

LEPTON NUMBER VIOLATION AND SCALAR SEARCHES AT THE LHC*

F. DEL AGUILA^{a†}, M. CHALA^{a‡}, A. SANTAMARIA^{b§}, J. WUDKA^{c¶}

^aDepartamento de Física Teórica y del Cosmos and CAFPE
Universidad de Granada, 18071 Granada, Spain

^bDepartament de Física Teórica and IFIC, Universitat de València — CSIC
Dr. Moliner 50, 46100 Burjassot (València), Spain

^cDepartment of Physics and Astronomy, University of California
Riverside, CA 92521-0413, USA

(Received October 25, 2013)

We review the SM extensions with scalar multiplets including doubly-charged components eventually observable as di-leptonic resonances at the LHC. Special emphasis is paid to the limits on LNV implied by doubly-charged scalar searches at the LHC, and to the characterization of the multiplet doubly-charged scalars belong to if they are observed to decay into same-sign charged lepton pairs.

DOI:10.5506/APhysPolB.44.2139

PACS numbers: 11.30.Fs, 12.60.Fr, 13.15.+g, 14.60.St

1. Introduction

The Standard Model (SM) provides a quite precise description of particle physics up to the electro-weak (EW) scale. In particular, ATLAS [1] and CMS [2] have recently established the Brout–Englert–Higgs mechanism [3, 4] providing to the SM fields a mass through the discovery of the Higgs boson, the last particle predicted by the SM. Moreover, not only no vestige of new physics (NP) has showed up at the LHC up to now, but EW precision data

* Presented by F. del Aguila at the XXXVII International Conference of Theoretical Physics “Matter to the Deepest” Ustroń, Poland, September 1–6, 2013, and at “From Majorana to LHC: Workshop on the Origin of Neutrino Mass”, Trieste, Italy, October 2–5, 2013, and by M. Chala at “Scalars 2013”, Warszawa, Poland, September 12–16, 2013.

† faguila@ugr.es

‡ miki@ugr.es

§ Arcadi.Santamaria@uv.es

¶ jose.wudka@ucr.edu

(EWPD) are in good agreement with the SM predictions to the radiative correction level [5, 6]. The SM also has two accidental global symmetries, baryon (B) and lepton (L) number: both anomalous but not their difference $B - L$. One may then wonder if they are also exactly realized in Nature, as predicted by the SM (up to non-perturbative effects). If broken, they are only very tiny violated. In fact, if the proton decays, B number would be broken, but we know that the proton mean life is extremely long $\tau_p > 2.1 \times 10^{29}$ yr at 90% C.L. [7]. The observed B asymmetry of the universe is also quite small $\eta \sim 10^{-10}$ [7], as it is the B number violation required to explain it if this is actually its origin. Similarly, L number (LN) is only very tiny broken if it is not exact. The only low energy process which might provide conclusive evidence of LN violation (LNV), neutrinoless double beta decay ($0\nu\beta\beta$), has not been undoubtedly observed $\tau_{\frac{1}{2}}(^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^-) > 1.9 \times 10^{25}$ yr at 90% C.L. [7]. Besides, if the B asymmetry is due to leptogenesis [8] and hence to LNV, its amount at low energy should be rather small, too.

As a matter of fact, the only conclusive departure from the SM is neutrino oscillations [7], what is explained by introducing neutrino masses in the model, but neutrinos may be Dirac, and LN conserved, or Majorana and, in this case, neutrino masses would provide the only conclusive evidence of LNV up to now. But this should be eventually tested through the consistency with other related experimental results as, for instance, the observation of $0\nu\beta\beta$ with half-life $\gtrsim 10^{26}$ yr for $|m_{ee}| \sim \text{few per cent eV}$ [9]. Although even if $0\nu\beta\beta$ is observed, the main contribution to this process could have a different source [10, 11]. However, at the LHC era the question is which is the LHC potential for observing LNV signals, as no B number violating (model independent) signature can emerge at the LHC.

