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Subjective refraction using power vectors by updating a conventional phoropter with a Stokes lens for continuous astigmatic power generation

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Abstract

Purpose: To implement a pure power vector method for monocular subjective refraction using a regular phoropter with the only modification being the inclusion of a Stokes lens. The proposed methodology was tested with three different Stokes lenses, and the results were compared with conventional clinical refraction procedures.

Methods: Power vector subjective refraction was performed by attaching a Stokes lens to the Risley prism holder. Stokes lenses allow for pure astigmatic compensation in the form of the J_0, J_{45} components while the spherical lenses in the phoropter allow determination of the spherical component in the form of M (spherical equivalent). The proposed routine is presented step-by-step using three Stokes lenses having different astigmatic powers.

Results: Monocular subjective refraction was performed on 26 healthy subjects with a mean age of 44 ± 16 years, mean spherical equivalent of −0.56 D (range −5.50 to +2.38 D) and refractive astigmatism ≤1.50 D. No differences were found between the results obtained with the conventional technique versus the vector-based procedure for the spherical equivalent ($p = 0.28$) or astigmatic components ($p = 0.34$). In addition, visual acuity (VA) was equivalent through the refractions measured with the conventional and vector procedures ($p = 0.12$). Repeatability coefficients for J_0 and J_{45} with the new vector methodology were <0.38 D.

Conclusions: The proposed routine could be helpful for cases where it is difficult to get a valid starting point for conventional refraction (e.g., irregular corneas and media opacities), for testing facilities with limited resources/equipment and/or for motivated clinicians who wish to know about alternative methods of refractive error determination.

KEYWORDS
astigmatic decomposition, phoropter, power vectors, subjective refraction

INTRODUCTION

In most cases, subjective refraction provides the final step when measuring refractive error in the clinic, and its foundations have remained unchanged for decades. Monocular subjective refraction involves a series of changes in the spherical and cylindrical lens powers (and orientation) guided by improvements in the patient's visual acuity (VA) or subjective judgement of visual quality. The final goal is to obtain a optical prescription resulting in optimal VA.[1] This optical prescription is obtained and expressed for clinical purposes in the classical sphero-cylindrical form, for example, −2.50 DS −1.50 DC × 30 in an eye requiring a refractive correction of −2.50 D in the 30° principal meridian and a refractive correction of −4.00 D (−2.50 + −1.50) in the 120° principal meridian. The sphere power (−2.50 DS) fully compensates for the refractive error in the 30° principal meridian and partially compensates for the refractive error in the 120° principal meridian (which still remains 1.50 D myopic), and the additional cylinder power (−1.50 DC × 30°) compensates for the remaining error in the 120° principal meridian. Monocular subjective refraction procedure could start without prior clinical data, but the process would be more time-consuming and prone to errors. To avoid this, objective refraction methods (such as autorefractometry, retinoscopy or even keratometry) are used to obtain a good starting point for the subjective refraction. Retinoscopy is a very accurate technique when performed by a trained optometrist,[2] but it needs a degree of training to become proficient,[3] especially in the presence of astigmatism.[4] Autorefractors produce a fast and repeatable measurement of refractive error,[5,6] although retinoscopy seems to be more accurate when performed by an experienced clinician, thus providing a better starting point to noncycloplegic refraction.[2] However, most clinicians do not prescribe spectacles based exclusively on the results from autorefraction, and retinoscopy together with subjective refraction are normally used as the gold standard method.[7]

Any optical prescription or refractive error can be expressed in the abovementioned sphero-cylindrical notation. However, in more advanced physiological optics, other mathematical formats are more suitable for statistics and investigation purposes.[8] In particular, Thibos et al.[9] described refractive error as a triplet of independent values (M, J_0, J_{45}) known as 'power vectors', with an M component representing the spherical equivalent (the spherical power that would place the circle of least confusion [COLC] on the retina), and two components defining astigmatism in the form of two Jackson cross cylinders (JCC) with axes at 0° and 90° (J_0) and 45° and 135° (J_{45}), respectively.

Power vectors provide satisfactory management of refractive error when considering mathematical operations such as addition, subtraction and averaging.[10] Therefore, they are recommended to solve basic problems such as over-refraction, toric contact lens misalignment, assessment of toric intraocular lens rotation,[8] as well as statistical analysis of sphero-cylindrical lenses, refractive errors or refractive surgery outcomes.[11-13] Moreover, they provide an intuitive graphical way to reinterpret some commonly used techniques during subjective refraction, such as the stenopaeic slit[14] and the adjustment of astigmatic compensation using fixed JCC lenses.[15] However, their most extended use comes from determining dioptric power in sphero-cylindrical form, transforming these findings to power vector notation and applying further analysis. Few attempts have been described to measure the refraction in vector format directly,[16] ranging from the Humphrey Vision Analyzer[17] (HVA) based on the astigmatic decomposition proposed by Gartner,[18] to the more recent methodology using Thibos's power vectors.[19]

Key points

- Refractive error can be expressed in sphero-cylindrical format or using power vector notation with values for the spherical equivalent (M) and astigmatic components (J_0 and J_{45}).
- Power vectors are usually obtained mathematically from the sphero-cylindrical notation for statistical purposes but can be measured directly with a subjective protocol using variable- and continuous-power Jackson cross cylinders in the form of a low-cost Stokes lens attached to a conventional phoropter.
- The power vector refraction procedure yields similar results to a conventional subjective refraction in healthy subjects, without the need for an objective starting point.

