

SEESAW MAJORON MODEL OF NEUTRINO MASS AND NOVEL SIGNALS IN HIGGS BOSON PRODUCTION AT LEP

Marco A. Díaz, M. A. García-Jareño,
Diego A. Restrepo and José W. F. Valle *

*Departamento de Física Teórica, IFIC-CSIC, Universidad de Valencia
Burjassot, Valencia 46100, Spain*

We perform a careful study of the neutral scalar sector of a model which includes a singlet, a doublet, and a triplet scalar field under $SU(2)$. This model is motivated by neutrino physics, since it is simply the most general version of the seesaw model of neutrino mass generation through spontaneous violation of lepton number. The neutral Higgs sector contains three CP-even and one massive CP-odd Higgs boson A , in addition to the massless CP-odd majoron J . The weakly interacting majoron remains massless if the breaking of lepton number symmetry is purely spontaneous. We show that the massive CP-odd Higgs boson may invisibly decay to three majorons, as well as to a CP-even Higgs H boson plus a majoron. We consider the associated Higgs production $e^+e^- \rightarrow Z \rightarrow HA$ followed by invisible decays $A \rightarrow JJJ$ and $H \rightarrow JJ$ and derive the corresponding limits on masses and coupling that follow from LEP I precision measurements of the invisible Z width. We also study a novel $b\bar{b}b\bar{b}\cancel{p}_T$ signal predicted by the model, analyse the background and perform a Monte-Carlo simulation of the signal in order to illustrate the limits on Higgs boson mass, couplings and branching ratios that follow from that.

*E-mail: mad@flamenco.ific.uv.es, miguel@flamenco.ific.uv.es, restrepo@flamenco.ific.uv.es, and valle@flamenco.ific.uv.es

1 Introduction

One of main puzzles in particle physics is the origin of mass in general as well as the problem of neutrino mass in particular. In the Standard Model (SM) the spontaneous breaking of the gauge symmetry through the expectation value of a scalar $SU(2) \otimes U(1)$ doublet is the origin of the masses of the fermions as well as those of the gauge bosons. The main implication for this scenario is the existence of the Higgs boson [1], not yet found [2, 3]. Many of the extensions of the Standard Model Higgs sector postulated in order to generate mass for neutrinos are characterised by the spontaneous violation of a global $U(1)$ lepton number symmetry by an $SU(2) \otimes U(1)$ singlet vacuum expectation value $\langle \sigma \rangle$ [4]. These models contain a massless Goldstone boson, called majoron (J), which interacts very weakly with normal matter [5]. Although the interactions of the majoron with quarks, leptons, and gauge bosons is naturally very weak, as required also by astrophysics [6], it can have a relatively strong interaction with the Higgs boson [7, 8]. It has been noted that the main Higgs boson decay channel is likely to be *invisible*, e.g.

$$H \rightarrow JJ, \tag{1}$$

where J denotes the majoron field. This feature also appears in variants of the minimal supersymmetric model in which R parity is broken spontaneously [9]. The phenomenological implications of the invisible CP-even Higgs boson decays for various collider experiments have been extensively discussed [10, 11, 12, 13, 14, 15].

In the seesaw model [16, 17] one adds an $SU(2) \otimes U(1)$ isosinglet right-handed neutrino associated with each generation of isodoublet neutrinos. In addition to the standard lepton number conserving *isodoublet* mass term analogous to those responsible for the charged fermion masses, there is also a Majorana mass term for the right handed neutrinos and left-handed neutrinos. The neutrino mass matrix takes the form

$$\begin{pmatrix} \nu & \nu^c \end{pmatrix}^T \sigma_2 \begin{pmatrix} M_L & D \\ D^T & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} \tag{2}$$

where σ_2 is the charge conjugation matrix and the entries obey the hierarchy $M_R \gg D \gg M_L$ [18]. In a model where neutrinos acquire mass only from the spontaneous violation of lepton number the entries M_L and M_R arise from vacuum expectation values (VEV) of $SU(2) \otimes U(1)$ triplet and singlet Higgs scalars Δ and σ [19], while the Dirac mass term D follows from the Standard Model doublet VEV. In this model the light neutrino masses arise from diagonalizing out the heavy right-handed neutrinos and has a contribution from the small triplet VEV.

In this paper we show that, for a wide choice of parameters, the complete version of the seesaw majoron model of neutrino mass containing $SU(2) \otimes U(1)$ doublet, singlet as well as triplet Higgs multiplets (called **123** model in ref.[19]) implies that the massive pseudoscalar Higgs boson can also decay invisibly, either directly as

$$A \rightarrow 3J, \quad (3)$$

or indirectly as

$$A \rightarrow HJ \text{ with } H \rightarrow JJ \quad (4)$$

when $m_A > m_H$. This feature has not been noted in any of the discussions given so far [10, 11, 13, 14], as it is absent in a number of models, for example the all the models discussed in [7].

Massive invisibly decaying CP-odd Higgs bosons may occur in the minimal supersymmetric standard model, where the decay involves a heavy fermion pair, $A \rightarrow \chi^0 \chi^0$, with χ^0 stable due to R-parity conservation. Similarly, it can also occur in the supersymmetric model with spontaneously broken R-parity considered in ref. [9]. In the latter case one could have, e.g. $A \rightarrow \chi^0 \chi^0$ or $A \rightarrow \nu \chi^0$, with $\chi^0 \rightarrow \nu J$, where J denotes the majoron. However, all these decays require a kinematical condition $m_A > 2m_{\chi^0}$ or $m_A > m_{\chi^0}$ which is avoided here due to the majoron being massless.

We carefully study the scalar potential of the model and derive from it the relevant CP-even as well as CP-odd Higgs boson mass matrices. In the next section, we discuss the parameterisation of Higgs bosons couplings relevant for their production at LEP, both for the ZH as well as AH production channels. Next we use these theoretical results in order to derive restrictions on the relevant Higgs boson parameters from the precision measurements of the invisible width of the Z boson at LEP I. The associated production $e^+e^- \rightarrow HA$ with $A \rightarrow HJ$ and $H \rightarrow b\bar{b}$ also leads to a novel $b\bar{b}b\bar{b}p_T$ signal topology that could be detectable at LEP II. We have performed a detailed analysis of the background and carried out a Monte-Carlo simulation of the signal in order to illustrate the limits on Higgs boson mass, couplings and branching ratios that follow from four-jet + missing momentum signal topology.

2 The Scalar Potential

The model we consider here is the one proposed in ref. [19] as a generalisation of the triplet [20] and singlet [5] majoron models. The Higgs sector of the model contains the usual $SU(2)$ Higgs complex doublet ϕ of the SM, with lepton number $L = 0$, and an

$SU(2)$ complex triplet Δ , with lepton number $L = -2$,

$$\Delta = \begin{bmatrix} \Delta^0 & \Delta^+/\sqrt{2} \\ \Delta^+/\sqrt{2} & \Delta^{++} \end{bmatrix}, \quad \phi = \begin{bmatrix} \phi^0 \\ \phi^- \end{bmatrix}, \quad (5)$$

where we have used the 2×2 matrix notation for the Higgs triplet given in ref. [21]. The Higgs sector is completed with a complex $SU(2) \otimes U(1)$ singlet scalar, denoted σ , carrying lepton number $L = 2$.

