

# Study of the $f_2(1270)$ , $f_2'(1525)$ , $\bar{K}_2^*(1430)$ , $f_0(1370)$ and $f_0(1710)$ production from $\psi(nS)$ and $\Upsilon(nS)$ decays

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Based on previous studies that support the important role of the  $f_2(1270)$ ,  $f_2'(1525)$ , and  $\bar{K}_2^*(1430)$  resonances in the  $J/\psi[\psi(2S)] \rightarrow \phi(\omega)VV$  decays, we make an analysis of the analogous decays of  $\Upsilon(1S)$  and  $\Upsilon(2S)$ , taking into account recent experimental data. In addition, we study the  $J/\psi$  and  $\psi(2S)$  radiative decays and we also made predictions for the radiative decay of  $\Upsilon(1S)$  and  $\Upsilon(2S)$  into  $\gamma f_2(1270)$ ,  $\gamma f_2'(1525)$ ,  $\gamma f_0(1370)$  and  $\gamma f_0(1710)$ , comparing with the recent results of a CLEO experiment. We can compare our results for ratios of decay rates with eight experimental ratios and find agreement in all but one case, where experimental problems are discussed.

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## I. INTRODUCTION

Properties of mesons are key issues for understanding the confinement mechanism of QCD. Within the traditional constituent quark models, mesons are described as quark-antiquark ( $q\bar{q}$ ) states. This picture could explain successfully the properties of the ground states of the flavor SU(3) vector meson nonet. However, there are many meson (or mesonlike) states that could not be explained as  $q\bar{q}$  states. Depending on their coupling to specific production mechanisms and their decay pattern, these states are interpreted as molecular-type excitations or as tetraquark states. But, there is debate on their exotic structure, unlike for states that carry spin-exotic quantum numbers, e.g.  $J^{PC} = 1^{-+}$ , and hence cannot be  $q\bar{q}$  states. One has an example in the sector of light mesons with mass below 1 GeV, where the scalar mesons  $\sigma(500)$ ,  $a_0(980)$  and  $f_0(980)$  [1] have been largely debated. Long ago it was suggested that the  $f_0(980)$  and  $a_0(980)$  resonances could be weakly bound states of  $K\bar{K}$  [2]. The advent of chiral unitary theory has brought new light into this issue and by now the  $\sigma(500)$ ,  $a_0(980)$  and  $f_0(980)$  are accepted as dynamically generated states from the interaction of coupled channels  $\pi\pi$ ,  $K\bar{K}$ ,  $\eta\eta$ ,  $\pi\eta$  in  $S$  wave [3–9].

Similarly, in Refs. [10, 11], the former work of Ref. [12] on the  $\rho\rho$  interaction was extended to SU(3) using the local hidden gauge formalism for vector-vector interaction and a unitary approach in coupled channels. This interaction generates resonances, some of which can be associated to known resonances, namely the  $f_2(1270)$ ,

$f_2'(1525)$ , and  $\bar{K}_2^*(1430)$ , as well as the  $f_0(1370)$  and  $f_0(1710)$ . The results obtained in those former works gave support to the idea of the  $f_2(1270)$ ,  $f_2'(1525)$ ,  $\bar{K}_2^*(1430)$ ,  $f_0(1370)$  and  $f_0(1710)$  as being quasimolecular states of two vector mesons. In reactions producing these resonances, a pair of vector mesons are primary produced and these two vector mesons rescatter after the production, giving rise to the resonances that can be observed in the invariant mass distributions.

