

Time Reversal Violation from the entangled $B^0\bar{B}^0$ system

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ABSTRACT:

We discuss the concepts and methodology to implement an experiment probing directly Time Reversal (T) non-invariance, without any experimental connection to CP violation, by the exchange of *in* and *out* states. The idea relies on the $B^0\bar{B}^0$ entanglement and decay time information available at B factories. The flavor or CP tag of the state of the still living neutral meson by the first decay of its orthogonal partner overcomes the problem of irreversibility for unstable systems, which prevents direct tests of T with incoherent particle states. T violation in the time evolution between the two decays means experimentally a difference between the intensities for the time-ordered ($\ell^+ X$, $J/\psi K_S$) and ($J/\psi K_L$, $\ell^- X$) decays, and three other independent asymmetries. The proposed strategy has been applied to simulated data samples of similar size and features to those currently available, from which we estimate the significance of the expected discovery to reach many standard deviations.

Contents

1	Introduction	2
2	Time Reversal from entangled $B^0\bar{B}^0$ decays	4
3	Methodology and T-violating parameters	6
4	Simulation study	8
4.1	Generation of samples	8
4.2	Sensitivity results	11
4.3	Asymmetries	11
4.4	Significance of results	13
5	Conclusions	14
6	Acknowledgments	15
A	The $B_-\rangle$ and $B_+\rangle$ states	15

1 Introduction

Violation of the CP symmetry has been observed in K and B meson decays [1–4]. In the Standard Model (SM) the mechanism of CP violation in weak interactions arises from the joint effect of the three non-vanishing mixing angles and the single irreducible phase in the 3-family Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The existence of this matrix conveys the fact that the quarks that participate in weak processes are a linear combination of mass eigenstates. This mechanism has been confirmed to be the dominant by experiments probing CP violation, particularly with studies involving B decays [5].

In the context of local quantum field theories with Lorentz invariance and Hermiticity, the CPT theorem [6] ensures a theoretical constraint between the CP and T symmetries. Although all present tests of CPT invariance confirm the validity of this symmetry, particularly in the neutral-kaon system where there are strong limits [7, 8], the theoretical connection between CP and T does not imply an experimental identity, except for processes which are identical under CPT transformation. Moreover, it is worthwhile to search for direct evidence of T non-invariance, with neither experimental nor theoretical connection to CP violation and CPT invariance. There is at present no existing result that clearly shows T violation in this sense [9].

In the case of transition processes, due to the antiunitarity of the operator implementing the symmetry, T invariance requires that the likelihood for reaction $in \rightarrow out$ equals that of $out \rightarrow in$ once the initial configurations, namely in in one case and out in the other, have been precisely prepared. However, the likelihood of the time reversed version of the process to happen is very low or impractical in some cases, like for unstable systems. This explains why T is much more difficult to study directly than P , C , and CP .

Since the SM is CPT invariant, it predicts T -violating effects in parallel to each CP -violation effect that arises due to the interference of amplitudes with different weak phases. These may appear in three different ways: T violation in decay; T violation in the mixing of neutral states; and T violation that arises from the interference between decay with and without mixing.

T violation matched to CP violation in decay has not been observed, because the difficulties of the preparation of the time reversed decay process. Let us see the example [10] of the rare weak decay of neutral- B mesons to $K^+\pi^-$, for which direct CP violation is well established [4] (different decay rates R_1 and R_2 for $B^0 \rightarrow K^+\pi^-$ and $\bar{B}^0 \rightarrow K^-\pi^+$, respectively) due to the ability of B factories to produce hundreds of millions of B mesons. T violation as implied by this result, combined with CPT invariance, tells us that the rates for the inverse processes $K^+\pi^- \rightarrow B^0$ and $K^-\pi^+ \rightarrow \bar{B}^0$ should be R_2 and R_1 , respectively. However, there is little chance to measure these inverse rates and check directly this prediction since the weak interaction production mechanism is highly suppressed ($B^0 \rightarrow K^+\pi^-$ branching ratio of order 10^{-5}) and the strong interaction would completely swamp the feeble weak process.

T violation associated to CP violation in the mixing has been experimentally analyzed in K [11] and B mesons [12]. Here one looks whether the rate for a neutral- K (B) meson tagged at its production as K^0 (B^0) and identified afterwards as \bar{K}^0 (\bar{B}^0) is equal to the

rate for the neutral particle tagged at its production as \bar{K}^0 (\bar{B}^0) and identified later as K^0 (B^0). Any difference in this case is both CP and T violating, because CP and T are experimentally identical for this process. The experimental results for kaons yielded a T -violating difference in these rates. Such a difference is proportional not only to the T -violating term of the $K^0\bar{K}^0$ matrix that defines the mass eigenstates in terms of the flavor eigenstates, but also to the width difference $\Delta\Gamma$ between the two mass eigenstates, thus T non-invariance would not be present in the limit $\Delta\Gamma \rightarrow 0$. Therefore, this asymmetry shows T violation proportional to $\Delta\Gamma$, time independent, experimentally identical to CP violation, thus it is not an independent T non-invariance test as one might like [9]. In the $B^0\bar{B}^0$ system no asymmetry has been yet found, as expected within the SM since in this case $\Delta\Gamma$ almost vanishes [13].

The largest CP -violating asymmetry in Nature has been found between the rate for $B^0 \rightarrow J/\psi K_S$ (and other similar CP -odd $c\bar{c}K_S$ final states, e.g., $\psi(2S)K_S$, $\chi_{c1}K_S$, or the CP -even $J/\psi K_L$ final state) and the CP -conjugate rate for \bar{B}^0 to decay to the same CP -eigenstate, which is originated in the interference between the time-dependent decay amplitudes with and without mixing [3, 5]. Here, CP violation arises because the mixing phase, i.e., the relative phase between the complex parameters defining the mass eigenstates in terms of the flavor eigenstates [14] minus the relative phase of the ratio of the amplitude for the decay and its CP conjugate, does not vanish. In the SM this phase difference is 2β , where β is the angle between the $V_{cd}V_{cb}^*$ and $V_{td}V_{tb}^*$ sides of the CKM unitarity triangle. At B factories the experimental studies are performed using the two B mesons produced in the antisymmetric coherent state from the $\Upsilon(4S)$ decay. One neutral- B meson decays into a definite flavor state and the other is reconstructed in the CP -eigenstate final state of interest with a given decay time difference $\Delta t = t_{CP} - t_{\text{flavor}}$. The measured asymmetry is large, proportional to $\sin 2\beta \approx 0.7$, a time-odd dependent function that reverses sign between B^0 and \bar{B}^0 tagged events, and between $B^0 \rightarrow J/\psi K_S$ and $B^0 \rightarrow J/\psi K_L$ events with the same flavor tag. This is the observed CP -violating effect. However, since there is no reversal of *in* and *out* states, this time asymmetry cannot be interpreted as genuine T violation.