In the following, we want to argue first that although the observation of LNV at the LHC [12] may be problematic, it is quite plausible in definite models and parameter space regions. As a matter of fact, LHC is especially sensitive to doubly-charged scalars with two-body decays into leptons [13, 14], which is the case we will concentrate on. Then, we review the expected limits on LNV in the first and in the next LHC run, and how to determine the type of multiplet the doubly-charged scalar belongs to.

In general, due to the smallness of LNV in low energy processes involving only SM particles LNV effects must be banished to high energy or to a secluded sector, which may or not gauge LN^1 . In the usual scenario with a new heavy sector with large LNV couplings, the effective operators obtained

¹ An interesting type of models gauging B and L is described in [15]. They include two new Z 's coupling to quarks and leptons, respectively, but their interactions do not violate LN in the SM sector. Right-handed (RH) neutrinos get Majorana masses, not necessarily heavy, through the vacuum expectation value (VEV) of a scalar singlet breaking LN; whereas LNV remains small in the SM processes.

by integrating the heavy modes out and hence describing the NP but only involving SM particles are suppressed by the corresponding power of the large effective scale Λ^n . The lowest dimensional and formally dominant operator being the Weinberg operator $\mathcal{O}^{(5)} = (\overline{L}_L^c \tilde{\phi}^*)(\tilde{\phi}^\dagger L_L)$ [16]² which only gives Majorana masses to the SM neutrinos and couples the Higgs to neutrino pairs with total $|\text{LN}| = 2$, although with very small LNV effective couplings.

The simplest SM extensions generating this operator are the see-saw of type I [17], II [18] and III [19]. Type I and III are mediated by fermions transforming as an EW singlet and triplet, respectively, and type II by a scalar triplet. If these messengers have masses near the TeV, appropriate combinations of LNV couplings must be effectively small, conspiring not to provide the SM neutrinos too large a mass. As the production mechanism cannot be suppressed if the extra heavy particles have to be produced copiously enough at the LHC, LN must be violated in their decays. This, in turn, implies that there must be at least two different channels, and none of them can dominate if LNV has to be observable. This restricts the model; what, in general, may appear to require fine tuning. But there are models where this seems not to be severe and, in any case, we must assume this to be the case if we want to search for genuine LNV signals.

For instance, in the type II case the scalar triplet Δ is (pair and associated) produced with EW strength for it transforms non-trivially under the $\text{SU}(2)_L \times \text{U}(1)_Y$ transformations; and can decay into lepton and boson pairs for it couples to two identical (neglecting family replication) lepton doublets (which defines its $|\text{LN}| = 2$) and to gauge boson pairs (with $\text{LN} = 0$) if its neutral component gets a VEV, $\langle \Delta^0 \rangle \neq 0$, breaking LN. Thus, if the Yukawa coupling is too large, the triplet components always decay into two leptons (diagram (a) in Fig. 1) [20]; and if it is very small and $\langle \Delta^0 \rangle$ large enough, then their decay is always into two gauge bosons (diagram (b) in Fig. 1).

The LNV process with each of the two pair (associated) produced scalars decaying in a different mode (diagram (c) in Fig. 1) is highly suppressed in both extreme cases. As in Ref. [21], we plot in Fig. 2 (left) the two-body branching ratios for the doubly-charged scalar component as a function of the effective di-boson coupling (equal to $g^2 \langle \Delta^0 \rangle$ if the scalar multiplet is a triplet) properly normalized (divided by g^2). As it is apparent, only near $\langle \Delta^0 \rangle \approx 5.5 \times 10^{-5}$ both decay rates are comparable and hence genuine LNV signals eventually observable at the LHC. This value is fixed in the see-saw of type II because the neutrino masses are proportional to both the Yukawa couplings and to $\langle \Delta^0 \rangle$, and hence can be related to the messenger (doubly-charged scalar) decays into leptons, and into bosons if we require

² $L_L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$ and $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ are the SM lepton and Higgs doublets, respectively.