In Ref. [13], the important role of the  $f_2(1270)$ ,  $f_2'(1525)$ , and  $\bar{K}_2^*(1430)$  in the  $J/\psi \rightarrow \phi(\omega)VV$  decays was studied based on the vector-vector molecular structure of those resonances. Related work was also done in Ref. [14] interpreting the  $J/\psi$  radiative decay into these resonances. Those latter works were then extended in Ref. [15] to study the decay of  $\psi(2S)$  into  $\omega(\phi)$  and  $f_2(1270)$ ,  $f_2'(1525)$  and  $\psi(2S)$  into  $K^{*0}$  and  $\bar{K}_2^{*0}(1430)$ . At the same time, in this latter work the ideas of Ref. [14] in the radiative decay were extended to the decay of  $\Upsilon(1S)$ ,  $\Upsilon(2S)$  and  $\psi(2S)$ . These hadronic and radiative decays for  $J/\psi$  have also been addressed within a scheme where the  $f_0(1370)$ ,  $f_0(1500)$  and  $f_0(1710)$  states emerge as a result of glueball quarkonia mixing [16]. With the steady accumulation of experimental data, new results are now available that can test these theoretical ideas and an update of the theoretical predictions has become timely. In particular the very recent CLEO data on  $J/\psi$ ,  $\psi(2S)$  and  $\Upsilon(1S)$  radiative decays [17] are most welcome.

In the present work, we make a reanalysis of those decays taking into account the recent report of the CLEO data. In addition to the  $J/\psi$  and  $\psi(2S)$  decays, we also made predictions for the radiative decay of  $\Upsilon(1S)$  and  $\Upsilon(2S)$  into  $\phi(\omega)f_2(1270)$ ,  $\phi(\omega)f_2'(1525)$ , or  $K^{*0}$  and  $\bar{K}_2^{*0}(1430)$ . We evaluate ratios of decay rates and can compare with eight experimental ratios. The agreement

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found with experiment is good, with one exception that will require future test due to present experimental difficulties.

## II. HADRONIC DECAY

### A. Formalism for $\Upsilon(1S)$ decay into $\omega(\phi)VV$

We extend here the formalism used in Refs. [13, 15] to study the decay of  $\Upsilon(1S)$  into  $\omega(\phi)$  and two interacting vectors,  $VV$ , that lead to the tensor state. The mechanism is depicted in Fig. 1. We follow the approach of Ref. [18] and write the  $\phi$  and  $\omega$  as a combination of a singlet and an octet of SU(3) states

$$\begin{aligned}\omega &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) = \sqrt{\frac{2}{3}}V_1 + \sqrt{\frac{1}{3}}V_8 \\ \phi &= s\bar{s} = \sqrt{\frac{1}{3}}V_1 - \sqrt{\frac{2}{3}}V_8\end{aligned}\quad (1)$$

The two  $VV$  states combine to  $I = 0$ , either with a  $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$  or  $s\bar{s}$  SU(3) structure to match the SU(3) singlet nature of the  $b\bar{b}$  state. One obtains matrix elements for the  $\Upsilon(1S) \rightarrow \omega\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \rightarrow \omega VV$  and  $\Upsilon(1S) \rightarrow \omega s\bar{s} \rightarrow \omega VV$  amplitudes with the results

$$\frac{2}{3}T^{(1,1)} + \frac{1}{3}T^{(8,8)} \quad \text{and} \quad \frac{\sqrt{2}}{3}T^{(1,1)} - \frac{\sqrt{2}}{3}T^{(8,8)} \quad (2)$$

respectively, where  $T^{(1,1)}$  is the  $T$  matrix for the singlet of  $\phi$  and the one of  $VV$  giving the vacuum and  $T^{(8,8)}$  the corresponding part for the octet.

Similarly, for the  $\Upsilon(1S) \rightarrow \phi\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \rightarrow \phi VV$  and  $\Upsilon(1S) \rightarrow \phi s\bar{s} \rightarrow \phi VV$ , we obtain

$$\frac{\sqrt{2}}{3}T^{(1,1)} - \frac{\sqrt{2}}{3}T^{(8,8)} \quad \text{and} \quad \frac{1}{3}T^{(1,1)} + \frac{2}{3}T^{(8,8)} \quad (3)$$

respectively.

It was found in Ref. [13] that in terms of  $VV$  the  $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$  and  $s\bar{s}$  components could be written as

$$\begin{aligned}\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) &\rightarrow \frac{1}{\sqrt{2}}(\rho^0\rho^0 + \rho^+\rho^- + \rho^-\rho^+ + \omega\omega \\ &\quad + K^{*+}K^{*-} + K^{*0}\bar{K}^{*0}), \quad (4) \\ s\bar{s} &\rightarrow K^{*-}K^{*+} + \bar{K}^{*0}K^{*0} + \phi\phi. \quad (5)\end{aligned}$$

Only the terms in Fig. 1 where the  $VV$  interact lead to the tensor resonance. Then we remove the first diagram of Fig. 1 corresponding to the tree level and this leads to the diagram depicted in Fig. 2.