In this paper we propose and describe a methodology that makes use of the Einstein-Podolsky-Rosen (EPR) entanglement [15] available at current B factories to overcome the problem of irreversibility and perform a direct observation of Time Reversal Violation. The method relies precisely on the possibility for preparing the quantum mechanical individual state of the neutral- B meson by the observation of particular decay channels of its orthogonal entangled partner, and studying the time evolution of the filtered state of the still living meson. This strategy allows the interchange *in* \leftrightarrow *out* states for a given process. Whereas the basic ideas have been presented previously [16] and scrutinized later [9, 10, 17], here we discuss for the first time the steps to implement these concepts into an actual experiment able to produce the desired results. In addition, using a simulation of data samples of similar size and properties to those currently available at B factories, we illustrate the application of the methodology and estimate the significance of the expected discovery. The proposed analysis is independent of the underlying theory, CPT invariant or not, for describing the relevant transitions.

2 Time Reversal from entangled $B^0\bar{B}^0$ decays

Bose statistics, for a $B^0\bar{B}^0$ system with $C = -$, requires these two mesons to be in an antisymmetric state of the system for any pair of orthogonal individual states. These can be either the B states projected by flavor,

$$|i\rangle = \frac{1}{\sqrt{2}}[B^0(t_1)\bar{B}^0(t_2) - \bar{B}^0(t_1)B^0(t_2)], \quad (2.1)$$

or the ones projected by CP final states [19],

$$|i\rangle = \frac{1}{\sqrt{2}}[B_+(t_1)B_-(t_2) - B_-(t_1)B_+(t_2)], \quad (2.2)$$

where t_1 and t_2 are the labels to specify the states “1” and “2” of each neutral- B meson by means of the time of its future decay, with $\Delta\tau = t_2 - t_1 > 0$. The antisymmetry remains invariant with the time evolution, including mixing, before the first decay at t_1 . In Eq. (2.2), B_- is the neutral- B state filtered by its decay to $J/\psi K_+$, K_+ being the neutral- K state filtered by its decay to $\pi\pi$, and B_+ is orthogonal to B_- , not connected to $J/\psi K_+$. Similarly, in Eq. (2.1), \bar{B}^0 is the neutral- B state filtered by its decay to $\ell^- X$, for example a semileptonic decay with a negatively charged lepton or a hadronic final state containing a D^+ or D^{*+} meson. We note that treating $J/\psi K_S$, $K_S \rightarrow \pi\pi$ final states as $J/\psi K_+$, i.e., neglecting CP violation in the neutral-kaon system, introduces effects that are small and thus can be neglected. Similarly, treating hadronic final states containing $D^{(*)+}$ mesons as \bar{B}^0 , i.e., neglecting CP violation due to interference from doubly CKM-suppressed decays, introduces effects small enough that can be considered as corrections. The same applies to B_+ as the state connected to $J/\psi K_-$ and treated as $J/\psi K_L$, and B^0 as the state connected to hadronic final states containing $D^{(*)-}$ mesons. The identification of B_- , B_+ as those states filtered by the decay to CP eigenstates is illustrated in Appendix A.

Therefore, in addition to the *flavor tagging* used in standard CP studies at B factory experiments [3, 18] we can apply a *CP tagging* [19] to one of the B mesons decaying into the CP -odd final state $J/\psi K_S$, preparing the orthogonal B meson as B_+ state at the initial time t_1 . Afterwards it decays at t_2 and is reconstructed in the flavor (B^0 or \bar{B}^0) final state of interest ($\ell^+ X$ or $\ell^- X$, respectively). This combination of flavor and CP tags allows to filter initial and final states to compare, for example, the rate for a \bar{B}^0 evolving to B_- ($\ell^+ X$ decay product of the B partner first, $J/\psi K_S$ final state later) with the rate for a B_- evolving to \bar{B}^0 ($J/\psi K_L$ decay product of the B partner first, $\ell^- X$ final state later). The relation between these T -conjugated transitions and the reconstructed final states in the experimental B factory scheme is illustrated in Fig. 1. The comparison between the time evolution of the neutral- B meson from its preparation as \bar{B}^0 until its decay as B_- , $\bar{B}^0(t_1) \rightarrow B_-(t_2)$, and its T transformed $B_-(t_1) \rightarrow \bar{B}^0(t_2)$ is our proposed test of Time Reversal symmetry.

There are other three independent comparisons between T -conjugated processes, as summarized in Table 1. A non-vanishing asymmetry in the rates for any pair of T -conjugated transitions thus constitutes a direct and independent observation of T violation, in the sense discussed previously.

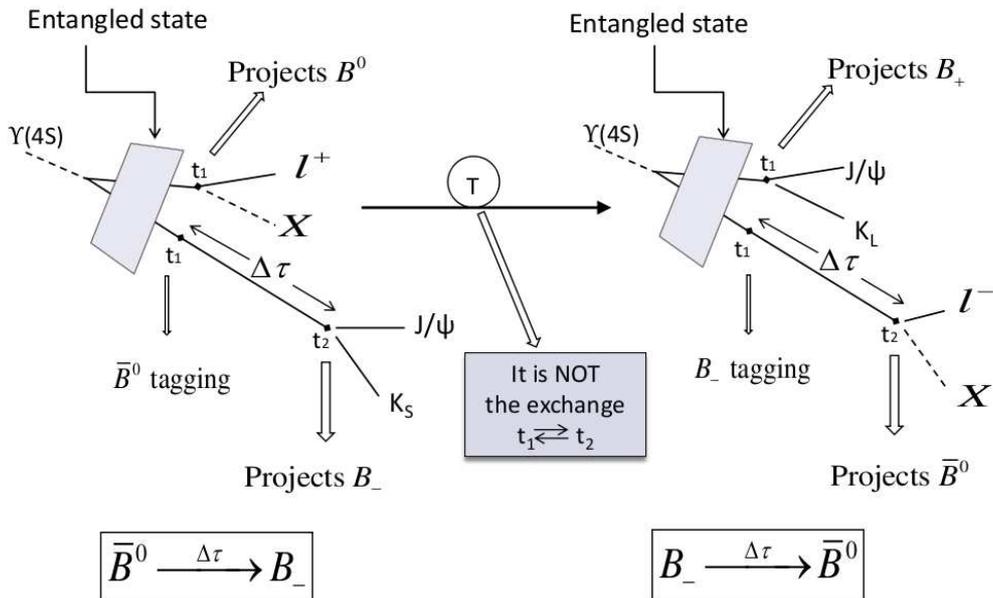


Figure 1. Sketch of two T -conjugate transitions in an experimental B factory scheme. The observation of the final states associated to the T -transformed transitions is divided into three well defined steps. We first observe the decay of one of the entangled B particles, produced in the $\Upsilon(4S)$ decay, into a definite flavor (or a definite CP) decay products at t_1 , preparing the state of the other entangled B particle, which has not yet decayed at t_1 , into its orthogonal state. This tagged B meson state evolves in time to finally decay at $t_2 > t_1$ into a CP (or a flavor) final state. It should be noted that T asymmetry is clearly different from the Δt ($t_1 \leftrightarrow t_2$) exchange and CP asymmetries. In fact, in the former we require to compare the reference transition $\bar{B}^0 \rightarrow B_-$, flavor-tagged by $\ell^+ X$ and decayed to $J/\psi K_S$ ($\ell^+ X, J/\psi K_S$), to the transition $B_- \rightarrow \bar{B}^0$, CP -tagged by $J/\psi K_L$ and decayed to $\ell^- X$ ($J/\psi K_L, \ell^- X$), whereas for Δt and CP asymmetries the reference decay products must be compared to $(J/\psi K_S, \ell^+ X)$ and $(\ell^- X, J/\psi K_S)$, respectively.