In previous work by Revert et al.,[19] it was demonstrated that refraction can be measured directly in power vector notation using a handheld JCC and trial lenses. This clinical procedure was less agile when compared with the use of a phoropter, thereby slowing down the procedure and increasing the probability of errors in alignment when changing from one handheld JCC to another. To ameliorate both drawbacks, the present work aimed to obtain the refraction in power vector notation directly, without an objective method as a starting point, by using a modified phoropter with a self-built, continuously variable astigmatic Stokes lens, and to compare the results with conventional subjective refraction in a sample of healthy subjects. Moreover, three different powered, continuously variable Stokes lenses will also be compared. As was the case in previous studies where laboratory-made Stokes lenses were built and placed into the Risley prism mount,[20,21] these three Stokes lenses were also presented in this manner.[22] This study reports on the use of these types of lenses as astigmatic compensation devices adapted to a commercial phoropter for refractive error determination.

METHODS

Measurement device

While the reader can find a full approach to the Stokes lenses in Albero-Moreno et al.,[22] we have included a brief description here for the sake of completeness. Figure 1 shows an exploded view of one of the constructed Stokes lenses used for the experimental measurements. The Risley prism was removed from the phoropter and used to construct the variable-power Stokes lens from two cylinders, as described and tested in the earlier work.[22] Figure 1 shows one of the three Stokes lenses built for the study, composed of two cylindrical lenses having

FIGURE 1 Exploded view of the built ±1.75 D Stokes lens with variable power. The top row on the left side of the picture shows an exploded view of the elements comprising the Risley mount, from left to right: screws to fix the mount to the phoropter, lock washer, black rotating ring, metallic cap to close the mount and the holder main body (with the toothed gear in place). The middle row shows, from left to right: a regular −1.75 D cylinder lens from a trial case, the same lens out of its frame and cut to the size of the mount, one of the prisms forming the Risley prism (that will be stored and not used in this work) and one of the gear wheels previously occupied by a prism that will now house the −1.75 D cylinder lens. The bottom row shows, from left to right: a regular +1.75 D cylinder lens from a trial case, the same lens out of its frame and cut to the size of the mount, one of the prisms forming the Risley prism (that will be stored and not used in this work), and one of the gear wheels previously occupied by a prism that will now house the +1.75 D cylinder lens. The right side of the picture shows a regular Risley prism inside its holder just removed from the phoropter (top) and the final result after conversion into a Stokes lens (bottom; notice the angular dial attached to the mount for direct determination of the rotation angle and the QR code).

the same power but opposite sign. In the example illustrated in Figure 1, we used two trial lens cylinders, namely a +1.75 DC (the trial lens in the lower left corner in Figure 1) and −1.75 DC (the trial lens in the middle-left side in Figure 1). Those cylindrical lenses were inserted into the Risley mount (substituting for the prism) so that they could rotate symmetrically with respect to each other inside the Risley mount, changing from a zero degree angle between the axes to a position where the axes were perpendicular to one another. When the plus and minus cylinder axes are parallel, the power of the Stokes lens is 0.00 D. As the plus and minus axes are counted-rotated, the Stokes lens power increases sinusoidally, reaching a maximum when each has been rotated by 45° (so their axes are 90° apart). This maximum power when the cylinder axes are perpendicular to one another will be the sum of the absolute value of the cylinder lenses. For a Stokes lens built with a +0.75 DC and a −0.75 DC, the maximum power will be 1.50 D. Similarly, lens constructed from ±1.25 and ±1.75 DC will measure astigmatism up to 2.50 and 3.50 D, respectively. We will refer to these as ±0.75, ±1.25 and ±1.75 D Stokes lenses. Each was labelled with a QR code to prevent easy identification and avoid bias during data collection.

These lenses were assembled into a Nidek RT-600 commercial phoropter (www.nidek-intl.com/), replacing the rotary prism. Figure 2 shows front and side views of the laboratory-made Stokes lenses incorporated into the phoropter.

Refractive examination procedure

All subjects underwent a full ophthalmological and optometric examination, including subjective refraction and VA testing using an Early Treatment of Diabetic Retinopathy Study (ETDRS) chart in LogMAR notation (Topcon CC-100, topconhealthcare.eu/en_EU). This ETDRS test incorporated random letter sequence presentation to avoid memorisation effects (since each subject was measured four times) and single letter scoring was adopted. Ophthalmic examinations included corneal topography, slit-lamp and funduscopic examination and macular optical coherence tomography.

One experienced optometrist performed objective (retinoscopy and autorefraction) followed by subjective refraction using a conventional protocol, guided by the maximum plus lens to obtain maximum VA, and using the standard ±0.25 D JCC in the phoropter to refine the axis and magnitude of astigmatism. Refractive error and VA for each subject were recorded and anonymised.

A second experienced optometrist performed the power vector refractions following the procedure described by Revert et al.[19] In that earlier work, the vector refractive procedure was tested clinically using a set of handheld JCCs with discrete powers from ±0.125 to ±1.00 D in ±0.125 D steps. As noted previously, the present study trialled three phoropter-mounted, laboratory-made Stokes lenses with refractive powers of ±0.75, ±1.25 and ±1.75 D, respectively. Once one of the Stokes lenses was in place, the following methodology was applied. We first determined the spherical component for the power vector refraction (the equivalent sphere M) using a conventional procedure including fogging to reduce the VA and determining the maximum plus (or minimum negative) spherical power that provided optimal VA. With this lens, the COLC is assumed to be positioned on