both branching ratios to be comparable [21, 22]:

$$\sum_{i,j=e,\mu,\tau} \Gamma(\Delta^{\pm\pm} \rightarrow l_i^{\pm} l_j^{\pm}) = \frac{m_{\Delta}}{8\pi} \frac{\sum_{i=1,2,3} m_{\nu_i}^2}{4 \langle \Delta^0 \rangle^2},$$

$$\Gamma(\Delta^{\pm\pm} \rightarrow W^{\pm} W^{\pm}) = \frac{g^4 \langle \Delta^0 \rangle^2}{32\pi} \frac{m_{\Delta}^3}{m_W^4} \quad \text{in the limit } m_W^2 \ll m_{\Delta}^2. \quad (1)$$

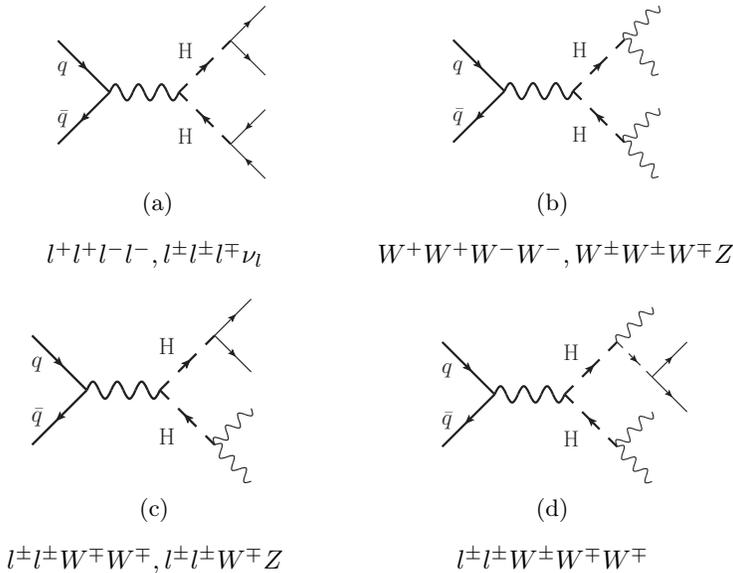


Fig. 1. (a), (b), (c): Doubly-charged scalar pair ($H^{++}H^{--}$) and associated ($H^{\pm\pm}H^{\mp}$) production diagrams. (d): Associated production with a triply-charged scalar ($H^{\pm\pm\pm}H^{\mp\mp\mp}$) when the multiplet also has a component with charge $|Q| = 3$.

Thus, having fixed the scalar mass $m_{\Delta^{\pm\pm}} = 500$ GeV and the sum of the three SM neutrino masses $\sum_{i=1,2,3} m_{\nu_i}^2 = 0.1^2$ eV², and assuming both decay rates equal, we can determine for which $\langle \Delta^0 \rangle$ both curves cross in Fig. 2 (left).

These relations need not apply in more elaborated SM extensions but realistic models must accommodate the correct SM neutrino masses which, in general, constrains the size of LNV in the SM sector. In the see-saw of type II, it is enough to satisfy the limits on neutrino masses to fulfil the bounds on $0\nu\beta\beta$, but this is not the case in general [10, 11]. The corresponding constraint implied by the limit on $0\nu\beta\beta$, as well as the EW limit on $\langle \Delta^0 \rangle$ not to upset the bounds on the ρ parameter, or the requirement of producing enough leptogenesis if this is the origin of the observed B number asymmetry of the universe, must be also satisfied by realistic SM extensions.

