

Radiative Υ decays and a light pseudoscalar Higgs in the NMSSM

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Abstract

We study possible effects of a light CP-odd Higgs boson on radiative Υ decays in the Next-to-Minimal Supersymmetric Standard Model. Recent constraints from CLEO on radiative $\Upsilon(1S)$ decays are translated into constraints on the parameter space of CP-odd Higgs boson masses and couplings, and compared to constraints from B physics and the muon anomalous magnetic moment. Possible Higgs- $\eta_b(nS)$ mixing effects are discussed, notably in the light of the recent measurement of the $\eta_b(1S)$ mass by Babar: The somewhat large $\Upsilon(1S) - \eta_b(1S)$ hyperfine splitting could easily be explained by the presence of a CP-odd Higgs boson with a mass in the range 9.4 - 10.5 GeV. Then, tests of lepton universality in inclusive radiative Υ decays can provide a visible signal in forthcoming experimental data.

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1 Introduction

The Next-to-Minimal Supersymmetric Standard Model (NMSSM) [1] provides the simplest solution to the μ problem of the MSSM [2]. Its phenomenology can differ in various respects from the MSSM. Notably, as emphasized in [3], a light CP-odd scalar A_1 can appear in the Higgs spectrum.

In the case where the CP-even Higgs boson H decays dominantly into a pair of CP-odd scalars [3–10], LEP constraints on CP-even Higgs masses [11] are alleviated considerably [5–10]. For $m_{A_1} \lesssim 10.5$ GeV, where the A_1 decay into $B\bar{B}$ is forbidden, this scenario could even explain the 2.3σ excess in the $e^+e^- \rightarrow Z+2b$ channel for $M_{2b} \sim 100$ GeV [6] (where the two b quarks would result from a CP-even Higgs H with $M_H \sim 100$ GeV and a branching ratio $\mathcal{B}(H \rightarrow b\bar{b}) \sim 0.08$, but $\mathcal{B}(H \rightarrow A_1A_1) \sim 0.9$). Also at hadron colliders the search for CP-even scalars would be particularly difficult [4, 6, 8–10] if they decay dominantly into A_1A_1 with m_{A_1} below the $B\bar{B}$ threshold. In this case, however, the A_1 can have important effects on Υ decays [12–18]. Notably a Super B factory can then play an important and complementary role [19] via its potential sensitivity to $\Upsilon \rightarrow \gamma A_1$ decays.

Whereas a light A_1 is also possible in the MSSM with a CP-violating Higgs sector [20, 21], scenarios with more than one gauge singlet [22], little Higgs models and non-supersymmetric two Higgs doublet models (see [23] for an overview), we concentrate subsequently on the simplest version of the NMSSM with a scale invariant superpotential.

$\Upsilon \rightarrow \gamma A_1$ decays have been investigated in the NMSSM before in [16] for $m_{A_1} \lesssim 9.2$ GeV, where the signal relies on a narrow peak in the photon spectrum. Recent results from CLEO on radiative $\Upsilon(1S)$ decays [24] (assuming a A_1 width $\lesssim 10$ MeV) constrain this domain of m_{A_1} strongly. One of our aims is to translate the CLEO results into constraints on the NMSSM parameter space (see also [25]) as X_d , the (reduced) coupling of A_1 to b quarks, and to compare them with constraints from B physics [26–28] and the anomalous magnetic moment of the muon [25, 29].

For $m_{A_1} \gtrsim 9$ GeV, various corrections to the $\Upsilon \rightarrow \gamma A_1$ decay rate become relatively large and uncertain [30], which makes it difficult to translate experimental constraints on the decay rate into constraints on X_d . Consequently, this region of m_{A_1} is hardly constrained by CLEO results.

Very recently, a CP-odd state with a mass of about 9389 MeV has been observed in $\Upsilon(3S)$ decays by BaBar [31], showing up as a peak with a significance of 10 standard deviations in the photon energy spectrum. At first sight, this state can be interpreted as the long-awaited $\eta_b(1S)$. However, in the presence of a CP-odd Higgs with a mass in the same region, the observed mass has to be interpreted as an eigenvalue of a 2×2 mixing matrix, and would differ correspondingly from $m_{\eta_{b0}}$, the mass of the $\eta_b(1S)$ in the absence of a CP-odd Higgs ($m_{\eta_{b0}}^2$ is now one of the diagonal entries of the mass matrix) [12]. In this case, a second peak in the photon spectrum should possibly be visible; however, the search for such a second peak would require a dedicated consideration of the various background contributions (notably from the ISR and $\chi_{bJ}(2P)$), which should be performed in the future.