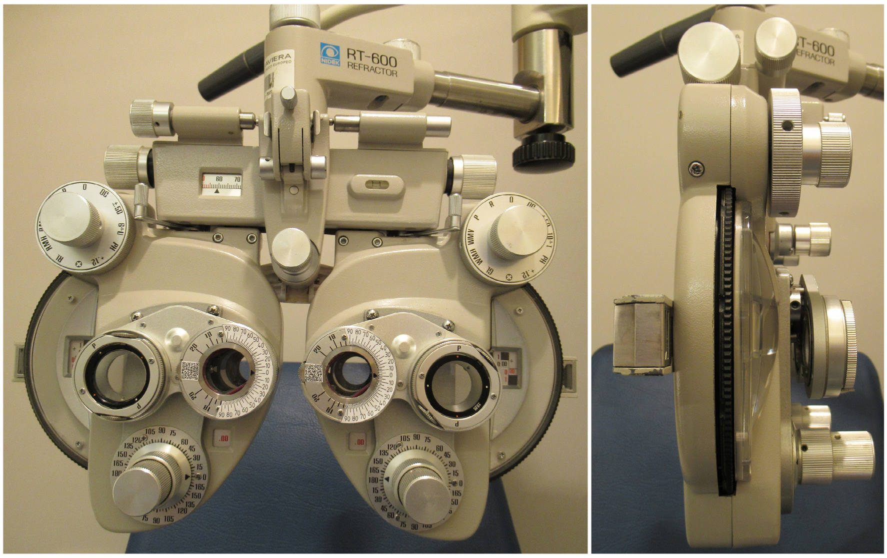

FIGURE 2 Front (left image) and side (right image) views of one of the laboratory-made Stokes lenses incorporated within the phoropter.

the retina. Next, we introduced the Stokes lens, and the astigmatic components were measured over the spherical equivalent lens before continuing to Steps 2 and 3 (measurements of J_0 and J_{45}). While J_0 was determined first followed by J_{45}, this order has no importance given the orthogonality of the vector components of astigmatism. It must be noted that at the time of measuring the M component, no information about astigmatism was available. The M component refers to a spherical power providing the best VA without considering cylindrical power, and thus it includes both sphere and cylinder (in fact, half of the cylinder, since the M component is the spherical equivalent refraction that puts the COLC on the retina). The M component is measured with spherical lenses until no further improvement in VA is obtained, and its value does not differentiate the spherical from the cylindrical components. After measuring the M component, and once J_0 and J_{45} (in subsequent steps) were measured, we could separate the information from M into its spherical and astigmatic parts.

In the astigmatic determination, one of the Stokes lenses was randomly selected and fitted into the phoropter with its axes at 0°/90° (Stokes lens handle along the 45°/225° meridian) and turned to its maximum power by rotating the cylinders in any direction (moving their axes apart) and asking the patient for their subjective perception of the presented optotype. If the patient did not report that their vision was becoming worse, the cylinders were rotated in the opposite direction, until a decline in vision was indicated by the patient. Starting from the configuration yielding the poorest visual perception, the cylindrical lenses were rotated to achieve the optimal VA by a procedure of limits; rotating the lenses until the best VA was obtained, ensuring a decline in vision with additional rotation, and then bringing the cylinder axes back towards their original orientation until the best VA was achieved. The procedure was repeated with the cylinder axes at 45°/135°. It must be clarified that for each of the two positions of our Stokes lens, the body of the Stokes lens (i.e., the Risley mount) was not rotated, to maintain power in the J_0 or J_{45} component. The plus and minus cylinder axes were rotated symmetrically inside the Stokes lens with respect to one another. This counter-rotation only changes the power of the Stokes lens, with no effect on its axis.

Once the best VA was achieved for each astigmatic component (J_0 and J_{45}), the angle between the cylindrical lenses was identified and used to compute the J component value in dioptres. For that purpose, the dial on the Risley prism was replaced with a scale having a range of ±90° (Figure 1, left side). For example, if we obtained a result of +10° when measuring the J_0 component with a ±1.75 D Stokes lens, this indicates that the plus cylinder inside the Stokes lens was +1.75 DC × 50° (45 + 5), while the minus cylinder inside the Stokes lens was −1.75 DC × 40° (45 − 5). The total J_0 astigmatic component.

($J_{0+cyl} + J_{0-cyl}$) was +0.3038 D, calculated as

$$J_0(+1.75\ \text{DC} \times 50°): J_0(+\text{cyl}) = -\frac{\text{Cyl}}{2} * \cos(2 * a) = -\frac{(+1.75)}{2} * \cos(100) = +0.1519\ \text{D}$$

$$J_0(-1.75\ \text{DC} \times 40°): J_0(-\text{cyl}) = -\frac{\text{Cyl}}{2} * \cos(2 * a) = -\frac{(-1.75)}{2} * \cos(80) = +0.1519\ \text{D}$$

$$\text{Measured } J_0: J_0(+\text{cyl}) + J_0(-\text{cyl}) = (+0.1519) + (+0.1519) = +0.3038\ \text{D}$$

It is important to note that both J power vector components can have positive or negative values. Thus, if measuring J_0 we recorded a rotation in the Stokes lens dial of −10° instead of +10° (meaning the +1.75 DC and −1.75 DC lenses were placed at 40° and 50°, respectively), then the J_0 astigmatic component would be −0.3038 D (given that both J_0 cylindrical components were −0.1519 D).

It can also be shown using the previous configuration that when measuring J_0, the J_{45} component will equal zero as calculated below:

$$J_{45}\ \left(+1.75\ \text{DC}\times 50^\circ\right): J_{45}(+\text{cyl}) = -\frac{\text{Cyl}}{2} * \sin(2*a) = -\frac{(+1.75)}{2} * \sin(100) = -0.8617\ \text{D}$$

$$J_{45}\ \left(-1.75\ \text{DC}\times 40^\circ\right): J_{45}(-\text{cyl}) = -\frac{\text{Cyl}}{2} * \sin(2*a) = -\frac{(-1.75)}{2} * \sin(80) = +0.8617\ \text{D}$$

$$\text{Measured } J_{45}: J_{45}(+\text{cyl}) + J_{45}(-\text{cyl}) = (-0.8617) + (+0.8617) = 0.0000\ \text{D}$$

These calculations were performed automatically using a Google spreadsheet (google.com). This overall procedure was performed with each of the three Stokes lenses (starting with the same value of M). A web-based tool (www.codigos-qr.com) was used to generate a QR code for each Stokes lens, linked to a Google sheet. Three small QR codes were printed, and each was glued to its corresponding Stokes lens (see bottom right corner of Figure 1). After obtaining the vector values M, J_0 and J_{45}, the optometrist used a cell phone to scan each Stokes lens QR code and saved the values in the corresponding spreadsheet, resulting in a blinded procedure, since the spreadsheets were randomly named by another practitioner who was unaware of the power of the Stokes lens.