The final transition matrix for  $\Upsilon(1S) \rightarrow \omega(\phi, K^{*0})R$ , with  $R$  the resonance under consideration, is given by

$$t_{\Upsilon(1S) \rightarrow \omega R} = \sum_j W_j^{(\omega)} G_j g_j, \quad (6)$$

with  $W_j^{(\omega)}$  the weights given in Ref. [13],  $G_j$  the  $VV$  loop functions and  $g_j$  the couplings of the resonance considered to the corresponding  $VV$  channel  $j$ . We proceed similarly for  $\Upsilon(1S) \rightarrow \phi R$  or  $\Upsilon(1S) \rightarrow K^{*0}R$  and all values of  $W, G, g$  are tabulated in Ref. [13].

The  $\Upsilon(1S)$  partial decay width is then given by

$$\Gamma = \frac{1}{8\pi} \frac{1}{M_{\Upsilon(1S)}^2} |t|^2 q \quad (7)$$

with  $q$  the momentum of the  $\omega(\phi, K^{*0})$  in the  $\Upsilon(1S)$  rest frame.

We apply the formalism to the same decay channels but from the  $\Upsilon(2S)$  state. In terms of quarks the  $\Upsilon(2S)$  state is the  $b\bar{b}$  state with the same quantum numbers as the  $\Upsilon(1S)$ , with a radial excitation to the  $2S$  state of a  $b\bar{b}$  potential. We then expect the same behavior as for the  $\Upsilon(1S)$ , which stands as the  $1S$   $b\bar{b}$  state. In the derivation we have only used two properties from the  $\Upsilon(1S)$  concerning these decays. First that it is an SU(3) singlet, which can also be said of the  $\Upsilon(2S)$  state. The other dynamical feature is related to the Okubo-Zweig-Iizuka (OZI) violation and the weight going into  $\phi s\bar{s}$  or  $\phi\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$  in its decay, that we parametrize in terms of  $\nu = T^{(1,1)}/T^{(8,8)}$  [13, 15]. Given the fact that this is also a dynamical feature not related to the internal excitation of the  $b\bar{b}$  quarks in the potential well, we shall also assume that the  $\nu$  parameter is the same for  $\Upsilon(2S)$  as for  $\Upsilon(1S)$ . The normalization of the  $W_j^{(\omega)}$  weights can be different but this will cancel in the ratios. With these two reasonable assumptions we can make predictions for the following four ratios that are discussed in the next subsection.

### B. Numerical results of hadronic decay

We collect the new results on these two sets of reactions, which are  $\Upsilon(1S)$  and  $\Upsilon(2S)$  hadronic decays, respectively.

In Ref. [15] results were given for the ratios  $\widehat{R}_1$ ,  $\widehat{R}_2$ ,  $\widehat{R}_3$ , and  $\widehat{R}_4$ , for  $\psi(2S) \rightarrow \omega(\phi, K^{*0})R$ , and the ratios were found compatible with experiments. We generalize them here to the  $\Upsilon(1S)$  decay.