First, this mixing effect could explain the fact that the observed mass is somewhat lower than expected, if m_{A_1} is somewhat above $m_{\eta_{b0}}$. Second, the off-diagonal element of the mass matrix can be estimated and turns out to be proportional to X_d [12, 17]. Assuming a reasonable range for $m_{\eta_{b0}}$, the observed value of ~ 9389 MeV for one of the eigenvalues implies an upper bound on X_d as a function of m_{A_1} , which is, however, particularly strong only for $m_{A_1} \sim 9389$ MeV and will be derived below.

At present, a direct detection of a CP-odd Higgs with a mass in the particularly interesting region $9.2 \text{ GeV} \lesssim m_{A_1} \lesssim 10.5 \text{ GeV}$ via a peak in the photon spectrum seems to be quite difficult. Fortunately, an alternative signal for an A_1 state below the $B\bar{B}$ threshold can be a breakdown of

lepton universality (LU) in $\Upsilon \rightarrow (\gamma) l^+ l^-$ decays (via an intermediate A_1 state), since A_1 would decay practically exclusively into $\tau^+ \tau^-$ [13, 15, 17]. Note that, to this end, the photon does not have to be detected. Present tests of lepton universality in $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ decays [32] have error bars in the 5–10% range. Remarkably, however, a general trend (at the 1σ level) seems to point towards a slight excess of the $\tau^+ \tau^-$ branching ratios, as expected in the presence of a A_1 state. Another aim of the present paper is to investigate corresponding sensitivities of forthcoming experimental data, assuming a possible reduction of the errors to the 2% range.

The layout of this paper is as follows: In section 2 we review the domains of the NMSSM parameter space which lead to a light CP-odd Higgs with strong couplings to down type quarks (and leptons). In section 3 we derive constraints on the NMSSM parameter space from recent CLEO results, using quite conservative estimates for the corrections to the $\Upsilon \rightarrow \gamma A_1$ decay rate which lead to quite conservative upper bounds on X_d as a function of m_{A_1} . (These upper bounds on X_d will be included in future versions of the code NMSSMTools [33].) In section 4, we discuss the mixing effects of A_1 with $\eta_b(nS)$ following [12, 13, 17]. In section 5, we derive constraints on X_d from the measured $\eta_{b_{obs}}$ mass by BaBar and from (conservative) assumptions on $m_{\eta_{b0}}$, and discuss quantitatively the possible mixing-induced shift of the measured $\eta_{b_{obs}}$ mass. In section 6 we compare these CLEO and BaBar constraints with constraints from LEP, B physics and the muon anomalous magnetic moment. This analysis is performed with the help of the updated NMSSMTools package. In section 7 we reconsider A_1 masses between 9.2 and 10.5 GeV and show that (for less conservative estimates of the corrections to the $\Upsilon \rightarrow \gamma A_1$ decay rate) a breakdown of lepton universality in $\Upsilon \rightarrow (\gamma) l^+ l^-$ decays can become an important observable for the detection of a CP-odd Higgs in this mass range. We present formulas for the relevant branching ratios including possible $A_1 - \eta_b(nS)$ mixings, and study future sensitivities on X_d from lepton universality breaking. Section 8 contains conclusions and an outlook.

2 A light CP-odd Higgs in the NMSSM

In this section we show that the parameter space of the NMSSM can accommodate a light CP-odd Higgs, which is strongly coupled to down-quarks and leptons (see also [3, 6, 7]). We consider the simplest version of the NMSSM with a scale invariant superpotential

$$W = \lambda S H_u H_d + \frac{1}{3} \kappa S^3 + \dots \quad (1)$$

and associated soft trilinear couplings

$$V_{soft} = (\lambda A_\lambda S H_u H_d + \frac{1}{3} \kappa A_\kappa S^3) + h.c. + \dots \quad (2)$$

in the conventions of [33]. A vev of the singlet field $s \equiv \langle S \rangle$ generates an effective μ -term, and it is convenient to define also an effective B -term:

$$\mu_{\text{eff}} = \lambda s, \quad B_{\text{eff}} = A_\lambda + \kappa s. \quad (3)$$

The Higgs sector of the NMSSM contains six independent parameters, which can be chosen as

$$\lambda, \kappa, A_\lambda, A_\kappa, \tan \beta, \mu_{\text{eff}}. \quad (4)$$