Visual acuity was measured from each eye using the measured subjective refractions (the conventional sphero-cylindrical prescription and the three vector values). For power vector refractions, the corresponding sphero-cylinder was placed in the phoropter to measure VA, rounding the sphere and cylinder power to the nearest 0.25 D power and the cylinder axis to the nearest 5° (the spreadsheets were programmed to make this conversion automatically). The VA values were also stored in the spreadsheet. Data from only one randomly selected eye per subject were considered here to avoid bias.[23]

Subjects

Inclusion criteria involved healthy subjects >18 years old, with refractive cylinder ≤1.50 D (so that all could be measured with the three constructed Stokes lenses). Exclusion criteria were: contact lens wear within 2 weeks prior to measurements, astigmatism >1.50 D, amblyopia, previous ocular surgery and a history of ocular disease.

Informed consent was obtained from the subjects after a detailed explanation of the nature of the study. The study was performed in accordance with the principles of the Declaration of Helsinki. Institutional review board approval was obtained from the Clinica Baviera Medico-Legal Committee prior to the start of the study. Before data recording and measurement, 15 subjects were used to 'train' the optometrist in the clinical procedure until he was comfortable with the process of power vector refraction measurement and anonymisation of data. These values were not included in the study.

Statistical analysis

Refractive findings from the conventional or standard subjective refraction protocol were obtained in the usual clinical sphero-cylindrical notation: sphere, (negative) cylinder and axis; whereas subjective power vector refractions with the Stokes lens were directly obtained in power vector components (M, J_0, J_{45}). Sphero-cylinders were converted to power vector notation for statistical comparison. The formulas for these conversions can be found elsewhere.[8,19] VA was recorded in LogMAR notation.

Power vector refraction with the three Stokes lenses differs only in the astigmatic measurement, so the M component was measured just once, and the same value was used for all three astigmatic determinations. Therefore, statistical comparison of the M component between conventional versus power vector refraction was tested with a paired *t*-test (or a Wilcoxon rank sum test, if normality testing with the Shapiro–Wilk test failed).

However, a more complex analysis was needed for the astigmatic components. Given that four different refraction measurement techniques were compared for the two components (J_0, J_{45}), the study has a 4 × 2 factorial design. Differences in astigmatic refraction between methods, and/or interactions between methods and astigmatic power vector components of refraction were evaluated with a two-way repeated measures analysis of variance (2-w RM ANOVA), with the measured astigmatic refraction being the dependent variable, and 'method' and 'vector component' being the two factors (with four levels for the former, i.e., conventional, ±0.75, ±1.25 and ±1.75; and two levels for the latter, J_0 and J_{45}). To apply this 2-w RM-ANOVA, normality and sphericity assumptions must be confirmed by means of the corresponding Shapiro–Wilk normality test and Mauchly's test for sphericity. In case of failure of sphericity testing, two separate 1-w RM-ANOVA tests were applied to each of the two astigmatic components, or its corresponding non-parametric one-way Friedman repeated measures analysis of variance on ranks if normality cannot be assumed.

For the VA data, a factorial design with VA as the dependent variable and 'method' being the factor (with the same four levels described above) was checked by means of a 1-w RM-ANOVA test, or its corresponding non-parametric one-way Friedman repeated measures analysis of variance on ranks if normality or sphericity could not be assumed. Post hoc testing with the Tukey test was applied to locate differences as necessary.

Given that the three forms of vector refraction used the same method and differed only in the power of the Stokes lens, data from the ±0.75 Stokes lens were considered as a 'test' set, and data from the ±1.25 Stokes lens were considered as a 're-test' for assessment of repeatability. Repeatability analysis of the power vector refraction procedure was performed by the calculation of the coefficient of repeatability (CoR), that is, 1.96 times the standard deviation of the differences between test and retest measurements.

Statistical analysis and graphics were performed using Rstudio version 4.2.0 (posit.co/download/rstudio-desktop)[24] and the graphic package ggplot2 (ggplot2.tidyverse.org).[25] The significance level was set at 5% ($\alpha = 0.05$).

RESULTS

Thirty subjects were recruited for the study. Of these, four were not considered because they had refractive astigmatism >1.50 D and power vector refraction with the ±0.75 D Stokes lens indicated that they needed more power than the cylinder was able to deliver in any of the tested orientations. Data from the remaining 26 subjects were included in the study, with a mean of 44 ± 16 years of age (range 23–75 years).

Differences between methods/spherical component

Table 1 summarises information regarding the M component obtained with both the conventional and power vector refraction procedures.

Given that the M component was measured twice rather than four times, once for the conventional refraction and once for the three vector procedures (the same value of M was used as the starting point for the three Stokes lenses), a two-group comparison test was performed to examine differences between the conventional and power vector techniques. No outliers were identified in either of the groups. A paired *t*-test was applied after checking for normality (Shapiro–Wilk, $p = 0.40$ for the conventional group and $p = 0.51$ for the vector group) and equal variances (F-Snedecor test, $p = 0.98$). The result revealed no significant difference in M between the two techniques (paired *t*-test, $p = 0.28$).