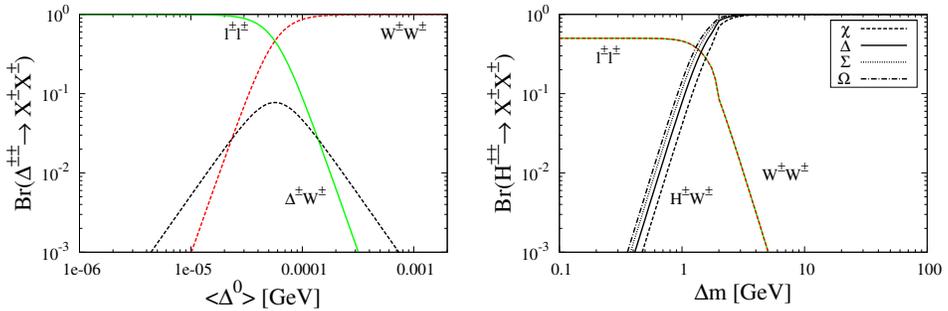


Fig. 2. Left: Scalar branching ratios for the triplet Δ as a function of $\langle \Delta^0 \rangle$ for $\sum_{i=1,2,3} m_{\nu_i}^2 = 0.1^2 \text{ eV}^2$ and $m_{\Delta^{\pm\pm}} = 500 \text{ GeV}$, and $\Delta m = 1 \text{ GeV}$ for the decay into $\Delta^{\pm}W^{\pm*} \rightarrow \Delta^{\pm}e^{\pm}\nu_e, \dots$. Right: Scalar branching ratios for different H multiplets as a function of $\Delta m = m_{H^{\pm\pm}} - m_{H^{\pm}}$ for $m_{H^{\pm\pm}} = 500 \text{ GeV}$ and $\Gamma(H^{\pm\pm} \rightarrow l^{\pm}l^{\pm}) = \Gamma(H^{\pm\pm} \rightarrow W^{\pm}W^{\pm})$ and the $HW_{\mu}W^{\mu}$ coupling equal to $5.5 \times 10^{-5}g^2 \text{ GeV}$.

In order to observe LNV at the LHC, not only the two types of couplings involved in the process must be of the same order but no other messenger decay can be much larger. This further constrains the model, restricting the mixing of the new heavy multiplets with other scalars to small values. Taking again the see-saw of type II as an example, we plot in Fig. 2 (right) the $\Delta^{\pm\pm}$ branching ratios into $l^{\pm}l^{\pm}$ and $W^{\pm}W^{\pm}$ assuming that they are equal and into $\Delta^{\pm}W^{\pm*}$ as a function of the mass splitting between contiguous components $\Delta m = m_{\Delta^{\pm\pm}} - m_{\Delta^{\pm}}$, which completely dominates for differences larger than the GeV [21, 23]³. On the left, we assume $\Delta m = 1 \text{ GeV}$ (and $m_{\Delta^{\pm\pm}} = 500 \text{ GeV}$). Then, in general, the mixing between different scalar multiplets must be rather small.

These considerations have different consequences when the mediator is a heavy Majorana neutrino transforming as a singlet or as a member of a triplet. In both cases, the two different decay modes are conjugated from each other and hence of equal size. The required suppression for a TeV messenger comes in this case from small mixing angles or large interferences as in the quasi-Dirac case [24]. As singlets must be produced through mixing, the LHC reach is very much reduced [25]⁴. In any case, the picture can be modified as soon as a new layer of NP is reached. For example, if parity is restored at higher energy [28] and RH charged gauge bosons with several TeV

³ $\Gamma(H^{\pm\pm} \rightarrow H^{\pm}W^{\pm*} \rightarrow H^{\pm}e^{\pm}\nu_e, \dots) = n \frac{g^4 T}{240\pi^3} \frac{\Delta m^5}{m_W^4}$ for small Δm , where n is the number of open lepton and quark channels (5 for $\Delta m < m_{\tau}, \dots$) and T the total isospin (0 for $\kappa, \frac{1}{2}$ for $\chi, 1$ for Δ, \dots).

⁴ Even if vector-boson fusion contributions are large [26], present limits on lepton mixing make difficult to observe this process [6, 27].

masses are produced, their LNV decays can be unsuppressed and observable [29]. The phenomenological constraints on neutrino oscillations, $0\nu\beta\beta$ and leptogenesis are still present but the suppression factors combine several effects which can enter differently in the LHC production process.

