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Music ontology. A construction from the semantic hylo-realism of Mario Bunge

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Abstract. In this research we intend to build the bases of a musical ontology, starting from the philosophical principles of scientific realism developed by the philosopher Mario Bunge. Bungean hylo-realist philosophy is based on several postulates, and these allow us to build a framework to analyze certain phenomena and facts of music. It is a very common and widespread opinion that music transmits feelings, for our part, we will answer this question from our ontology.

Keywords. Ontology, Semantic, Expressionism, Emotivism.

Ontología musical. Una construcción desde el hilorrealismo semántico de Mario Bunge

Resumen. En esta investigación pretendemos construir las bases de una ontología musical, partiendo de los principios filosóficos del realismo científico desarrollados por el filósofo Mario Bunge. La filosofía hilorrealista bungeana se sustenta en varios postulados, y estos nos permiten construir un marco para analizar determinados fenómenos y hechos de la música. Es una opinión muy común y extendida que la música transmite sentimientos, por nuestra parte, responderemos a esta pregunta desde nuestra ontología.

Palabras clave. Ontología, Semántica, Expresionismo, Emotivismo.

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1. Introduction

How should the work of art be studied? Like the other concrete products of human activity, such as machines, organizations, and languages? Or rather as

> abstract products, such as theories and ideologies? Or in a totally different way, because artistic appreciation is not totally objective or cross-cultural?[1]

Behind these questions, the philosopher Bunge did not see the answers to his questions as optimistic. In fact, Bunge treats aesthetics as something possible to construct, which could be called analytical aesthetics. This would be in charge of analyzing and relating to each other aesthetic categories such as those of work of art, form and style. But when asked if this was a proper discipline, Bunge replied that "for now, no. But nothing prevents it from becoming so", and in this way he harbored hopes that the categories and methods to build it would be developed.

For our part, we propose to develop some bases of what we could call musical ontology, based on the philosophical principles of Bungean scientific realism. The bungean hylorealist philosophy is based on several postulates and these will allow us to build an interesting framework to analyze certain phenomena and facts of music.

It is a very common and widespread opinion that music transmits feelings. For our part, once our explanatory framework has been developed, we will give our own answer to this question.

2. Statements, sentences, and propositions

Bunge (1980) argues that it is necessary to distinguish a proposition from the sentences that designate (express, formulate) it, just as it is necessary to distinguish a sentence from its various utterances (oral, written, or by gesture) (p. 66).

For our part, we can similarly affirm that a musical proposition is a musical fact with some theoretical meaning that cannot be confused with the phrase that designates it. Nor can the phrase be confused with the different ways of interpreting it in a performance.

The enunciation (whether written or verbal) and the perception of the enunciation (reading or listening) are psychophysical processes[2].

Analogously, we can maintain that the interpretation of a musical work, that is, the enunciation, and its perception, whether reading or listening to it, are psycho neurophysiological processes.

In Bunge's theory of reference, sentences—oral or written—are a kind of statement. Sentences can be stated in many ways. Despite this, they can produce equivalent effects, and other times pronounced in different contexts they can produce disparate effects.

[1] Bunge, M. (1999). La ciencia de lo bello, lo bonito y lo sublime, *Diario La Nación*. https://www.lanacion.com.ar/opinion/la-ciencia-de-lo-bello-lo-bonito-y-lo-sublime-nid142347

[2] Bunge, M. (1980). *Epistemología: curso de actualización…*, p.66.

From the above we can sustain that the musical content, from the phrases of a work, are a kind of interpretation. The phrases can be reproduced, that is, interpreted in as many ways as interpreters address them, but despite this diversity they refer to or are equivalent to the same work.

Propositions are a singular type of sentences that designate something. From this it follows that, whatever the proposition, it can be expressed by different sentences, but this relation is not symmetric: given a sentence it cannot be equivalent to different propositions.

So too musical propositions are special cases of phrases designating theoretical content. Hence any given proposition can be expressed by different sentences.

It is a linguistic phenomenon that there are sentences that do not constitute propositions. In this same sense, in music there can be the concatenation of musical symbols but as phrases they do not constitute musical propositions (they make sense within a musical theory).

The Bungean theory of reference states that there are three classes of linguistic objects (statements, sentences and propositions) and, between these, two relationships (enunciation and expression or designation)[3].

$$\textit{Statements} \xrightarrow{\textit{enunciate}} \textit{Sentences} \xrightarrow{\textit{express}} \textit{Propositions}$$

$$\textit{Interpretation} \xrightarrow{\textit{interpret}} \textit{Phrases} \xrightarrow{\textit{express}} \textit{Propositions}$$

Let Sta = Statements, Sen = Sentences, P = Propositions, I = Interpretation, and Ph = Phrases.

The enunciation function is of the many-to-one type and referenced by the sign (E) can be written $E : Sta \rightarrow Sen$

The interpretation function is of the many-to-one type and referenced by the sign (I) can be written $I : I \rightarrow Ph$

The designation function is also of the many-to-one type and represented by (D) can be symbolized by $D : Sen \rightarrow P$

The expression function is also of the many-to-one type and represented by (E) can be symbolized by $E : Ph \rightarrow P$

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3 *Ibid.*, p.67.