TABLE 1 Summary of M values (D) obtained with the two compared methods (Conv, conventional refraction procedure vs. Vect, power vector refraction procedure).

M	Min	Max	Mean	SD	Mean (abs)
Conv	−5.50	2.38	−0.86	2.22	1.88
Vect	−5.50	2.50	−0.89	2.20	1.88

Abbreviations: M, spherical equivalent of power vector refraction; Max, maximum value; Min, minimum value; Mean (abs), mean of the absolute values; SD, standard deviation.

Differences among methods/astigmatic components

Tables 2 and 3 summarise information for J_0 (Table 2) and J_{45} (Table 3) astigmatic components obtained with the four procedures.

J_0 data obtained with the four procedures passed normality testing (Shapiro–Wilk, $p \geq 0.20$ in the four groups). No outliers were identified in any of the groups. All J_{45} data also passed normality testing (Shapiro–Wilk, $p \geq 0.05$ in the four groups). Nine outliers were identified for the J_{45} component (1, 2, 4 and 2 in the conventional refraction, ±0.75, ±1.25 and ±1.75 groups, respectively), but none were rated as extreme.

Considering the J_0 and J_{45} data in a 4 × 2 factorial model (4 refraction methods × 2 astigmatic refractive components), data passed Mauchly's test for sphericity, for both the refraction method factor ($p = 0.12$) and the interaction between the refraction method and the astigmatic component ($p = 0.16$). A two-way repeated measures analysis of variance indicated no significant difference between the astigmatic components for the four refraction methods ($p = 0.34$), and no significant interaction between the refraction method and astigmatic component ($p = 0.64$).

Differences among methods/visual acuity

The VA data did not pass the normality testing (Shapiro–Wilk, $p < 0.0003$ in the four groups), so non-parametric statistics were applied to look for differences between the four measured refractions. The Friedman repeated measures analysis of variance on ranks indicated no significant difference between the VA obtained for the four refractions

TABLE 2 Summary of J_0 values (D) obtained with the four procedures.

J_0	Min	Max	Mean	SD	Mean (abs)
Conv	−0.54	0.63	−0.05	0.35	0.30
±0.75	−0.65	0.74	−0.02	0.38	0.32
±1.25	−0.63	0.67	−0.02	0.35	0.28
±1.75	−0.67	0.74	−0.01	0.39	0.31

Note: ±0.75, ±1.25 and ±1.75 indicate power vector refraction procedure using the respective Stokes lenses.

Abbreviations: Conv, conventional refraction procedure; Max, maximum value; Mean (abs), mean of the absolute values; Min, minimum value; SD, standard deviation.

under study ($p = 0.12$). Median VA was 0.00 LogMAR for the four groups, with a very low dispersion (the highest interquartile range was 0.02 for the vector refraction with the ±0.75 D Stokes lens).

Agreement among methods

The intraclass correlation coefficient for the M component (ICC [M]) = 0.83, indicated good agreement between conventional and power vector refraction. The ICC was also high for the astigmatic components, with values for J_0 and J_{45} of 0.93 and 0.81, respectively.

Figure 3 shows a composite Bland–Altman plot for the J_0 and J_{45} components, comparing the standard or conventional subjective refraction results with the power vector refraction findings.

TABLE 3 Summary of J_{45} values (D) obtained with the four procedures.

J_{45}	Min	Max	Mean	SD	Mean (abs)
Conv	−0.31	0.62	0.05	0.22	0.17
±0.75	−0.46	0.59	0.04	0.26	0.18
±1.25	−0.43	0.72	0.08	0.24	0.17
±1.75	−0.38	0.74	0.07	0.24	0.18

Note: ±0.75, ±1.25 and ±1.75 indicate power vector refraction procedure using the respective Stokes lenses.

Abbreviations: Conv, conventional refraction procedure; Max, maximum value; Mean (abs), mean of the absolute values; Min, minimum value; SD, standard deviation.

Repeatability analysis of power vector refraction

The mean difference for the J_0 component between the ±0.75 and the ±1.25 Stokes lens measurements (re-test) was 0.00 ± 0.13 D, with a CoR of 0.26 D. For J_{45}, the equivalent mean difference was −0.04 ± 0.13 D, with a CoR of 0.25 D.

Repeatability analysis could be performed by comparing any other two sets of measurements. The poorest results were obtained when comparing data from the ±0.75 Stokes lens as the 'test', and from the ±1.75 Stokes lens as the 're-test', with a mean difference of −0.01 ± 0.15 D for J_0 (CoR = 0.29 D) and −0.03 ± 0.20 D for J_{45} (CoR = 0.38 D).

DISCUSSION

In this study, we compared results obtained from conventional monocular subjective refraction with a regular phoropter (gold standard) versus a subjective power vector technique using a modified phoropter with a continuously variable Stokes lens. No significant differences were found between the four measurement groups. In other words, the refractive error obtained with conventional subjective refraction and the proposed power vector procedure seem to be equivalent for both sphere and cylinder as shown in Tables 1–3.