We can also apply this methodology for similar tests of CP violation and CPT invariance, providing a proof that T non-invariance is compensated by CP violation. Tables 2 and 3 summarize all the possible comparisons of CP - and CPT -conjugated transitions, together with the corresponding final states. As anticipated, the transitions involved in the experimental tests of CP and T symmetries are different. For example, a test of CP symmetry can be done with the $J/\psi K_S$ final state only. On the contrary, a test of T invariance necessarily involves both $J/\psi K_S$ and $J/\psi K_L$ final states. Furthermore, one may check that none of all comparisons between T -, CP -, or CPT -conjugated transitions in Tables 1, 2, and 3, respectively, corresponds to exchange of t_1 and t_2 .

Reference		T -conjugate	
Transition	Final state	Transition	Final state
$\bar{B}^0 \rightarrow B_-$	$(\ell^+ X, J/\psi K_S)$	$B_- \rightarrow \bar{B}^0$	$(J/\psi K_L, \ell^- X)$
$B_+ \rightarrow B^0$	$(J/\psi K_S, \ell^+ X)$	$B^0 \rightarrow B_+$	$(\ell^- X, J/\psi K_L)$
$\bar{B}^0 \rightarrow B_+$	$(\ell^+ X, J/\psi K_L)$	$B_+ \rightarrow \bar{B}^0$	$(J/\psi K_S, \ell^- X)$
$B_- \rightarrow B^0$	$(J/\psi K_L, \ell^+ X)$	$B^0 \rightarrow B_-$	$(\ell^- X, J/\psi K_S)$

Table 1. Possible comparisons between T -conjugated transitions and the associated time-ordered decay products in the experimental B factory scheme.

Reference		CP -conjugate	
Transition	Final state	Transition	Final state
$\bar{B}^0 \rightarrow B_-$	$(\ell^+ X, J/\psi K_S)$	$B^0 \rightarrow B_-$	$(\ell^- X, J/\psi K_S)$
$B_+ \rightarrow B^0$	$(J/\psi K_S, \ell^+ X)$	$B_+ \rightarrow \bar{B}^0$	$(J/\psi K_S, \ell^- X)$
$\bar{B}^0 \rightarrow B_+$	$(\ell^+ X, J/\psi K_L)$	$B^0 \rightarrow B_+$	$(\ell^- X, J/\psi K_L)$
$B_- \rightarrow B^0$	$(J/\psi K_L, \ell^+ X)$	$B_- \rightarrow \bar{B}^0$	$(J/\psi K_L, \ell^- X)$

Table 2. Possible comparisons between CP -conjugated transitions and the associated time-ordered decay products in the experimental B factory scheme.

Reference		CPT -conjugate	
Transition	Final state	Transition	Final state
$\bar{B}^0 \rightarrow B_-$	$(\ell^+ X, J/\psi K_S)$	$B_- \rightarrow B^0$	$(J/\psi K_L, \ell^+ X)$
$B_+ \rightarrow B^0$	$(J/\psi K_S, \ell^+ X)$	$\bar{B}^0 \rightarrow B_+$	$(\ell^+ X, J/\psi K_L)$
$B^0 \rightarrow B_-$	$(\ell^- X, J/\psi K_S)$	$B_- \rightarrow \bar{B}^0$	$(J/\psi K_L, \ell^- X)$
$B_+ \rightarrow \bar{B}^0$	$(J/\psi K_S, \ell^- X)$	$B^0 \rightarrow B_+$	$(\ell^- X, J/\psi K_L)$

Table 3. Possible comparisons between CPT -conjugated transitions and the associated time-ordered decay products in the experimental B factory scheme.

3 Methodology and T -violating parameters

We can now proceed to a partition of the complete set of final states with definite flavor and CP content into 8 pairs, defined by the first decaying B at t_1 and preparing the tagging state of the still living meson, i.e., B^0, \bar{B}^0, B_-, B_+ , as a function of $\Delta\tau = t_2 - t_1 > 0$. Each of these 8 processes has a time-dependent decay rate $g_{\alpha,\beta}^{\pm}(\Delta\tau)$, where indexes $\alpha \in \{\ell^+, \ell^-\}$ and $\beta \in \{K_S, K_L\}$ run over the final states with definite flavor ($\ell^+ X, \ell^- X$) and CP eigenstates ($J/\psi K_S, J/\psi K_L$), respectively, and the upper index $+$ or $-$ indicates if the decay to the flavor final state α occurred before or after to the CP -eigenstate final state β . Thus a $+$ \rightarrow $-$ replacement corresponds to Δt exchange, which means experimentally the

exchange of the two decay products at t_1 and t_2 .

From only quantum mechanics each decay rate can be written as function of $\Delta\tau$

$$g_{\alpha,\beta}^{\pm}(\Delta\tau) \propto e^{-\Gamma\Delta\tau} \{ C_{\alpha,\beta}^{\pm} \cos(\Delta m\Delta\tau) + S_{\alpha,\beta}^{\pm} \sin(\Delta m\Delta\tau) + D_{\alpha,\beta}^{\pm} \cosh(\Delta\Gamma\Delta\tau) + E_{\alpha,\beta}^{\pm} \sinh(\Delta\Gamma\Delta\tau) \}, \quad (3.1)$$

where Γ is the average decay width, Δm and $\Delta\Gamma$ are the mass and width differences between the mass eigenstates, and $C_{\alpha,\beta}^{\pm}$, $S_{\alpha,\beta}^{\pm}$, $D_{\alpha,\beta}^{\pm}$ and $E_{\alpha,\beta}^{\pm}$ are generic coefficients. This construction makes no assumptions about neither CPT invariance nor CP or T violation. Assuming $\Delta\Gamma = 0$ in the time dependence and renormalizing to the coefficient of the $\cosh(\Delta\Gamma\Delta\tau)$ term, Eq. (3.1) simplifies to

$$g_{\alpha,\beta}^{\pm}(\Delta\tau) \propto e^{-\Gamma\Delta\tau} \{ 1 + C_{\alpha,\beta}^{\pm} \cos(\Delta m\Delta\tau) + S_{\alpha,\beta}^{\pm} \sin(\Delta m\Delta\tau) \}. \quad (3.2)$$

The sine term in Eq. (3.2) results from the interference between amplitudes with and without mixing, whereas the cosine term arises from the interference between decay amplitudes with different weak and strong phases. Effects due to any small lifetime difference in the time dependence, and the renormalization of all coefficients to the $\cosh(\Delta\Gamma\Delta\tau)$ term for each subsample separately, introduce small corrections.