Thus the denotation ($\boldsymbol{\Delta}$*) is represented in the expression* $\boldsymbol{\Delta}: D \wedge E : Sta \rightarrow P$

Thus the expression ($\boldsymbol{\Delta}$*) is represented in the expression* $\boldsymbol{\Delta}: E \wedge I: I \rightarrow P$

Every proposition has at least one statement. Similarly, at least one interpretation corresponds to each proposition. The denoting function implies that an utterance—verbal or written—has a proposition attached to it. The expression function implies that an interpretation —made in performance or in a score— is accompanied by a proposition.

3. Meaning, sense and reference

All predicates, propositions, and theories have as their subject—refer to—something, be it concrete or abstract. A proposition out of context has no precise meaning. To determine the referents of a proposition, it is necessary to analyze the predicates that appear in it, and in turn the reference class of a predicate is determined by its domain of definition. A predicate can be analyzed as a function from individuals (or n-tuples of individuals) to the set of all propositions containing the predicate in question[4].

> [...] a construct has a content whereas a signal, such as a written sentence, *conveys* (carries, transmits) information to *someone*. In other words, whereas intension is defined on the set of constructs, information is defined on the set of pairs signal-subject, where a subject is a viewer or hearer competent to decode the signal. Information, to be received, requires a receiver, i.e. a system equipped with suitable receiving and decoding devices.[5]

The sign *x* designates the concept (or proposition) y; *x* could be ♪, ♯, ○, ⅜, I, Eb Major, V13, VII6/5, D, T, SD, presto, forte, etc.; *x* could be "eighth note", "natural", "whole note", "composite compass", "major chord", "diminished fifth", "diminished chord", "tonic", "fifth degree with added thirteen", "seventh chord on first inversion", "metronome velocity", "intensity of performance", etc. Examples of exposed signs are part of the musical communication system.

The musical sign represents a physical object (the tempered note, the value of the note, the intensity of the pulsation, the speed of the metronome, etc.) and a conceptual object (for example, the tempered note can be a major third of a chord). Musical signs are typical of the musical notation system and can be concatenated with each other to produce more complex signs, and in this way their use is a means of communication about certain musical processes, mechanisms or state of affairs. A musical chord is an object that has two forms of existence, one conceptual and the other material. Its conceptual existence is the product of the fact that it acquires ontological status from harmonic theory, and its material existence happens when there is a copy of its execution or performance.

4 *Ibid*., pp.69-72; Bunge, M. (1974a & 1974b).

5 Bunge, 1974a, p. 135.

> Any system of coded signals used for communication purposes qualifies as a language. Moreover, the coding and decoding need not be accompanied by understanding: they can be automatic - as is often the case within humans.[6]

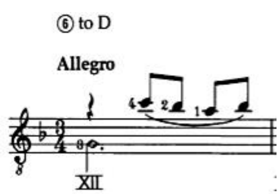
In particular, the interpreters in music, since the performance can occur with the simple fact of following the guidelines of instrumental pulsation from the score without this entailing a harmonic understanding or the formal structures of the work.

4. Linguistic categories and conceptual categories

Music theory has its own terminology. There are different kinds of constructs in it. We can basically state two types of propositions in music, a strictly musical proposition, which is the same as saying made in object language, a musical proposition refers to a musical fact with some theoretical meaning that cannot be confused with the phrase that designates it, just as it does not the phrase can be confused with the different ways of interpreting it in a performance (its execution).

> The smallest structural unit is the phraseological fragment, a kind of musical molecule consisting of a number of integrated musical facts, possessing some sense of whole idea, as well as adaptable to combination with other similar units. The term phraseological fragment means, structurally, an approximate unit that could be sung in a single breath. Its ending suggests a form of punctuation such as a comma".[7]

Here is an example to clarify our speech. As an example of a musical phrase, we can bring this first measure of the passage from Vals No. 3 by Agustín Barrios Mangoré.



Example 01: Burley, Raymond (1991) Agustín Barrios Mangoré 18 Concert Pieces, Vol. 2, Schott Music Ltd., London, p. 23.

In this musical writing, on the one hand, we have a treble clef, a flat sign in the key signature, an indication of time (allegro) and another that the sixth string of the guitar must be tuned in D, a silence of quarter note, a technique indication (a harmonic on the third string at the 12th fret), the notes C, B, and A with a slur indication. On the other hand we can say, taking as parameters the notes B flat in the melodic line and G (in harmonic) as the main constituents of the harmony, that the passage is subsumed into several possible chords, G minor [G (fundamental) and if flat (minor third)], or E diminished without root [E (implied root), G (minor third) and B flat (minor fifth), C major seventh with the added

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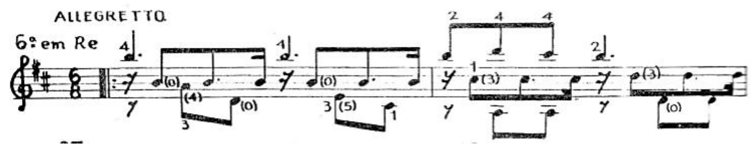
[6] Bunge, 1974a, pp. 8-9.