The full scalar potential is given by [22]

$$\begin{aligned} V(\phi, \Delta, \sigma) = & \mu_2^2 \phi^\dagger \phi + \mu_3^2 \text{tr}(\Delta^\dagger \Delta) + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 [\text{tr}(\Delta^\dagger \Delta)]^2 \\ & + \lambda_3 \phi^\dagger \phi \text{tr}(\Delta^\dagger \Delta) + \lambda_4 \text{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta) + \lambda_5 (\phi^\dagger \Delta^\dagger \Delta \phi) \\ & + \mu_1^2 \sigma^\dagger \sigma + \beta_1 (\sigma^\dagger \sigma)^2 + \beta_2 (\phi^\dagger \phi) (\sigma^\dagger \sigma) \\ & + \beta_3 \text{tr}(\Delta^\dagger \Delta) \sigma^\dagger \sigma - \kappa (\phi^T \Delta \phi \sigma + \text{h.c.}) \end{aligned} \quad (6)$$

where μ_i , $i = 1, 2, 3$, are mass parameters, and λ_i , $i = 1, \dots, 5$, β_i , $i = 1, 2, 3$, and κ are dimensional-less couplings. The first two lines in eq. (6) correspond to the Gelmini–Roncadelli model [20], and the last two lines are new terms involving the scalar σ . The term in κ has been introduced in ref. [19] and plays an important role in our present discussion.

The singlet σ , as well as the neutral components of the fields ϕ and Δ , acquire vacuum expectation values v_1 , v_2 , and v_3 respectively. According to this, we shift the fields in the following way

$$\begin{aligned} \sigma &= \frac{v_1}{\sqrt{2}} + \frac{R_1 + iI_1}{\sqrt{2}} \\ \phi^0 &= \frac{v_2}{\sqrt{2}} + \frac{R_2 + iI_2}{\sqrt{2}} \\ \Delta^0 &= \frac{v_3}{\sqrt{2}} + \frac{R_3 + iI_3}{\sqrt{2}} \end{aligned} \quad (7)$$

We assume that the three vacuum expectation values are real. The scalar potential contains the following linear terms

$$V_{linear} = t_1 R_1 + t_2 R_2 + t_3 R_3, \quad (8)$$

where

$$\begin{aligned} t_1 &= v_3 (\mu_1^2 + \lambda_2 v_3^2 + \frac{1}{2} \lambda_3 v_2^2 + \lambda_4 v_3^2 + \frac{1}{2} \lambda_5 v_2^2 + \frac{1}{2} \beta_3 v_1^2) - \frac{1}{2} \kappa v_1 v_2^2 \\ t_2 &= v_1 (\mu_2^2 + \beta_1 v_1^2 + \frac{1}{2} \beta_2 v_2^2 + \frac{1}{2} \beta_3 v_3^2) - \frac{1}{2} \kappa v_2^2 v_3 \\ t_3 &= v_2 (\mu_3^2 + \lambda_1 v_2^2 + \frac{1}{2} \lambda_3 v_3^2 + \frac{1}{2} \lambda_5 v_3^2 + \frac{1}{2} \beta_2 v_1^2 - \kappa v_1 v_3) \end{aligned} \quad (9)$$

The conditions for a extreme of the potential are $t_i = 0$, $i = 1, 2, 3$. Therefore, the $t_i = 0$ vanish at the minima of the potential. We will verify explicitly below that, for many choices of its parameters, the potential has indeed minima for nonzero values of v_1 , v_2 and v_3 .

3 Neutral Higgs Mass Matrices

Taking into account the fact that this model contains one doubly-charged and one singly-charged scalar boson, in addition to the two charged unphysical $SU(2) \otimes U(1)$ Goldstone modes (longitudinal W), it follows that the neutral Higgs sector of this model is composed by six real fields. Due to CP invariance they split into two unmixed sectors of three CP-even and three CP-odd fields. Their mass matrices are contained in the quadratic scalar potential which includes:

$$V_{quadratic} = \frac{1}{2} [R_1, R_2, R_3] \mathbf{M}_R^2 \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} + \frac{1}{2} [I_1, I_2, I_3] \mathbf{M}_I^2 \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} + \dots \quad (10)$$

The CP-even Higgs mass matrix, which is in agreement with ref. [22], is given by

$$\mathbf{M}_R^2 = \begin{bmatrix} 2\beta_1 v_1^2 + \frac{1}{2} \kappa v_2^2 \frac{v_3}{v_1} + \frac{t_1}{v_1} & \beta_2 v_1 v_2 - \kappa v_2 v_3 & \beta_3 v_1 v_3 - \frac{1}{2} \kappa v_2^2 \\ \beta_2 v_1 v_2 - \kappa v_2 v_3 & 2\lambda_1 v_2^2 + \frac{t_2}{v_2} & (\lambda_3 + \lambda_5) v_2 v_3 - \kappa v_1 v_2 \\ \beta_3 v_1 v_3 - \frac{1}{2} \kappa v_2^2 & (\lambda_3 + \lambda_5) v_2 v_3 - \kappa v_1 v_2 & 2(\lambda_2 + \lambda_4) v_3^2 + \frac{1}{2} \kappa v_2^2 \frac{v_1}{v_3} + \frac{t_3}{v_3} \end{bmatrix} \quad (11)$$

where it is safe to take $t_i = 0$, $i = 1, 2, 3$, unless $v_1 = 0$ or $v_3 = 0$ in which case the expression of the corresponding extremization condition (“tadpole equation”) in eq. (9) must be replaced in the mass matrix \mathbf{M}_R^2 . The physical CP-even mass eigenstates H_i , $i = 1, 2, 3$, are related to the corresponding weak eigenstates R_i as

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = O_R \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}. \quad (12)$$

where the 3×3 matrix O_R is the matrix which diagonalizes the CP-even mass matrix such that

$$O_R \mathbf{M}_R^2 O_R^T = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2) \quad (13)$$

and where by definition we take $m_{H_1} \leq m_{H_2} \leq m_{H_3}$.

The CP-odd Higgs mass matrix \mathbf{M}_I^2 is given by [19]

$$\mathbf{M}_I^2 = \begin{bmatrix} \frac{1}{2} \kappa v_2^2 \frac{v_3}{v_1} + \frac{t_1}{v_1} & \kappa v_2 v_3 & \frac{1}{2} \kappa v_2^2 \\ \kappa v_2 v_3 & 2\kappa v_1 v_3 + \frac{t_2}{v_2} & \kappa v_1 v_2 \\ \frac{1}{2} \kappa v_2^2 & \kappa v_1 v_2 & \frac{1}{2} \kappa v_2^2 \frac{v_1}{v_3} + \frac{t_3}{v_3} \end{bmatrix}. \quad (14)$$

If $v_1 \neq 0$ and $v_3 \neq 0$ we can safely set the tadpoles equal to zero in eq. (14), in whose case we can see that the matrix \mathbf{M}_I^2 has two zero eigenvalues. One of them is the unphysical Goldstone boson and the other is the physical Majoron. The physical CP-odd mass eigenstates A_i , $i = 1, 2, 3$, are related to the corresponding weak eigenstates I_i as

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \equiv \begin{bmatrix} J \\ G \\ A \end{bmatrix} = O_I \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}. \quad (15)$$

where the 3×3 matrix O_I is the matrix which diagonalizes the CP-odd mass matrix such that

$$O_I \mathbf{M}_I^2 O_I^T = \text{diag}(0, 0, m_A^2) \quad (16)$$

and the CP-odd Higgs mass is given by

$$m_A^2 = \frac{1}{2} \kappa \frac{v_2^2 v_1^2 + v_2^2 v_3^2 + 4v_3^2 v_1^2}{v_3 v_1} \quad (17)$$

Note that $m_A \rightarrow 0$ as $\kappa \rightarrow 0$. The diagonalization matrix O_I can be found analytically

$$O_I = \begin{bmatrix} cv_1 V^2 & -2cv_2 v_3^2 & -cv_2^2 v_3 \\ 0 & \frac{v_2}{V} & -2\frac{v_3}{V} \\ b\frac{v_2}{2v_1} & b & b\frac{v_2}{2v_3} \end{bmatrix}, \quad (18)$$

where V , c , and b are the normalisation constants for the eigenvectors G , J , A respectively. They are given by

$$\begin{aligned} V^2 &= v_2^2 + 4v_3^2 \\ c^{-2} &= v_1^2 V^4 + 4v_2^2 v_3^4 + v_2^4 v_3^2 \\ b^2 &= \frac{4v_1^2 v_3^2}{v_2^2 v_1^2 + v_2^2 v_3^2 + 4v_3^2 v_1^2} \end{aligned} \quad (19)$$

We now briefly discuss three special cases, motivated by the cases when tadpoles cannot be trivially set to zero in the scalar mass matrices \mathbf{M}_R^2 and \mathbf{M}_I^2 .