Previous work[19] demonstrated that power vector refraction can be measured directly using a set of handheld JCCs with discrete powers, and not just computed from spherocylindrical prescriptions. It was concluded that the next

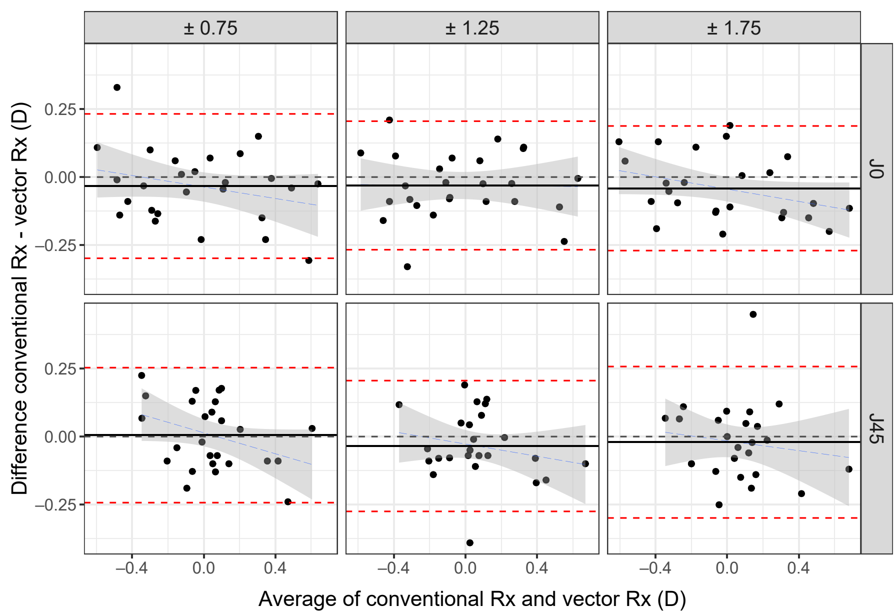

FIGURE 3 Bland–Altman plots showing the difference between values of the J_0 (top graphs) and J_{45} (bottom graphs) components obtained with the conventional and power vector refraction methods versus the average of the two. Horizontal dotted red lines: ±1.96 × SD of the differences; horizontal dotted black line: zero-difference line; horizontal continuous black line: bias line; dashed blue line: linear data tendency; grey zone: 95% confidence interval around the tendency line.

step would be to develop a device with variable astigmatic power and the capability to be attached to a phoropter to make the power vector refraction procedure more convenient and precise (by achieving continuous variation in astigmatic power rather than discrete steps). For that purpose, we developed and validated[22] a set of three continuously variable Stokes lenses with three different powers, ranging from 0.00 to ±0.75, ±1.25 and ±1.75 D. This is not the first attempt to develop new methodologies for the direct measurement of power vector components in monocular refraction and to compare the findings with standard subjective routines. In that sense, repeatability as well as efficiency, precision and accuracy of directly manipulating the power vectors have been reported for the HVA in the late 1970s/early 1980s,[26-28] a novel clinical refractor introduced by Raasch in the 2000s[29] and a phoropter introduced by Essilor (Vision-R 800; essilorusa.com) incorporating tunable lenses for continuous variation of the spherical and cylindrical powers.[30] However, we present here an intermediate solution which is cost-effective (thus ameliorating one of the drawbacks of the HVA as well as the complex power vector phoropters such as the Vision-R 800) while providing similar results in comparison with standard monocular subjective refraction. Moreover, we are describing the full step-by-step methodology (something hidden in previous work) while providing an exhaustive statistical analysis from the obtained results.

The refractive error prescription, regardless of how it is expressed, can be considered to be composed of two parts or components: one spherical and one astigmatic. The M component measured for both the power vector procedure and conventional refraction were almost equal (Table 1) with no significant differences. The ICC (0.83) indicates a very good agreement between the conventional and vector techniques. This result is not surprising since the measurement of the M component for each procedure was guided by the same principle, namely to place the COLC on the retina and maximise VA after an initial fogging procedure.

Regarding the astigmatic components of refraction (J_0 and J_{45}) measured after the M component was obtained and incorporated into the phoropter, there were no significant differences between the methods of measurement for any of the Stokes lens powers tested, for either astigmatic component (Tables 2 and 3). Furthermore, there were no significant interactions between the method used and the astigmatic component, so one can conclude that both the power vector and conventional refraction yielded similar values of astigmatism in the healthy subjects tested, irrespective of the power of the Stokes lens. The Bland–Altman plot for J_0 (Figure 3, top) shows good agreement between conventional refraction and power vector refraction for the ±1.25 D lens, with a bias line very close to zero, and an almost parallel linear tendency line very close to that. The limits of agreement for the ±1.25 D Stokes lens were close to those obtained with the ±1.75 D lens, and better (narrower) than those observed with the ±0.75 D lens. For the J_{45} component (Figure 3, bottom), the narrowest limits of agreement was also obtained with the ±1.25 D Stokes lens, whereas the lowest slope of the linear tendency line was seen with the ±1.75 D Stokes lens and the minimum difference between the bias line and the zero-difference line was found with the ±0.75 D Stokes lens. For both the J_0 and J_{45} components, it seems that the ±0.75 D Stokes lens yielded the poorest results in terms of comparison with the standard refraction outcomes. The ICC for the astigmatic components revealed good agreement amongst methods, although better for the J_0 component.

Even though the overall behaviour of the J_0 and J_{45} astigmatic components was similar for both methods of testing, it should be noted that no outliers were found when examining the distribution of differences between methods for J_0, whereas some outliers were detected for the J_{45} component. This can be explained by the significant presence of either with-the-rule or against-the-rule astigmatism in our sample, with only 6 out of 26 subjects having oblique astigmatism (19.2%). However, the orientation of cylindrical defocus could also play a role, given the subtle difference in the subjective component underlying each method of measurement. In the conventional procedure, the JCC adjustment requires the subject to decide which of the two presented images is clearer, or if they are similar.[31] However, in the power vector method, there is continuous image blur so that one astigmatic component is assessed (e.g., J_0) in the presence of the blur from the other non-compensated component (J_{45} in this example). Currently, there is no clear consensus in the literature regarding how the orientation of astigmatic blur affects VA. While Kobashi et al.[32] found a significant relationship between these two factors, this was not the case for Remón et al.,[33] and so we can conclude that further studies are required. In a previous investigation examining power vector refraction and tumbling E tests,[19] no significant relationship was observed between the measured astigmatic vector and the orientation of the tumbling Es.