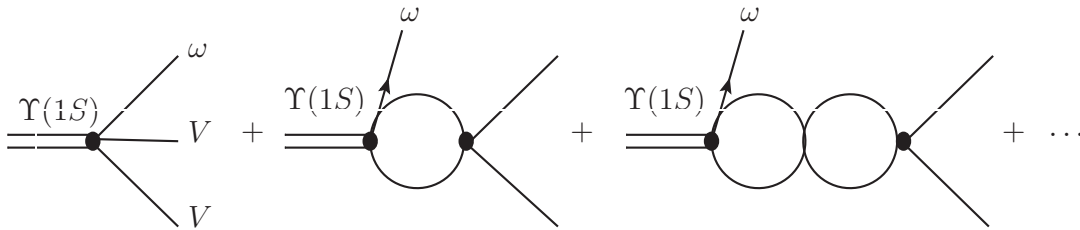
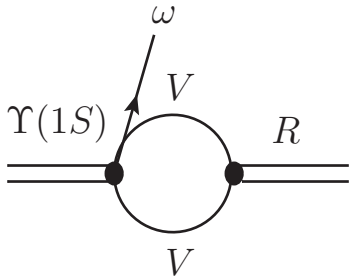
$$\widehat{R}_1 \equiv \frac{\Gamma_{\Upsilon(1S) \rightarrow \phi f_2(1270)}}{\Gamma_{\Upsilon(1S) \rightarrow \phi f_2'(1525)}}, \quad (8)$$

$$\widehat{R}_2 \equiv \frac{\Gamma_{\Upsilon(1S) \rightarrow \omega f_2(1270)}}{\Gamma_{\Upsilon(1S) \rightarrow \omega f_2'(1525)}}, \quad (9)$$

$$\widehat{R}_3 \equiv \frac{\Gamma_{\Upsilon(1S) \rightarrow \omega f_2(1270)}}{\Gamma_{\Upsilon(1S) \rightarrow \phi f_2(1270)}}, \quad (10)$$

$$\widehat{R}_4 \equiv \frac{\Gamma_{\Upsilon(1S) \rightarrow K^{*0}\bar{K}_2^{*0}(1430)}}{\Gamma_{\Upsilon(1S) \rightarrow \omega f_2(1270)}}. \quad (11)$$

Here we bring new data for new decays reported in Ref. [19] on  $\Upsilon(1S)$  and  $\Upsilon(2S)$  decays. Concretely,

FIG. 1: Production mechanism of  $\omega$  and two interacting vector mesons.FIG. 2: Selection of diagrams of Fig. 1 that go into resonance formation, omitting the coupling to  $VV$  without interaction.

$Br[\Upsilon(1S) \rightarrow \phi f_2'(1525)]$ ,  $Br[\Upsilon(1S) \rightarrow \omega f_2(1270)]$ ,  $Br[\Upsilon(1S) \rightarrow K^{*0}(892)\bar{K}_2^{*0}(1430)]$ , and the same decays for  $\Upsilon(2S)$ . In Table I we show the new numbers for the  $\Upsilon(1S)$  decays. The criteria used to obtain the theoretical errors are the same as in Ref. [15]. For this case, we have three data and can obtain two ratios, and as we can see, we find agreement with experiment within errors.

TABLE I: Numerical results of  $\Upsilon(1S)$  decays.  $\widehat{R}_1 \cdot \widehat{R}_3$  provides the ratio  $\Gamma_{\Upsilon(1S) \rightarrow \omega f_2(1270)} / \Gamma_{\Upsilon(1S) \rightarrow \phi f_2'(1525)}$ . Experimental data are taken from Ref. [19]. The numbers in parentheses are the theoretical values with upper and lower errors. The numbers before them indicate the band of theoretical values considering the former errors.

	Theory	Experiment
$\widehat{R}_1$	0.11 – 0.52 (0.24 $^{+0.28}_{-0.13}$ )	
$\widehat{R}_2$	2.58 – 11.99 (5.19 $^{+6.80}_{-2.61}$ )	
$\widehat{R}_3$	5.64 – 17.46 (9.67 $^{+7.77}_{-4.06}$ )	
$\widehat{R}_4$	0.94 – 2.37 (1.50 $^{+0.87}_{-0.56}$ )	1.75-10.9
$\widehat{R}_1 \cdot \widehat{R}_3$	0.73 – 5.61 (2.32 $^{+3.29}_{-1.59}$ )	0.0-8.76