In the NMSSM, two physical pseudoscalar states appear in the spectrum, which are superpositions of the MSSM-like state A_{MSSM} (the remaining $SU(2)$ doublet after omitting the Goldstone

boson) and the singlet-like state A_S . In the basis (A_{MSSM}, A_S) , the 2×2 mass square matrix for the CP-odd Higgs bosons has the following matrix elements [33]

$$\begin{aligned} M_{11}^2 &= \frac{2\mu_{\text{eff}} B_{\text{eff}}}{\sin 2\beta}, & M_{12}^2 &= \lambda v(A_\lambda - 2\kappa s) \\ M_{22}^2 &= \frac{\lambda^2 v^2 \sin 2\beta}{2\mu_{\text{eff}}} (A_\lambda + 4\kappa s) - 3\kappa s A_\kappa \end{aligned} \quad (5)$$

where $v^2 = 1/(2\sqrt{2}G_F)$. The masses of the CP-odd eigenstates $A_{1,2}$ are

$$m_{A_{1,2}}^2 = \frac{1}{2}[M_{11}^2 + M_{22}^2 \mp \Delta M^2] \quad (6)$$

with $\Delta M^2 = \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4(M_{12}^2)^2}$.

The lighter CP-odd state A_1 can be decomposed into (A_{MSSM}, A_S) according to

$$A_1 = \cos \theta_A A_{MSSM} + \sin \theta_A A_S, \quad (7)$$

where the mixing angle θ_A is

$$\cos 2\theta_A = \frac{M_{22}^2 - M_{11}^2}{\Delta M^2}. \quad (8)$$

To a good approximation (for moderate A_λ , small A_κ and large $\tan \beta$), the mass of the lightest CP-odd Higgs boson and $\cos \theta_A$ can be written as [3, 7]

$$m_{A_1}^2 \simeq 3\kappa s \left(\frac{3\lambda^2 v^2 A_\lambda \sin 2\beta}{2\mu_{\text{eff}} B_{\text{eff}} - 3\kappa s A_\kappa \sin 2\beta} - A_\kappa \right), \quad (9)$$

$$\cos \theta_A \simeq -\frac{\lambda v(A_\lambda - 2\kappa s) \sin 2\beta}{2\mu_{\text{eff}} B_{\text{eff}} + 3\kappa s A_\kappa \sin 2\beta}. \quad (10)$$

(The approximate equation for $\cos \theta_A$ ceases to be valid if the second term in the denominator is large compared to the first one.)

The reduced coupling X_d of the light physical A_1 Higgs boson to down-type quarks and leptons (normalized with respect to the coupling of the CP-even Higgs boson of the Standard Model) is given by

$$X_d = \cos \theta_A \tan \beta. \quad (11)$$

Interesting phenomena in the Υ -system take place for large values of X_d , i.e. large values of $\tan \beta$ without $\cos \theta_A$ being too small. (A possible enhancement of X_d [18] can occur due to the radiatively generated $\tan \beta$ -enhanced Higgs-singlet Yukawa couplings [34]. However, in the case of a sizable value of $\cos \theta_A$ already at tree level as considered below, this effect is small.)

At first sight eq. (10) seems to imply (from $\sin 2\beta \sim 2/\tan \beta$ for large $\tan \beta$) that $\cos \theta_A$ decreases indeed with $\tan \beta$ – this would be the case in the PQ-symmetry-limit ($\kappa \rightarrow 0$) or R-symmetry-limit ($A_\kappa, A_\lambda \rightarrow 0$), where the second term in the denominator of (10) tends to zero. On the other hand, it follows from the minimization equations of the scalar potential of the NMSSM (as in the MSSM), for fixed soft Higgs mass terms and μ_{eff} , that $\tan \beta$ is proportional to $1/|\mu_{\text{eff}} B_{\text{eff}}|$ for large $\tan \beta$ [35], hence large values of $\tan \beta$ are associated to small values of $|B_{\text{eff}}|$ (since $|\mu_{\text{eff}}| \gtrsim 100$ GeV from the lower bound on chargino masses). It is useful to replace $|B_{\text{eff}}|$ by the parameter

$$M_A^2 \equiv M_{11}^2 = \frac{2\mu_{\text{eff}} B_{\text{eff}}}{\sin 2\beta}, \quad (12)$$

which sets the scale for the masses of the complete $SU(2)$ multiplet of Higgs states including a scalar, a pseudoscalar and a charged Higgs as in the MSSM (in our case, the corresponding pseudoscalar is the heavier one A_2). In terms of M_A^2 , X_d can be written approximately as

$$X_d \simeq -\frac{\lambda v(A_\lambda - 2\kappa s)}{M_A^2 + 3\kappa s A_\kappa} \times \tan \beta, \quad (13)$$

and it is reasonable to examine the large $\tan \beta$ region keeping M_A fixed.

