Right-handed neutrino magnetic moments

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Abstract

We discuss the phenomenology of the most general effective Lagrangian, up to operators of dimension 5, build with standard model fields and interactions including right-handed neutrinos. In particular we find there is a dimension 5 electroweak moment operator of right-handed neutrinos, not discussed previously in the literature, which could have interesting phenomenological consequences.

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1. INTRODUCTION

Since the first hints on neutrino masses [1, 2], the physics of neutrinos is coming of age with a significant amount of new and increasingly precise data and a variety of new experiments. Though a significant number of parameters in the neutrino sector have been recently measured [3, 4, 5, 6, 7, 8, 9, 10] (for a recent global fit see [11] and for a recent review see [12]), many questions remain. In particular it is not known whether neutrinos are (dominantly) Dirac or Majorana fermions, what is their absolute mass scale and whether they have the electromagnetic properties predicted by the Standard Model. In this paper we will concentrate mostly on the latter issue (for a very recent review see [13]).

Given that our knowledge of neutrino interactions is limited, it is sensible to study neutrino properties using a framework that includes possible non-SM interactions in a systematic way. This is most easily done using an effective Lagrangian. The application of this formalism to the neutrino system exhibits novel complications since the complete set of low-energy degrees of freedom is not definitively known. For example, the appropriate description of the light neutrino masses may require the introduction of new relatively light ($\lesssim$GeV) degrees of freedom\(^1\), which might be convenient to include in the low-energy theory, and the approach must be sufficiently general to allow for this possibility.

The effective Lagrangian approach is reliable only at energies significantly below the scale of new physics [15, 16, 17, 18, 19] that will be denoted by $M_{NP}$. In addition we will assume that the underlying physics is decoupling [20], so that the effective theory can be expanded in powers of $1/M_{NP}$. The use of effective theories in neutrino physics is far form new [21, 22] (for recent applications see, for instance, [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]); despite this we find that when right-handed neutrinos are included in the low-energy theory, not all the interactions allowed by gauge invariance have been adequately studied in the literature [36].

The first-order corrections (in powers of $1/M_{NP}$) to the SM interactions correspond to dimension 5 operators, which in our case fall into three classes: those contributing to the Majorana mass matrices for the left and right-handed neutrinos, and those describing a magnetic moment coupling for the right-handed neutrinos; it is this last term that has been

\(^1\) For instance, to explain baryon asymmetry and dark matter in the universe one may need light right-handed neutrinos [14].
largely ignored.

In the following we will investigate several properties and consequences of this new electroweak interaction and discuss its origin, experimental constraints and possible effects both in collider experiments and in various areas of astrophysics and cosmology.

II. DIMENSION 5 EFFECTIVE LAGRANGIAN

When considering the low energy effects of a (hypothesized) heavy physics that is not directly probed, it is convenient to parametrize all new physics effects using a series of effective vertices involving only light fields \[ \nu, e, L \]. These vertices are constrained only by the gauge invariance of the light theory \[ 38 \]. Assuming that the physics underlying the Standard Model (SM) is decoupling, the heavy-physics corrections to the SM processes will be suppressed by powers of the heavy scale \(^2 \) \( M_{\text{NP}} \).

Concerning the light degrees of freedom, we will assume these consist of all the SM excitations together with 3 right-handed neutrinos \( \nu'_R \), assumed to be gauge singlets (the prime indicates that these are not mass eigenstates). Should the scale of the \( \nu'_R \) be \( \gtrsim M_{\text{NP}} \), these excitations will disappear from the low-energy theory; the effective theory in this case is obtained from the expressions below by simply erasing all Lagrangian terms containing the \( \nu'_R \).

The most general form of the effective Lagrangian including up to dimension 5 terms is

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\nu_R} + \mathcal{L}_5 + \cdots
\]

\[
\mathcal{L}_{\text{SM}} = i\bar{\ell} D \ell + i\bar{e}_R D e_R - (\bar{\ell} Y_\ell e_L \phi + \text{h.c.}) + \cdots
\]

\[
\mathcal{L}_{\nu_R} = i\bar{\nu'_R} \partial \nu'_R - \left( \frac{1}{2} \bar{\nu'^*_R} M \nu'_R + \text{h.c.} \right) - (\bar{\ell} Y_\ell \nu'_R \, \tilde{\phi} + \text{h.c.})
\]

\[
\mathcal{L}_5 = \bar{\nu'^*_R} \xi \sigma^{\mu\nu} \nu'_R B_{\mu\nu} + (\bar{\ell} \phi)(\phi^+ \ell) - (\phi^+ \phi) \bar{\nu'^*_R} \xi \nu'_R + \text{h.c.}
\]

where \( \ell = (\nu'_L, e_L) \) denotes the left-handed lepton isodoublet, \( e_R \) and \( \nu'_R \) the corresponding right-handed isosinglets, and \( \phi \) the scalar isodoublet (family and gauge indices will be suppressed

\(^2 \) Though there are corrections that grow with \( M_{\text{NP}} \) these can always be absorbed in the renormalization of the SM parameters. Even if formally unobservable, these contributions are of interest when the naturality of the theory is studied.
when no confusion can arise); we will assume three right-handed neutrino flavors. The charge-conjugate fields are defined as $e_R^c = C e_R^T$, $\nu_R^c = C \nu_R^T$ and $\tilde{\ell} = \epsilon C \tilde{\ell}^T$, $\tilde{\phi} = \epsilon \phi^*$ where $\epsilon = i \sigma_2$ acts on the $SU(2)$ indices. The hypercharges assignments are $\phi : 1/2$, $\ell : -1/2$, $e_R : -1$, $\nu_R : 0$. The $SU(2)$ and $U(1)$ gauge fields are denoted by $W$ and $B$ respectively (gluon and quarks fields will not be needed in the situations considered below). The Yukawa couplings $Y_e$ and $Y_\nu$ are completely general $3 \times 3$ matrices in flavor space; $M$, $\chi$, and $\xi$ are complex symmetric $3 \times 3$ matrices in flavor space that generate the most general neutrino mass matrix, while $\zeta$ is a complex antisymmetric matrix proportional to the right-handed neutrino electroweak moments. Without loss of generality, $Y_e$ and $M$ can be taken diagonal with positive and real elements.

The term involving $M$ is the usual right-handed neutrino Majorana mass. The term involving $\chi$ was first described by Weinberg \[21\] and provides a Majorana mass for the left-handed neutrino fields plus various lepton-number-violating neutrino-Higgs interactions; this type of effective operator is the same that is obtained when considering generic seesaw models. The term involving $\zeta$ has been mostly ignored in the literature; it describes electroweak moment couplings of the right-handed neutrinos. We will dedicate a significant part of this paper to the study of some of the consequences this operator might have on various collider, astrophysical and cosmological observables. Note that Dirac-type neutrino magnetic moments (involving $\ell$ and $\nu_R^c$) are generated by operators of dimension $\geq 6$, while Majorana-type magnetic moments for left-handed neutrinos (involving only the $\ell$) require operators of dimension $\geq 7$. One can easily see that these effects are subdominant when compared to those produced by the term containing $\zeta$ in $\mathcal{L}_5$. In addition, Majorana-type and Dirac-type magnetic moment operators contribute, at the loop level, to neutrino masses \[34, 39\] and, therefore, are strongly constrained.

The couplings $\chi$, $\xi$, $\zeta$ have dimension of inverse mass, which is associated with the scale of the heavy physics responsible for the corresponding operator. Though we will refer to this scale generically as $M_{\text{NP}}$ it must be kept in mind that different types of new physics might be responsible for the various dimension 5 operators and that the corresponding values of $M_{\text{NP}}$ might be very different. One common characteristic of all these scales is that they should all be much larger than the electroweak scale $v \sim 0.25 \text{ TeV}$, by consistency of the approach being used. Below we discuss the possible types of new physics that can generate these operators and the natural size for the corresponding coefficients.
A. Heavy-Physics content of the effective vertices.

As mentioned previously there are various kinds of heavy physics that can generate $\mathcal{L}_5$ at low energies; we will briefly discuss the various possibilities.

1. $\nu_L$ Majorana mass term.