It then follows that asymmetries in decay rates for any pair of T -conjugated transitions (Table 1) would be apparent through differences between their respective best fit $S_{\alpha,\beta}^{\pm}$ and $C_{\alpha,\beta}^{\pm}$ coefficients. For example, a significant difference between the S_{ℓ^+,K_S}^+ and S_{ℓ^-,K_L}^- coefficients would imply observation of T violation. In general, a net difference of $S_{\alpha,\beta}^{\pm}$ or $C_{\alpha,\beta}^{\pm}$ parameters between two T -, CP -, or CPT -transformed processes (Tables 1, 2, or 3) would be a proof of T , CP , or CPT violation, respectively.

The standard CP violation studies performed by the B factory experiments [18] extract a single set of best fit S and C coefficients, reversing the sign of S under $\Delta t \leftrightarrow -\Delta t$, or $B_+ \leftrightarrow B_-$, or $B^0 \leftrightarrow \bar{B}^0$ exchanges, and reversing the sign of C only under $B^0 \leftrightarrow \bar{B}^0$ exchange,

$$S = \frac{2Im(\lambda)}{1+|\lambda|} = S_{\ell^+,K_S}^+ = -S_{\ell^-,K_S}^+ = -S_{\ell^+,K_S}^- = S_{\ell^-,K_S}^- = -S_{\ell^+,K_L}^+ = S_{\ell^-,K_L}^+ = S_{\ell^+,K_L}^- = -S_{\ell^-,K_L}^-, \quad (3.3)$$

$$C = \frac{1-|\lambda|}{1+|\lambda|} = C_{\ell^+,K_S}^+ = -C_{\ell^-,K_S}^+ = C_{\ell^+,K_S}^- = -C_{\ell^-,K_S}^- = C_{\ell^+,K_L}^+ = -C_{\ell^-,K_L}^+ = C_{\ell^+,K_L}^- = -C_{\ell^-,K_L}^-. \quad (3.4)$$

This construction is valid under the assumptions of CPT invariance and $\Delta\Gamma = 0$ [16]. If $\Delta\Gamma = 0$, automatically there is no CP violation in the mixing. Under these assumptions the $\Delta t \leftrightarrow -\Delta t$ (or equivalently $t_1 \leftrightarrow t_2$) exchange, which is not a T -symmetry operation, becomes related to T (exchange of the *in* and *out* neutral- B states), and to CP (exchange of B^0 and \bar{B}^0 states), $CP \leftrightarrow T \leftrightarrow \Delta t$ [20]. In other words, the resulting statement that

particle \leftrightarrow antiparticle invariance test is related to the exchange $\Delta t \leftrightarrow -\Delta t$. In the SM $\lambda = e^{i2\beta}$, therefore

$$\begin{aligned} S &= \sin 2\beta \\ C &= 0. \end{aligned} \tag{3.5}$$

As most of the currently available data on neutral- B mesons are well described by the SM, whether our proposed T -violating observables are also well accounted for in the SM or not, the direct observation of T violation would provide a proof of principle.

It is now convenient to introduce the asymmetry parameters $\Delta S_{\text{T}}^{\pm}$, $\Delta C_{\text{T}}^{\pm}$ (and similarly $\Delta S_{\text{CP}}^{\pm}$, $\Delta C_{\text{CP}}^{\pm}$, and $\Delta S_{\text{CPT}}^{\pm}$, and $\Delta C_{\text{CPT}}^{\pm}$), defined in Table 4, as the difference of $S_{\alpha,\beta}^{\pm}$, $C_{\alpha,\beta}^{\pm}$ coefficients between the corresponding symmetry-transformed processes for two reference Δt -exchanged transitions, for example S_{ℓ^+,K_S}^+ and S_{ℓ^+,K_S}^- . Using these parameters rather than the $S_{\alpha,\beta}^{\pm}$, $C_{\alpha,\beta}^{\pm}$ coefficients has the main advantage that the symmetry violation will be apparent through a non-vanishing value of any of the associated four parameters. In other words, if $\Delta S_{\text{T}}^+ \neq 0$ or $\Delta S_{\text{T}}^- \neq 0$ or $\Delta C_{\text{T}}^+ \neq 0$ or $\Delta C_{\text{T}}^- \neq 0$, then there is T violation, and similarly for CP and CPT symmetries. Table 5 summarizes the values and relations among the asymmetry parameters in the case of invariance under the three space-time discrete symmetry transformations. We would like to emphasize that the definition of these parameters requires to choose as reference two Δt -exchanged samples, in our case S_{ℓ^+,K_S}^{\pm} since $J/\psi K_S$ events are cleaner and more efficiently reconstructed than $J/\psi K_L$ [18].

4 Simulation study

We show in this section how the proposed methodology can be applied to an actual experiment to produce the desired results. The experimental samples are generated using a Monte Carlo simulation technique and are similar in size and properties to those currently available at B factories and used for standard CP -violation studies [18]. Thereby this study will provide an up to date estimate of the significance of the expected observation.

4.1 Generation of samples

Samples of events with a B^0 or \bar{B}^0 state (i.e., $\ell^+ X$ or $\ell^- X$) and a CP -eigenstate $J/\psi K_S$, $\psi(2S)K_S$, $\chi_{c1}K_S$, denoted generally as $c\bar{c}K_S$, or $J/\psi K_L$, are generated taking into account their relative branching ratios, reconstruction efficiencies and misidentification rates. We consider about 8000 $c\bar{c}K_S$ and 6000 $J/\psi K_L$ events, with purities around 90% and 60%, respectively [18]. The signal component uses the probability density function (PDF) given in Eq. (3.2), with values for $\Delta m = 0.507 \text{ ps}^{-1}$, $1/\Gamma = 1.519 \text{ ps}$ [8], and the coefficients $S_{\alpha,\beta}^{\pm}$ and $C_{\alpha,\beta}^{\pm}$ as given in Table 4 assuming the SM. The misidentification of CP final states is accounted for through a signal probability that depends on the two usual kinematic variables at B factory experiments, the energy of the $J/\psi K_L$ and the mass of the $c\bar{c}K_S$ candidates, including the different individual background sources [18]. Mistakes in the flavor identification (mistags) are included by modifying the time-dependent PDF as follows,

$$h_{\alpha,\beta}^{\pm}(\Delta\tau) \propto (1 - \omega_{\alpha})g_{\alpha,\beta}^{\pm}(\Delta\tau) + \omega_{\alpha}g_{\alpha,\beta}^{\mp}(\Delta\tau), \tag{4.1}$$