Having discussed the difficulties for observing LNV directly at the LHC, let us follow the optimistic approach and consider how large the LHC reach can be. If the new sector does not have strong interactions and mixes little, the LNV mediator has to be pair produced and hopefully with EW strength. Moreover, in order to ease its reconstruction, it must resonate in two-body channels because then final products will have larger momenta, much less probable within the SM. These conditions uniquely characterize doubly-charged scalars, but not the full multiplet H they belong to. They can be part of a triplet, as in the see-saw of type II, or of an EW multiplet with arbitrary isospin [30, 31]. In the following, we compare the LHC potential for the different EW multiplet quantum number assignments. For a general multiplet, the larger the total isospin T and hypercharge Y , the larger is the charge $Q = T_3 + Y$ that the components can have. We will restrict ourselves to multiplets for which the highest charge is 2 ($|Q_{\max}| = 2$): $T_Y = \mathbf{0}_2$ (singlet κ), $\frac{1}{2}_{\frac{3}{2}}$ (doublet χ), $\mathbf{1}_1$ (triplet Δ), $\frac{3}{2}_{\frac{1}{2}}$ (quadruplet Σ), $\mathbf{2}_0$ (quintuplet Ω). Multiplets with higher charges [32] can have striking signatures, $H^{\pm\pm\pm} \rightarrow H^{\pm\pm}W^\pm \rightarrow \ell^\pm\ell^\pm W^\pm$ (see, for example, Fig. 1 (d)) but the momenta of the final products are smaller and one must identify doubly-charged resonances in any case, although the total cross-section is, in general, also larger for larger T (it also depends on T_3 and Y [31]). Not only the production cross-sections but the decays within the multiplet depend on the component quantum numbers. In Fig. 2 we also plot the $H^{\pm\pm} \rightarrow H^\pm W^{\pm*}$ branching ratio, which grows with T , for the different multiplets the doubly-charged scalar can belong to above.

2. LHC bounds on doubly-charged scalar masses and their LNV signals

CMS [13] and ATLAS [14] have searched for doubly-charged scalars decaying into electrons and muons, setting stringent bounds on their mass. These, however, are very much dependent on the doubly-charged scalar branching ratios, as it is apparent from the CMS analyses allowing for $\Delta^{\pm\pm}$ decays into tau leptons. Obviously, this is even more dramatic if the gauge boson channel $W^\pm W^\pm$ is also sizeable, which is compulsory in order to observe LNV, as emphasized above. To perform searches for LNV signals mediated by doubly-charged scalars at the LHC, $pp \rightarrow H^{\pm\pm}H^{\mp\mp}$, $H^{\pm\pm}H^\mp \rightarrow \ell^\pm\ell^\pm W^\mp W^\mp$, $\ell^\pm\ell^\pm W^\mp Z$, we have implemented the corresponding models in MadGraph5 [33], which can be obtained under request [34].

(A full description of the Monte Carlo implementation and the analysis described in this section can be found in Ref. [31].) Using it, we can mimic the CMS analyses and estimate the corresponding bounds. In Table I, we collect in the first row, for $\sqrt{s} = 7$ TeV and $\mathcal{L}_{\text{int}} = 4.9 \text{ fb}^{-1}$, the corresponding limit on $m_{\Delta^{\pm\pm}} \sim 400$ GeV, assuming that doubly-charged scalars decay 100 % of the time into $\ell^\pm \ell^\pm$, $\ell = e, \mu$. Obviously, this bound depends on the (pair) production cross-section which in turn depends on the EW multiplet the doubly-charged scalar belongs to. In the same row, we quote the corresponding bounds for the scalar multiplets H with no components of higher charge. As the cross-section grows with T , so does the limit. In the other two rows below, we estimate the bounds for $\sqrt{s} = 8$ and 14 TeV and $\mathcal{L}_{\text{int}} = 20$ and 100 fb^{-1} , respectively [31]. For the three energies, we apply the same cuts as CMS for 7 TeV. Certainly, these cuts will be optimized by the LHC collaborations for higher energies and hence the corresponding bounds improved, but they should not differ much from our estimates if no event excess is observed.