Using appropriate Fierz transformations we can re-write the operator containing $\chi$ as follows ($i$ and $j$ denote family indices):

$$\left(\bar{\ell}_i \phi\right) \left(\phi^\dagger \ell_j\right) = -\left(\bar{\ell}_i \sigma \phi\right) \cdot \left(\phi^\dagger \sigma \ell_j\right) = \frac{1}{2} \left(\bar{\ell}_i \sigma \ell_j\right) \cdot \left(\phi^\dagger \sigma \phi\right).$$

It follows that this operator can be generated perturbatively at tree level by the exchange of (i) a scalar isorotplet of hypercharge 1, (ii) a zero hypercharge fermion isorotplet, or (iii) a fermion isosinglet also of zero hypercharge (note that these are the quantum numbers of the $\nu_R$, which are required in many extensions of the SM). For weakly coupled heavy physics we then expect

$$\chi \sim \lambda^2/M_{\text{NP}}, \quad (6)$$

where $M_{\text{NP}}$ denotes the mass of the corresponding heavy particle and $\lambda$ the coupling constants of the heavy fermions to $\phi \ell$, or of the heavy scalar to $\phi \phi$ and $\ell \ell$. When generated by a scalar (fermion) isorotplet this interaction can realize the type II (III) see-saw mechanism \cite{40, 41, 42, 43, 44, 45, 46, 47, 48}; when generated by singlet neutrinos it realizes type I see-saw \cite{49, 50, 51, 52, 53}.

2. $\nu_R$ Majorana mass term.

The operator

$$\left(\phi^\dagger \phi\right) \overline{\nu_{iR}^c} \nu_{jR}^c$$

can be generated at tree level by (i) a scalar isosinglet of vanishing hypercharge, or (ii) a fermion isodoublet of hypercharge 1/2. We again expect

$$\xi \sim \lambda^2/M_{\text{NP}}, \quad (8)$$
where $M_{NP}$ again denotes the mass of the heavy particles, and $\lambda$ the coupling of the heavy fermion to $\phi \nu$ or the heavy scalar to $\phi^\dagger \phi$ and $\nu \nu$. Except for the neutrino-Higgs interactions, the effects of this operator can be absorbed into a redefinition of the Majorana mass $M$. The terms that do involve the Higgs particle may open a new decay channel $H \rightarrow NN$ for the Higgs boson (provided it is kinematically allowed). We will discuss this possibility in section III.

3. $\nu_R$ electroweak coupling.

Finally, the operator

$$ (\bar{\nu}_R \sigma^{\mu \nu} \nu_R) B_{\mu \nu} \tag{9} $$

can be generated only at the one loop level by (i) a scalar-fermion pair $\{\omega, E\}$, with opposite (non-zero) hypercharges that have couplings $\omega \bar{E} \nu_R$ and $\omega \bar{E} \nu_R^c$, or (ii) a vector-fermion pair $\{W'_\mu, E\}$, with opposite (non-zero) hypercharges that have couplings $W'_\mu \bar{E} \gamma^\mu \nu_R$ and $W'_\mu \bar{E} \gamma^\mu \nu_R^c$. Then

$$ \zeta \sim \frac{g' y \lambda^2}{16\pi^2 \max(m_{\text{fermion}}^2, m_{\text{boson}}^2)} \frac{m_{\text{fermion}}^2}{16\pi^2 m_{\text{fermion}}} \tag{10} $$

where $\lambda$ denotes the coupling of the two heavy particles to the $\nu_R$, and $y$ the hypercharge of the heavy boson or fermion. A specific example is provided in appendix A.

We should mention that these coefficient estimates need not hold in case the underlying physics is strongly coupled. In this case one can obtain a natural estimate for the various coefficients using naive dimensional analysis (NDA) [54, 55]. The resulting values are

$$ \chi, \xi \sim \frac{16\pi^2}{M_{NP}}; \quad \zeta \sim \frac{1}{M_{NP}}, \tag{11} $$

where $M_{NP}$ is, in this case, the scale of the strong interactions; it is important to note that these estimates are based on the assumption that $\ell, \nu$ and $\phi$ participate in these strong interactions. It is also worth noting that these estimates revert to the previous ones [6], [8] and [10] upon replacing $M_{NP} \rightarrow (4\pi)^2 M_{NP}$.

In the following we will denote by $\Lambda_{NP}$ the scale associated with $\zeta$, so that

$$ \Lambda_{NP} \sim \frac{1}{\zeta} \sim \begin{cases} 16\pi^2 M_{NP} & \text{weakly-coupled and decoupling heavy physics} \\ M_{NP} & \text{strongly coupled heavy physics (NDA estimate)} \end{cases}. \tag{12} $$
B. The Lagrangian in terms of mass eigenfields

From $\mathcal{L}$ it is straightforward to obtain the neutrino and lepton mass matrices and electroweak moments after SSB. Replacing $\phi \to \langle \phi \rangle = (v/\sqrt{2})(0, 1)$ yields the following mass terms for the leptons

$$\mathcal{L}_m = -\bar{e} L e_R - \bar{\nu}_L M_D \nu_R' - \frac{1}{2} \bar{\nu}_L' M_L \nu_L - \frac{1}{2} \bar{\nu}_R' M_R \nu_R' + \text{h.c.} \quad (13)$$

$$M_R = M + \xi v^2, \quad M_L = \chi v^2, \quad M_D = Y_\nu \frac{v}{\sqrt{2}}, \quad M_e = Y_e \frac{v}{\sqrt{2}}; \quad (14)$$

it is worth noting that, up to possible coupling-constant factors, $M_D \sim v$ while $M_L \sim v^2 / M_{\text{NP}}$. Various situations obtain depending on the hierarchy between $M_R$, $M_D$ and $M_L$: the standard (type I) see-saw scenario results from $M_R \gg M_D \gg M_L$; types II and III see-saw are indistinguishable at the level of the dimension 5 effective Lagrangian and correspond to $M_L \gg M_D^2 / M_R$. For these cases there is no conserved or approximately conserved fermion number and the mass eigenstates are Majorana fermions. In contrast, when $M_D \gg M_{R,L}$ there is an approximately conserved fermion number and the mass eigenstates will be Dirac fermions up to small admixtures (pseudo-Dirac case).

When $M_R \gg M_D \gg M_L$ the mass matrices can approximately be diagonalized in blocks leading to two $3 \times 3$ Majorana mass matrices

$$\text{heavy : } \mathcal{M}_N \approx M_R, \quad (15)$$

$$\text{light : } \mathcal{M}_\nu \approx M_L - M_D^* \frac{1}{M_R^*} M_D^\dagger. \quad (16)$$

These matrices can subsequently be diagonalized by using the unitary matrices $U_N$ and $U_\nu$, $M_N = U_N^T \mathcal{M}_N U_N$ and $M_\nu = U_\nu^T \mathcal{M}_\nu U_\nu$ with $M_N$ and $M_\nu$ diagonal matrices with positive elements (in general one can choose $\mathcal{M}_N$ diagonal, in which case $U_N = 1$). Thus, the mass terms (13) can be rewritten in terms of mass eigenfields as (without loss of generality we can also take $M_e$ real and diagonal with positive elements)

$$\mathcal{L}_m = -\bar{e} M_e e - \frac{1}{2} \bar{\nu} M_\nu \nu - \frac{1}{2} \bar{\nu} N M_N N,$$

and the $\nu'_{L,R}$ have simple expressions in terms of the light ($\nu$) and heavy ($N$) mass-eigenstate Majorana fields ($\nu = \nu^c$ and $N = N^c$)

$$\nu'_L = P_L (U_\nu \nu + \varepsilon U_N N + \cdots); \quad (17)$$

$$\nu'_R = P_R (U_N N - \varepsilon U_\nu \nu + \cdots); \quad (18)$$
with $P_{L,R} = (1 \mp \gamma_5)/2$ the usual chirality projectors, and
\[
\varepsilon \approx M_D M_R^{-1}
\]
a $3 \times 3$ matrix characterizing the mixing between heavy and light neutrinos. Note that barring cancellations in $\mathcal{M}_\nu$, the elements of the mixing matrix $\varepsilon$ in eqs. (17,19) obey generically ($m_\nu$ is a mass of the order of the light neutrino masses and $m_N$ a mass of the order of the heavy neutrino masses)
\[
|\varepsilon_{ij}| \lesssim \sqrt{\frac{m_\nu}{m_N}};
\]
leading to a strong suppression of all mixing effects in most scenarios.

Substituting eq. (18) in eq. (4) and using the well known expression of $B_\mu$ in terms of the photon and the $Z$ field, we obtain the relevant interactions in terms of the mass eigenfields. For instance from the right-handed electroweak moment interaction we obtain
\[
\mathcal{L}_\xi = \left( \overline{N} U^\dagger N - \overline{\nu} U^\dagger \nu \right) \sigma^{\mu\nu} (\xi P_R + \xi^\dagger P_L) \left( U_N N - \varepsilon^T U_\nu \nu \right) (c_W F_{\mu\nu} - s_W Z_{\mu\nu}),
\]
where $F_{\mu\nu}$ and $Z_{\mu\nu}$ are the Abelian field strengths of the photon and the $Z$-gauge boson respectively, and $c_W = \cos \theta_W$, $s_W = \sin \theta_W$ with $\theta_W$ the weak mixing angle.