Coefficient	Assumed value	Fit value
$\Delta S_{\text{T}}^+ = S_{\ell^-, K_L}^- - S_{\ell^+, K_S}^+$	-1.4	-1.57 ± 0.15
$\Delta S_{\text{T}}^- = S_{\ell^-, K_L}^+ - S_{\ell^+, K_S}^-$	1.4	1.25 ± 0.19
$\Delta C_{\text{T}}^+ = C_{\ell^-, K_L}^- - C_{\ell^+, K_S}^+$	0.0	-0.07 ± 0.14
$\Delta C_{\text{T}}^- = C_{\ell^-, K_L}^+ - C_{\ell^+, K_S}^-$	0.0	-0.09 ± 0.14
$\Delta S_{\text{CP}}^+ = S_{\ell^-, K_S}^+ - S_{\ell^+, K_S}^+$	-1.4	-1.65 ± 0.11
$\Delta S_{\text{CP}}^- = S_{\ell^-, K_S}^- - S_{\ell^+, K_S}^-$	1.4	1.54 ± 0.13
$\Delta C_{\text{CP}}^+ = C_{\ell^-, K_S}^+ - C_{\ell^+, K_S}^+$	0.0	0.03 ± 0.10
$\Delta C_{\text{CP}}^- = C_{\ell^-, K_S}^- - C_{\ell^+, K_S}^-$	0.0	-0.09 ± 0.10
$\Delta S_{\text{CPT}}^+ = S_{\ell^+, K_L}^- - S_{\ell^+, K_S}^+$	0.0	-0.25 ± 0.22
$\Delta S_{\text{CPT}}^- = S_{\ell^+, K_L}^+ - S_{\ell^+, K_S}^-$	0.0	0.04 ± 0.13
$\Delta C_{\text{CPT}}^+ = C_{\ell^+, K_L}^- - C_{\ell^+, K_S}^+$	0.0	-0.04 ± 0.15
$\Delta C_{\text{CPT}}^- = C_{\ell^+, K_L}^+ - C_{\ell^+, K_S}^-$	0.0	-0.06 ± 0.13
S_{ℓ^+, K_S}^+	0.7	0.92 ± 0.10
S_{ℓ^+, K_S}^-	-0.7	-0.70 ± 0.06
C_{ℓ^+, K_S}^+	0.0	0.08 ± 0.07
C_{ℓ^+, K_S}^-	0.0	0.06 ± 0.06

Table 4. Definition of the T -, CP -, and CPT -asymmetry parameters. These parameters are defined as the differences between the $S_{\alpha, \beta}^{\pm}$, $C_{\alpha, \beta}^{\pm}$ coefficients for two reference Δt -exchanged processes and those of the corresponding symmetry-transformed transitions. In the central column we show the expected values based on the SM CP violation studies at B factory experiments, as given in Eqs. (3.3) and (3.4). The right column reports the fit results from one of the 350 simulated experiments described in Sec. 4.

where the $\bar{\alpha}$ index denotes the other flavor state to that given by α and ω_{α} represents the fraction of flavor states reconstructed as $\ell^- X$ being a B^0 (or as $\ell^+ X$ being a \bar{B}^0), with $g_{\alpha, \beta}^{\pm}(\Delta\tau)$ given by Eq. (3.2).

The experiments have employed flavor identification algorithms that analyze tracks not associated to the completely reconstructed CP -eigenstate to assign a given category. These assignments are determined from different inclusive or semi-inclusive signatures, such as isolated primary leptons, kaons, and pions from B decays to final states containing D^* mesons, and high momentum charged particles from B decays. For this study we use different categories with efficiencies ϵ (mistags ω) ranging between 9% and 17% (3% and 42%), with a total effective flavor identification efficiency of $Q = \epsilon(1 - 2\omega)^2 \approx 31\%$.

The resolution in the reconstruction of the decay time difference by the experiment

T invariance	CP invariance	CPT invariance
$\Delta S_{\text{T}}^+ = 0$	$\Delta S_{\text{CP}}^+ = 0$	$\Delta S_{\text{CPT}}^+ = 0$
$\Delta S_{\text{T}}^- = 0$	$\Delta S_{\text{CP}}^- = 0$	$\Delta S_{\text{CPT}}^- = 0$
$\Delta S_{\text{CP}}^+ = \Delta S_{\text{CPT}}^+$	$\Delta S_{\text{T}}^+ = \Delta S_{\text{CPT}}^+$	$\Delta S_{\text{T}}^+ = \Delta S_{\text{CP}}^+$
$\Delta S_{\text{CP}}^- = \Delta S_{\text{CPT}}^-$	$\Delta S_{\text{T}}^- = \Delta S_{\text{CPT}}^-$	$\Delta S_{\text{T}}^- = \Delta S_{\text{CP}}^-$
$\Delta C_{\text{T}}^+ = 0$	$\Delta C_{\text{CP}}^+ = 0$	$\Delta C_{\text{CPT}}^+ = 0$
$\Delta C_{\text{T}}^- = 0$	$\Delta C_{\text{CP}}^- = 0$	$\Delta C_{\text{CPT}}^- = 0$
$\Delta C_{\text{CP}}^+ = \Delta C_{\text{CPT}}^+$	$\Delta C_{\text{T}}^+ = \Delta C_{\text{CPT}}^+$	$\Delta C_{\text{T}}^+ = \Delta C_{\text{CP}}^+$
$\Delta C_{\text{CP}}^- = \Delta C_{\text{CPT}}^-$	$\Delta C_{\text{T}}^- = \Delta C_{\text{CPT}}^-$	$\Delta C_{\text{T}}^- = \Delta C_{\text{CP}}^-$

Table 5. Expected values and relations among the asymmetry parameters under invariance of one of the three discrete space-time symmetry transformations.

introduces a smearing such that the observed $\Delta\tau$ might take negative values. Therefore, the sign of Δt , which indicates if the decay to the flavor final state occurred before or after the CP final state (as described in Sec. 3), cannot be used directly to disentangle between $+$ and $-$ events. To overcome this problem, we describe the observed decay rate as a function of the reconstructed Δt , Δt_{rec} , as

$$\begin{aligned} \mathcal{H}_{\alpha,\beta}(\Delta t_{\text{rec}}) \propto & h_{\alpha,\beta}^+(\Delta t)H(\Delta t) \otimes \mathcal{R}(\delta t; \sigma_{\Delta t_{\text{rec}}}) + \\ & h_{\alpha,\beta}^-(-\Delta t)H(-\Delta t) \otimes \mathcal{R}(\delta t; \sigma_{\Delta t_{\text{rec}}}), \end{aligned} \quad (4.2)$$

