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Multifrequency nonlinear pulse propagation

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Abstract: The nonlinear coefficient dependence on multiple frequencies is rigorously incorporated into the propagation equation so that the resulting nonlinear term is still straightforwardly computed. Readily observable consequences due to this multifrequency disper-sion are predicted.

Frequency-dependent nonlinearities are catching a growing attention because of the major impact that they may have on pulse propagation [1,2]. For example, supercontinuum (SC) instabilities in the ps regime can be cancelled due to the dispersion of the nonlinear coefficient [2]. This result, in particular, has aroused considerable interest [3] as SC driven by ps pulses had been considered intrinsically incoherent during two decades [4]. Accouting for the nonlinear-coefficient dispersion rigorously, however, is not trivial because of its multifrequency character. Consequently, the nonlinear term in the propagation equation could not be calculated using Fourier transforms $\mathscr{F}$, leading to a much higher computational load [5] or a formal solution with a limited applicability [6].

Two approximations have been proposed so far to deal with the nonlinearity dispersion maintaining an efficient modelling. The generalized nonlinear Schrödinger equation (GNLSE) treats the nonlinear coefficient γ as a function of a single frequency, similarly to the propagation constant β [4]. This model is widely employed, although it does not generally conserve the photon number and the energy [7]. These flaws can be amended by factorizing heuristically the multifrequency dispersion of the nonlinearity [7]. However, lacking a derivation, this method does not possess a well-defined scope, nor paves it the way for any further improvement.

The propagation equation of a single-mode electric field undergoing four-wave mixing (FWM) interactions can be written as

$$\partial_z \tilde{\Psi}(z,\Omega) = i\beta_p(\Omega)\tilde{\Psi} + i\frac{\eta(\Omega)}{(2\pi)^2}\int \mathrm{d}^3\Omega\, \Gamma^{(3)}(\Omega,\Omega_1,\Omega_2,\Omega_3)\tilde{\Psi}(\Omega_1)\tilde{\Psi}^*(\Omega_2)\tilde{\Psi}(\Omega_3)\,\delta(\Omega-\Omega_1+\Omega_2-\Omega_3), \quad (1)$$

where $\tilde{\Psi} = (N(\omega_0)/N(\omega))^{1/2}\mathscr{F}A(z,T)$, with A the complex envelope of the pulse [4] and N the normalization constant of the mode [5,6], ω_0 is the carrier frequency, $\Omega = \omega - \omega_0$, $\beta_p = \beta(\omega) - \beta(\omega_0) - \mathrm{d}\beta/\mathrm{d}\omega|_{\omega_0}\Omega$, $\eta = (\omega/c)N(\omega_0)/N(\omega)$, $\mathrm{d}^3\Omega = \mathrm{d}\Omega_1\mathrm{d}\Omega_2\mathrm{d}\Omega_3$, and $\Gamma^{(3)} = 3\,\varepsilon_0 c/(4N^2(\omega_0))\int \mathrm{d}^2\mathbf{x}_t\, \chi^{(3)}_{ijkl}\, e_i(\omega)e_j(\omega_1)e_k(\omega_2)e_l(\omega_3)$, with $e_i = e_i(\mathbf{x}_t,\omega)$ the i-component of the mode (assumed to be transverse and real) [8]. When the frequency dependence of $\Gamma^{(3)}$ is neglected, the multiple integral in Eq. (1) can be efficiently computed as $\mathscr{F}(\Psi^*\Psi^2)$ [5]. Here a rigorous description of the multifrequency $\Gamma^{(3)}$ allowing a practical evaluation of the nonlinear term is built.

If the fields in $\Gamma^{(3)}$ are expanded in Taylor series to first order around the *mean frequency of each FWM* contributing to Eq. (1), $\Omega_m = (\Omega+\Omega_1+\Omega_2+\Omega_3)/4 = (\Omega+\Omega_2)/2$ (where energy conservation has been applied), then $\Gamma^{(3)}(\Omega,\Omega_1,\Omega_2,\Omega_3) \approx \Gamma^{(3)}(\Omega_m,\Omega_m,\Omega_m,\Omega_m)$ since the first-order term is null [8]. Note that neither a global linear mode dispersion is here assumed nor is this approach necessarily restricted to a first-order local accuracy. Under this approximation, Eq. (1) can be recast as the multifrequency nonlinear Schrödinger equation (MFNLSE) [8],

$$\partial_z \tilde{\Psi}(z,\Omega) = i\beta_p(\Omega)\tilde{\Psi} + i\eta(\Omega)\mathscr{F}\,\Psi^*\,\mathscr{F}^{-1}\Gamma^{(3)}\Big(\frac{\Omega'}{2}\Big)\mathscr{F}\Psi^2, \quad (2)$$

where the notation $\Gamma^{(3)}(\Omega) = \Gamma^{(3)}(\Omega,\Omega,\Omega,\Omega)$ has been introduced, and Ω' indicates the frequency of the Fourier transforms enclosing $\Gamma^{(3)}$. Besides keeping the computational advantages of the GNLSE, the MFNLSE also conserves the number of photons (even in the presence of Raman scattering) and the pulse energy [8].