We see that the $\nu'_R$ electroweak moment operator generates a variety of couplings when expressed in terms of mass eigenstates. These vertices include a tensor coupling of the $Z$-boson and magnetic moment couplings for both $N$ and $\nu$, as well as $N - \nu$ transition moments. Note, however, that there is a wide range in the magnitude of the couplings, in particular heavy-light couplings are suppressed by $\varepsilon$ and light-light couplings are suppressed by $\varepsilon^2$.

Similarly, if we substitute eqs. (17,18) in the last term of eq. (4) and choose the unitary gauge, we obtain, in addition to a contribution to the $N$ mass, a Higgs-heavy neutrino interaction:
\[
\mathcal{L}_\xi = -v H \overline{N} (\xi P_R + \xi^\dagger P_L) N + \cdots,
\]
where we again took $U_N = 1$ and the dots represent other interactions generated by this operator: $HHNN$ vertices as well as $N - \nu$ and $\nu - \nu$ interactions that are suppressed by the mixing $\varepsilon$; these vertices are also generated by the neutrino Yukawa coupling in $\mathcal{L}_{\nu_R}$ and are also suppressed.
Finally we should mention that when eq. (17) is substituted in the SM weak interaction terms $\bar{\nu}_L \gamma^\mu \nu'_L Z_\mu$ and $\bar{e}_L \gamma^\mu \nu'_L W_\mu$, one obtains $N-\nu-Z$, $N-e-W$ couplings, which, although suppressed by $\varepsilon$, are important for the decays of the lightest of the heavy neutrinos.

III. COLLIDER EFFECTS

The new heavy particles responsible for the right-handed electroweak moment must be charged under the electroweak group and are then expected to have standard couplings to the photon and the $Z$ gauge bosons. Since they have not been produced at LEP2 or Tevatron we can conclude that $M_{NP} > 100 \text{ GeV}$. As discussed above, if the new physics is perturbative, the associated effective scale in the coupling $\zeta$ is $1/\zeta = (4\pi)^2 M_{NP} > 15 \text{ TeV}$, and its effects will be suppressed. However, it is possible for the new interactions to be generated in the strong coupling regime, in which case $\zeta$ can be much larger and may have interesting effects at near-future colliders such as the LHC. It is then worth studying the effects of the new interactions for this scenario; accordingly, following the estimates in eq. (12), we will take $\zeta = 1/\Lambda_{NP}$ and study the impact of the new interactions at LEP, LHC and ILC. The results for the perturbative regime can be recovered by taking $\Lambda_{NP} = (4\pi)^2 M_{NP}$.

As discussed previously, the mixing between light and heavy neutrinos is $\varepsilon \sim \sqrt{m_\nu/m_N}$, so that all effects $\propto \varepsilon$ will be negligible unless $m_N$ is very small, but for light $m_N$, $m_N < 10 \text{ KeV}$, we have very stringent bounds on the coupling from astrophysical considerations which will render the effects at colliders negligible (see section IV). Thus, in most cases all mixing effects can be ignored. The main exception occurs when studying the decays of the lightest $N$ which becomes stable when $\varepsilon = 0$.

A. Decay rates and decay lengths

Before discussing the impact of the new interactions in past and future colliders, we would like to discuss briefly the dominant decay modes of the new neutral fermions and

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3 One also generates a $Z-N-N$ coupling suppressed by $\varepsilon^2$.

4 This scenario is in many aspects similar to the case of excited neutral fermions which has been largely considered in the literature (for limits from LEP1 and LEP2 see for instance [56, 57] and [58, 59], and for prospects at future colliders see [60, 61]), with the difference that in our case we have right-handed neutrinos which do not have standard weak interactions.
their decay lengths for the relevant experiments. Although in principle we could have three or more right-handed neutrinos, for simplicity we will only consider the two lightest ones, $N_1$ and $N_2$ (with $m_1 < m_2$). The extension to more heavy neutral fermions is straightforward.

If the magnetic-moment-type interactions are strong enough to produce the new particles, the dominant decay modes of the heaviest neutrino, $N_2$, will be $N_2 \rightarrow N_1 \gamma$, and $N_2 \rightarrow N_1 Z$ if the $N_2$ is heavy enough. For relatively heavy $N_2$, $m_2 > 10 \text{ GeV}$, the produced photons will be hard and can be measured. The lifetime will be very small and the decay length very short; for example, we find that for $N_2$ produced at center of mass (CM) energies ranging from $100 - 1000 \text{ GeV}$, the decay lengths of the $N_2$ are well below $10^{-8} \text{ m}$ unless $m_2 \approx m_1$.

In contrast, the lightest heavy neutrino, $N_1$, must decay into SM particles. As discussed above, this means that $N_1$ decays will always be suppressed by the mixing parameter $\varepsilon$ (which we take as $\varepsilon = 10^{-6}$ for our estimates), and the corresponding decay lengths will be much longer.

![Image](image_url)

Figure 1: Decay branching ratios of $N_1$. Solid for $N_1 \rightarrow \nu \gamma$ and dashed for $N_1 \rightarrow eW^* \rightarrow e+\text{fermions}$, $N_1 \rightarrow \nu Z^* \rightarrow \nu+\text{fermions}$ and $N_1 \rightarrow \nu H$ (see text). We take $\varepsilon \sim 10^{-6}$, $\Lambda_{NP} = 10 \text{ TeV}$ and $m_H = 130 \text{ GeV}$.

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5 If $N_2$ and $N_1$ are almost perfectly degenerate these decays will be suppressed. In that case decays to SM particles like $N_2 \rightarrow \nu \gamma$, $N_2 \rightarrow eW$, $N_2 \rightarrow \nu Z$ or $N_2 \rightarrow \nu H$, although suppressed by $\varepsilon$, could be relevant.
Since all the decay widths of the $N_1$ are proportional to $\varepsilon$, the branching ratios will depend weakly on the heavy-light mixing parameters; they will, however, be sensitive to the strength of the new magnetic moment interaction. An example is presented in figure I for $\Lambda_{NP} = 10$ TeV: if $m_1 < m_W$ the decay is dominated by $N_1 \to \nu \gamma$ although for larger masses of the $N_1$ the tree-body decay $N_1 \to eW^* \to e + \text{fermions}$ could also be important.

![Figure 2: $N_1$ decay lengths for a $N_1$ produced together with a $N_2$ at CM. We present results for CM energies of $\sqrt{s} = 100$ GeV (solid), 500 GeV (dashed), and 1 TeV (dotted); we took $m_2 = 2m_1$, $\Lambda_{NP} = 10$ TeV and $\varepsilon = 10^{-6}$.

For $m_1$ above $m_W$ the decays are dominated by the two body decay $N_1 \to \ell W$ and for masses above $m_Z$ the decay $N_1 \to \nu Z$ is also important\(^6\). If $m_1 > m_H$, the $N_1$ can also decay into a real Higgs boson (in the figure we have taken $m_H = 130$ GeV), however for these masses the Higgs boson width is very small, therefore virtual production is suppressed and the branching ratio drops rapidly once $m_1 \lesssim m_H$. Notice that for $m_1 \gg m_H$, the decay widths $\Gamma(N_1 \to \nu Z)$ and $\Gamma(N_1 \to \nu H)$ are equal and half of $\Gamma(N_1 \to eW)$, as required by the equivalence theorem [63, 64] (see also the discussion in appendix [B3]). Notice also that in figure I we have taken $\Lambda_{NP} = 10$ TeV and the decay width $\Gamma(N_1 \to \nu \gamma)$ is suppressed by $1/\Lambda_{NP}^2$ while the decays to weak gauge bosons are not. Thus, for relatively small $\Lambda_{NP}$,

\(^6\) In this and several other points we disagree with the results presented in [62].
$\Lambda_{NP} \sim 1\text{TeV}$, the decay $N_1 \rightarrow \nu\gamma$ could also be relevant even above the threshold of production of weak gauge bosons.

In figure 2 we present an estimate of the $N_1$ decay lengths as a function of its mass. We assume that the $N_1$ is produced through the new electroweak moment interaction together with a $N_2$ (for instance $e^+e^- \rightarrow N_1N_2$) at CM, and subsequently decays into the allowed channels, $N_1 \rightarrow \nu V$ ($V = \gamma, W, Z$). Decay lengths are presented as a function of the $N_1$ mass for different values of the CM energy for $m_2 = 2m_1$, $\Lambda_{NP} = 10\text{TeV}$ and $\varepsilon = 10^{-6}$. We observe that the decay lengths of the $N_1$ will be very small for masses above $100\text{GeV}$. However, for masses below $100\text{GeV}$ the decay lengths could range from a few millimeters to a few kilometers, depending on the $N_1$ and the $N_2$ masses, the heavy-light mixing, the electroweak coupling and the kinematical configuration of the experiment. In particular there is an intermediate range of masses for which the $N_1$ could be identified through the presence of a displaced photon vertex [65, 66, 67].