- $v_1 = 0, v_3 = 0$.

In this case there is no breaking of lepton number, as in the Standard Model and, as a result there is no massless Majoron. The unphysical Goldstone boson is purely doublet. One of the CP-even Higgs bosons is also pure doublet with a mass $m_H^2 = 2\lambda_1 v_2^2$. The remaining two CP-even Higgs bosons are massive and are a mixture of singlet and triplet. There are also two massive CP-odd Higgs bosons and they are degenerate with the CP-even Higgs bosons.

- $v_1 \neq 0, v_3 = 0$. In this case the third minimization equation forces to have $\kappa = 0$. There is a Majoron with $m_J = 0$ which is purely singlet, as in the simplest **12** model considered in ref. [19], and the unphysical Goldstone boson is purely doublet. The real and imaginary parts of the neutral component of the triplet field are degenerate and form a complex field with mass $m_{\Delta^0}^2 = \mu_3^2 + \frac{1}{2}(\lambda_3 + \lambda_5)v_2^2 + \frac{1}{2}\beta_3 v_1^2$. There are two additional massive CP-even Higgs bosons, mixture of singlet and doublet.

- $v_1 = 0, v_3 \neq 0$.

In this case the first tadpole equation forces to have $\kappa = 0$. There is a Majoron with $m_J = 0$ which has zero component along the singlet, and is therefore experimentally

ruled out by the LEP data. Here the situation is analogous to the simplest **23** model of ref. [19] and it is for this reason that the presence of the singlet field σ with non-zero VEV is mandatory.

Thus we conclude that the situation of interest for us is the general one in which all three VEVs v_1 , v_2 and v_3 take on nonzero values. In our numerical calculations reported in section V we must take into account an important astrophysical constraint on these VEVs that follows from stellar cooling by majoron emission which severely restricts the majoron electron coupling to be less than about 10^{-12} or so. This is discussed in detail in section V.

4 Higgs Boson Production and Decays

In this section we derive the couplings relevant for Higgs boson production at e^+e^- colliders and for their invisible decays. The two production mechanisms we consider are the emission of a CP-even Higgs H by a Z -boson, and the associated production consisting of a Z -boson decaying into a CP-even Higgs H and a CP-odd Higgs A . In order to derive the couplings ZZH and ZHA , we need the kinetic part of the scalar Lagrangian contained in

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}_\mu\phi)^\dagger\mathcal{D}^\mu\phi + \text{tr}[(\mathcal{D}_\mu\Delta)^\dagger\mathcal{D}^\mu\Delta] + \partial_\mu\sigma^\dagger\partial^\mu\sigma - V(\phi, \Delta, \sigma), \quad (20)$$

where the covariant derivative is defined by

$$\mathcal{D} = \partial^\mu + ig\mathbf{T} \cdot \mathbf{W}^\mu + \frac{i}{2}g'YV^\mu \quad (21)$$

and g and g' are the gauge couplings corresponding to the $SU(2)$ and $U(1)$ groups respectively. On the scalars fields, the generators act as follows

$$\begin{aligned} \mathbf{T}\phi &= \frac{1}{2}\vec{\tau}\phi, & \mathbf{T}\Delta &= -\frac{1}{2}\vec{\tau}\Delta - \frac{1}{2}\Delta\vec{\tau}^* \\ Y\phi &= -1\phi, & Y\Delta &= 2\Delta, \end{aligned} \quad (22)$$

and with these definitions we have $T_3\phi^0 = \frac{1}{2}\phi^0$ and $T_3\Delta^0 = -1\Delta^0$.

The Higgs singlet does not contribute to the gauge boson masses, therefore, they depend only on v_2 and v_3 and are given by [18]

$$m_Z^2 = \frac{g^2}{4\cos^2\theta_W}(v_2^2 + 4v_3^2), \quad m_W^2 = \frac{g^2}{4}(v_2^2 + 2v_3^2) \quad (23)$$

and from the measurement of the ρ parameter one has a restriction on v_3 , namely [23]

$$\rho = 1 + \frac{2v_3^2}{v_2^2 + 2v_3^2} = 1.001 \pm 0.002. \quad (24)$$

which implies in practice that $v_3 \leq 8$ GeV.

We now calculate the relevant couplings for the production of Higgs bosons at e^+e^- colliders. Using eq. (20), we determined the $H AZ$ couplings to be

$$\mathcal{L}_{HAZ} = \frac{g}{2c_w} Z^\mu \left[R_2 \overleftrightarrow{\partial}^\mu I_2 - 2 R_3 \overleftrightarrow{\partial}^\mu I_3 \right] = \frac{g}{2c_w} Z^\mu \left[O_{32}^I O_{a2}^R - 2 O_{33}^I O_{a3}^R \right] H_a \overleftrightarrow{\partial}^\mu A. \quad (25)$$

where $c_W \equiv \cos \theta_W$ and H_a is any of the three CP-even neutral Higgs bosons. The quantity defined by

$$\epsilon_A = O_{32}^I O_{12}^R - 2 O_{33}^I O_{13}^R \quad (26)$$

essentially represents the strength of the coupling $H_1 AZ$ [see the second squared parenthesis in eq. (25)].

From eq. (20) we find that the HZZ coupling is

$$\mathcal{L}_{HZZ} = \frac{g}{4c_W^2} m_Z Z^\mu Z_\mu \left[\frac{v_2}{V} O_{a2}^R + \frac{4v_3}{V} O_{a3}^R \right] H_a, \quad (27)$$

and again we define the following quantity

$$\epsilon_B = \frac{v_2}{V} O_{12}^R + \frac{4v_3}{V} O_{13}^R \quad (28)$$

as a measure of the strength of the $H_1 ZZ$ coupling. Therefore, the Bjorken production mechanism (Fig. 1a) and the associated production mechanism (Fig. 1b) are determined by the parameters ϵ_A and ϵ_B respectively.