For the case of  $\Upsilon(2S)$  hadronic decays, we define the

equivalent four ratios,

$$\overline{R}_1 \equiv \frac{\Gamma_{\Upsilon(2S) \rightarrow \phi f_2(1270)}}{\Gamma_{\Upsilon(2S) \rightarrow \phi f_2'(1525)}}, \quad (12)$$

$$\overline{R}_2 \equiv \frac{\Gamma_{\Upsilon(2S) \rightarrow \omega f_2(1270)}}{\Gamma_{\Upsilon(2S) \rightarrow \omega f_2'(1525)}}, \quad (13)$$

$$\overline{R}_3 \equiv \frac{\Gamma_{\Upsilon(2S) \rightarrow \omega f_2(1270)}}{\Gamma_{\Upsilon(2S) \rightarrow \phi f_2(1270)}}, \quad (14)$$

$$\overline{R}_4 \equiv \frac{\Gamma_{\Upsilon(2S) \rightarrow K^{*0}\bar{K}_2^{*0}(1430)}}{\Gamma_{\Upsilon(2S) \rightarrow \omega f_2(1270)}}. \quad (15)$$

The values of these ratios are shown in Table II. For the case of  $\Upsilon(2S)$  the datum for  $\omega f_2(1270)$  with negative width and large errors cannot be used for ratios and hence we can only construct one ratio. We can see that we find agreement of the theoretical numbers with the only experimental ratio that we can form.

TABLE II: Numerical results of  $\Upsilon(2S)$  decays.  $\overline{R}_1 \cdot \overline{R}_3 \cdot \overline{R}_4$  provides the ratio  $\Gamma_{\Upsilon(2S) \rightarrow K^{*0}\bar{K}_2^{*0}(1430)} / \Gamma_{\Upsilon(2S) \rightarrow \phi f_2'(1525)}$ . Experimental data are taken from Ref. [19]. The numbers in parentheses are the theoretical values with upper and lower errors. The numbers before them indicate the band of theoretical values considering the former errors.

	Theory	Experiment
$\overline{R}_1$	0.11 – 0.51 (0.24 $^{+0.27}_{-0.13}$ )	
$\overline{R}_2$	2.58 – 11.99 (5.19 $^{+6.79}_{-2.61}$ )	
$\overline{R}_3$	5.63 – 17.45 (9.69 $^{+7.76}_{-4.06}$ )	
$\overline{R}_4$	0.94 – 2.37 (1.50 $^{+0.87}_{-0.56}$ )	
$\overline{R}_1 \cdot \overline{R}_3 \cdot \overline{R}_4$	0.77 – 8.71 (3.49 $^{+5.22}_{-2.72}$ )	0.0 – 7.92

### III. RADIATIVE DECAY

#### A. Formalism for $\psi(nS)$ and $\Upsilon(nS)$ decay into $\gamma VV$

Another successful test on the vector-vector nature of the  $f_2(1270)$  and  $f_2'(1525)$  was done in Ref. [14] by looking at the decay of  $J/\psi$  into  $\gamma T$ , where  $T$  is any of these two tensor resonances. A justification was given in Ref. [14] for the photon being radiated from the initial  $c\bar{c}$  state. The remaining  $c\bar{c}$  gave rise to a pair of vector mesons which upon rescattering produced the tensor resonances. The only dynamical assumption made in Ref. [14] was that the photon was radiated from the  $c\bar{c}$  state, not from the final  $VV$  state. Translated to the present problem, the argument is based on the dominance of the diagram of Fig. 3(a) over the one of Fig. 3(b), which require two, versus three, gluon exchange as discussed in Ref. [27] for the  $J/\psi$  case.

In the present work the  $b\bar{b}$  state is assumed to be an SU(3) singlet in  $\Upsilon$  decay, like in the case of the  $c\bar{c}$  state. Both assumptions hold equally here and hence the only difference in the results stems from an overall normalization, which disappears when ratios are made, and the momenta  $q$  in the formula of the width, since now the  $\Upsilon$  mass is different to the one of the  $J/\psi$ . It is easy to extend the results of  $J/\psi$  to the decay of the  $\Upsilon$ . The mechanism is depicted in Fig. 4 for  $\Upsilon(1S)$  radiative decay. When this is taken into account, we can evaluate the same ratios for  $\Upsilon(nS)$  radiative decay as those for  $J/\psi$  case.