[7] Schoenberg, A. (1970). *Fundamentals of musical composition*, p.3.

thirteenth [C (implied root), E (major third implied), G (minor fifth), B flat (minor seventh), D (implied major ninth) and la (trecena), etc.

A single note, or a musical rest, expressed with some musical symbol, are atomic propositions in music. The phraseological fragment, that is, a concatenation of musical symbols, constitutes a musical molecular proposition, as our previous example of Mangoré's Waltz No. 3 illustrates very well. A musical fact can be found in the musical phenomenon of the melody, since its components imply a harmony, a temporal duration, therefore they are associated with some musical figure.

Musical ideas can be written in different ways, and all of them can be equivalent provided that their invariants are maintained. In the following examples, although the two passages are written in different measurments and thus the symbols of the melodic figurations do not correspond, they are still the same referenced object in both.



Example 02: Paraguayan Dance by Agustín Barrios Mangoré in the version made by Raymond Burley

Example 03: Paraguayan dance by Agustín Barrios Mangoré in the version made by Romeo Di Giorgio

Mathematically speaking, if we operate by doubling the values of example 02 of all the figures, we obtain the figures of example 03, the same if we divide by 2 the measurement of example 2, we obtain the equivalent of example 03. The rule would be if we divide by 2 the measurement, it operates reciprocally by multiplying all the figures of the compass (or the work) by 2. With these operations they will change their scriptural form, but they will not change the harmonies or the structural functions of these.

Despite the previous elegant operation, as time in music as performance is psychological, and its perception depends on how we group the notes, thus any musical idea can be transcribed in any measurement. Musical writing is conventional and is given to facilitate reading for the interpreter, but this does not annul the possibility of writing the same idea under all possible measurements (despite how difficult it is for the interpreter to read them). In music there is no such thing as that there is only one correct and true way of writing this or that work, all possible ways of writing it will be equivalent if the essential structural relationships remain invariant. As time is a variable subject to the interpreter's psyche, their performance will always be very personal and can hardly be similar to that of others.

Every musical fact can be reduced to its metrizable aspects[8]. In this sense we can point out that music in its material aspect is sound (or its absence as constitutive), but that sound to which we refer is the musical "sound" that we call musical "note" and this is closely related (insofar as its organization in well-defined structures) to what we call musical "scale".

In the tempered scale, used in our common musical practice (in this writing we are not going to discuss the physical or formal possibilities of creating and using theoretical scales), frequencies are assigned to specific intervals, these are what we call musical notes, these frequencies start from a well-defined one, in our case (western music) the note “A”, whose oscillation is at 440 Hz, from this specific note, either dividing or multiplying by integer powers of 12√2, we obtain the other notes of the musical scale[9].

Each note has a unique frequency, but there are notes that turn out to be powers of 2, even if these are x, $2x$, $4x$, ... , $x/2$, $x/4$, ... , the results of this operation are notes that receive the same name but they are separated by what we call an octave (7 notes apart, or what is equal 11 half steps), this characteristic is known as a cyclic behavior.

For example, notes with a frequency of 440 Hz or 880 Hz or 1760 Hz or 220 Hz or 110 Hz are called “A”. Given two notes $x1$ and $x2$ of the tempered scale, whose frequencies are $f1$ and $f2$ respectively, the pitch distance from $x1$ to $x2$ is nominally the value of t in the expression $2k(12\surd 212)t$ [10].

By taking the frequency of any note, multiplying said frequency by 12√2, thus obtaining the frequency of a new note, and measuring the tonal distance between these two notes, it will be obtained that there is a semitone from the first note to the second.

From the quotient $f2/f1$, such that $t\geq 0$ and the expression is irreducible, that is, t takes the minimum possible positive value (or, failing that, zero). We will denote the distance from $x1$ to $x2$ as $d(x1\rightarrow x2)$ and it will be measured in semitones. If we take a C with a frequency of 220(12√2)15 and an D with a frequency of 880(12√2)−7, the value of d(C→D) will be 2 and d(D→C)=10[11].

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[8] Oxley, Víctor M. (2018). Invariant equivalents and their representation in the symbolic musical notation: a case in the musical culture of Paraguay…

[9] Pachón Casas, A. (2015). *Matemáticas y música: una mirada a la armonía desde la teoría de grupos…*, p. 42.

[10] *Ibid.*, p. 43.

[11] *Ibid.*, p. 45.

Multiplying the frequency of one note by $(12\sqrt{2})t$, with $t \in \{0,1,2,3,4,5,6,7,8,9,10,11\}$, to get the frequency of another note , and measure the distance from the original note to the new one, the distance will be t semitones. (Pachón, 2015: 46) Multiplying a frequency by $12\sqrt{2}$ consecutively allows us to obtain the frequency of the other notes, regardless of where it starts, this idea leads us to think about the action of $(\mathbb{Z}12,+)$ on the set of notes (call it N) because finally semitone raising (seen as consecutive multiplication by the factor $12\sqrt{2}$), behaves cyclically, then a function $\oplus$ can be defined as an external operation $\oplus$: $N \times \mathbb{Z}_{12} \rightarrow N$

The interpretation of this action is given as the increase of a number of semitones from a given note, for example, $E\oplus 3$ is equivalent to taking the E note and increasing it by 3 semitones, from this the G note is obtained. This function effectively fulfills the action conditions, for the first condition it is guaranteed by the order given to the notes.