where $H(\Delta t)$, $H(-\Delta t)$ are Heaviside step functions, $h_{\alpha,\beta}^+(\Delta t)$, $h_{\alpha,\beta}^-(-\Delta t)$ are given by Eq. (4.1), and the symbol \otimes indicates the mathematical convolution with the resolution function $\mathcal{R}(\delta t; \sigma_{\Delta t_{\text{rec}}})$, where $\delta t = \Delta t_{\text{rec}} - \Delta t$ and $\sigma_{\Delta t_{\text{rec}}}$ is the estimate of the Δt_{rec} uncertainty obtained by the reconstruction algorithms. The first term of the sum is related to $\Delta t > 0$ (i.e., the tagging B , identified as ℓ^+X or ℓ^-X , decayed before the other B , reconstructed as $c\bar{c}K_S$ or $J/\psi K_L$ state), while the second term is related to $\Delta t < 0$ (the neutral- B meson decayed to a flavor state later). With this construction the distribution $\mathcal{H}_{\alpha,\beta}(\Delta t_{\text{rec}})$ for $\Delta t_{\text{rec}} > 0$ will be dominated by $\Delta t > 0$ events, but will also contain events with $\Delta t < 0$ due to the limited Δt resolution. Similarly, the distribution $\mathcal{H}_{\alpha,\beta}(\Delta t_{\text{rec}})$ for $\Delta t_{\text{rec}} < 0$ will contain predominantly events having $\Delta t < 0$ with contribution from $\Delta t > 0$ events. The need of distinguishing between $\Delta t < 0$ and $\Delta t > 0$ in the presence of Δt resolution is one of the main complications of this study in comparison to the standard CP violation analyses at B factories. This is the reason of the Heaviside step function in Eq. (4.2) and the \pm index in the upper part of the $C_{\alpha,\beta}^\pm$ and $S_{\alpha,\beta}^\pm$ parameters.

The resolution function $\mathcal{R}(\delta t; \sigma_{\Delta t_{\text{rec}}})$ is modeled by the sum of three Gaussian functions [18] with means and widths proportional to $\sigma_{\Delta t_{\text{rec}}}$. The mean scale factor for the main component (which includes about 90% of the events) ranges between 0 for flavor identification categories containing isolated primary leptons to -0.2 for those categories

without leptons, to account for the typical offset in the resolution function observed by the experiments.

4.2 Sensitivity results

We generate a total of 350 experiments containing all 8 pairs of flavor- CP events, each of which is then fitted using an unbinned maximum likelihood procedure with the same PDF as that employed for the generation of the samples. Each fit determines the best 8 asymmetry parameters of Table 4, with all other parameters of the PDF, like those describing the resolution function, mistags and backgrounds, kept fixed. This is a realistic scenario for an experimental analysis since all these parameters are usually determined by the experiments using either control samples or detailed simulations of the detectors. In order to increase sensitivity and account for the significantly different reconstruction efficiencies between the different CP final states (especially between $J/\psi K_L$ and other events) the normalization of the PDF in the fit is performed separately for the different CP final states, but simultaneously for $\ell^+ X$, $\ell^- X$, $\Delta t_{\text{rec}} > 0$ and $\Delta t_{\text{rec}} < 0$ events.

Table 6 summarizes the standard deviation (root mean square, or r.m.s) of the residual distribution (fitted minus generated values), the mean of the fit uncertainty, and the standard deviation of the fit uncertainty, for each of the asymmetry parameters from the 350 experiments. The expected sensitivities for the T -violating parameters are 0.15, 0.20, 0.14 and 0.15, for ΔS_{T}^+ , ΔS_{T}^- , ΔC_{T}^+ , and ΔC_{T}^- , respectively. While the sensitivity for ΔC_{T}^+ and ΔC_{T}^- is similar, it is significantly worse for ΔS_{T}^- in comparison to ΔS_{T}^+ , since the associated decay rates involve time-dependent distributions with larger tails. The uncertainties obtained directly by the fit (parabolic errors) provide a good estimator of the true resolution given by the r.m.s of the residual distributions, while no statistically significant biases are observed, concluding that all asymmetry parameters have good Gaussian behavior. To illustrate the case of an actual experimental analysis, in Table 4 we give the complete fit results from one of the simulation experiments. As it can be observed, all asymmetry parameters are consistent with the generated values (indicated in the same Table), with fit errors in good agreement with the expected sensitivities.

4.3 Asymmetries

The difference in the rates for any pair of symmetry-transformed transitions normalized to their sum is usually used as observable to probe the symmetry violation. In practice, we construct raw asymmetries using the number of events for each pair of transitions in bins of $\Delta t_+ \equiv |\Delta t_{\text{rec}}|$, normalized to the total number of events of the given subsample. This normalization is particularly relevant for T (and CPT) asymmetries since it involves comparison of $c\bar{c}K_S$ and $J/\psi K_L$ states, which overall have different reconstruction efficiencies. Since for a given discrete symmetry there are four possible comparisons between conjugated processes, we have four independent asymmetries. For example, for the $\bar{B}^0 \rightarrow B_-$ reference transition the raw T asymmetry would explicitly be defined as

$$A_T(\Delta t_+) = \frac{\mathcal{H}_{\ell^-, K_L}^-(\Delta t_+) - \mathcal{H}_{\ell^+, K_S}^+(\Delta t_+)}{\mathcal{H}_{\ell^-, K_L}^-(\Delta t_+) + \mathcal{H}_{\ell^+, K_S}^+(\Delta t_+)}, \quad (4.3)$$

Parameter	r.m.s residual	mean error	r.m.s error
ΔS_{T}^+	0.149	0.148	0.009
ΔS_{T}^-	0.201	0.201	0.009
ΔC_{T}^+	0.139	0.134	0.008
ΔC_{T}^-	0.150	0.141	0.009
ΔS_{CP}^+	0.120	0.106	0.005
ΔS_{CP}^-	0.123	0.120	0.005
ΔC_{CP}^+	0.103	0.094	0.004
ΔC_{CP}^-	0.103	0.098	0.003
ΔS_{CPT}^+	0.209	0.213	0.017
ΔS_{CPT}^-	0.137	0.131	0.009
ΔC_{CPT}^+	0.143	0.147	0.009
ΔC_{CPT}^-	0.128	0.128	0.007
S_{ℓ^+, K_S}^+	0.101	0.090	0.004
S_{ℓ^+, K_S}^-	0.060	0.057	0.003
C_{ℓ^+, K_S}^+	0.069	0.066	0.002
C_{ℓ^+, K_S}^-	0.058	0.057	0.002

Table 6. Root mean square (r.m.s.) of the residual distribution (fit minus generated values), mean of the fit uncertainty, and the r.m.s of the fit uncertainty, for each of the asymmetry parameters from the 350 simulation experiments.