**B. Heavy neutrinos in $e^+e^-$ colliders**

As mentioned previously, if $N_1$ and $N_2$ are sufficiently light, the fact that these particles were not observed at LEP1 [68, 69, 70, 71] and LEP2 [72, 73, 74] places strong bounds on their couplings. The most conservative bound is obtained by assuming that both $N_1$ and $N_2$ escape undetected. This is likely for a relatively light $N_1$ because it can only decay through heavy-light mixing and, as discussed above, the corresponding decay length could be very large. The $N_2$, however, will decay into $N_1$ and $\gamma$, with the energetic photon providing a potentially clear signature. In that case stronger bounds can be set but those bounds will depend on the details of the spectrum\textsuperscript{7}. Instead of providing an exhaustive description of all possible scenarios we will limit ourselves to the interesting case of the bounds that can be derived from the LEP data when it is assumed that the $Z$ decays invisibly into $N_1, N_2$; then, at the end of this section, we will comment on the bounds that could be derived from visible $N_2$ decays.

The decay width $\Gamma (Z \rightarrow N_1N_2)$ is given in appendix 11 and it is proportional to $|\zeta_{12}|^2$. Assuming that only the standard decays $Z \rightarrow \nu\bar{\nu}_\ell$ ($\ell = e, \mu, \tau$) and $Z \rightarrow N_1N_2$ contribute

\textsuperscript{7} For instance, if the $N_1$ and $N_2$ are almost degenerate the photon will be too soft to provide a viable signal.
to the invisible width of the $Z$-boson, $\Gamma_{\text{inv}}$, we can obtain a bound on $|\zeta_{12}|$. Using the experimental values $\Gamma^0$, we have

$$\Gamma_{\text{inv}} = 3\Gamma_{\nu\bar{\nu}}^S + \Gamma(Z \to N_1N_2) = 499.0 \pm 1.4 \text{ MeV} . \quad (23)$$

Using also the charged lepton $Z$ boson width, $\Gamma_{\ell\ell} = 83.984 \pm 0.086 \text{ MeV}$ and the ratio of the neutrino and charged leptons partial widths calculated within the SM, $\Gamma_{\nu\bar{\nu}}^S / \Gamma_{\ell\ell}^{SM} = 1.991 \pm 0.001$, we find

$$\Gamma(Z \to N_1N_2) = \Gamma_{\text{inv}} - 3 \left( \frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{\ell\ell}} \right)^{SM} \Gamma_{\ell\ell} \approx -2.6 \pm 1.5 \text{ MeV} . \quad (24)$$

Since $\Gamma(Z \to N_1N_2)$ is positive and the mean value is negative, we use the Feldman & Cousins prescription $\Gamma_{\nu\bar{\nu}}$ to estimate the 90% CL bound

$$\Gamma(Z \to N_1N_2) < 0.48 \times 1.5 \text{ MeV} = 0.72 \text{ MeV} \quad 90\% \text{ CL} ,$$

which in our case implies that

$$\Lambda_{NP} = \frac{1}{|\zeta_{12}|} > 7\sqrt{f_Z(m_2, m_1, m_2)} \text{ TeV} , \quad (25)$$

where $f_Z(m_z, m_1, m_2)$ is a phase space factor given in the appendix $\Gamma$, normalized in such a way that $f_Z(m_2, 0, 0) = 1$. For example, $\Lambda_{NP} > 1.9 \text{ TeV}$ if $m_1 = m_2 = 35 \text{ GeV}$.

If the right-handed neutrino electroweak moment is large enough to allow significant production of $N_1, N_2$ pairs at LEP energies, the dominant decay of $N_2$ will also be $N_2 \to N_1\gamma$, unless the mass of the $N_1$ is very close to the $N_2$ mass. Then, the resulting photons could be detected and separated from the background if $E_{\gamma} > 10 \text{ GeV}$. In fact, searches for this type of processes (some searches for excited neutrinos also fall in this class of processes) have been conducted at LEP1 [56, 68, 69, 70] and at LEP2 [72, 73, 74]. If the mass of the heavy neutrino is below $\sim 90 \text{ GeV}$ one typically obtains upper bounds on the production branching ratio $BR(Z \to N_1N_2)$ of the order of $2 \times 10^{-6} - 8 \times 10^{-6}$ (see for instance [56, 68]) depending on the masses of $N_1$ and $N_2$ (these results also assume that $BR(N_2 \to N_1\gamma) = 1$ and that $m_2 < 5 \text{ GeV}$). Using these data one can set much stronger bounds. For instance if $m_1 = 0$ and $m_2$ is relatively light, $10 \text{ GeV} < m_2 < m_Z$, we can use the conservative limit $BR(Z \to N_1N_2) < 8 \times 10^{-6}$ and obtain $\Lambda_{NP} = 1/|\zeta_{12}| > 40 \text{ TeV}$. Data from LEP2 can also be used to place limits [72, 73, 74] on the couplings for masses up to 200 GeV. For typical values of $m_{1,2}$ one can set upper bounds on the production cross section of the order of
0.1 pb (for $\sqrt{s} = 207 \text{ GeV}$) which translate into bounds on $1/|\zeta_{12}|$ of the order of a few TeV. LEP bounds based on visible $N_2$ decays depend more strongly on the $N_1$ and $N_2$ masses (for instance, they are completely lost lost if $m_2 - m_1 \lesssim 10 \text{ GeV}$) but they could be important if some signal of this type is seen at the LHC.

In figure 3 we give the cross section for $e^+e^- \rightarrow N_1N_2$ as a function of $m_2$ (for illustration we took $m_1 = 0$ and $\Lambda_{NP} = 10 \text{ TeV}$) for the center of mass energies of LEP1 and LEP2 (we plotted values for $\sqrt{s} = 200 \text{ GeV}$). We also included results for $\sqrt{s} = 500 \text{ GeV}$ and $\sqrt{s} = 1000 \text{ TeV}$ in view of the proposals for future $e^+e^-$ colliders as the International Linear Collider (ILC). We see that, except for collisions at the $Z$ peak, which are enhanced by about two orders of magnitude, or close to the threshold of production, which are suppressed by phase space, cross sections are quite independent on the CM energy and are of the order of 0.1 pb for $\Lambda_{NP} = 10 \text{ TeV}$.

C. Neutral heavy lepton production at the LHC

The right-handed electroweak moment can help to produce the heavy neutrinos at hadron colliders. In particular, heavy neutrinos will be produced at the LHC through the Drell-Yan
Figure 4: $pp \to N_1 N_2 + X$ cross section at the LHC ($\sqrt{s} = 14$ TeV) as a function of the mass of $N_2$. We took $\Lambda_{NP} = 10$ TeV and drew three curves for few representative masses of the $N_1$.

The differential cross section for proton-proton collisions can be computed in terms of the the partonic cross sections (for a very clear review see for instance [124])

$$d\sigma(pp \to N_1 N_2 + X) = \sum_q \int_0^1 dx_1 \int_0^1 dx_2 \left( f_q(x_1, \hat{s}) f_{\bar{q}}(x_2, \hat{s}) + (q \leftrightarrow \bar{q}) \right) d\hat{\sigma}(q\bar{q} \to N_1 N_2, \hat{s}),$$

where $\hat{s} = x_1 x_2 s$ is the partonic center of mass invariant square mass, $\hat{\sigma}$ is the partonic cross section and $f_q(x_1, \hat{s})$, $f_{\bar{q}}(x_2, \hat{s})$ are the parton distribution functions for the proton. Taking the partonic cross sections given in appendix B and performing the convolution over the parton distribution functions we find the total cross section as a function of the heavy neutrino masses\(^8\).

The cross section depends on the masses and the coupling $\zeta_{12} = 1/\Lambda_{NP}$. In figure 11 we represent the total cross section for $\Lambda_{NP} = 10$ TeV as a function of the $N_2$ mass for $\sqrt{s} = 14$ TeV. We give results for three representative values of $m_1$. We see that cross

---

\(^8\) We have used the CTEQ6M parton distribution sets [78]. One could also include next-to-leading-order corrections by multiplying by a $K$-factor which typically would change cross sections by $10-20\%$. Results have been checked against the CompHEP program [79, 80].
sections above 100 fb are easily obtained but only for \( m_1 + m_2 \lesssim m_Z \), where LEP bounds apply. For larger masses the cross section decreases very fast.