We now turn to the couplings relevant for the invisible decay of the Higgs bosons H_1 and A . We need to find first the trilinear couplings HJJ , $H AJ$, and $H AA$, thus we start with the cubic part of the scalar potential which involve one (unrotated) CP-even Higgs field R_i and two (unrotated) CP-odd Higgs fields I_j . This part of the potential is given by

$$\begin{aligned} V_{RII} = & \lambda_1 v_2 I_2^2 R_2 + (\lambda_2 + \lambda_4) v_3 I_3^2 R_3 + \frac{1}{2} (\lambda_3 + \lambda_5) (v_2 I_3^2 R_2 + v_3 I_2^2 R_3) \\ & + \beta_1 v_1 I_1^2 R_1 + \frac{1}{2} \beta_2 (v_2 I_1^2 R_2 + v_1 I_2^2 R_1) + \frac{1}{2} \beta_3 (v_1 I_3^2 R_1 + v_3 I_1^2 R_3) \\ & + \kappa [v_1 (\frac{1}{2} I_2^2 R_3 + I_2 I_3 R_2) + v_3 (\frac{1}{2} I_2^2 R_1 + I_1 I_2 R_2) + v_2 (I_1 I_2 R_3 + I_3 I_1 R_2 + I_2 I_3 R_1)] \end{aligned} \quad (29)$$

Making the substitution $I_i I_j \rightarrow O_{1i}^I O_{1j}^I J^2$ in eq. (29), we find the coupling $H_a J J$ in terms of the mass m_{H_a} and the rotation matrices O^R and O^I

$$\begin{aligned} V_{HJJ} = & \frac{1}{2} \left[\frac{O_{12}^I{}^2}{v_2} (M_R^2)_{1a} + \frac{O_{13}^I{}^2}{v_3} (M_R^2)_{2a} + \frac{O_{11}^I{}^2}{v_1} (M_R^2)_{3a} \right] R_a J^2 \\ = & \frac{1}{2} \left[\frac{O_{12}^I{}^2}{v_2} (O^R)_{2a}^T + \frac{O_{13}^I{}^2}{v_3} (O^R)_{3a}^T + \frac{O_{11}^I{}^2}{v_1} (O^R)_{1a}^T \right] m_{H_a}^2 H_a J^2. \end{aligned} \quad (30)$$

where eq. (13) has been used.

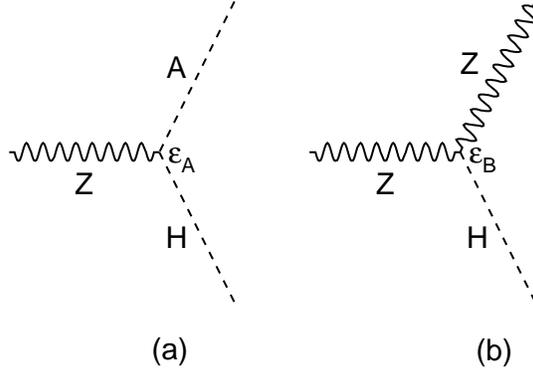


Figure 1: Feynman rules relevant to Bjorken and Associated Higgs Production.

In a similar way, the terms in the Lagrangian relevant for the vertices $H AJ$ and $H AA$ can be found from eq. (29) making the following substitutions

$$\begin{aligned} V_{HAJ} &= V_{RII}(I_i I_j \rightarrow [O_{3i}^I O_{1j}^I + O_{1i}^I O_{3j}^I] AJ), \\ V_{HAA} &= V_{RII}(I_i I_j \rightarrow O_{3i}^I O_{3j}^I A^2). \end{aligned} \quad (31)$$

These terms are not explicitly displayed.

Finally we turn to the quartic coupling responsible for the invisible decay $A \rightarrow 3J$. The relevant piece of the quartic scalar potential is

$$V_{I^4} = \frac{1}{4}[\lambda_1 I_2^4 + (\lambda_2 + \lambda_4) I_3^4 + (\lambda_3 + \lambda_5) I_2^2 I_3^2 + \beta_1 I_1^4 + \beta_2 I_2^2 I_1^2 + \beta_3 I_3^2 I_1^2 - 2\kappa I_1 I_2^2 I_3] \quad (32)$$

and after making the following substitution

$$I_i I_j I_k^2 \longrightarrow (O_{3i}^I O_{1j}^I O_{1k}^{I2} + O_{1i}^I O_{3j}^I O_{1k}^{I2} + 2O_{1i}^I O_{1j}^I O_{3k}^I O_{1k}^I) A J^3 \quad (33)$$

we find the term AJ^3 in the scalar potential:

$$\begin{aligned} V_{AJ^3} &= \left[\lambda_1 O_{12}^I O_{32}^I + (\lambda_2 + \lambda_4) O_{13}^I O_{33}^I + \frac{1}{2}(\lambda_3 + \lambda_5) O_{12}^I O_{13}^I (O_{32}^I O_{13}^I + O_{33}^I O_{12}^I) \right. \\ &\quad \left. \beta_1 O_{11}^I O_{31}^I + \frac{1}{2}\beta_2 O_{11}^I O_{12}^I (O_{31}^I O_{12}^I + O_{32}^I O_{11}^I) + \frac{1}{2}\beta_3 O_{11}^I O_{13}^I (O_{31}^I O_{13}^I + O_{33}^I O_{11}^I) \right. \\ &\quad \left. - \kappa(O_{31}^I O_{13}^I O_{12}^{I2} + O_{11}^I O_{12}^I O_{13}^I O_{32}^I) \right] A J^3 \end{aligned}$$

which complete all the relevant information necessary to calculate the production and invisible decay of the Higgs bosons.

5 Numerical Expectations of the Model

In this section we describe the expectations of our Model for the various Higgs boson masses and couplings relevant for our discussion. In order to do this we numerically diagonalize the mass matrix in eq. (11) and impose the minimisation conditions $t_i = 0$, $i = 1, 2, 3$, where the tadpoles are in eq. (9), and check the positivity of the three CP-even and CP-odd eigenvalues. As seen explicitly, these matrices are determined in terms of the 9 dimension-less coupling constants and the three VEVs characterising the Higgs potential, as the three mass parameters μ_i have all been eliminated. Note, however that there is a restriction that arises from the W mass constraint that relates v_2 and v_3 through eq. (23) that can be written as

$$\sqrt{v_2^2 + 2v_3^2} \simeq 246 \text{ GeV} \quad (35)$$

Moreover, v_3 must be smaller than about 8 GeV in order to obey the experimental value of the ρ parameter defined in eq. (24).

A more stringent constraint on v_3 follows from astrophysics, due to the stellar cooling argument, already mentioned. Indeed, if produced in a stellar environment via the Compton-like process $\gamma + e \rightarrow J + e$, the majoron would escape leading to excessive energy loss [6]. In order to suppress this one must severely restrict the coupling of the Majoron to the electrons. Such coupling arises from the projection of the majoron J onto the doublet, leading to

$$|\langle J | \phi \rangle| = \frac{2|v_2|v_3^2}{\sqrt{v_1^2(v_2^2 + 4v_3^2)^2 + 4v_2^2v_3^4 + v_2^4v_3^2}} \lesssim 10^{-6} \quad (36)$$

In order to have an idea of the parameter ranges involved we have randomly varied over the parameters $0 < \lambda_i < 4$, $0 < \beta_i < 4$, (in order to guarantee a perturbative regime) and the three v_i subject to the above restrictions, with the lepton number violating v_1 varied in the range $0 < v_1 < 1000$ GeV. The resulting v_1 - v_3 region allowed by the model is seen in Fig. 2. The shape of the region can be understood easily from eq. (36) noting that, since the lepton number violating VEV v_3 is small and v_2 is almost fixed by the W mass constraint, then the boundary of the allowed region satisfies $v_3 \sim \sqrt{v_1}$.

The lowest-lying Higgs boson masses in our model may be similarly determined after imposing the above restrictions. For example, it is instructive to display the results as a function of the parameter κ characterising the Higgs potential. The corresponding plots are shown in Figs. 3a–b. In Fig. 3a we plot the correlation between the massive pseudoscalar mass and the parameter κ , for different values of v_1 . Each curve corresponds to the boundary of a scatter plot with the solutions concentrated above it. Its shape can be understood from eq. (17) and eq. (36) where we find $m_A \sim \sqrt{\kappa}$. In order to have m_A below a certain value one requires an upper bound on κ , which tightens with larger v_1 .

