].

[23] L. Di Luzio et al., *Solar axions cannot explain the XENON1T excess*, *Phys. Rev. Lett.* **125** (2020) 131804 [arXiv:2006.12487] [INSPIRE].

[24] A. Caputo, G. Raffelt and E. Vitagliano, *Muonic boson limits: supernova redux*, *Phys. Rev. D* **105** (2022) 035022 [arXiv:2109.03244] [INSPIRE].

JHEP01(2024)078

[25] A. Jodidio et al., *Search for right-handed currents in muon decay*, *Phys. Rev. D* **34** (1986) 1967 [*Erratum ibid.* **37** (1988) 237] [INSPIRE].

[26] M. Hirsch, A. Vicente, J. Meyer and W. Porod, *Majoron emission in muon and tau decays revisited*, *Phys. Rev. D* **79** (2009) 055023 [*Erratum ibid.* **79** (2009) 079901] [arXiv:0902.0525] [INSPIRE].

[27] BELLE-II collaboration, *Search for lepton-flavor-violating τ decays to a lepton and an invisible boson at Belle II*, *Phys. Rev. Lett.* **130** (2023) 181803 [arXiv:2212.03634] [INSPIRE].

[28] A. Pilaftsis, *Radiatively induced neutrino masses and large Higgs neutrino couplings in the standard model with Majorana fields*, *Z. Phys. C* **55** (1992) 275 [hep-ph/9901206] [INSPIRE].

[29] A. Pilaftsis, *Lepton flavor nonconservation in H^0 decays*, *Phys. Lett. B* **285** (1992) 68 [INSPIRE].

[30] H.H. Patel, *Package-X 2.0: a Mathematica package for the analytic calculation of one-loop integrals*, *Comput. Phys. Commun.* **218** (2017) 66 [arXiv:1612.00009] [INSPIRE].

[31] P. Minkowski, *$\mu \to e\gamma$ at a rate of one out of 10^9 muon decays?*, *Phys. Lett. B* **67** (1977) 421 [INSPIRE].

[32] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, *Conf. Proc. C* **7902131** (1979) 95 [INSPIRE].

[33] R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912 [INSPIRE].

[34] M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories*, *Conf. Proc. C* **790927** (1979) 315 [arXiv:1306.4669] [INSPIRE].

[35] J. Schechter and J.W.F. Valle, *Neutrino masses in* $SU(2) \times U(1)$ *theories*, *Phys. Rev. D* **22** (1980) 2227 [INSPIRE].

[36] W. Grimus and L. Lavoura, *The seesaw mechanism at arbitrary order: disentangling the small scale from the large scale*, *JHEP* **11** (2000) 042 [hep-ph/0008179] [INSPIRE].

[37] S. Centelles Chuliá, R. Srivastava and A. Vicente, *The inverse seesaw family: Dirac and Majorana*, *JHEP* **03** (2021) 248 [arXiv:2011.06609] [INSPIRE].

[38] A. Abada and M. Lucente, *Looking for the minimal inverse seesaw realisation*, *Nucl. Phys. B* **885** (2014) 651 [arXiv:1401.1507] [INSPIRE].

[39] S. Boulebnane, J. Heeck, A. Nguyen and D. Teresi, *Cold light dark matter in extended seesaw models*, *JCAP* **04** (2018) 006 [arXiv:1709.07283] [INSPIRE].

[40] A.J. Cuesta, M.E. Gómez, J.I. Illana and M. Masip, *Cosmology of an axion-like Majoron*, *JCAP* **04** (2022) 009 [arXiv:2109.07336] [INSPIRE].

[41] Z.-J. Tao, *Radiative seesaw mechanism at weak scale*, *Phys. Rev. D* **54** (1996) 5693 [hep-ph/9603309] [INSPIRE].

[42] E. Ma, *Verifiable radiative seesaw mechanism of neutrino mass and dark matter*, *Phys. Rev. D* **73** (2006) 077301 [hep-ph/0601225] [INSPIRE].

[43] K.S. Babu and E. Ma, *Singlet fermion dark matter and electroweak baryogenesis with radiative neutrino mass*, *Int. J. Mod. Phys. A* **23** (2008) 1813 [arXiv:0708.3790] [INSPIRE].

[44] P. Escribano and A. Vicente, *An ultraviolet completion for the Scotogenic model*, *Phys. Lett. B* **823** (2021) 136717 [arXiv:2107.10265] [INSPIRE].

JHEP01(2024)078