In figure 5, we present the differential cross section for the process \( pp \rightarrow N_1 N_2 + X \) (with respect to the transverse momentum) for different sets of neutral heavy lepton masses. For \( m_1 + m_2 < m_Z \) we see clearly the peak of the \( Z \) gauge boson.

D. Higgs decays into heavy neutrinos

In this paper we are mainly interested in the effects of a possible magnetic moment of right-handed neutrinos. However, as discussed before, among the three possible dimension five operators there is one which gives a correction to the right-handed neutrino Majorana mass. Moreover, it also gives new Higgs boson couplings which could be relevant for Higgs boson searches at the LHC/ILC. In particular, it could induce new additional decays of the Higgs into right-handed neutrinos which could be dominant in some region of parameters, particularly if the Higgs mass is in the range \( m_H \sim 100 - 160 \text{ GeV} \) and if the right-handed neutrinos are light enough to be produced in Higgs decays. Let us discuss briefly the possible effects of this operator.
In subsection \[11\] we derived the relevant interactions induced by the new operators. In particular the Higgs boson interaction with heavy neutrinos is given in eq. \[22\], where couplings $H-\nu-N$ and $H-\nu-\nu$, which are suppressed, have been neglected.

From eq. \[22\] we compute the decay width of the Higgs boson into two heavy neutrinos which is given in appendix \[B\]. Then, we can compare with the SM decay rates of the Higgs boson. In figure 6 we represent the decay branching ratios into the different channels for the new physics scale given by \[9\] $M_{NP\xi} = 1/\xi = 10\text{ TeV}$. For simplicity we neglected heavy neutrino masses. For heavier neutrinos there are some phase space suppression factors given in appendix \[B\]. We see that if $m_H$ lies below the $WW$ threshold, right-handed neutrinos dominate Higgs decays (if kinematically allowed). In fact, for low enough $M_{NP\xi}$, these decays could be significant even when the $WW$ and $ZZ$ channels are open. This also means that the branching ratios to other interesting channels in this region, as for instance $H \rightarrow 2\gamma$, are suppressed and could make its detection more difficult. However, the effect of this new interaction is not necessarily so bad since the produced $N'$s have to decay. If the magnetic-moment interaction of right-handed neutrinos is also present the heaviest neutrinos can decay

\[9\] Notice that this interaction can be generated at tree level; therefore, up to possible small couplings, $1/\xi$ can be identified directly with the masses of the new physics particles in the perturbative regime.
into lighter ones and photons, and those photons could be detected. Moreover, the lightest of the heavy neutrinos will decay into light neutrinos and photons. As discussed in section [11] this is suppressed by the mixing heavy-light, therefore the $N_1$ could be rather long-lived and produce non-pointing photons which could be detected. If the magnetic moment interaction is not present, the heavy neutrinos will have three-body decays ($N_1 \rightarrow W^* \nu$ or $N_1 \rightarrow Z^* \nu$) suppressed by the heavy-light mixing\textsuperscript{10}.

### IV. ASTROPHYSICAL AND COSMOLOGICAL CONSIDERATIONS

In this section we consider several astrophysical and cosmological systems and processes that may be affected by the presence of a magnetic coupling of the neutrinos. Neither the calculations nor the list are intended to be exhaustive; we will instead focus on some of the most interesting effects.

#### A. Astrophysical effects

Among the various astrophysical processes that are affected by neutrino magnetic couplings the cooling of red giant stars plays a prominent role because it provides a very tight bound on the magnitude of the magnetic moments – provided the masses of the neutrinos involved are sufficiently small. This limit is based on the observation that in a plasma photons acquire a temperature-dependent mass (and are then referred to as plasmons); any electromagnetic neutrino coupling will then open a decay channel for the plasmon into a neutrino pair, unless kinematically forbidden. If produced, the neutrinos leave the star, resulting in an additional cooling mechanism that is very sensitive to the size of the magnetic moment [81, 82, 83, 84, 85, 86, 87]; this can be used to impose stringent upper limit on this moment.

The electroweak moment couplings of mass eigenstates derived from $\mathcal{L}_5$ is given in eq. (21). In particular the electromagnetic coupling of heavy neutrino eigenstates is (we already took

\textsuperscript{10} Recall that the interesting mass range is $m_H < 160 \text{ GeV}$ and such a light Higgs boson cannot decay into real $W$’s or $Z$’s. However, if one of the heavy neutrinos is light enough, then the heavier one could still decay into real $W$’s and $Z$’s and produce interesting signals.
\[ U_N = 1. \]

\[ \mathcal{L}_{EM} = c_W N \sigma^{\mu \nu} \left( \zeta P_R + \zeta^\dagger P_L \right) N F_{\mu \nu}. \]

In a nonrelativistic nondegenerate plasma the emissivity of neutrinos is dominated by transverse plasmons [88], which have an effective mass equal to the plasma frequency \( \omega_P \). A calculation shows that the decay width of these plasmons into two neutrino species, labeled by \( i \) and \( j \) and satisfying \( m_i + m_j < \omega_P \), is

\[ \Gamma(\text{plasmon} \to N_i N_j) = \frac{2c_W^2 |\zeta_{ij}|^2 \omega_P^4}{3\pi} \frac{\omega_P}{\omega} f_Z(\omega_P, m_i, m_j), \]

where \( \omega \) is the plasmon energy in plasma rest frame, and \( f_Z \) has been defined in eq. [88]. The total decay rate is then

\[ \Gamma(\text{plasmon} \to NN) = \frac{\mu_{\text{eff}}^2 m_P^4}{24\pi} \omega, \]

\[ \mu_{\text{eff}}^2 = 16c_W^2 \sum_{\text{all}} |\zeta_{ij}|^2 f_Z(\omega_P, m_i, m_j), \]  

(28)

and the sum runs over all allowed channels, \( i > j \) such that \( m_i + m_j < \omega_P \). The observational limits from red giant stars cooling then imply [88]

\[ \mu_{\text{eff}} < 3 \times 10^{-12} \mu_B, \]

(29)

where \( \mu_B \) is the Bohr magneton. This translates into a bound on the couplings \( \zeta_{ij} \) provided the sum of the associated neutrino masses lies below \( \omega_P \), for example, for \( \zeta_{ij} \) real,

\[ |\zeta_{ij}| < 8.5 \times 10^{-13} \mu_B; \quad m_{i,j} \ll \omega_P \approx 8.6 \text{ KeV}. \]

(30)

This then gives \( \Lambda_{NP} \gtrsim 4 \times 10^6 \text{ TeV} \); this bound is degraded somewhat when the neutrino masses are comparable to \( \omega_P \).

It is clear from eq. [21] that the photon (plasmon) can also decay into \( N-\nu \) and \( \nu-\nu \). However, the relevant couplings for these processes are suppressed by \( \varepsilon \) and \( \varepsilon^2 \), respectively, which are small numbers (for instance, if \( m_\nu \sim 0.1 \text{ eV} \) and \( m_N \sim 1 \text{ keV} \), \( \varepsilon \sim 0.01 \), see [20]). Therefore, this mixing can only affect plasmon decays for extremely light \( N, m_N \sim m_\nu \); in this case all neutrino masses can be neglected compared to the plasma frequency \( \omega_P \sim 10 \text{ keV} \), and since photons only couple to right-handed neutrinos, our result still applies (taking \( m_i = m_j = 0 \)). Alternatively, if \( m_N > \omega_P \sim 10 \text{ keV} \) the heavy neutrinos cannot be produced in plasmon decay and the only bound comes from plasmon \( \to \nu \nu \); however, the
amplitude for this process is suppressed by $\varepsilon^2$ which is very small if $m_N \gg \omega_P \sim 10\text{keV}$, so that the bounds derived from this process are weak (if we take $\varepsilon^2 \sim m_\nu/m_N$ we roughly expect $\Lambda_{NP} \gtrsim (m_\nu/m_N) \times 4 \times 10^6\text{TeV} \sim 40\text{TeV}$ for $m_\nu = 0.1\text{eV}$ and $m_N = 10\text{keV}$ and drops below a TeV already for $m_N > 0.4\text{MeV}$).

The same type of reasoning can be applied to other astrophysical objects. This might be of interest because the corresponding plasma frequency $\omega_P$ will be larger in denser objects, so that the corresponding limits will apply to heavier neutrino states; unfortunately the limits themselves are much poorer. As an example, we consider the case of a neutron star whose plasma frequency in the crust is $\omega_P \sim 1\text{MeV}$. This could allow us to extend the magnetic moment bounds to higher neutrino masses; however, the much weaker limit, $\mu_{\text{eff}} < 5 \times 10^{-7}\mu_B$ [89] implies $\Lambda_{NP} \gtrsim 23\text{TeV}$ when $m_{i,j} \lesssim 1\text{MeV}$ which is not competitive with bounds derived below from $\gamma + \nu \rightarrow N$ in supernovas, which also apply in this range of masses. Limits derived for plasmon decays from solar and supernova data are also not competitive [88, 90].