As noted above, the measurement of one of the astigmatic components (e.g., J_0) following the determination of M is performed in the presence of astigmatic blur from the other non-compensated astigmatic vector component (J_{45} in this case). A modification of this procedure could improve the measurement of astigmatism. For example, once M and J_0 are measured, then the astigmatic correction corresponding to the J_0 measurement could be introduced into the phoropter together with the M value, before proceeding to measure J_{45}. For instance, if the value of M is −5.00 DS, then J_0 will be measured through this lens. If the resulting measurement of J_0 equals +0.50, then this component would be converted into the corresponding sphero-cylinder, that is, +0.50 DS −1.00 DC × 180 and added to the M lens, to equal −4.50 DS −1.00 DC × 180, before proceeding to the measurement of J_{45}. However, we believe this makes the clinical procedure more complicated, and may add little to the measurement of J_{45}. Moreover, if J_{45} is measured with J_0 corrected, then *a posteriori* readjustment of J_0 will be needed, especially for axes other than 0°/90° or 45°/135°.

No significant differences in VA were found between the conventional and power vector refractions when the latter was transformed from vector form into sphero-cylindrical prescriptions to the nearest 0.25 D (for sphere and cylinder) and 5° (for astigmatic axis). As an example, a power vector refraction of $M = -0.50$ D, $J_0 = +0.13$ D, $J_{45} = -0.46$ D corresponds to a sphero-cylindrical prescription of −0.02 −0.96 × 143, which was approximated to Pl −1.00 × 145. Had these converted results been refined further with a conventional JCC procedure, then VA might have improved further. This was not tested here because the goal was to compare the power vector refraction with the standard protocol.

Regarding the repeatability of the proposed vector refraction methodology, we obtained good CoR for both astigmatic power vector components, with CoR values for J_0 and $J_{45} < 0.29$ and <0.38 D, respectively. Leinonen et al.[34] studied the repeatability of refractive error assessment in clinical settings with cataractous, pseudophakic and healthy eyes, reporting differences between test–retest as a three-dimensional vector value. They obtained CoR values between 0.57 D (for the vertical component) and 0.71 D (for the horizontal component) in eyes having VA > 0.7 decimal equivalent. Huang et al.[35] used the same vector approach adopted in the present investigation, computing spherocylindrical refraction as power vectors (M, J_0, J_{45}) to study the repeatability of tele-controlled subjective refraction in 65 subjects between 9 and 40 years of age. They reported CoR values of 0.57 D for J_0 and 0.79 D for J_{45}. Rosenfield and Chiu[36] noted a CoR for subjective refraction in the horizontal meridian of 0.27 D in nine visually normal subjects. Later, Raasch et al.[37] studied the repeatability of subjective refraction in both normal myopic and keratoconic eyes using a vector approach and reported repeatability coefficients of 0.23 D for J_0 and 0.16 D for J_{45} in the normal group (with much higher values for keratoconic eyes). Thus, the repeatability of the vector refraction procedure described here is at least within the limits or better than that previously reported for conventional subjective refraction.

To the best of our knowledge, this is the first work reporting power vector refraction results using a modified and low-cost Stokes lens attached to a phoropter. The use of a phoropter throughout the procedure, with a continuous astigmatic powered Stokes lens, makes the procedure time efficient while minimising subject's fatigue. Our results are very close to those obtained by Revert et al.[19] who found that the agreement between the manual power vector refraction and the conventional subjective procedure presented ICC values of 1.0 for M, between 0.93 and 0.94 for J_0 and between 0.87 and 0.96 for J_{45}. Thus, adding a modified Stokes lens did not reduce in the validity of the methodology, but accelerated the entire process. In comparison with the standard refraction procedure, the vector-based refraction generally took less time once the examiner had completed the initial training process. However, subjects with difficulties understanding the test, or those struggling to appreciate the subtle differences in visual quality produced by small rotations of the Stokes lens may have needed more time than was the case for the conventional subjective routines. Fortunately, this only occurred in a minority of cases evaluated here.

The proposed power vector methodology involved a minor modification to a conventional phoropter, thereby making it possible to measure clinical refraction directly in power vector notation, without the need for starting with autorefraction or retinoscopy (although they might make the process easier). Conventional subjective refraction protocol determines and corrects the spherical error before attempting to find astigmatism, even though most practitioners will go back and recheck the sphere a second time once the cylindrical power and axis have been verified. Since a change in cylindrical power while readjusting the astigmatic correction (using the JCC procedure for astigmatism refinement, for instance) will alter the position of the COLC, a corresponding change in sphere equal to half the magnitude of the cylinder change (and with the opposite sign) is necessary to maintain the COLC on the retina. Thus, conventional subjective refraction is an incremental feedback procedure that asymptotically approaches the final prescription. By comparison, the proposed power vector refraction method is a straightforward, single-pass method consisting of three independent steps: that is, correct M, correct J_0 and correct J_{45}, with no need for readjustment of a component after a subsequent measurement, given the orthogonality of power vector components.

We do not propose this power vector procedure as a substitute for conventional sphero-cylindrical refraction but rather to complement it in certain situations. A possible application is when examining eyes with irregular corneas, perhaps caused by keratoconus or corneal scars, where a valid starting point for conventional refraction by means of autorefractometry, retinoscopy or corneal topography could be difficult to obtain, or have limited validity. Other possible uses of power vector refraction might be for local staff in humanitarian cataract surgery campaigns in developing countries, since the proposed approach does not require a large investment, thus making it suitable for modest refractive centres with limited resources/equipment. Once patients have undergone surgery and adequate healing has taken place, then a local trained auxiliary can obtain a refraction (even an approximation) in the absence of the surgeons or optometric teams. We believe that power vector refraction needs less training time and less optics-optometric knowledge than conventional refraction. These potential uses of power vector refraction must be validated with further research. Moreover, the proposed methodology is applicable to motivated clinicians who wish to expand their knowledge and implement new routines in refractive error determination.