Due to the way in which tonal distance is measured, when thinking about building a model that allows visualizing its order, it must be considered that for each pair of notes, the distance from one to the other is t and from the second to the first is $12 - t$, with $t > 0$, this consideration suggests a circular model for the notes, where every pair of different notes can have two different distances when measured clockwise and counterclockwise. This also endows the set of notes with a cyclic behavior under the idea of increasing semitones, the latter allows considering a new relationship between them and the sets $\mathbb{Z}n$ to which said cyclic behavior is also associated, since both sets behave in the same way. same way under their respective "operation" so it seems almost natural to think of an isomorphism between them. A bijective function between the elements in both sets equals $\mathbb{Z}n$ and $\mathbb{Z}12$[12].

The chromatic scale is based on the following scheme: C, C♯(or D♭), D, D♯(or E♭) E, F, F♯, G, G♯, A, A♯(or B♭), B, C.

Between each note there is a distance of ½ tone.

$$\begin{array}{c} 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \quad 1/2\ tone \\ note1 \leftrightarrow note2 \leftrightarrow note3 \leftrightarrow note4 \leftrightarrow note5 \leftrightarrow note6 \leftrightarrow note7 \leftrightarrow note8 \leftrightarrow note9 \leftrightarrow note10 \leftrightarrow note11 \leftrightarrow note12 \end{array}$$

By taking a note t of the tempered scalar, its major scale can be determined by doing: $\{t, t\oplus 2, t\oplus 4, t\oplus 5, t\oplus 7, t\oplus 9, t\oplus 11\}$

[12] *Ibid.*, pp. 46-47.

Which correspond to the elements of the scale of t major, after this we look for $f(t)$ to perform the increase of semitones as sums in $\mathbb{Z}12$, that is $\{f(t), f(t) + 2, f(t) + 4, f(t) + 5, f(t) + 7, f(t) + 9, f(t) + 11\}$

Taking into account that adding 1 to an element of $\mathbb{Z}12$ is equivalent to raising the note corresponding to that element by a semitone, adding 2 must musically correspond to raising a tone, which allows us to see the tonal distances between two consecutive notes in the major scale as: $+1\ tono\ f(t) \leftrightarrow f(t)+2$, $+1\ tono\ f(t)+2 \leftrightarrow (f(t)+2)+2=f(t)+4$, $+1/2 tono\ f(t)+4 \leftrightarrow (f(t)+4)+1=f(t)+5$, $+1\ tono\ f(t)+5 \leftrightarrow (f(t)+5)+2=f(t)+7$, $+1\ tono\ f(t)+7 \leftrightarrow (f(t)+7)+2=f(t)+9$, $+1\ tono\ f(t)+9 \leftrightarrow (f(t)+9)+2=f(t)+11$, $+1/2 tono\ f(t)+11 \leftrightarrow (f(t)+11)+1=f(t)$[13].

For example, if $t = E$ then to determine the E major scale, set $f(MI) = 4$, and replace $f(t)$ with 4: $\{4, 4+2, 4+4, 4+5, 4+7, 4+9, 4+11\} = \{4, 6, 8, 9, 11, 1, 3\}$

When looking for the notes corresponding to each element of $\mathbb{Z}12$ in the set that was obtained, that is, evaluating the function $f{-1}$ in each one of the elements of the subset of $\mathbb{Z}12$ determined in the last step, the scale of E major is found to be composed by the notes *E, F♯, G♯, A, B, C♯* and *D♯*, it can be easily verified that these notes indeed correspond to the scale that one wanted to find by looking at the tonal distances between them.

The scale, be it major or minor, gives rise to the tonality. The major scale is defined by the distances between each component note of it. The C major scale is made up of the following notes: C, D, E, F, G, A, B, C.

1 tone is the sum of two ½ tone:

$$C \xleftrightarrow{1\ tone} D \xleftrightarrow{1\ tone} E \xleftrightarrow{1/2\ tone} F \xleftrightarrow{1\ tone} G \xleftrightarrow{1 tone} A \xleftrightarrow{1\ tone} B \xleftrightarrow{1/2\ tone} C$$

The phrase that designates a musical event is materially speaking the written product of the musical syntax adjusted to the rules of the use of musical signs. The musical phrase will have as many forms of execution (performance) to the extent that so many interpreters (performers) take it to its material concretion (sound).

Linguistic and conceptual categories can be found in the language of music theory. For example, at the level of the linguistic category of "individual name", we have the conceptual category of a specified individual or the individual concept of "quaver" (a musical temporal figure).

[13] *Ibid.*, pp.49-50.

At the category level of "predicate symbol" there is, for example, the conceptual category of a specified predicate, e.g.: the concept of a major chord.

At the category level of "phrase" we have, for example, the conceptual category of a defined description. e.g.: “The retrograde inversion of the main motive”.

At the category level of "statement", we have, for example, the conceptual category of a specified statement. e.g..: “The note C sharp is a decrease of the ninth of the chord”.

At the category level of “language”, we have, for example, the conceptual category of a specified theory (theory of a model). For example, harmonic regions (Arnold Schoenberg theory).