The neutrino electromagnetic coupling would also affect other interesting processes. For example, it generates a new supernova cooling mechanism through $\gamma + \nu \rightarrow N$ (when kinematically allowed), with the $N$ escaping. Limits on this “anomalous” cooling [88] imply that the effective magnetic moment then must lie below $3 \times 10^{-12}\mu_B$ provided the heavy neutrino mass lies below $\sim 30\text{MeV}$ (which is of the order of the maximum neutrino energy in the supernova core). The coupling for this process is suppressed $\sim \zeta \varepsilon \sim \left(\sqrt{m_\nu/m_N}\right)/\Lambda_{NP}$, so we find $\Lambda_{NP} \gtrsim 4 \times 10^6 \times \sqrt{m_\nu/m_N}\text{TeV}$. Taking, for example, $m_\nu \sim 0.1\text{eV}$ we obtain $\Lambda_{NP} > 1.5 \times 10^4\text{TeV}$ for $m_N = 10\text{keV}$ and $\Lambda_{NP} > 390\text{TeV}$ for $m_N = 10\text{MeV}$. These limits are interesting in the region $10\text{keV} < m_N < 30\text{MeV}$, where red giant bounds do not apply.

It is also worth noting that if the $N$ mass is $m_N \sim 1\text{keV}$, these particles may contribute to the dark matter content of the universe [91, 92] (but see also [93, 94, 95, 96]). However, although the bounds on the right-handed neutrino magnetic moment coming from red giants apply, they could still have important effects in the analysis and further study is necessary.

B. CP asymmetries

The electroweak moments involving only the $\nu_R$ are also of interest because they generate lepton number violation and may contribute to the baryon asymmetry of the universe [97].
Though providing a complete description of these effects lies beyond the scope of the present paper we will provide a simplified discussion of the issues involved.

In the presence of the electroweak moments the relevant lepton-number-violating decays remain the standard\(^\text{11}\) \(N \rightarrow e^± \phi^±\) (here \(e^±\) denotes a charged lepton and \(\phi^±\) the charged scalar components of the Higgs doublet) which receive a contribution from this dimension five operator. The new graphs, however, necessarily involve a virtual heavy neutrino \(N'\) (see fig. \(\text{7}\)) and will generate a lepton asymmetry only if \(N'\) is lighter than \(N\). Because of this, this type of contributions may be relevant only when the lightest of the heavy neutrino states are degenerate or almost degenerate (for a recent review talk on these scenarios see \[^{10}\text{1}\])

The calculation of the contributions of the Majorana electroweak moments to the lepton-number-violating decay width of the \(N\) is straightforward. We will assume that \(m_N \gg v\) so that we can neglect electroweak symmetry breaking and assume that all gauge bosons, leptons and scalars are massless except the heavy neutrino which has a Majorana mass term. Also, for simplicity, we neglect Yukawa couplings for charged leptons. The relevant pieces of the Lagrangian are discussed in section \[\text{II}\] in particular in eqs. \((\text{2-4})\) and \((\text{22})\),

\[
\mathcal{L}_N = \frac{i}{2} \bar{N} \gamma^\mu \phi N - \frac{1}{2} \bar{N} M_N N - i Y_\nu \bar{N} N \phi - \bar{\phi} Y_\nu^\dagger P_L \ell + \bar{\phi} \sigma^{\mu \nu}(\zeta P_R + \zeta^\dagger P_L) N B_{\mu \nu},
\]

where \(N\) are Majorana fields and \(M_N\) is their mass matrix which, without loss of generality, can be taken diagonal. Since we ignore the charged lepton Yukawa couplings we can rotate the doublet fields \(\ell\) so that \(Y_\nu\) is Hermitian; there are no other possible field redefinitions so \(\zeta\) is, in general, antisymmetric and complex. For \(n\) generations both \(Y_\nu\) and \(\zeta\) contain \(n(n - 1)/2\) phases; in particular, for \(n = 3\) we will have a total of 6 phases. But even for \(n = 2\) we have two phases since both \(Y_{12}\) and \(\zeta_{12}\) can be complex. This is important because \(CP\)-violating observables should depend on those couplings; it also means that we can make our estimates in a model with just 2 generations, as we will do for simplicity.

Assuming 2 generations with \(N_2\) the heavier of the right-handed neutrinos, we consider the lepton-number-violating decays \(N_2 \rightarrow e^- \phi^+\) and \(N_2 \rightarrow e^+ \phi^-\). At tree level the amplitudes

\(^{11}\) For a review of leptogenesis together with references to the original literature see, for example, ref. \[^{99}\].

For new mechanism of leptogenesis involving neutrino magnetic moments see \[^{99}\] and for leptogenesis using composite neutrinos see \[^{100}\].
are simply
\[ A_0(N_2 \rightarrow e^- \phi^+) = Y_{e2} \tilde{u}(p_e) P_R u(p_2), \]
\[ A_0(N_2 \rightarrow e^+ \phi^-) = Y_{e2}^* \tilde{v}(p_2) P_L v(p_e) = -Y_{e2}^* \tilde{u}(p_e) P_L u(p_2), \]
where we used \( v(p) = u^c(p) \).

\[ \begin{array}{c}
\text{(a)} \\
\text{(b)}
\end{array} \]

Figure 7: 1-loop graphs involving electroweak moments contributing to \( L \)-violating heavy-neutrino decays

The one-loop corrections to these processes induced by the electroweak moment coupling \( \zeta \) are given in figure 7. Notice that if the external particle is \( N_2 \) then the antisymmetry of \( \zeta \) dictates that only \( N_1 \) can run in the loop. Thus, if \( m_2 > m_1 \) we expect (finite) imaginary contributions from these graphs. A straightforward but tedious calculation confirms this expectation. Explicitly we find the following CP-violating asymmetry in \( N_2 \) decays to be
\[ \epsilon_{CP} = \frac{\Gamma(N_2 \rightarrow e\phi^+) - \Gamma(N_2 \rightarrow e\phi^-)}{\Gamma(N_2 \rightarrow e^\phi^+) + \Gamma(N_2 \rightarrow e^\phi^-)} \]
\[ = -\frac{g'}{2\pi} (m_2^2 - m_1^2) \frac{m_1}{m_2^3} \text{Im} \left\{ \frac{Y_{e2} Y_{e1}^*}{|Y_{e2}|^2} (\zeta_{12}^* m_2 + \zeta_{12} m_1) \right\}. \]

For \( m_1 \ll m_2 \)
\[ \epsilon_{CP} = -\frac{g'}{2\pi} m_1 \text{Im} \left\{ \frac{Y_{e2} Y_{e1}^*}{|Y_{e2}|^2} \xi_{12} \right\} \sim -\frac{g'}{2\pi} \frac{m_1}{\Lambda_{NP}} \text{Im} \left\{ \frac{Y_{e2} Y_{e1}^*}{|Y_{e2}|^2} e^{-i\delta_{12}} \right\}, \]
where \( \delta_{12} \) is the phase of \( \zeta_{12} \).

We see that the Majorana electroweak moments do generate additional contributions to CP violating asymmetries in heavy neutrino decays. These, however, are relevant only for the decay of the heavier neutrinos and so could be relevant for leptogenesis only when \( m_1 \)
and $m_2$ are relatively close $[102, 103, 104, 105]$. In this limit the amplitude is proportional to $(m_2^2 - m_1^2)$; despite this suppression the possible relevance of these interactions requires a careful comparison of all contributions, and this lies beyond the scope of the present investigation.

V. SUMMARY OF BOUNDS, PROSPECTS AND CONCLUSIONS

As can be seen from the previous sections, the dimension 5 operators involving right-handed neutrinos open up observable effects in several scenarios of interest. The electroweak moment operator (first term in eq. (11)) provides the richest phenomenology, but contributions coming from the $(\phi^+ \phi) \nu_R^c \nu_R^c$ operator (last term in eq. (11)) can affect Higgs boson decays. After spontaneous symmetry breaking, this operator gives rise to several interaction vertices involving right-handed neutrinos and the Higgs boson, the strongest being a simple $H N_i N_j$ term, which provides new decay channels of the Higgs to $N$’s (if such a process is kinematically allowed). These decays could dramatically change the Higgs decay branching ratios (see figure 8), especially in the region $100 \text{ GeV} < m_H < 160 \text{ GeV}$ where the gauge boson channels are still closed. The new decays could result in an invisible Higgs, if the heavy neutrinos cannot be detected, or in new, enhanced detection channels if the right-handed neutrinos can be seen through their own decay channels, for instance $N_2 \rightarrow N_1 \gamma$, or $N_1 \rightarrow \nu \gamma$ and $N_1 \rightarrow eW$ with a displaced vertex.