TABLE I

Estimated limits on the cross-section and on the corresponding scalar mass $m_{H^{\pm\pm}}$ [GeV] as a function of the multiplet, it belongs to, from LHC searches for doubly-charged scalars. The $\ell^\pm \ell^\pm \ell^\mp \ell^\mp$ analysis assumes that $H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm$ 100% of the time; whereas the other two analyses assume a 50% branching ratio for each decay mode of $H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm, W^\pm W^\pm$ and of $H^\pm \rightarrow \ell^\pm \nu_\ell, W^\pm Z$.

\sqrt{s} [TeV], \mathcal{L}_{int} [fb ⁻¹]	Isospin _{hypercharge}				
	0₂	$\frac{1}{2}_{\frac{3}{2}}$	1₁	$\frac{1}{2}_{\frac{3}{2}}$	2₀
	$m_{H^{\pm\pm}}$ [GeV] bounds from $\ell^\pm \ell^\pm \ell^\mp \ell^\mp$				
7 , 4.9	340	350	395	450	490
8 , 20	480	490	550	610	665
14 , 100	900	915	1030	1140	1230
	$m_{H^{\pm\pm}}$ [GeV] bounds from $\ell^\pm \ell^\pm W^\mp W^\mp$				
7 , 4.9	< 200	< 200	< 200	230	335
8 , 20	240	260	340	415	475
14 , 100	540	570	720	850	940
	$m_{H^{\pm\pm}}$ [GeV] bounds from $\ell^\pm \ell^\pm W^\mp Z$				
7 , 4.9	—	< 200	< 200	< 200	< 200
8 , 20	—	210	280	330	360
14 , 100	—	470	620	720	780

This analysis can be extended to scalar decays into gauge bosons and hence to LNV signals. We collect the corresponding bounds on $\ell^\pm\ell^\pm W^\mp W^\mp$ and $\ell^\pm\ell^\pm W^\mp Z$ production in the second and third sets of rows of the table, respectively, for the different LHC runs and doubly-charged scalar multiplet assignments [31]. We assume for these analyses the same set of cuts applied by CMS for Δ decaying into $\ell\tau$ at 7 TeV, and that the number of observed events coincides with the background estimate. In both channels, we also assume that the heavy scalars decay 50% of the time in each of the two modes (electron or muon and gauge boson pairs), being all other possible decay modes negligible, especially the cascade decays within the scalar multiplet. These analyses are based on four and three-lepton samples [13]. Although the estimates for LNV only use the three-lepton sample, which is much more sensitive than the four-lepton one to final modes involving also gauge bosons for pair and associated scalar production, with efficiencies almost an order of magnitude larger. In the last three rows, there is no bound for the singlet because it does not have a singly-charged component and hence the doubly-charged scalar can be only pair produced. As no experimental analysis is available for scalar masses below 200 GeV, we write < 200 GeV in the table when there is not enough sensitivity to set a bound for larger masses.

3. Doubly-charged scalar multiplet determination at the LHC

Doubly-charged scalars are predicted by many SM extensions and may show up at the LHC even if no LNV signal can be ever established at colliders. Therefore, a resonance in the same-sign charged di-lepton channel can be detected and hence the question is if the EW multiplet it belongs to can be determined. As explained above, the production cross-section depends on the total isospin and hypercharge but the number of observed events in each final state is also proportional to the corresponding branching ratio. Then, a multi-sample analysis is mandatory. In [30], we have proposed how to measure the doubly-charged scalar pair production cross-section and hence how to determine the multiplet it belongs to, assuming that only two-body decays are sizeable and the two-lepton channel $H^{\pm\pm} \rightarrow \ell^\pm\ell^\pm$ is observable. However, only with a relatively large statistics and a large enough $H^{\pm\pm} \rightarrow \ell^\pm\ell^\pm$ branching ratio it is possible to obtain a crucial test (see Ref. [30] for a detailed quantitative discussion). For example, the production cross-sections for the different multiplets stay apart by at least 3σ if $H^{\pm\pm}$ only decays into $\ell^\pm\ell^\pm$ for $\sqrt{s} = 14$ TeV and $\mathcal{L}_{\text{int}} = 300 \text{ fb}^{-1}$. However, if $H^{\pm\pm}$ decays 50% of the time into $\ell^\pm\ell^\pm$ and $\ell^\pm\tau^\pm$, respectively, this integrated luminosity is not enough to separate the doublet from the triplet, and a longer run to accumulate 3000 fb^{-1} becomes necessary to distinguish the different cases.