Regarding the limitations of both the study and the power vector refraction technique presented herein, the first consideration is the amount of astigmatism that we were able to measure with the variable-powered Stokes lens. The highest power Stokes lens constructed for this study (±1.75 D) can measure up to 3.50 D astigmatism, but since we wanted to test for possible differences using three different powers, the limit of astigmatism was determined by the lowest power Stokes lens (±0.75 D), which could only measure up to 1.50 D of astigmatism. Even though greater amounts of astigmatism could have been quantified with the ±0.75 D Stokes lens by introducing mixed astigmatic compensation at the appropriate orientation (0°/90° for J_0 measurements and 45°/135° for J_{45} measurements) to act as a 'supplement', this could have introduced errors by imperfect alignment with the Stokes lens, and so it was not performed.

With regard to the compensation of astigmatism with the proposed vector-based procedure, the physical configuration of the phoropter must be taken into consideration. As can be seen in Figure 1, the plane of the Stokes lens is further from the eye (52 mm) than the plane containing the phoropter spheres and cylinders (12 mm). This is important when high-powered vector components are being measured, and vertex distance conversions will be required to correct these differences. However, in the present study, this effect was negligible given that we restricted our sample to astigmatism <1.50 D. This is demonstrated by considering a ±0.75 D Stokes lens placed at a vertex distance of 12 mm and configured to produce +0.75 and −0.75 D in the orthogonal meridians. The same lens positioned at a vertex distance of 52 mm would have meridional powers of +0.77 and −0.73 D that is, a negligible error of 0.02 D. But with a ±1.75 D Stokes lens for measuring 3.50 D of astigmatism, the +1.75 and −1.75 D meridional powers at 12 mm will become +1.88 and −1.64 D powers at 52 mm, resulting in a 0.12 D difference in sphere and a 0.02 D in cylinder.

An additional drawback to consider is the continuous variation in cylindrical blur when measuring the J_0 and J_{45} components, which can make interpretation difficult for some subjects, since it is harder to identify a subtle difference than a bigger step change in dioptric power. The clinician needs to learn how to explain the procedure to the subjects, instructing them how to judge the change in blur until the desired final point. In our experience, a number of subjects have experienced difficulties and the clinician (an experienced optometrist with more than 20 years of clinical practice) needed up to 15 'training subjects' to develop a valid protocol with clear instructions. On the contrary, this drawback was buffered by using a method of limits, and by repeating the Stokes lens measurement until the subject was sure about their final judgement and a coherent and repeatable answer was obtained. This became relatively easy once the subject understood the test. Furthermore, subjects must learn how to interpret the variation in blur properly and how to respond during measurement with the first Stokes lens. This may create possible bias for the subsequent measurements with the remaining Stokes lenses, although it was minimised by using a random lens sequence. A further limitation is in regard to the power of the Stokes lenses. While they were anonymised with a QR code, the optometrist performing the power vector refraction could still infer which Stokes lens power was being used by the degree of rotation necessary until the subject reported a change in blur and VA.

A final limitation is that the procedure required the Risley prism to be removed so that its mount could be used for Stokes lens. This meant that other important testing procedures, such as binocular balancing or heterophoria/vergence measurements, could not be carried out unless other prisms or alternative techniques were used. This is a specific restriction of the modified phoropter used here, but it could be corrected in more sophisticated refractors.

This study could be a precursor to further investigations of the power vector refraction technique. For example, its applicability to different populations (irregular corneas, pseudophakic eyes, misaligned toric or multifocal intraocular lenses, paediatric subjects, etc.) should be examined. It would also be interesting to determine how the orientation of cylindrical blur affects VA and whether this impacts the orientation of the measured astigmatic axis. For example, would a subject with a 45° axis of astigmatism respond better to the J_{45} testing than a subject with a 180° axis of astigmatism?

Cylinders in a conventional phoropter are typically available in powers ranging from 0.00 to −6.00 D in 0.25 steps, so that a subjective refraction of −2.50 DS −1.25 DC × 175 can be measured. With the power vector refraction proposed here, a set of components (M, J_0, J_{45}) with values (−3.25, +0.65, −0.15) can be measured, corresponding to a spherocylindrical refraction of −2.58 DS −1.33 DC × 174. This −1.33 DC cylinder power cannot be obtained nor implemented with a conventional phoropter (nor can a −2.58 D sphere) or trial lenses. In this work we rounded the spheres and cylinders derived by power vector refraction to the nearest 0.25 D to introduce these lenses into the phoropter for comparison with conventional subjective refraction. Direct comparison of the VA obtained with the power vector refractions (without rounding) versus conventional refractions with a non-conventional phoropter capable of introducing cylinders such as a −1.33 DC (such as the Essilor Vision-R 800 phoropter) is a possible future line of investigation. This will indicate whether patients do appreciate the superior lens precision. Finally, we plan to continue working to develop a clinically feasible power vector technique without the need to remove the Risley prism, given its importance in optometric practice.

AUTHOR CONTRIBUTIONS

José Ricardo Albero: Conceptualization (equal); data curation (equal); formal analysis (equal); investigation (lead); writing – review and editing (equal). **Vicente Micó:** Conceptualization (equal); methodology (lead); supervision (lead); writing – review and editing (equal). **César Albarrán Diego:** Conceptualization (equal); data curation (equal); formal analysis (equal); investigation (supporting); writing – review and editing (equal).

CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflicts of interest.

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