where $\mathcal{H}_{\alpha,\beta}^\pm(\Delta t_+) = \mathcal{H}_{\alpha,\beta}(\pm\Delta t_{\text{rec}})H(\Delta t_{\text{rec}})$. Neglecting mistag and proper-time resolution effects, and assuming Eqs. (3.3) and (3.4) in the denominator for simplicity, the raw T -asymmetry becomes proportional to $(\Delta C_{\text{T}}^+, \Delta S_{\text{T}}^+)$,

$$A_T(\Delta t_+) \approx \frac{\Delta C_{\text{T}}^+}{2} \cos \Delta m \Delta t_+ + \frac{\Delta S_{\text{T}}^+}{2} \sin \Delta m \Delta t_+. \quad (4.4)$$

The other three T asymmetries are constructed similarly using the association between transitions and time-ordered decay products for reference and T -conjugate processes given in Table 1, and are proportional to $(\Delta C_{\text{T}}^-, \Delta S_{\text{T}}^-)$, $(\Delta C_{\text{CP}}^- - \Delta C_{\text{CPT}}^-, \Delta S_{\text{CP}}^- - \Delta S_{\text{CPT}}^-)$, and $(\Delta C_{\text{CP}}^+ - \Delta C_{\text{CPT}}^+, \Delta S_{\text{CP}}^+ - \Delta S_{\text{CPT}}^+)$, respectively. The raw CP and CPT asymmetries are constructed following a similar procedure.

It should be noted, that with the methodology proposed, these asymmetries are only used with the purpose of illustrating the symmetry violation effect, through direct comparisons of the time-dependent raw asymmetries from data and the projections of the best fit results to the decay rates when we allow for both non-invariance and invariance under the symmetry transformation.

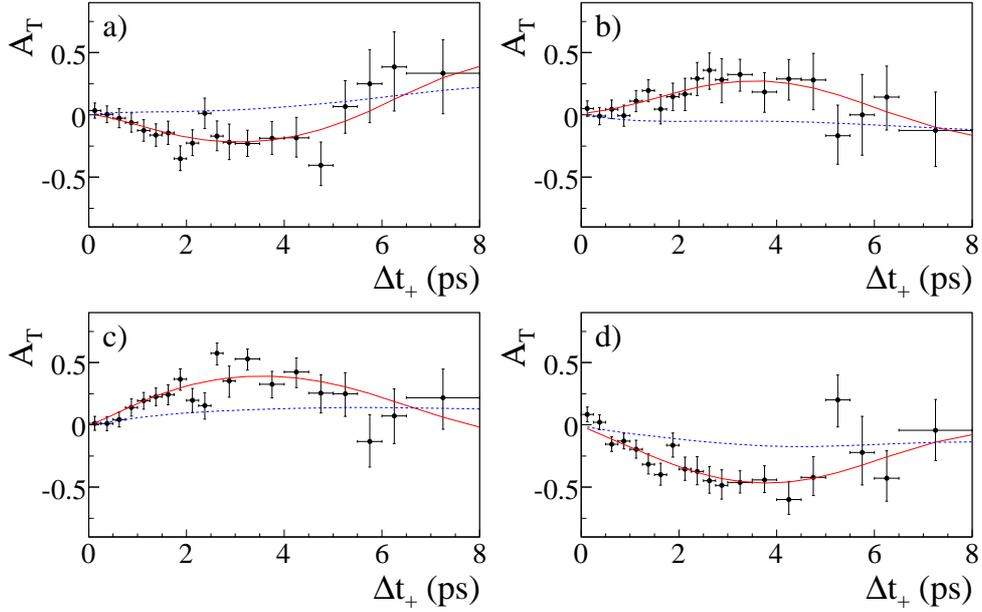


Figure 2. The four independent raw T -asymmetries corresponding to the four possible comparisons between T -conjugated and reference transitions a) $\bar{B}^0 \rightarrow B_- (\ell^+ X, c\bar{c}K_s)$, b) $B_+ \rightarrow B^0 (c\bar{c}K_s, \ell^+ X)$, c) $\bar{B}^0 \rightarrow B_+ (\ell^+ X, J/\psi K_L)$, and d) $B_- \rightarrow B^0 (J/\psi K_L, \ell^+ X)$ of Table 1, for one of the 350 simulation experiments and combining flavor categories with low mistag (isolated leptons and kaons), for a signal enriched region. The points with error bars represent the simulated data, the solid (red) curves represent the projections of the best fit results, while the dashed (blue) curves represent the projection of the best fit assuming T invariance.

Figure 2 shows the four independent raw T -asymmetries from data for one of the 350 simulation experiments, overlaid with the projection of the best fit results together with the projection of the best fit under the assumption of T invariance. The T invariance hypothesis implies to impose to our most general model in Eq. (3.2) used in the former case the 8 restrictions given in the left column of Table 5. This reduces the number of signal parameters left free in the fit from 16 to 8. The non-flat shape of the asymmetry curve with T invariance imposed is due to the offset of the resolution function discussed in Sec. 4.1.

4.4 Significance of results

The significance of T non-invariance can be evaluated by the variation of likelihood (s_T^2) in the space of asymmetry parameters. This variation is determined by the difference between the negative log-likelihood of the best fit with the most general model ($\ln \mathcal{L}$) and that of the best fit imposing the 8 restrictions related to T invariance listed in Table 5 (T -invariance point, $\ln \mathcal{L}_T$),

$$s_T^2 \equiv -2\Delta \ln \mathcal{L}_T = -2(\ln \mathcal{L}_T - \ln \mathcal{L}). \quad (4.5)$$

In the limit that the likelihood function takes a Gaussian shape in the space of asymmetry parameters, which is a good approximation in our case as discussed in Sec. 4.2 and sum-

marized in Table 6, s_T^2 is properly described by a χ^2 distribution with $\nu = 8$ degrees of freedom. The confidence level (C.L.) can then be obtained by computing the cumulative χ^2 probability that the value s_T^2 is exceeded [8].

For the same simulation experiment shown in Figure 2 the variation s_T^2 is 353 units, which corresponds to a C.L. equivalent to about 18 standard deviations (here we have adopted the convention that 1, 2, 3, ... standard deviations in 8 dimensions have $1 - \text{C.L.} = 0.3173, 4.55 \times 10^{-2}, 2.7 \times 10^{-3}, \dots$). This is the figure of merit that establishes the degree of inconsistency between the best fit result and the T invariance point, i.e., the significance that any of the 8 conditions in the left column of Table 5 is not satisfied, or equivalently, how significant is the combined difference between the solid (red) and dashed (blue) curves for the four asymmetries in Figure 2.

The significance of CP and CPT non-invariance can be evaluated similarly. For the same experiment as previously, the variations in s^2 for the CP - and CPT -invariance points are found 536 and 2 units, corresponding to about 22 and less than 1 standard deviations, respectively. These values are consistent with T non-invariance compensating CP violation so that CPT remains invariant, in agreement with the input parameters used in the generation of the simulated experiment.

5 Conclusions

This work concerns the study of microscopic Time Reversal Violation in the fundamental laws of physics. The observed time asymmetries in current macroscopic and microscopic phenomena are not connected to this problem and they can occur in theories with exact T invariance.