We thank A. Aparici and J. Santiago for useful discussions and the careful reading of the manuscript. This work has been supported in part by the Spanish Ministry of Economy and Competitiveness (MINECO), under the grant numbers FPA2006-05294, FPA2010-17915 and FPA2011-23897, by the Junta de Andalucía grants FQM 101 and FQM 6552, by the “Generalitat Valenciana” grant PROMETEO/2009/128, and by the U.S. Department of Energy grant No. DE-FG03-94ER40837. M.C. is supported by the MINECO under the FPU program.

REFERENCES

- [1] G. Aad *et al.* [ATLAS Collaboration], *Phys. Lett.* **B716**, 1 (2012) [arXiv:1207.7214 [hep-ex]].
- [2] S. Chatrchyan *et al.* [CMS Collaboration], *Phys. Lett.* **B716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].
- [3] F. Englert, R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964).
- [4] P.W. Higgs, *Phys. Rev. Lett.* **13**, 508 (1964).
- [5] F. del Aguila, J. de Blas, *Fortsch. Phys.* **59**, 1036 (2011) [arXiv:1105.6103 [hep-ph]].
- [6] J. de Blas, arXiv:1307.6173 [hep-ph].
- [7] J. Beringer *et al.* [Particle Data Group Collaboration], *Phys. Rev.* **D86**, 010001 (2012).
- [8] T. Hambye, *New J. Phys.* **14**, 125014 (2012) [arXiv:1212.2888 [hep-ph]].
- [9] F.T. Avignone III, S.R. Elliott, J. Engel, *Rev. Mod. Phys.* **80**, 481 (2008) [arXiv:0708.1033 [nucl-ex]].
- [10] F. del Aguila *et al.*, *J. High Energy Phys.* **1205**, 133 (2012) [arXiv:1111.6960 [hep-ph]].
- [11] F. del Aguila *et al.*, *J. High Energy Phys.* **1206**, 146 (2012) [arXiv:1204.5986 [hep-ph]]; F. del Aguila *et al.*, *PoS Corfu2012*, 028 (2013) [arXiv:1305.4900 [hep-ph]].
- [12] W.-Y. Keung, G. Senjanovic, *Phys. Rev. Lett.* **50**, 1427 (1983).
- [13] S. Chatrchyan *et al.* [CMS Collaboration], *Eur. Phys. J.* **C72**, 2189 (2012) [arXiv:1207.2666 [hep-ex]].
- [14] G. Aad *et al.* [ATLAS Collaboration], *Eur. Phys. J.* **C72**, 2244 (2012) [arXiv:1210.5070 [hep-ex]].
- [15] M. Duerr, P. Fileviez Perez, M.B. Wise, *Phys. Rev. Lett.* **110**, 231801 (2013) [arXiv:1304.0576 [hep-ph]]; P. Fileviez Perez, M.B. Wise, *Phys. Rev.* **D88**, 057703 (2013) [arXiv:1307.6213 [hep-ph]].
- [16] S. Weinberg, *Phys. Rev. Lett.* **43**, 1566 (1979).
- [17] P. Minkowski, *Phys. Lett.* **B67**, 421 (1977); T. Yanagida, *Conf. Proc.* **C7902131**, 95 (1979); M. Gell-Mann, P. Ramond, R. Slansky, *Conf. Proc.* **C790927**, 315 (1979) [arXiv:1306.4669 [hep-th]]; S.L. Glashow, *NATO Adv. Study Inst. Ser. B Phys.* **59**, 687 (1980); R.N. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980).