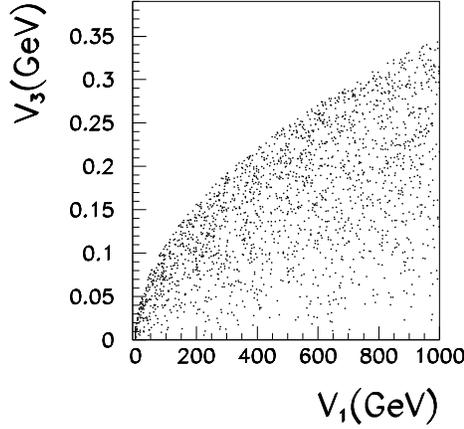


Figure 2: Allowed region in v_1 - v_3 space obtained when the parameters are varied as described in the text.

From Fig. 3b, we can see that, as a consequence of the smallness of v_3 , $m_{H_1} < m_A$, except for a narrow window in which $m_{H_1} \gtrsim m_A$. In such a small region the decay $H_1 \rightarrow AJ$ would be allowed, while the decay $H_1 \rightarrow AA$ is forbidden by phase space. This can be contrasted with the MSSM where the decay $h \rightarrow AA$ is allowed, though in a very small region in parameter space [24]. When κ is small H_1 is mostly triplet and almost degenerate with A and this corresponds to the horizontal lines in Fig. 3b. For larger κ the component of H_1 along the triplet decreases and H_1 becomes lighter than A , as seen in Fig. 3b.

We have verified explicitly that in our model the invisible branching ratios of H_1 and A given by ¹

$$\begin{aligned}
 B_{inv} &= BR(H_1 \rightarrow JJ) + BR(H_1 \rightarrow JA)BR(A \rightarrow JJJ), \\
 A_{inv} &= BR(A \rightarrow JJJ) + BR(A \rightarrow JH_1)BR(H_1 \rightarrow JJ),
 \end{aligned}
 \tag{37}$$

and their product $B_{inv} A_{inv}$, which will be needed in the next section, can be large and even 100 % over large regions of the parameter space. This can be seen in Fig. 4 where we are considering only points in parameter space where $B_{inv} A_{inv} > 0.9$. We also have verified that ϵ_A^2 can vary over all its range for all possible values of the invisible branching ratios. Thus, one may obtain plots similar to Fig. 4a for other possible values of the product $B_{inv} A_{inv}$. The solutions where ϵ_A^2 is larger than the label associated to a particular curve are concentrated in the region between the curve and the main diagonal. The points corresponding to $\epsilon_A^2 > 0.4$ are so close to the main diagonal that the width of the region cannot be seen with the naked eye. An alternative way to display this information is in terms of the associated production cross section which we choose to calculate at $\sqrt{s} = 205$ GeV. This is shown in Fig. 4b. In this figure the diagonal line corresponds to the maximum

¹We have neglected the invisible decay $A \rightarrow \nu\nu$ relative to $A \rightarrow 3J$, which is expected for reasonable choices for the quartic parameters in the scalar Higgs potential and for the lepton Yukawa couplings.

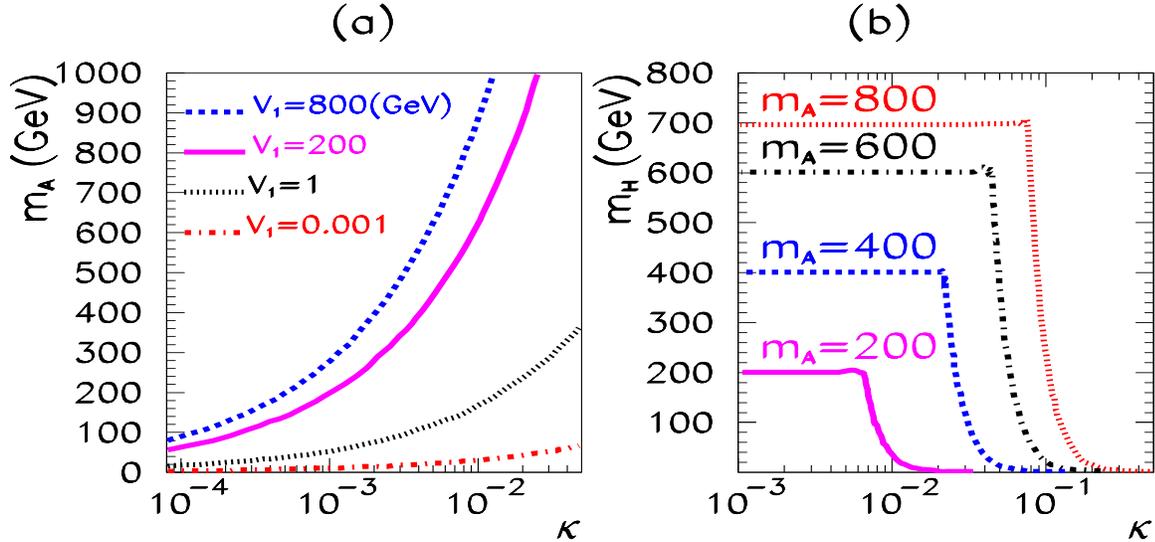


Figure 3: Lowest-lying Higgs boson masses allowed in our model when the parameters are varied as described in the text.

cross section $\sigma_{\max} \approx 0.5$ pb, while region I corresponds to points where the cross section lies between 0.1 pb and 0.5 pb. For region II we have $0.1 > \sigma \geq 0.01$ pb and for region III, $0.01 > \sigma \geq 0.001$ pb. Note that there are no points with $m_{H_1} > m_A$ except very near the line $m_A = m_{H_1}$. For completeness we also present in Fig. 5 the results for the Bjorken production. This plot displays the effective coupling strength parameter ϵ_B^2 versus m_{H_1} for different ranges of the invisible branching ratio B_{inv} . Note that B_{inv} is large only when the coupling of the lightest CP-even Higgs to the fermions is small. This coupling is determined by the projection of the lightest CP-even Higgs onto the doublet Higgs boson, O_{12}^R . Should it be small the corresponding value of $\epsilon_B \approx O_{12}^R$ which determines the Bjorken cross section is also small. This correlation can easily be seen from Fig. 5. As a result if the Higgs is produced via the Bjorken process it is likely to decay visibly, as in the SM.

6 Model-independent Analysis

In this section we perform a model independent study of the limits that can be set based on Higgs boson production in e^+e^- colliders and its subsequent decays, focussing on LEP.