A direct evidence for T violation means an experiment that, considered by itself, clearly demonstrates T violation independent of the results for CP violation. For transition processes, the antiunitarity of the operator implementing the symmetry transformation implies that a genuine test of T violation needs an asymmetry under the interchange of *in* and *out* states in the dynamical evolution of the system. Nowadays, there is no experimental result providing direct evidence of genuine T violation. The measured asymmetry among the probabilities $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$ cannot be interpreted as such since, being CPT -even transitions, CP and T are experimentally identical, no matter whether there is CPT invariance or not. Other experiments that could provide evidence involve a non-vanishing expected value of T -odd observables for stationary, non-degenerate states, like electric dipole moments, not yet observed. For unstable particles, where we might expect large effects, the associated irreversibility seems to prevent a direct test of T symmetry through the exchange of *in* and *out* states.

This paper discusses how to overcome this irreversibility problem and defines the precise steps for implementing an actual experiment able to obtain direct evidence of genuine T violation in the time evolution of a neutral- B meson, taking place between the two decays of the $B^0\bar{B}^0$ system produced at B factories. The essential ingredients are the quantum mechanical entanglement imposed by the EPR correlation, in particular, the first decay of one B prepares the quantum state of its living partner, and the experimental ability

to unfold the ordering and difference of the two decay times. Identifying the B decays into definite flavor ($\ell^+ X$ or $\ell^- X$) and CP -eigenstates ($J/\psi K_S$ or $J/\psi K_L$), the difference between the decay rates for ($\ell^+ X$, $J/\psi K_S$) and ($J/\psi K_L$, $\ell^- X$) transitions at the same decay time difference is our signature of T violation. There are three other independent signatures which can be constructed from the eight time-ordered pairs of B decays. These proofs are independent of the results for CP violation, as shown by the fact that, given a reference ($\ell^+ X$, $J/\psi K_S$) for example, the T -, CP -, CPT -transformed, and Δt -exchanged processes are all experimentally different.

To demonstrate the feasibility of the proposed methodology and how it can be applied for a real data experimental analysis, we have performed a realistic Monte Carlo simulation of data samples with similar size and properties to those currently available at B factories, taking into account misidentification of CP eigenstates, mistakes in the flavor identification and resolution in the reconstruction of the decay time difference. The need of resolving simultaneously the ordering and difference of the two decay times in the presence of resolution is one of the main challenges in comparison to standard CP violation studies. For the purpose of illustrating the T -violating effect, we have presented a direct comparison of the time-dependent raw asymmetries from data and the projections of the best fit results to the decay rates with and without T non-invariance.

Using a large number of generated experiments, we have inferred the expected sensitivities of the asymmetries in the parameters which determine the time-dependent decay rates, together with the expected significance of T non-invariance. A large significance, equivalent to about 18 standard deviations, is foreseen with the currently available data, providing the expectation of a discovery of direct, genuine Time Reversal Violation in the time evolution of a neutral- B meson.

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A The $|B_-\rangle$ and $|B_+\rangle$ states

The $|B_-\rangle$, $|B_+\rangle$ states are experimentally identified as those filtered by the observation of the decay to definite CP eigenstates. The only requirement needed for the analysis is their existence, independent of the underlying theory (CPT invariant or not), with the orthogonality property $\langle B_- | B_+ \rangle = 0$. In this Appendix we construct these states explicitly.

Neutral kaons decaying inside the geometrical acceptance of detectors surrounding their production point are usually reconstructed through their decay to two pions. Thus we can first define $|B_-\rangle$ as the state filtered by the decay into $J/\psi K_+$, K_+ being the neutral- K state decaying to $\pi\pi$, a pure CP -odd state. The state $|\tilde{B}_+\rangle$ is then defined as

the state orthogonal to $|B_-\rangle$, $\langle \tilde{B}_+ | B_-\rangle = 0$, which cannot decay into the $J/\psi\pi\pi$ final state, $\langle J/\psi\pi\pi | T | \tilde{B}_+\rangle = 0$, where T is the transition operator. The state $|\tilde{B}_+\rangle$ can be written in terms of flavor eigenstates as

$$|\tilde{B}_+\rangle \equiv \tilde{N}_+ [|B^0\rangle - \alpha |\bar{B}^0\rangle], \quad (\text{A.1})$$

where $\alpha = \frac{\langle J/\psi\pi\pi | T | B^0\rangle}{\langle J/\psi\pi\pi | T | \bar{B}^0\rangle}$ and \tilde{N}_+ is a normalization constant. Since $|B_-\rangle$ is orthogonal to $|\tilde{B}_+\rangle$ it then follows

$$|B_-\rangle = N_- [|\bar{B}^0\rangle + \alpha^* |B^0\rangle]. \quad (\text{A.2})$$

Analogously, we define $|B_+\rangle$ as the state filtered by the decay into $J/\psi K_L$, a CP -even state up to $\mathcal{O}(10^{-3})$ due to CP violation in the neutral-kaon system. We note that we could have defined $|B_+\rangle$ through its decay to $J/\psi K_-$, $K_- \rightarrow \pi^0\pi^0\pi^0$, in this case a pure CP -even state, but this final state cannot be reconstructed since long-lived neutral kaons tend to interact hadronically inside the detectors before they can undergo decay. Of course, both definitions become operationally identical when CP violation in neutral-kaons is neglected. The state $|\tilde{B}_-\rangle$, defined as its orthogonal state, $\langle \tilde{B}_- | B_+\rangle = 0$ and $\langle J/\psi K_L | T | \tilde{B}_-\rangle = 0$, is

$$|\tilde{B}_-\rangle \equiv \tilde{N}_- [|B^0\rangle - \beta |\bar{B}^0\rangle], \quad (\text{A.3})$$

where $\beta = \frac{\langle J/\psi K_L | T | B^0\rangle}{\langle J/\psi K_L | T | \bar{B}^0\rangle}$. Therefore the state $|B_+\rangle$ can be explicitly written as

$$|B_+\rangle = N_+ [|\bar{B}^0\rangle + \beta^* |B^0\rangle]. \quad (\text{A.4})$$

Let us note that here we keep separate the definitions of the states $|B_-\rangle$ and $|B_+\rangle$, which are observed through their decays to the $J/\psi\pi\pi$ and $J/\psi K_L$ final states, from the states $|\tilde{B}_-\rangle$ and $|\tilde{B}_+\rangle$, produced exploiting the EPR correlations in the entangled $B\bar{B}$ system. The two bases are the same when the orthogonality condition $\langle B_- | B_+\rangle = 0$ is fulfilled. As α and β correspond to opposite CP final states, we have $\alpha\beta^* = -1$ when only one weak amplitude is responsible of the decay ($|\alpha| = |\beta| = 1$). Consequently as all these considerations apply to our experimental framework,

$$\langle B_- | B_+\rangle = 1 + \alpha\beta^* = 0, \quad (\text{A.5})$$

which was to be demonstrated.

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