The extended formalism for the transition amplitudes of  $\Upsilon(1S)$  decay into  $\gamma T$  provides the amplitudes

$$t_{\Upsilon(1S)\rightarrow\gamma R} = \sum_j \widetilde{w}_j G_j g_j. \quad (16)$$

and the weights  $\widetilde{w}_j$  are the same as those obtained in Ref. [14] and given by

$$\widetilde{w}_i = c \begin{cases} -\sqrt{\frac{3}{2}} & \text{for } \rho\rho \\ -\sqrt{2} & \text{for } K^*\bar{K}^* \\ \frac{1}{\sqrt{2}} & \text{for } \omega\omega \\ \frac{1}{\sqrt{2}} & \text{for } \phi\phi \end{cases}. \quad (17)$$

where  $c$  is a normalization constant, which cancels in the ratios, and  $G_j$ ,  $g_j$  are again the loop functions of the intermediate  $VV$  states and the couplings of the resonance to these  $VV$  channels. All these quantities are given in Table 1 of Ref. [14]. The same theoretical framework allows us to evaluate the  $\Upsilon(1S)$  radiative decay into the scalar meson  $f_0(1370)$  and  $f_0(1710)$  which are also obtained from the interaction of  $VV$ , mostly  $\rho\rho$  and  $K^{*0}\bar{K}^{*0}$  respectively. The decay width is given again by Eq. (7) where  $q$  is the momentum of the photon in the  $\Upsilon(1S)$  rest frame.

#### B. Numerical results of radiative decays

For the radiative decays, we find in the PDG new results for  $\Upsilon(1S)$ . For the  $\Upsilon(2S)$  there are only upper bounds and we cannot compare ratios. There is also a new set of data on  $J/\psi \rightarrow \gamma T$  and  $\psi(2S) \rightarrow \gamma T$  from Ref. [17], and similarly going to  $\gamma S$ , where  $S$  is any of the scalar mesons  $f_0(1370)$ ,  $f_0(1710)$ .

We have taken advantage of the fact that these data for  $J/\psi$  and  $\psi(2S)$  radiative decays come from the same experiment, so they are advantageous for the evaluation of ratios since they usually cancel systematic uncertainties. The data of Ref. [17] are given for  $J/\psi[\psi(2S)] \rightarrow \gamma R \rightarrow \gamma\pi\pi$  or  $J/\psi[\psi(2S)] \rightarrow \gamma R \rightarrow \gamma K\bar{K}$ . In order to convert those numbers in partial decay widths we divide by the  $V \rightarrow \pi\pi$  or  $V \rightarrow K\bar{K}$  branching ratio and add relative errors in quadrature. In Table III we show the branching ratios that we used in the present work. Some of these branching ratios are not well known, and there are different values for them, such as for  $f_0(1370) \rightarrow K\bar{K}$  and  $f_0(1710) \rightarrow K\bar{K}$ . We take an approximate average value, compatible with the different results.

Then we evaluate the ratios,

$$R_T = \frac{\Gamma_{J/\psi \rightarrow \gamma f_2(1270)}}{\Gamma_{J/\psi \rightarrow \gamma f_2'(1525)}}, \quad (18)$$

$$R_S = \frac{\Gamma_{J/\psi \rightarrow \gamma f_0(1370)}}{\Gamma_{J/\psi \rightarrow \gamma f_0(1710)}}, \quad (19)$$

$$\widetilde{R}_T = \frac{\Gamma_{\psi(2S) \rightarrow \gamma f_2(1270)}}{\Gamma_{\psi(2S) \rightarrow \gamma f_2'(1525)}}, \quad (20)$$

$$\widetilde{R}_S = \frac{\Gamma_{\psi(2S) \rightarrow \gamma f_0(1370)}}{\Gamma_{\psi(2S) \rightarrow \gamma f_0(1710)}}, \quad (21)$$

$$\widehat{R}_T = \frac{\Gamma_{\Upsilon(1S) \rightarrow \gamma f_2(1270)}}{\Gamma_{\Upsilon(1S) \rightarrow \gamma f_2'(1525)}}, \quad (22)$$