As for the electroweak moment operator, figure 8 summarizes present bounds on the model parameters as well as two regions of potential interest, namely: the region relevant for the LHC and the region that can provide a relatively large CP asymmetry.

When expanded in terms of mass eigenstates the unique electroweak moment operator generates $N - N$, $N - \nu$ and $\nu - \nu$ magnetic moments, and $N - N$, $N - \nu$ and $\nu - \nu$ tensor couplings to the $Z$-bosons, eq. (21), giving rise to a very rich phenomenology which depends basically on three parameters: the coupling, $\zeta = 1/\Lambda_{NP}$, the heavy-light mixing $\varepsilon$, and the masses of the $N$. For our estimates in figure 8 we take $m_N = m_2$ and $\varepsilon \sim \sqrt{m_\nu/m_N}$ with $m_\nu = 0.1 \text{ eV}$, and neglect $m_1$. Then we represent the regions in the $\Lambda_{NP} - m_N$ plane forbidden by the red giant bound on the $N$ and $\nu$ magnetic moments, by the supernova bound on the transition magnetic moment $N - \nu$ and by the LEP bound from the “invisible” $Z$-boson decay width.
To test the new interactions at the LHC one should produce first the heavy neutrinos and then one should detect them. The analysis of the detection is complicated and depends on the details of the spectrum and the capabilities of the detectors, but at least one should produce them with reasonable rates. Thus we require that the cross section of \( pp \rightarrow N_1N_2X \) is at least 100 fb.

The new interactions we have introduced contain new sources of CP non-conservation which can modify the standard leptogenesis scenarios. In particular we have found that the electroweak moment operator gives additional contributions to the CP asymmetry in \( N_2 \rightarrow e^-\phi^+ \) decays. These could be relevant in leptogenesis if \( \epsilon_{\text{CP}} \sim (g'/2\pi)m_N/\Lambda_{NP} > 10^{-6} \) and \( m_N > 1 \text{ TeV} \), a region that has also been represented in figure 8.

Note that for the regions marked LHC and CP asymmetries the shadowed area represents the region of interest, in contrast to the previous ones, for which the shadowed area represents the excluded region.

Finally, the effective theory we use cannot be applied for all energies and all masses. Thus, to give graphically an idea of the regions where the EFT cannot be applied, we represent the regions with \( m_N > \Lambda_{NP} \), for the strong-coupling regime (EFTs) and \( m_N > (4\pi)^2\Lambda_{NP} \), for the weak-coupling regime (EFTw).

From figure 8 we can draw the following conclusions:

i) There are very tight bounds coming from red giants cooling for \( m_N \lesssim 10 \text{ keV} \), so strong as to require \( \Lambda_{NP} > 4 \times 10^9 \text{ GeV} \); in this scenario, obviously, any effect of the electroweak moment coupling would be totally negligible in any present or planned collider experiment.

ii) For \( 10 \text{ keV} \lesssim m_N \lesssim 10 \text{ MeV} \) supernova cooling produced by the magnetic moment transitions \( \gamma \nu \rightarrow N \), provides very strong bounds. These bounds, however, depend on the assumptions made on the heavy-light mixing parameter, \( \varepsilon \). For this mass range the magnetic moment limits from red giants are derived from plasmon decay into a \( \nu \) pair, which is proportional to \( \varepsilon^2 \) and yields less restrictive constraints.

iii) For \( m_N \lesssim m_Z \), the invisible \( Z \) decays impose \( \Lambda_{NP} \gtrsim 7 \times 10^3 \text{ GeV} \), depending on
Figure 8: Summary of bounds and prospects. The shaded areas labeled $\nu$, $N$ mag. moment, $N - \nu$ transition and LEP denote regions excluded by the corresponding observables; the areas marked EFTw and EFTs correspond to the regions where the EFT parametrization is inconsistent (for the weak- and strong-coupling regimes, respectively). Finally, shaded areas marked CP asym. and LHC denote the range of parameters where the dimension 5 electroweak moment might affect the corresponding observables. See text for details.

the details of the heavy neutrino spectrum\textsuperscript{12}.

iv) For $m_N \sim 1$–200 GeV and roughly $7$ TeV $< \Lambda_{NP} < 100$ TeV, heavy neutrinos could be produced at the LHC with cross sections above 100 fb. The heaviest two of them would decay rapidly to hard photons which could be detected. The lightest one is quite long-lived and, in part of the parameter space, would produce non-pointing photons which could be detected.

Above we have expressed our conclusions in terms of $\Lambda_{NP} = 1/\zeta$. Since our operator is a magnetic moment-type operator, this scale can only be interpreted as the mass of new

\textsuperscript{12} Most likely, searches for hard photons in the Galaxy X-ray background could impose tighter bounds for this mass range, but the precise constraint will depend on the details of the neutrino spectrum; a thorough examination of this issue lies outside the scope of the present paper.
particles in a non-perturbative context. If it is generated by perturbative physics it arises at one loop and one expects $\zeta \sim 1/(4\pi^2 M_{NP})$, where $M_{NP}$ are the masses of the particles running in the loop and coupling constants have been set to one. Thus, in this case, all the constraints discussed above still apply to $M_{NP} = \Lambda_{NP}/(4\pi)^2$. Then, if the new physics is weakly coupled, the interesting range for collider physics, $\Lambda_{NP} \sim 10-100$ TeV translates into $M_{NP} \sim 100-1000$ GeV. For such low masses the effective theory cannot be applied at LHC energies and one should use the complete theory that gives rise to right-handed neutrino electroweak moments. Those models should contain new particles carrying weak charges with masses $\sim 100-1000$ GeV which should be produced in the LHC via, for instance, the Drell-Yan process.

As for the future work around this effective theory, much work still remains to be done, especially concerning astrophysical and cosmological scenarios:

a) The magnetic coupling may have effects in the early universe because it can potentially alter the equilibrium conditions of the $N$ and their decoupling temperature.

b) Heavy neutrinos with masses $\sim 1$ keV could be a good dark matter candidate. The right-handed neutrino magnetic moments could change significantly the analysis of this possibility.

c) One should evaluate carefully the effects of the Majorana magnetic couplings on non-thermal leptogenesis.

d) For sufficiently large $\zeta$, this same coupling might lead to the trapping of the right-handed neutrinos in the supernova core.

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Appendix A: MODEL CALCULATION

The simplest model that can generate a magnetic moment for the right-handed neutrinos consists of adding to the standard model a vector-like fermion $E$ and a scalar $\omega$, both isosinglets of hypercharge $y$, with interactions

$$
\mathcal{L}_{\text{int}} = \sum_i \lambda_i^\dagger \nu_{iR} E \omega + \lambda_i \bar{E} \nu_i \omega + \text{h.c.}
$$

(A1)

where $i$ is a family index; we take $\lambda_i, \lambda_j^\dagger$ real. In this model the effective magnetic coupling is given by

$$
\zeta_{ij} = \frac{g' y (\lambda_i \lambda_j^\dagger - \lambda_j^\dagger \lambda_i)}{64\pi^2 M_{NP}}; \quad M_{NP} = m_E \frac{2(1-r)^2}{1-r + r \ln r}, \quad r = \left( \frac{m_\omega}{m_E} \right)^2.
$$

(A2)

This choice of $M_{NP}$ is, of course, somewhat arbitrary, since experiment only measures $\zeta$. We have chosen it so that $M_{NP} = m_E$ when $m_\omega = m_E$.