Consider the massive pseudoscalar A and the lightest CP-even scalar H_1 . If $m_A > m_{H_1}$ then A may decay in the standard way to $b\bar{b}$, or to $b\bar{b} + \cancel{p}_T$ via $A \rightarrow H_1 J$ with $H_1 \rightarrow b\bar{b}$, or invisibly into three majorons. From unitarity it follows that only two of these three branching ratios are independent. In addition, the lightest scalar boson H_1 can

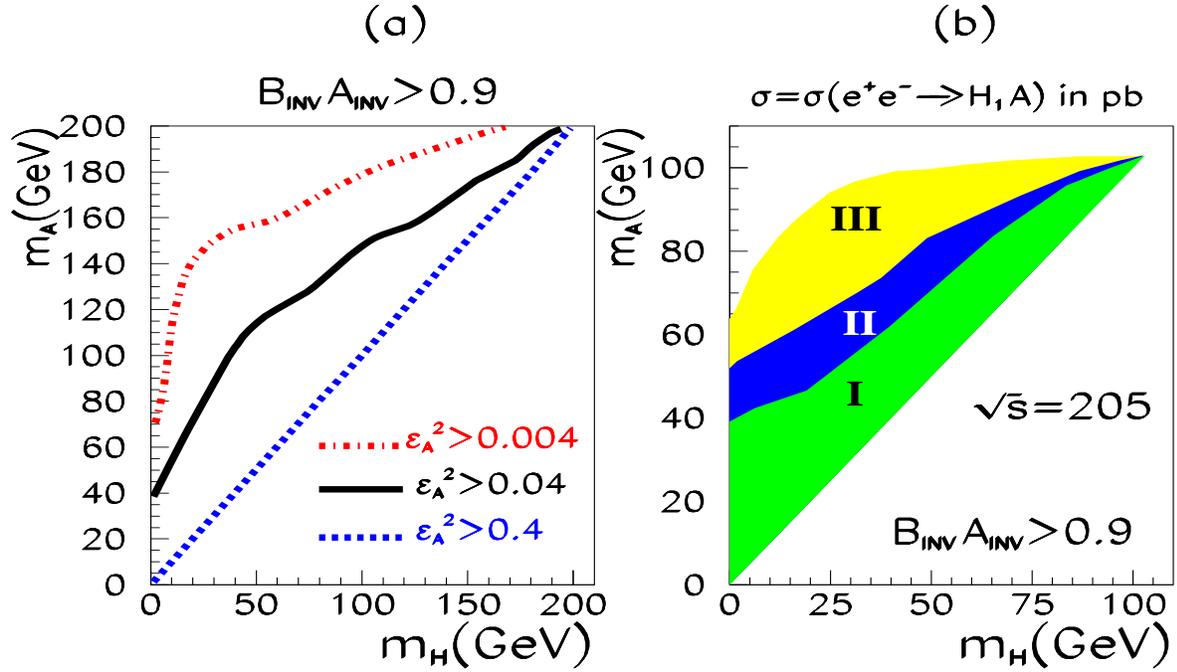


Figure 4: Lowest-lying Higgs boson masses versus effective coupling strength parameter (a) and associated production cross section (b).

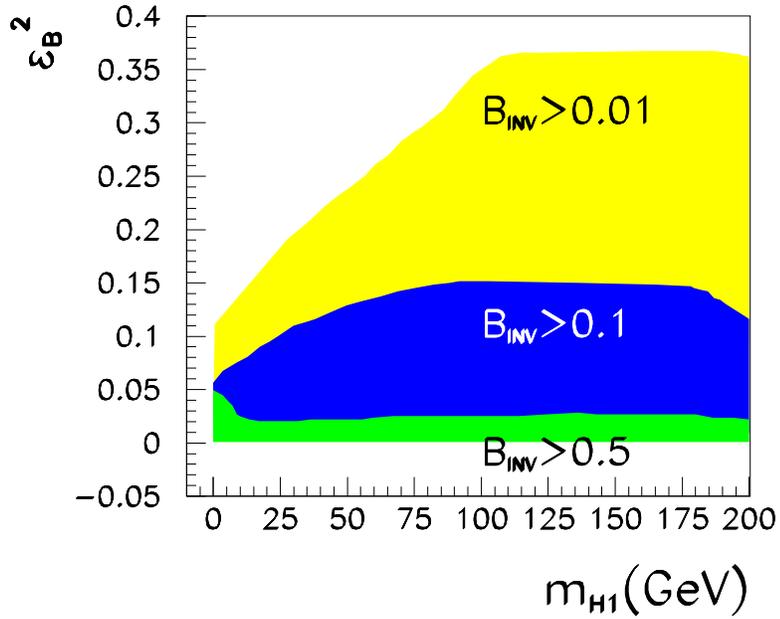


Figure 5: Bjorken production effective strength parameter versus m_{H_1} for different ranges of the parameter B_{inv} .

decay either to $b\bar{b}$ or invisibly to two majorons and only one of the two branching ratios is independent. Similarly, if $m_{H_1} > m_A$ we have in total three independent branching ratios. Thus, in order to make a model independent analysis, we need seven parameters to describe the implications of the production of Higgs bosons at the Z peak: two masses m_A and m_{H_1} , the two parameters ϵ_A and ϵ_B which determine the Bjorken and associated production cross sections and finally, three independent branching ratios (two for the heavier Higgs boson and one for the lightest). Table 1 shows the signatures expected in the model for the cases of Bjorken and Associated production.

In order to have an idea one may compare the above seven parameters with those needed in the simpler models considered before. In the majoron-less model in ref. [25] only five parameters would be relevant, as there is a unitarity relation $\epsilon_B^2 + \epsilon_A^2 = 1$ which does not hold in the present case because the admixture of the singlet Higgs bosons reduces the H and A couplings to the Z . The present model has the additional $A \rightarrow HJ$ branching ratio. Moreover in ref. [25] the A must decay either visibly (mainly to $b\bar{b}$) or invisibly to neutrinos. On the other hand in the majoron model considered in ref. [14] there are also five parameters: m_A , m_H , ϵ_B , ϵ_A and finally, the visible H decay branching ratio is an arbitrary parameter. Note that the A must decay visibly (to $b\bar{b}$ mainly) but there is no unitarity relation for the ϵ 's due to the admixture of the singlet Higgs bosons.

It is outside the scope of our present paper to perform an exhaustive study of restrictions on the parameters of the Higgs potential of this model, especially because of its complexity. However we analyse all signatures that can be engendered by Higgs boson production and its subsequent decays in this model. Although tedious, it is a straightforward task to convert the bare information we provide into restrictions on the model parameters. However we prefer not perform this in detail and use the underlying model only to motivate the analysis.

First we study the constraints arising from the invisible Z width, following closely the analysis performed in ref. [25], where a simpler model, with lepton number violation introduced explicitly and the Higgs bosons decaying to neutrinos rather than to majorons was analysed. As in the case of the model in ref. [25], the Bjorken process contribution to the invisible Z width $Z \rightarrow Z^*H_1$ is very small. Therefore we consider the limits that can be set on associated Higgs boson production at the Z peak, $e^+e^- \rightarrow Z \rightarrow H_1A$ when both CP-even (H_1) as well as CP-odd Higgs bosons (A) decay invisibly. The contribution to the invisible Z width in terms of invisible branching ratios B_{inv} and A_{inv} can be found in ref. [25], and the invisible branching ratios themselves are defined in eq. (37).

In Fig. 6 we show 95 % CL bounds on ϵ_A^2 in the m_{H_1} - m_A plane for a fixed illustrative value of the product $B_{inv}A_{inv} = 0.5$. Five curves labelled by a value of ϵ_A^2 are shown. No points below each of these curves are allowed with ϵ_A^2 larger than that value. The corresponding exclusion plot corresponding to $B_{inv}A_{inv} = 1$ has been given in ref. [25].