To evaluate the accuracy of this proposal under relatively large mode dispersion, the LP_{02} mode of a step-index fiber with a core diameter of $13\,\mu$m and a numerical aperture of 0.2 is considered. For a degenerate FWM

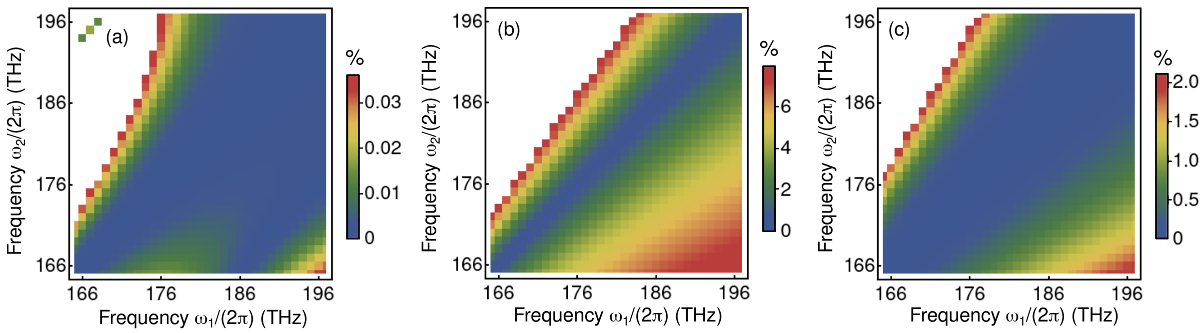

Fig. 1. Relative errors of (a) $\Gamma^{(3)}(\Omega_m)$, (b) $\Gamma^{(3)}(\Omega)$ and (c) $(\Gamma^{(3)}(\Omega)\Gamma^{(3)}(\Omega_1)\Gamma^{(3)}(\Omega_2)\Gamma^{(3)}(\Omega_1))^{1/4}$. Note the different scales of the color bars in each case.

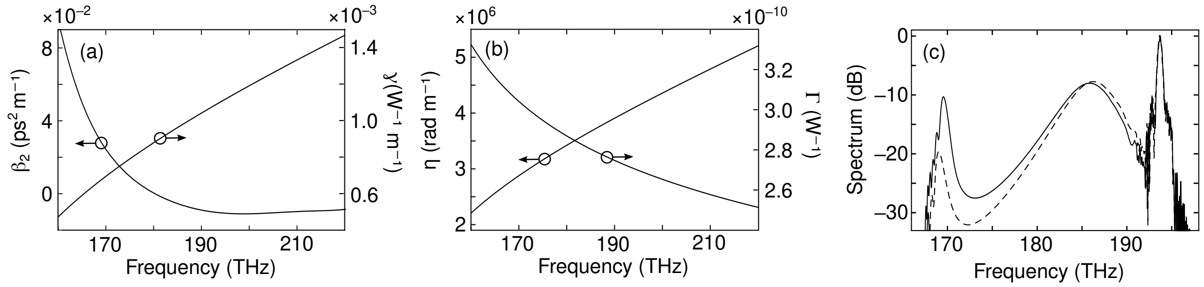

Fig. 2. (a) GVD and GNLSE nonlinear coefficient, and (b) MFNLSE nonlinear-term coefficients corresponding to the LP_{02} mode of the step-index fiber under test. (c) Spectra predicted by the GNLSE (dashed line) and the MFNLSE (solid line) at 22 m using the curves in (a) and (b).

$2\omega_1 \to \omega_2 + \omega$, the relative errors between $\Gamma^{(3)}(\Omega, \Omega_1, \Omega_2, \Omega_1)$ and $\Gamma^{(3)}(\Omega_m)$, see Fig. 1(a), $\Gamma^{(3)}(\Omega)$, see Fig. 1(b), and $(\Gamma^{(3)}(\Omega)\Gamma^{(3)}(\Omega_1)\Gamma^{(3)}(\Omega_2)\Gamma^{(3)}(\Omega_1))^{1/4}$, see Fig. 1(c), are calculated. Despite the color maps do not cover all frequencies to preserve meaningful pictures, they are representative as the mean relative errors including all frequencies are 0.024%, 45% and 3.2%, respectively. Similar conclusions follow from a nondegenerate FWM [8].

Finally, the impact of a multifrequency $\Gamma^{(3)}$ on pulse propagation is explored solving numerically both GNLSE and MFNLSE for a 0.5-ps-long sech-pulse with 1 kW of peak power pumped at 1.55 μm ($\sim$ 193 THz) in anomalous group-velocity dispersion (GVD) but with a zero GVD at $\sim$ 180 THz, see Fig. 2(a). The GNLSE and MFNLSE nonlinear coefficients are plotted in Fig. 2(a) and Fig. 2(b), respectively. Raman scattering is included in both cases [8]. Attending to Fig. 2(c), the multifrequency dispersion of $\Gamma^{(3)}$ can enhance significantly the efficiency of dispersive wave emission. Based on these results, Eq. (2) holds potential to displace the GNLSE and to become a very useful tool to explore the research opportunities offered by multifrequency nonlinearities.

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