$$\widehat{R}_S = \frac{\Gamma_{\Upsilon(1S) \rightarrow \gamma f_0(1370)}}{\Gamma_{\Upsilon(1S) \rightarrow \gamma f_0(1710)}}, \quad (23)$$

$$\overline{R}_T = \frac{\Gamma_{\Upsilon(2S) \rightarrow \gamma f_2(1270)}}{\Gamma_{\Upsilon(2S) \rightarrow \gamma f_2'(1525)}}, \quad (24)$$

$$\overline{R}_S = \frac{\Gamma_{\Upsilon(2S) \rightarrow \gamma f_0(1370)}}{\Gamma_{\Upsilon(2S) \rightarrow \gamma f_0(1710)}}. \quad (25)$$

The numerical results are summarized in Table IV compared with the experimental data. We note that the comparison with the experimental results is particularly valuable since the theoretical results were predictions done before (see Ref. [15]) that can be contrasted with data observed later. We see that we have agreement in all numbers except for the ratio of  $\widetilde{R}_T$  where even within errors there is a discrepancy of about a factor two. Actually, the reason for the large experimental value of this ratio is the small value for  $Br[\psi(2S) \rightarrow \gamma f_2'(1525)]$ . One can see in Ref. [17] that this rate is small in absolute value since the ratio  $\mathfrak{B}_2[\psi(2S)]/\mathfrak{B}_2[J/\psi]$  is of 4.1% (see more details of Table VI in Ref. [17]) and is the smallest one of the eight ratios tabulated there, diverg-

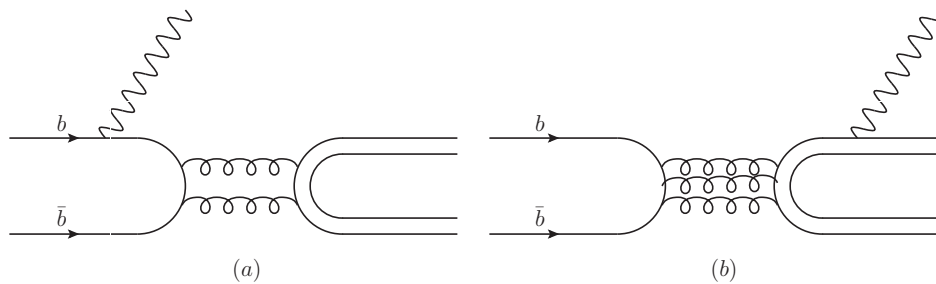


FIG. 3: Two mechanisms of the  $\Upsilon(1S)$  radiative decays.

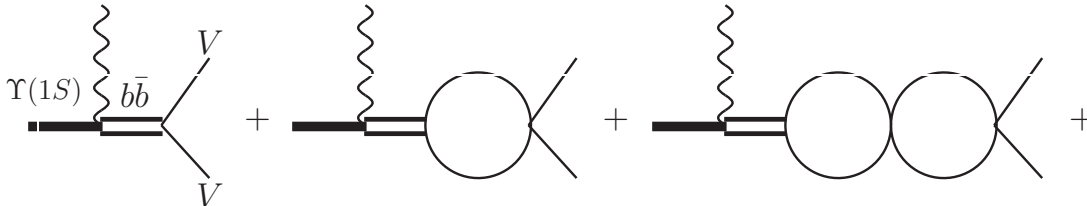


FIG. 4: Schematic representation of  $\Upsilon(1S)$  decay into a photon and one dynamically generated resonance.

ing significantly from the 13% rule for this ratio. We have consulted the authors of Ref. [17], who admit problems in this datum for the  $\psi(2S)$  transitions, as we can see in Fig. 9 of Ref. [17], where the relative strength of  $f'_2(1525)$  and  $f_0(1710)$  are quite different in the decay modes  $\psi(2S) \rightarrow \gamma K^+ K^-$  and  $\psi(2S) \rightarrow \gamma K_s K_s$ , when they should be the same.

The problem for this  $\psi(2S)$  transition, together with the discrepancy with the theoretical results, which have otherwise been successful in the other cases, should serve as incentive to have a further experimental look into this transition.