At the category level of "class name" there is, for example, the conceptual category of a specified (concrete) set, for example, the set of values of musical figures.

Bunge (1974a) holds that every symbol in Newton's formula for the gravitational force designates a concept, and the formula as a whole designates a proposition (which, in turn, represents an objective stable pattern) (p. 15). Similarly, we hold that every musical symbol written in a bar (or over several bars) designates a concept, and the signs that form a musical phrase (understood as the unit of the composition) designate a musical proposition.

Basically, two types of concepts can be distinguished. One "individual", the single note for example (or any musical sign), and another "collective", such as the chord (sequence of notes sounding simultaneously). Chords are class concepts (they refer to types: major, minor, seventh, diminished, augmented, etc.).

The concept of a mathematical function is understood as a correspondence between two sets A and B, where for every member x of A there is a single element y of B. A musical propositional function relates «individuals» to statements. Thus, for example, let it be the proposition 'i is a minor fifth' or 'i is a half note'. Thus, in the function $N: O \rightarrow S$, it can be understood that the domain O is, for example, the set of notes, and S a set of statements in which the concept N appears.

The single note of “A” can be interpreted in many ways, but by itself it says nothing: it can have the figuration of a quarter note, or a sixty-fourth note, etc., it can be constituted as a tonic (or fundamental), third (major or minor), fifth (major or minor), sixth, seventh (major or minor), ninth (augmented or diminished) etc. You can also define a chord type, for example, and from there you can interpret this chord as some tonal degree, and then that key as some region of some work, and so on.

It is true that natural language cannot be avoided to formulate statements about a musical work, but the words used in said statements must be defined by relating them to a particular musical theory. This is so because many terms are common currency in many musical theories, but in each one they acquire their own connotation, and being so the use of these terms will not have the same meaning in different theoretical contexts in which they appear with their consequent semantic derivations and logical.

In music three variables of a different nature can be distinguished simultaneously. In other words, the musical fact (phrase, chord, etc.) is an *object, property and space-time variable*.

In the linguistic expression that seeks to reference the musical phenomena of a specific work, the three classes of variables are presented in a single statement.

The musical analysis of a specific work will unfold on theoretical statements that have an objective reference (the theories of each case). Now, the analysis of a musical performance of a specific work will inevitably deal with how the cognitive subject (the musical performer) approaches or moves away from the "absolute" ideal (the written work and its lax references on a rigid physical time) based on in the relativity of its psychological pulse (tempo).

Musical formalism (the syntax of its grammar) is a concrete representation of musical facts and phenomena. This needs to be interpreted to define its references on a double level both as objectives (rhythmic patterns, metrical patterns in melody and harmony: all this constitutes transformations of the geometric or topological plane) and pragmatic (its execution or performance by the musician-interpret).

5. Discussion

Musical notation represents physical facts, that is, sounds or their absence; the musical sounds, that is, the musical notes (intervals) are the product of the convention concretized in a homogenized temperament. The musician systematizes and controls physical processes with sound, explores a feasible possibility.

The musician is ingenious because from musical rules he builds new structural solutions, by dint of coherence, of a logic that makes such concretion possible.

Music is a realistic art insofar as it models sound structures, but these are not figurative or referents of the reality of the world. In music, achievements or works that are materially impossible cannot be specified, since sound or its absence is a physical fact, and silence (due to lack of air) or total noise are not proper objects of music, since it creates its own (musical) objects and facts by convention.

Just as a mathematical object accompanied by a physical interpretation, for example, abandons what is strictly formal and pure, and becomes a construct with factual content, musical symbols refer to factual facts of sound (or their absence under specific rules). All musical fact can be modeled mathematically, this is possible because music and mathematics are based on common ideas, both developed their own language, now mathematics has achieved such sophistication, that today they can express any musical idea in a precise and clear way, hence today a whole specialized field of mathematized music, or if you prefer to call it musical mathematics, has flourished.

A musical work exists from the moment it is created, whether it is written (its writing potentially ensures its reproducibility) or performed (an improvisation, which can be recorded, for example). It is a creation on the fly. It ceases to exist when we don't think about it or hear it (when, for example, we read it in a score or engage in performance).

The existence of a musical work cannot be confused with its written or recorded copy. The score, as we have already said, only contains reference to how sounds (notes) or their absence should be produced: the writing of the same work can be done in different symbolic ways. The score is or contains the instructions on how musical phenomena or events are structured as stipulated by the author.

To affirm, for example, that "The Cathedral" by Agustín Barrios Mangoré[14] exists is a fiction. Musical works have the peculiarity that they are not suddenly apprehensible in their entirety, such as a pictorial painting or a photograph. In our memory we can evoke the passages of «La Catedral» bar to bar, or we can listen to those passages from a CD or the interpretation of a guitarist. To say that “La Catedral” exists is to pretend that it is so, since we will never see or hear “La Catedral” as a whole suddenly, but we will do it through a sequential and temporary process of acoustic events.

From this we can then say that a musical work exists (in the fictional sense) as long as there is a brain that can appreciate these acoustic phenomena as (conventional) musical facts. A musical work finds a substratum, on the one hand, as a form —as a message in a code (its writing)— and on the other hand, in the material, in its physical transmission.