Appendix B: DECAY RATES AND CROSS SECTIONS

Here we present the relevant formulas for decay rates and cross sections used in the text. Before we introduce some notation useful to to simplify the presentation of the formulas. First $s_W = \sin \theta_W$, $c_W = \cos \theta_W$ are the sine and cosine of the weak mixing angle. As usual we denote by $q_f$ the charge of fermion $f$ and its vector, $v_f = t_3(f) \left(1 - 4 |q_f| s_W^2 \right)$, and axial couplings, $a_f = t_3(f)$, with $t_3(f) = +1/2 (-1/2)$ for up-type (down-type) fermions. We will write the new couplings as $\zeta_{ij} = |\zeta_{ij}| e^{i\delta_{ij}}$. We will also define as usual the Källen’s Lambda function

$$
\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc.
$$

(B1)

1. $Z \to N_i N_j$

The decay width of the $Z$ boson into heavy neutrinos is
\[ \Gamma(Z \rightarrow N_iN_j) = \frac{2|\zeta_{ij}|^2}{3\pi} s_W^2 m_Z^3 f_Z(m_Z, m_i, m_j), \]  
\text{(B2)}

where \( f_Z(m_Z, m_i, m_j) \) is a kinematical factor \( f_Z(m_Z, 0, 0) = 1 \)

\[ f_Z(m_Z, m_i, m_j) = \frac{\sqrt{\lambda(m_Z^2, m_i^2, m_j^2)}}{m_Z^6} \left[ m_Z^2 \left( m_Z^2 + m_i^2 + m_j^2 - 6m_i m_j \cos 2\delta_{ij} \right) - 2 \left( m_i^2 - m_j^2 \right)^2 \right] . \]  
\text{(B3)}

2. \( N_2 \) decay rates

If the new interaction is strong enough the dominant decays of the heaviest neutral lepton proceed through the new interaction. The decay rates are

\[ \Gamma(N_2 \rightarrow N_1\gamma) = \frac{2}{\pi} e_W^2 |\zeta_{12}|^2 m_2^3 (1 - m_1^2/m_2^2)^3 , \]  
\text{(B4)}

\[ \Gamma(N_2 \rightarrow N_1Z) = \frac{2}{\pi} e_W^2 |\zeta_{12}|^2 m_2^3 f_2(m_Z, m_1, m_2) , \]  
\text{(B5)}

with

\[ f_2(m_Z, m_1, m_2) = -\frac{m_Z^6}{2m_Z^2} f_Z(m_Z, m_1, m_2) , \quad f_2(0, 0, m_2) = 1 . \]  
\text{(B6)}

3. \( N_1 \) decay rates

The lightest of the heavy neutrinos, \( N_1 \), can decay only due to mixing with the SM sector. If \( m_1 > m_Z \) the dominant decays proceed through SM interactions induced by the mixing of heavy-light neutrinos.

\[ \Gamma(N_1 \rightarrow \ell^- \ell^+W^+) = \frac{1}{16} \left| \epsilon_{W}^\ell \right|^2 \frac{\alpha m_1^3}{s_W^2 c_W^2} \left( 1 - \frac{m_W^2}{m_1^2} \right)^2 \left( 1 + 2 \frac{m_W^2}{m_1^2} \right) . \]  
\text{(B7)}

Here \( \beta \) is a flavour index and \( \epsilon_W \) characterizes the mixing of heavy-light neutrinos in \( W \) boson couplings, which is order \( \sqrt{m_\nu/m_N} \).

For \( N_1 \rightarrow \nu Z \) decays we obtain

\[ \Gamma(N_1 \rightarrow \nu \beta Z) = \frac{1}{16} \left| \epsilon_{Z}^\beta \right|^2 \frac{\alpha m_1^3}{s_W^2 c_W^2 m_Z^2} \left( 1 - \frac{m_Z^2}{m_1^2} \right)^2 \left( 1 + 2 \frac{m_Z^2}{m_1^2} \right) , \]  
\text{(B8)}

with \( \epsilon_Z \) is defined as above but for \( Z \) boson couplings. Notice that since \( m_W = c_W m_Z \) the two decay widths are equal up to phase space factors and differences in the mixing factors.
\( \varepsilon_Z \) and \( \varepsilon_W \). However, we have two decay channels into \( W \)'s, \( N_1 \to e^- W^+ \) and \( N_1 \to e^+ W^- \), and only one into \( Z \)'s (we already took into account that the \( \nu_\beta \) are Majorana particles; should we treat them as Weyl particles, we have two decay channels and the sum over them gives the same result).

If \( m_1 > m_H \) the \( N_1 \) can also decay into Higgs bosons, \( N_1 \to \nu H \) with a decay width given by

\[
\Gamma (N_1 \to \nu_\beta H) = \frac{|Y_\nu^\beta|^2 m_1}{32\pi} \left( 1 - \frac{m_H^2}{m_1^2} \right)^2 .
\]  

This looks quite different from \( \Gamma (N_1 \to \ell_\beta W^+) \) and \( \Gamma (N_1 \to \nu_\beta Z) \); however, we can use that \( \varepsilon \approx M_D M_N^{-1} \), \( M_D = Y_\nu v/\sqrt{2} \) and \( \alpha/(s_W^2 m_W^2) = 1/(\pi v^2) \) to rewrite \( |\varepsilon|^2 \alpha m_1^3/(s_W^2 m_W^2) \sim |Y_\nu|^2 m_1/(2\pi v^2) \) and see that, in the limit \( m_1 \gg m_H, m_W, m_Z \), the three decay widths are identical. This is required by the equivalence theorem \[63, 64\] which states that, in this limit, the calculation could have been performed in the theory before spontaneous symmetry breaking; in that theory, all the fields except the \( N \) are massless, there is no heavy-light mixing and the \( N \)'s decay into the doublet of leptons and the Higgs scalar doublet through the standard model Yukawa couplings. However, for moderate \( m_1 \), since \( m_H > m_Z > m_W \), the phase space factors are important; in particular \( \Gamma(N_1 \to \nu_\beta H) \) decreases rapidly when approaching the threshold of production.

If \( m_1 < m_W \) the dominant decay is the decay into a light neutrino and a photon. It requires the new interaction and light-heavy mixing.

\[
\Gamma (N_1 \to \nu_\beta \gamma) = \frac{2}{\pi} \left| \varepsilon_\beta^1 \right|^2 c_\beta^2 m_1^3 ,
\]

where \( \varepsilon_\gamma \) is a parameter that characterizes the strength of the \( N_1-\nu_\beta-\gamma \) interaction and it is of the order of \( (1/\Lambda_{NP})\sqrt{m_\nu/m_N} \).

4. \( e^+ e^- \to N_1 N_2 \) cross section

By neglecting the heavy-light mixing, the LEP and ILC cross section is given by

\[
\sigma (e^+ e^- \to N_1 N_2) = \frac{2\alpha}{3} |\zeta_1|^2 f_Z(\sqrt{s}, m_1, m_2) \eta_f(s) \quad (B10)
\]

with \( f = e \),

\[
\eta_f(s) = 4q_f^2 c_W^2 - 4q_f v_f \text{Re}\{\chi(s)\} + \frac{v_f^2 + a_f^2}{c_W^2} |\chi(s)|^2 ,
\]  

(\( B11 \))
and
\[ \chi(s) = \frac{s}{s - m_Z^2 + im_Z \Gamma_Z} . \]

5. Partonic cross sections for \( pp \rightarrow N_1N_2X \)

To compute the \( pp \rightarrow N_1N_2 + X \) cross section we need the different partonic cross sections \( q\bar{q} \rightarrow N_1N_2 \) which proceed through the new interaction and are dominated by \( \gamma \) and \( Z \) exchange.

\[
\frac{d\sigma}{d\Omega} (q\bar{q} \rightarrow N_1N_2) = \frac{\alpha}{6\pi} |\xi_{12}|^2 \eta_q(\hat{s}) \frac{\sqrt{\lambda(\hat{s}, m_1^2, m_2^2)}}{\hat{s}^3} \\
\times \left[ (m_1^2 + m_2^2)(\hat{s} + 2\hat{t}) - 2\hat{t}(\hat{s} + \hat{t}) - (m_1^4 + m_2^4) - 2\hat{s} m_1 m_2 \cos 2\delta_{12} \right],
\]

(B12)

with \( \hat{s} \) and \( \hat{t} \) the Mandelstam variables for the partonic collision in the center of mass frame of the quarks, and \( \eta_q(\hat{s}) \) is defined in eq. (B11) with the quantum numbers appropriate to the quarks. The total partonic cross section is obtained by integration of the angular variables and leads to the result in eq. (B10) with an additional factor 1/3 due to color and with \( q_f, a_f, v_f \) appropriate for \( f = u, d \).

6. Higgs boson decays into right-handed neutrinos \( H \rightarrow N_1N_2 \)

Above we have discussed only cross sections and decays induced by the electroweak moment interaction or by standard model interactions and heavy-light mixing. The last term in eq. (II) also have interesting consequences, in particular if the \( N \)'s are light enough it can induce new decay modes for the Higgs boson. We found

\[
\Gamma (H \rightarrow N_1N_2) = \frac{v^2}{2\pi m_H^3} |\xi_{12}|^2 \sqrt{\lambda(m_H^2, m_1^2, m_2^2)} \left[ (m_H^2 - m_1^2 - m_2^2) - 2m_1 m_2 \cos 2\delta_{12} \right],
\]

(B13)

where \( \xi_{ij} = |\xi_{ij}| e^{i\theta_{ij}} \) and \( v = \sqrt{2}\langle \phi^{(0)} \rangle \).