- [18] M. Magg, C. Wetterich, *Phys. Lett.* **B94**, 61 (1980); T.P. Cheng, L.F. Li, *Phys. Rev.* **D22**, 2860 (1980); G.B. Gelmini, M. Roncadelli, *Phys. Lett.* **B99**, 411 (1981); G. Lazarides, Q. Shafi, C. Wetterich, *Nucl. Phys.* **B181**, 287 (1981); R.N. Mohapatra, G. Senjanovic, *Phys. Rev.* **D23**, 165 (1981).
- [19] R. Foot, H. Lew, X.G. He, G.C. Joshi, *Z. Phys.* **C44**, 441 (1989); E. Ma, *Phys. Rev. Lett.* **81**, 1171 (1998) [arXiv:hep-ph/9805219].
- [20] A. Hektor *et al.*, *Nucl. Phys.* **B787**, 198 (2007) [arXiv:0705.1495 [hep-ph]].
- [21] P. Fileviez Perez *et al.*, *Phys. Rev.* **D78**, 015018 (2008) [arXiv:0805.3536 [hep-ph]].
- [22] F. del Aguila, J.A. Aguilar-Saavedra, *Nucl. Phys.* **B813**, 22 (2009) [arXiv:0808.2468 [hep-ph]].
- [23] J.A. Grifols, A. Mendez, G.A. Schuler, *Mod. Phys. Lett.* **A4**, 1485 (1989); A. Djouadi, J. Kalinowski, P.M. Zerwas, *Z. Phys.* **C70**, 435 (1996) [arXiv:hep-ph/9511342].
- [24] F. del Aguila, J.A. Aguilar-Saavedra, *Phys. Lett.* **B672**, 158 (2009) [arXiv:0809.2096 [hep-ph]].
- [25] T. Han, B. Zhang, *Phys. Rev. Lett.* **97**, 171804 (2006) [arXiv:hep-ph/0604064]; F. del Aguila, J.A. Aguilar-Saavedra, R. Pittau, *J. High Energy Phys.* **0710**, 047 (2007) [arXiv:hep-ph/0703261]; A. Atre, T. Han, S. Pascoli, B. Zhang, *J. High Energy Phys.* **0905**, 030 (2009) [arXiv:0901.3589 [hep-ph]].
- [26] P.S.B. Dev, A. Pilaftsis, U.-k. Yang, arXiv:1308.2209 [hep-ph].
- [27] F. del Aguila, J. de Blas, M. Perez-Victoria, *Phys. Rev.* **D78**, 013010 (2008) [arXiv:0803.4008 [hep-ph]].
- [28] J.C. Pati, A. Salam, *Phys. Rev.* **D10**, 275 (1974) [*Erratum ibid.* **D11**, 703 (1975)]; R.N. Mohapatra, J.C. Pati, *Phys. Rev.* **D11**, 2558 (1975); G. Senjanovic, R.N. Mohapatra, *Phys. Rev.* **D12**, 1502 (1975).
- [29] S.N. Gninenko, M.M. Kirsanov, N.V. Krasnikov, V.A. Matveev, *Phys. Atom. Nucl.* **70**, 441 (2007); F. del Aguila, J.A. Aguilar-Saavedra, J. de Blas, *Acta Phys. Pol. B* **40**, 2901 (2009) [arXiv:0910.2720 [hep-ph]]; P.S.B. Dev, C.-H. Lee, R.N. Mohapatra, arXiv:1309.0774 [hep-ph].
- [30] F. del Aguila, M. Chala, A. Santamaria, J. Wudka, *Phys. Lett.* **B725**, 310 (2013) [arXiv:1305.3904 [hep-ph]]; arXiv:1307.0510 [hep-ph].
- [31] F. del Aguila, M. Chala, arXiv:1311.1510 [hep-ph].
- [32] K.S. Babu, S. Nandi, Z. Tavartkiladze, *Phys. Rev.* **D80**, 071702 (2009) [arXiv:0905.2710 [hep-ph]].
- [33] J. Alwall *et al.*, *J. High Energy Phys.* **1106**, 128 (2011) [arXiv:1106.0522 [hep-ph]].
- [34] <http://cafpe.ugr.es/index.php/pages/other/software>