It should be noted that the interpretation of a musical work occurs in two forms: a symbolic one, that is, in its syntactic and semantic origins (its musical writing), made possible through human cognition that will make the musician read it and at best understand it; at this stage the musical work is at its informative-structural level.

At the performance level, its origins respond to pragmatic criteria on the psychophysiological basis of the musician, an act in which he shows his sensations

[14] Oxley, 2010.

and emotions (his facial or body expressions could be symptoms of his momentary feelings).

On the other hand, we can also find musical propositions about music, that is, we refer to musical questions at the level of metalanguage. We limit ourselves to this level when we do musical criticism or philosophy of music, or we simply point out new terms or constructs to refer to certain musical phenomena or when we try to capture musical phenomena through metaphysical categories about it.

It is often said that musical pieces, or their interpretations, are happy, sad, etc. The classic referents that support expressionism in art and frequently cited are Tolstoy (1898), Dewey (1934) and Collingwood (1938).

> Thus Tolstoy in What Is Art? asserts that "art is a human activity consisting in this, that one man consciously by means of certain external signs, hands on to others feelings he has lived through, and that others are infected by these feelings and also experience them"[15].

When it comes to expressionism in music, the cliché that music is "the language of emotions" comments Kania (2017), is often considered as a possible starting point for a theory of musical expressiveness. The idea combines the attractive simplicity of conventionality that associationism makes the basis of music's meaning with the formalist notion that music's order is to be understood in terms of syntax. (See Lerdahl & Jackendoff 1983 for a theory along the latter lines.) However, although Deryck Cooke (1959) and Leonard Meyer (1956) are often cited as proponents, it is not clear that anyone holds a full-blown version of the theory. The central problem is the great disparities between language and music, in terms of the ways in which each is both syntactic and semantic (Jackendoff 2011). Several theorists have defended accounts of musical expressiveness known variously as resemblance, contour, or appearance theories (e.g., Kivy 1989, though see Kivy 2002: 31–48 for recent qualms; Budd 1995: 133–54; S. Davies 1994: 221–67). The central idea is that music's expressiveness consists in the resemblance between its dynamic character and the dynamic character of various aspects of people experiencing emotions. The aspects appealed to include the phenomenology of the experience of the emotion, the emotion's typical facial expression, the contour of vocal expression typical of a person experiencing the emotion, and the contour of bodily behavior typical of such a person, including "gait, attitude, air, carriage, posture, and comportment"[16]. Stephen Davies argues that such theories hold music to be expressive in a literal albeit secondary sense of the term. We say that a piece of music is sad in the same sense in which we say that a weeping willow is sad[17]. Such uses are no more metaphorical than a claim that a chair has arms. Jerrold Levinson agrees that there is an important

[15] Tolstoy, (1930), p.123; Grahan, (2013), p. 106.
[16] S. Davies, (2006), p. 182.
[17] *Ibid*., p. 183.

resemblance between the contour of music expressive of an emotion and the contour of typical behavioral expressions of that emotion. He objects, however, that such an account cannot be the whole, or even the most fundamental part of the story[18]. (https://plato.stanford.edu/archives/fall2017/entries/music/)

> Susanne Langer (1953) starts her discussion of expression in her book Feeling and Form by acknowledging that the terms "feeling" and "form" are themselves often taken to be antithetical to one another. She argues says Alperson (2004), that if we construe expression in music as the self-expression of the composer, we are faced with the implausible consequence that it is only the composer who could judge the expressive value of the work of art, since only he or she could judge the accuracy of the expression. If we say that expression consists in the evocation of psychological states in the listener, we reduce musical composition to a psychological ploy whose aims are not far removed from those of the advertising agent. We seem to be left with the paradoxical claim that the work itself is expressive of emotion (Langer 1953: chs 2 and 3) [...] she argues that music, like all art, functions symbolically to refer to ideas about the life of feeling [...] That is to say, music functions as a "presentational symbol": its tonally moving forms are a semblance or illusion of the feeling of time. The musical forms and tensions that we hear in music (tensions in tempi, loudness, orchestration, harmony and dissonance, counterpoint, etc.) share the same "logical form" with the moving patterns of physical, emotional, and intellectual tensions by means of which we experience the passage of time. Music, then, presents what Langer calls "musical" or "virtual" time, a directly felt audible illusion or image of "duration," or "psychological" time. On Langer's view, a piece of music just is "an organically developed illusion of time in audible passage" (Langer 1953: 134), and the performer is someone who makes crucial decisions about the final audible form of the work. The commanding form of the composition falls short of the full realization of the work. The performer completes the work, marshalling not only what Langer calls "sonorous imagination" and the "muscular imagination of tone" but also a qualified sense of "self-expression," the performer's "ardor" for the emotive import conveyed. These are all features of the performer's sense of "utterance" that contribute to the presentation of the illusion begotten by the sounds of the music. These last comments also provide an account of how it is that performers come to have particular repertoires, ranges of music for which the performer is temperamentally suited.[19]

On the one hand, we can say that children at the beginning of their school mathematics education, between the ages of 4 and 5, when they have the task of writing numerical references, do so according to their skills at the time. Thus, for example, Idiosyncratic responses will be given, pictographic, iconic, symbolic, symbolic-conventional and mixed notations.