#### IV. CONCLUSION

A further test on the molecular nature of the  $f_2(1270)$ ,  $f'_2(1525)$  and  $K_2^*(1430)$  has been made, using the decay of  $\Upsilon(nS)$  into  $\phi(\omega)$  and any of the  $f_2(1270)$ ,  $f'_2(1525)$  resonances, or  $K^*(892)$  and  $K_2^*(1430)$ . We have also studied the same decays from the  $\psi(nS)$  state. The theory only makes use of the fact that both  $\psi(nS)$  and  $\Upsilon(nS)$  are singlets of SU(3). A dynamical factor for the OZI violation into the strange and nonstrange sectors, the  $\nu$  parameter, is taken from other experiments. The needed modifications due to kinematics with respect to the analogous cases of  $J/\psi$ ,  $\psi(2S)$  decays have been done and results for the decays of the  $\Upsilon(nS)$  are found in agreement with experiment for the cases where experimental information is available.

We also analyzed the radiative decay  $J/\psi$ ,  $\psi(2S)$ ,  $\Upsilon(1S)$  and  $\Upsilon(2S)$  into a photon and a tensor  $f_2(1270)$ ,  $f'_2(1525)$  or a photon and a scalar  $f_0(1370)$ ,  $f_0(1710)$ .

New data on these decays has been reported recently from the CLEO Collaboration which has allowed us to compare with predictions for these decays made prior to the experiment. The agreement found with experiment is good in all cases except in one ratio involving the  $Br[\psi(2S) \rightarrow \gamma f'_2(1525)]$  decay which was found exceptionally small in the experiment and was admitted as problematic there. Those problems and the discrepancy with the theory, which otherwise is in agreement with the data, calls for a further reanalysis of this datum.

The overall agreement found with the data on different experiments provides extra support for the picture in which the tensor states  $f_2(1270)$ ,  $f'_2(1525)$ ,  $\bar{K}_2^*(1430)$ , as well as the scalar ones  $f_0(1370)$  and  $f_0(1710)$  are dynamically generated states from the vector-vector interaction.

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TABLE III: Values of some of the parameters used in this work.

Ref.	Decay channels					
	$f_2(1270) \rightarrow \pi\pi$	$f_2'(1525) \rightarrow K\bar{K}$	$f_0(1370) \rightarrow K\bar{K}$		$f_0(1710) \rightarrow K\bar{K}$	
	[1]	[1]	[20, 21]	[22]	[22]	[23] [10]
Branching ratios (%)	$84.8^{+2.4}_{-1.2}$	$88.7 \pm 2.2$	$35 \pm 13$	$\sim 20$	$36 \pm 12$	$38^{+9}_{-19}$ 55
Adopted value (%)	$84.8 \pm 1.3$	$88.7 \pm 2.2$	$30 \pm 15$		$35 \pm 15$	

TABLE IV: Ratios of the molecular model and in comparison with data from Refs. [17, 24–26].

	Molecular picture	Data
$R_T (J/\psi)$	$2 \pm 1$ (Ref. [14])	$3.18^{+0.58}_{-0.64}$ (Refs. [24–26]) $2.57 \pm 0.5$ (Ref. [17])
$R_S (J/\psi)$	$1.2 \pm 0.3$	$0.51 \pm 0.41$ (Ref. [17])
$\widetilde{R}_T (\psi(2S))$	$1.94 \pm 0.97$	$8.6 \pm 3.0$ (Ref. [17])
$\widetilde{R}_S (\psi(2S))$	$1.14 \pm 0.28$	$0.81^{+0.84}_{-0.73}$ (Ref. [17])
$\widehat{R}_T (\Upsilon(1S))$	$1.84 \pm 0.92$	$2.66 \pm 0.67$ (Ref. [17])
$\widehat{R}_S (\Upsilon(1S))$	$1.05 \pm 0.26$	
$\overline{R}_T (\Upsilon(2S))$	$1.83 \pm 0.92$	
$\overline{R}_S (\Upsilon(2S))$	$1.05 \pm 0.26$	

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