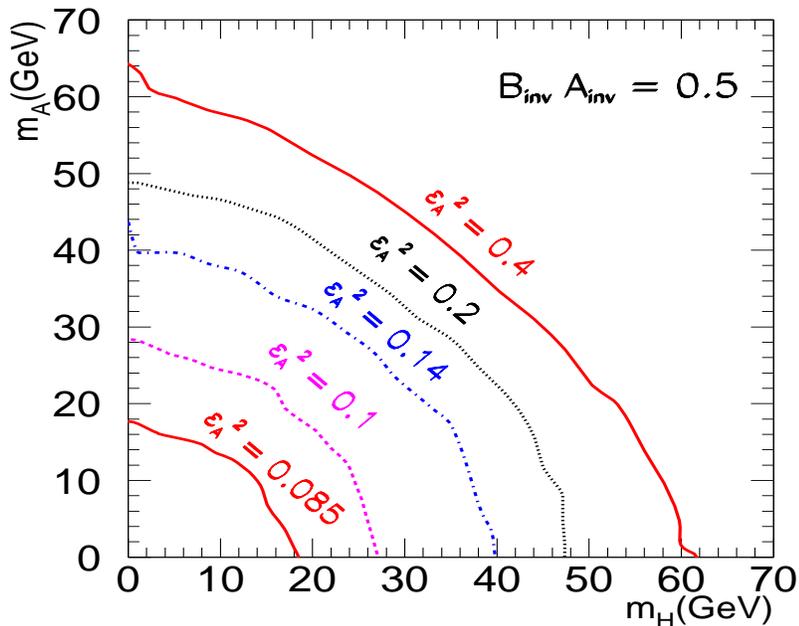


Figure 6: 95 % CL bounds on ϵ_A^2 in the m_H - m_A plane for the the indicated value of the invisible branching ratios

We see from these plots that simply by using the neutrino counting at the Z peak one can already impose important constraints on the parameters of the model. For example, for H_1 and A masses around 20 GeV the upper bound on ϵ_A^2 is a few times 10^{-2} .

Apart from an additional contribution to the invisible Z width, the model produces the variety of signals shown in table 1. Most of these are exactly the same as analysed in [14]. Though the analysis presented in ref. [14] was in a different context, those results are applicable here. They are summarized in Figs 4 and 5 of ref. [14]. These plots may be regarded as particular cases of our model when $A_{inv} \rightarrow 0$. With appropriate re-interpretation they can be adapted to our case. However, as we mentioned, we will not enter into that.

We now consider the various final state topologies that can be produced in e^+e^- collisions at LEP, for example those exhibiting $b\bar{b}$ or $\ell^+\ell^-$ ($\ell = \mu$ or e) pairs and missing energy. The complete table of signatures is reproduced in table 1.

A lot of information follows from the detailed study of these signals. Of the topologies considered in table 1, all have been previously analysed in ref. [14] by carefully evaluating the signals and backgrounds, and by choosing appropriate cuts to enhance the discovery limits. There is only one exception: the present model contains a completely novel signature, namely four b-jets plus missing momentum. This is not present in [14]

Associated production	Bjorken production
$b\bar{b}b\bar{b}$	$b\bar{b}\cancel{p}_T$
$b\bar{b}\cancel{p}_T$	$b\bar{b}l^+l^-$
$b\bar{b}b\bar{b}\cancel{p}_T$	$b\bar{b}q\bar{q}$
\cancel{p}_T	$l^+l^-\cancel{p}_T$
	$l^+l^-\cancel{p}_T$
	\cancel{p}_T

Table 1: Final signals arising from associated production (left column) as well Bjorken production (right column).

nor in [25]. As far as we know it is the first extension of the Higgs sector with this feature. Therefore, from now on we concentrate on this $b\bar{b}b\bar{b}\cancel{p}_T$ signal. The main background comes from $e^+e^- \rightarrow Z\gamma Z\gamma$, $e^+e^- \rightarrow WW$ and $e^+e^- \rightarrow Z\gamma$. This background has been analysed in other contexts, such as chargino production at LEP II [26, 27], where two charginos are produced decaying each one into a neutralino plus a W boson, where the neutralino is stable or decays invisibly. This also gives the 4 jets + missing momentum signature. With appropriate cuts in the \cancel{p}_T , number of jets and invariant mass distributions the background could be removed keeping high signal efficiencies. For our illustrative purposes, we imposed the following cuts in order to remove the background:

- when dealing with hadronic events we only select those events with at least 12 charged particles in the final state.
- In to avoid high energy initial state radiation Z events we reject events with a photon with an energy of more than 35 GeV
- We only accept events with at least four jets.
- We reject an event if the sum of the energy of the two less energetic jets is less than 10 GeV, in order to remove events like $e^+e^- \rightarrow q\bar{q}g$, and $e^+e^- \rightarrow (Z \rightarrow q\bar{q})(\gamma^* \rightarrow q\bar{q})$ characterised by two very energetic jets plus two less energetic ones.
- We reject events with a missing transverse momentum smaller than 10 GeV, in order to avoid events with particles going down the beam pipe.
- We finally impose the invariant mass of the event to $m_{inv} > 100$ GeV. This cut is essential to kill the Z background, which has a large cross section.

Applying all these cuts we eliminate the background, that has been simulated using JETSET [28]. We did a Monte-Carlo study of the signal for our model, which allowed

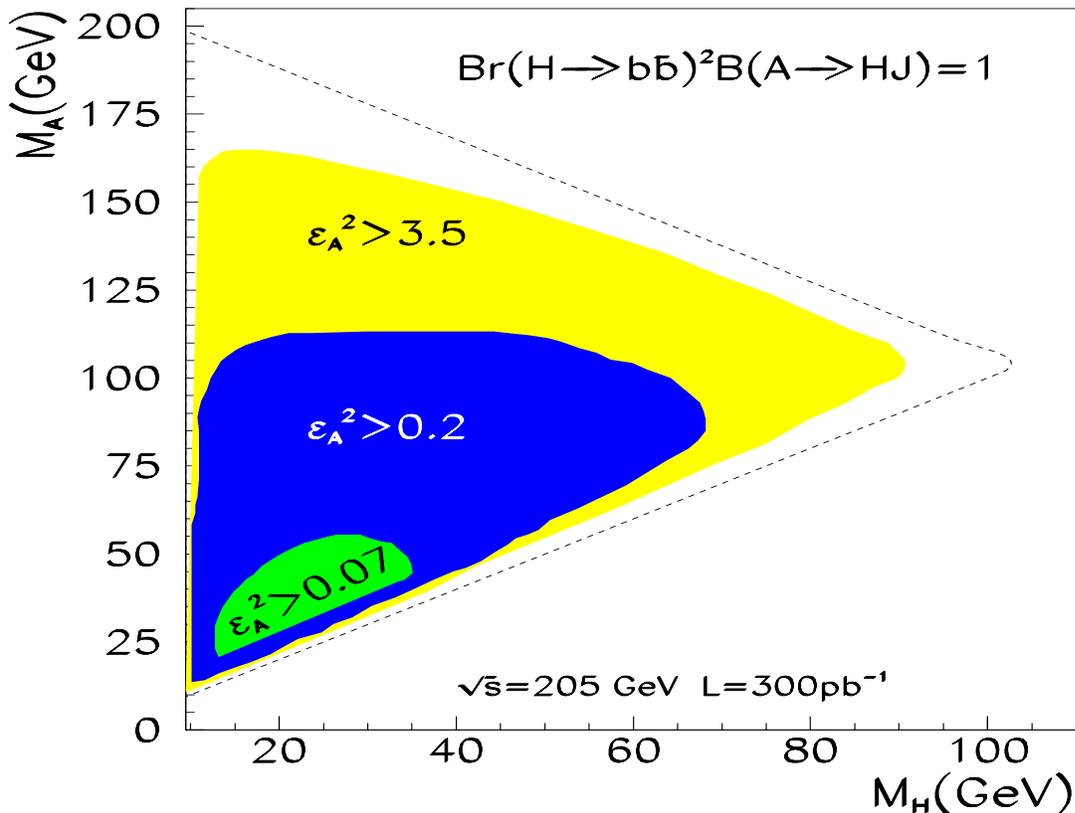


Figure 7: 95 % CL bounds on ϵ_A^2 in the m_H - m_A plane that follow from the 4jets + missing transverse momentum analysis.

us to calculate the efficiency for the signature after implementing the above mentioned cuts. Our results are given as a 95 % CL exclusion plot in the $M_H - M_A$ plane shown in Fig. 7. We have assumed a LEP II integrated luminosity of 300 pb^{-1} . These results are complementary to those arising from the invisible width only, and also complement those that can be derived by appropriate rescaling of the plots shown in [14] corresponding to the other signals in table 1.