[18] Levinson (1996a, 2006b).

[19] Langer 1953: chs 8 and 9; Langer 1953: chs 3, 4, and 7; Alperson, 2004: pp. 264-265.

idiosincrásicas	pictográficas	icónicas	simbólica		Simbólica e icónica	simbólico-convencionales	notaciones mixtas
1	2	3	4	5	6	7	8

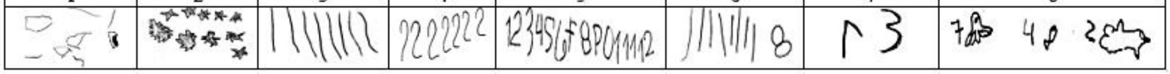


Table 01

Source: own elaboration from Cañellas & Rassetto (2013).

> Conventional notations increase with age, but even at age 8 not all notations are correct. The results indicate that a variety of numerical and notational ideas intervene in: the understanding of cardinality, the counting sequence, the correspondences between the oral mode and the written mode, the notational regularities, the place value. Transcription rules are slowly and painstakingly elaborated.[20]

On the other hand, following our argument, intuitively, -the mathematical expert will tell us- natural numbers are those obtained by adding 1 to itself: $1, 1 + 1 = 2, 1 + 1 + 1 = 3$, etc. The process does not stop and produces larger and larger numbers: $1 < 2 < 3 < \ldots$ A set $A \subset \mathbb{R}$ is said to be inductive when it satisfies the following two conditions: (i) $1 \in A$ (ii) $x \in A \Rightarrow x + 1 \in A$. Thus, for example, $\mathbb{R}$ and $\mathbb{R}+$ are inductive sets, $\mathbb{R}-$ and $\mathbb{R}*$ are not. The set $\mathbb{N}$ of natural numbers is, by definition, the intersection of all inductive subsets of $\mathbb{R}$. $\mathbb{N}$ is an inductive set. On the one hand, 1 belongs to all the inductive subsets of $\mathbb{R}$, so $1 \in \mathbb{N}$. On the other hand, if $n \in \mathbb{N}$ and A is an inductive subset of $\mathbb{R}$, we have that $n \in \mathbb{N} \subset A$, then also $n + 1 \in A$, since A is inductive. We thus see that $n + 1$ belongs to all the inductive subsets of $\mathbb{R}$, that is, $n + 1 \in \mathbb{N}$, as desired. We could say that $\mathbb{N}$ is the smallest of all the inductive subsets of $\mathbb{R}$, since it is contained in all of them.

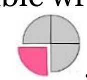
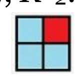
How much distance there is between the crude symbols and their stammering uses in the precocious child and the magnanimous affirmation of the sophisticated use of mathematical formalisms by the expert who dominates the mathematical language, and not to mention the qualitative depth of the ideas, that behind those signs that refer are found, both in some and in others. Contrasting novice mathematics apprentices with those who dominate their territories and borders, many questions assail us. How do the professional mathematician and the primary school child understand the number? What is the process in which the use of the language of mathematical science is acquired? Questions like the above and many more we could ask ourselves, and trying to answer them will immerse us in the complex universe of Philosophy in terms of mathematical ontology and epistemology, as well as what these philosophical disciplines imply in terms of what they have to say to Mathematics Education. Now, on the other hand, how do

[20] Cañellas & Rassetto (2013), pp. 12-13.

the professional musician and the ordinary passive listener understand the meaning of music?

According to Duval (1993), a semiotic system can be a register of representation, if it allows three cognitive activities related to semiosis: 1) The presence of an identifiable representation, 2) The treatment of a representation that is the transformation of the representation within the same register where it has been formulated, 3) The conversion of a representation that is the transformation of the representation into another representation of another register in which all or part of the meaning of the initial representation is preserve. That is, with two types of dissimilar registers, with different representations.

Thus, we can write: $r^m =_{df}$ m-th semiotic register. $R^m{}_i\ (A) =_{df}$ semiotic representation of a concept A in the semiotic register r^m (m = 1, 2, 3, ...; i = 1, 2, 3, ...). Concept A to represent. Choice of distinctive features of A. Representation of A. $R^m{}_i\ (A)$ in a given semiotic register r^m. New representation ($h \neq i$, $h \neq j$): $R^m{}_j\ (A)$. In another semiotic register r^m ($n \neq m$) (m, n, i, j, h = 1, 2, 3, ...)[21].

You can have different semiotic representations of an object. Thus, for example, the concept of 'fractional number'. This for example can be referenced in r^1: common language ($R^1{}_1$: quarter, $R^1{}_2$: half of half), r^2: arithmetic language [$R^2{}_1$: ¼ (fractional writing), $R^2{}_2$: 0,25 (decimal writing), $R^2{}_3$: 25×10^{-2} (exponential writing)], r^3: Algebraic language [$R^3{}_1$: $\{x \in Q^+ / 4x - 1 = 0\}$ (ensemble writing), $R^3{}_2$: $y = f(x)$: $x \to 4\,x$ (functional writing)], r^4: graphic schemes ($R^4{}_1$: , $R^4{}_2$: ) etc.