7 Conclusions

We have studied the neutral Higgs sector of the general seesaw majoron-type model of neutrino mass generation with spontaneous violation of lepton number. This sector contains three massive CP-even Higgs bosons H_i , $i = 1, 2, 3$, one massive CP-odd A , and one massless CP-odd J called the majoron. We show that H_1 and A can decay invisibly into majorons and determine the constraints that arise from the invisible decay width of the Z gauge boson. The existence of such novel invisible pseudoscalar Higgs boson decays discussed in this paper should be taken into account when determining the Higgs boson

discovery prospects at LEP II and other colliders, such as the LHC and NLC.

We have also noted that the existence of the new decay channel $A \rightarrow H_1 + J$ leads to a novel four-jet + missing momentum signature in associated Higgs boson production $e^+e^- \rightarrow H_1 A$ when $A \rightarrow H_1 J$ and $H_1 \rightarrow b\bar{b}$. This could be detectable at LEP II. We have studied the background and performed a Monte-Carlo simulation of the signal in order to determine the limits that follow from that. Although the structure of the Higgs sector is quite rich one has already important restrictions on Higgs boson mass, couplings and branching ratios that should be taken into account in relation to possible new studies at future colliders such as such as the LHC [29] and the NLC [12] or at a possible muon collider.

Acknowledgements

We thank A. Joshipura, J. J. Hernandez and S. Navas for useful discussions. Special thanks to Oscar Eboli for very helpful discussions related to the study presented in section VI. This work was supported by DGICYT under grant PB95-1077 and by the TMR network grant ERBFMRXCT960090 of the European Union. M. A. D. was supported by a DGICYT postdoctoral grant, D. A. R. was supported by Colombian COLCIENCIAS fellowship, while M. A. G-J was supported by a Spanish MEC FPI fellowship.

References

- [1] P. W. Higgs, *Phys. Lett.* **12** (1964) 132; F. Englert and R. Brout, *Phys. Rev. Lett.* **13** (1964) 321; G. Guralnik and C. K. Hagen, *Phys. Rev. Lett.* **13** (1964) 585.
- [2] F. Richard, in *Elementary Particle Physics: Present and Future*, Proceedings of Valencia 95, ed. A. Ferrer and J. W. F. Valle, ISBN 981-02-2554-7 (World Scientific, 1996), p. 3-31
- [3] *Higgs Physics*, convenors M. Carena and P. Zerwas *et al.*, Proceedings of LEP2 Workshop, CERN Yellow Report, eds. G. Altarelli *et al.*
- [4] For a review see J. W. F. Valle, *Prog. Part. Nucl. Phys.* **26** (1991) 91 and references therein.
- [5] Y. Chikashige, R. Mohapatra, and R. Peccei, *Phys. Lett.* **98B** (1980) 265.
- [6] J. E. Kim, *Phys. Rep.* **150** (1987) 1 and references therein.
- [7] A. Joshipura and J. W. F. Valle, *Nucl. Phys.* **B397** (1993) 105 and references therein.
- [8] A. S. Joshipura and S. Rindani, *Phys. Rev. Lett.* **69** (1992) 3269.
- [9] F. de Campos and J. W. F. Valle, *Phys. Lett.* **B292** (1992) 329;
A. Masiero and J. W. F. Valle, *Phys. Lett.* **B251** (1990) 273;
J. C. Romão, C. A. Santos, and J. W. F. Valle, *Phys. Lett.* **B288** (1992) 311 .
- [10] A. Lopez-Fernandez, J. C. Romão, F. de Campos and J. W. F. Valle, *Phys. Lett.* **B312** (1993) 240.
- [11] B. Brahmachari, A. S. Joshipura, S. Rindani, D. P. Roy, K. Sridhar, *Phys. Rev.* **D48** (1993) 4224
- [12] O. J. P. Eboli, M. C. Gonzalez-Garcia, A. Lopez-Fernandez, S. F. Novaes, J. W. F. Valle, O. J. P. Éboli, *et al.*, *Nucl. Phys.* **B421** (1994) 65 [hep-ph/9312278];
F. de Campos *et al.*, Working Group on e^+e^- Collision at 500 GeV: The Physics Potential, edited by P. Zerwas (1993) 55.
- [13] F. de Campos, M. A. Garcia-Jareño, A. S. Joshipura, J. Rosiek, D. P. Roy and J. W. F. Valle, *Phys. Lett.* **B336** (1994) 446.
- [14] F. de Campos, O. J. P. Eboli, J. Rosiek, J. W. F. Valle, *Phys. Rev.* **D55** (1997) 1316
- [15] For a recent LEP analysis see M. Acciarri *et al.* (L3 Collaboration) CERN-PPE-97-097, Jul 1997, Submitted to Phys.Lett.B

- [16] M. Gell-Mann, P. Ramond, R. Slansky in Supergravity, ed by D. Freedman *et al.*, , (North Holland, 1979), T. Yanagida, KEK lectures, ed O. Sawada et al (1979)
- [17] R. Mohapatra, G. Senjanovic, *Phys. Rev.* **D23** (1981) 165
- [18] J. Schechter, J. W. F. Valle, *Phys. Rev.* **D22** (1980) 2227
- [19] J. Schechter and J. W. F. Valle, *Phys. Rev.* **D25** (1982) 774
- [20] G. B. Gelmini and M Roncadelli, *Phys. Lett.* **B99** (1981) 411
- [21] H. M. Georgi, S. L. Glashow, and S. Nussinov, *Nucl. Phys.* **B193** (1981) 297
- [22] A. S. Joshipura, *Int. J. Mod. Phys.* **A7** (1992) 2021
- [23] A. Pich, in *Physics Beyond the Standard Model: Present and Future*, Proceedings of Valencia 97, ed. I. Antoniadis, L. E. Ibáñez and J. W. F. Valle, to be published.
- [24] A. Djouadi, J. Kalinowski, and P.M. Zerwas, *Z. Phys.* **C57** (1993),569 (1992).
- [25] M.A. Díaz, M.A. Garcia-Jareño, D.A. Restrepo, and J.W.F. Valle, hep-ph/9712487, Dec 1997.
- [26] DELPHI Coll., P. Abreu *et al*, *Eur. Phys. J.* **C1**(1998)1
- [27] F. de Campos, O.J.P. Eboli, M. A. García-Jareño, and J.W.F. Valle, hep-ph/9710545
Ref [28]:
- [28] T. Sjöstrand, *Comp. Phys. Comm.* **39**(1986)347; T. Sjöstrand, PYTHIA 5.6 and JET-SET 7.3, CERN-TH/6488-92.
- [29] J. Gunion, *Phys. Rev. Lett.* **72** (1994) 199;
D. Choudhury and D. P. Roy, *Phys. Lett.* **B322** (1994) 368;
J. C. Romão, F. de Campos, L. Diaz-Cruz, and J. W. F. Valle, *Mod. Phys. Lett.* **A9** (1994) 817;
S. G. Frederiksen, N. P. Johnson, G. L. Kane, J. H. Reid, *Phys. Lett.* **D50** (1994) 4244