Assuming the pragmatic sense of the construction of meaning, that is, the process by which an organism attributes a meaning to a sign, we understand that the meaning that music can have for a common listener or a professional musician is not the same, hence that it is not lawful to impose a "particularized" vision of incomplete meaning, such as the one that a common music listener has in comparison with the "generalized" conception of music that a professional musician will have, take into account our example of the concept of number from the school child who initially confuses the material object (the representation) with the numbers (the object that is represented) and from the professional mathematician, it is evident that we would not ask the child: What is mathematics or what is mathematics about? and then generalize his answer as an adequate description of mathematics as a cultural object, for more than obvious reasons.

6. Conclusion

On the other hand, for analytical purposes we can assume that a propositional function is a function whose values (members of the second set) are propositions. A predicate or attribute can be analyzed as a function that matches individuals of

[21] D'Amore, (2006) p. 10.

one class with propositions of another, and this other set is the set of all propositions that contain the attribute or predicate that we postulate or define. Following Mario Bunge[22] and exemplifying the above: Changing *C*: Concrete objects → Propositions containing *C* (e.g, "The atmosphere is changing") Adapted *A*: Set of all pairs (organism, environment) → Statements containing *A* (e.g, "Penguins are adapted to Antarctic"). In the cited examples the following relationships are given. If a belongs to *A* (or to ∈ *A*), then a is an individual and *A* is a set. (It does not matter that a may itself be a set: with respect to *A* it will be an individual.) If *A* and *B* are sets, then *R* is a relation from *A* to *B* if and only if, for every element or member *a* of *A*, there is at least one member *b* of *B* such that *R* links a to *b*, that is, such that the proposition "*Rab*" is true. The set of such pairs, that is, $E(R)= \{<a, b> \in A \times B \mid Rab\}$, is called the extension of *R*. In general, the function can be defined as follows: let *f* be a member of the class of relations from *A* to *B*. Then *f* is a function from *A* to *B* if and only if *f* assigns to each member *a* of *A* exactly one member of *B*, namely *f*(*a*). Assuming that we can say that material and conceptual objects exist, we can say that any attribution of properties that correspond to conceptual objects to physical objects, and attributing properties that correspond to physical objects to conceptual ones is a procedure that falls into a lack of consistency, since ontologically these Objects are not on the same plane, so the properties of one or the other are misattributed or ontologically misformed predications. Examples such as stating that "an electromagnetic field is an antisymmetric tensor" instead of "an electromagnetic field can be represented by an antisymmetric tensor" or "natural numbers are very old" instead of "man invented natural numbers in prehistory" they fall into what we call misattributed or ontologically ill-formed predicates, or in our analysis saying "music is an expression of feelings" is predicative nonsense based on an inadequate ontological understanding, since music is not a human being, only these can have complex emotions.

If we call r^{m}_{i} (*M*) $=_{df}$ m-th semiotic representation of a concept *M* (*M* can refer to the concept of music) in the semiotic register r^{m}_{i} ($m = 1, 2, 3, \ldots; i = 1, 2, 3, \ldots$), y r^{1}_{1} (*M*) to the semiotic register of the common music listener, based on common sense (the least common of the senses) expressed in common language, and r^{2}_{1} (*M*) to the semiotic register of the professional musician we can distinguish with clarity, how both can express what the music transmits to them. There is no doubt that the professional musician will find many resources in the language of music for all that structural complexity that comes to him through sounds and will be able to describe them in detail as the meaning that he finds in the musical work; Now, what response can the common listener try on the same work, he can only expand from what he manages, the common language and the analogies that this allows him, which is the same as saying that he expressed himself in emotional senses, as a familiar form of expression your experience. From the above it is very clear that r^{1}_{1} (*M*) is erroneous, because if *a* is the music construct *M* and *b* the attribute of

[22] Bunge (1980) p. 58.

being some emotion *E*, as we saw the *Rab* is false for attributing a predicate that does not correspond ontologically, since emotion is a state psychobiological characteristic of human beings, and this property is not attributable to music.

The musical work does not transmit emotions, it transmits "information" about how musical ideas are structured within itself, how it is itself. This statement does not deny that it is capable of producing "emotions" in the receiver, but as a subjective experience in him, not as something that reaches him objectively (music is not a sentient being, much less a being that can infect us psychobiological pathologies), from the sounds he perceives in a work.

R.A. Sharpe (2000) suggests that our emotional responses to music are a much smaller component of our understanding experience of it than the philosophical literature on the topic would suggest -Kania (2017) comments-. (see also Zangwill 2004). Peter Kivy (1999) arguing that those who report emotional reactions to music are confusing the pleasure they take in the beauty of the music, in all its expressive individuality, with the feeling of the emotion expressed. (https://plato.stanford.edu/archives/fall2017/entries/music/).

Given these results and addressing the very common and widespread opinion that music conveys feelings, we can say that the error of the "emotivists" in music in general is that, by emphasizing the physical medium of how the work is presented, her performance —as the “message” was said— gets lost in the confusion of levels of “interpretation”. The "emotivists" mistakenly only focus on how the message is physically presented and forget the meaning of the message, that is, what is "what is said".

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