It follows from eq. (9) that there exist always values of A_κ of the same sign as A_λ (typically both negative) where m_{A_1} is small [9], while $\cos \theta_A \simeq 0.1 - 0.6$ and hence X_d is not suppressed. This requires a moderate fine-tuning of A_κ (or M_A); on the other hand the authors of Ref. [16] stress that such values for A_κ , which allow a light SM-like Higgs to decay into two A_1 with $m_{A_1} < 2m_b$, correspond to the smallest degree of fine-tuning in the entire parameter space of the NMSSM. For corresponding values of A_κ , the denominator of X_d in (13) is dominated by M_A^2 . For $M_A^2 \lesssim |\kappa A_\kappa s|$, even larger values of $\cos \theta_A \simeq 0.6 - 1.0$ are possible while m_{A_1} remains small. In this regime, the approximations leading to eqs. (9), (10) and (13) are no longer valid, however.

To summarize, the following conditions can be fulfilled simultaneously in the NMSSM, which yield possibly observable effects in Υ decays:

- $m_{A_1} \lesssim 10.5$ GeV from, e.g., appropriate values of A_κ ;
- a large value of X_d , if $\tan \beta$ is large while M_A in the denominator of (13) remains moderate.

The numerical results in section 6 confirm the analytical estimates above.

3 Constraints from CLEO

Recently, the CLEO collaboration presented results on Higgs searches from $\Upsilon(1S)$ decays [24]. $21.5 \cdot 10^6$ $\Upsilon(1S)$ decays had been collected and, for the $\Upsilon(1S) \rightarrow \gamma + (A_1 \rightarrow \tau^+ \tau^-)$ search, the photon energy spectrum in events with missing energy and one identified μ^\pm or e^\pm (allegedly from $\tau \rightarrow e\nu\nu$ or $\tau \rightarrow \mu\nu\nu$) had been examined. For the $A_1 \rightarrow \mu^+ \mu^-$ search, both muons were identified.

No narrow peaks (of a width below ~ 10 MeV) in the photon energy spectrum are observed (except for $\Upsilon(1S) \rightarrow \gamma J/\Psi \rightarrow \gamma \mu^+ \mu^-$), which allows to place stringent upper limits between 10^{-4} and 10^{-5} on the branching ratio $\mathcal{B}(\Upsilon(1S) \rightarrow \gamma(A_1 \rightarrow \tau^+ \tau^- / \mu^+ \mu^-))$ for $m_{A_1} \lesssim 9.2$ GeV [24].

The $\mathcal{B}(\Upsilon(1S) \rightarrow \gamma A_1)$ is given by the Wilczek formula [36, 37]

$$\frac{\mathcal{B}(\Upsilon(1S) \rightarrow \gamma A_1)}{\mathcal{B}(\Upsilon(1S) \rightarrow \mu^+ \mu^-)} = \frac{G_F m_b^2 X_d^2}{\sqrt{2} \pi \alpha} \left(1 - \frac{m_{A_1}^2}{m_{\Upsilon(1S)}^2} \right) \times F \quad (14)$$

where α denotes the fine structure constant and X_d is given in (11). F is a correction factor, which includes three kinds of corrections to the leading-order Wilczek formula (the relevant formulas are summarized in [30]): bound state, QCD and relativistic corrections. Bound state effects have a quite different behaviour for a scalar or a pseudoscalar Higgs, increasing the ratio (14) by $\sim 20\%$ in the latter case [38–40]. QCD corrections reduce the ratio (14) by a similar amount [41, 42]. Relativistic corrections can generate an important reduction, and were calculated in [43].

These relativistic corrections depend quite strongly on the b quark mass m_b , and become unreliable at least for Higgs masses m_{A_1} above 8 GeV [43] where they can generate a vanishing (or even negative) correction factor F . Frequently, the approximation $F \sim 0.5$ for all m_{A_1} is employed in the literature [26, 37]. However, in order to derive conservative bounds on the NMSSM parameters

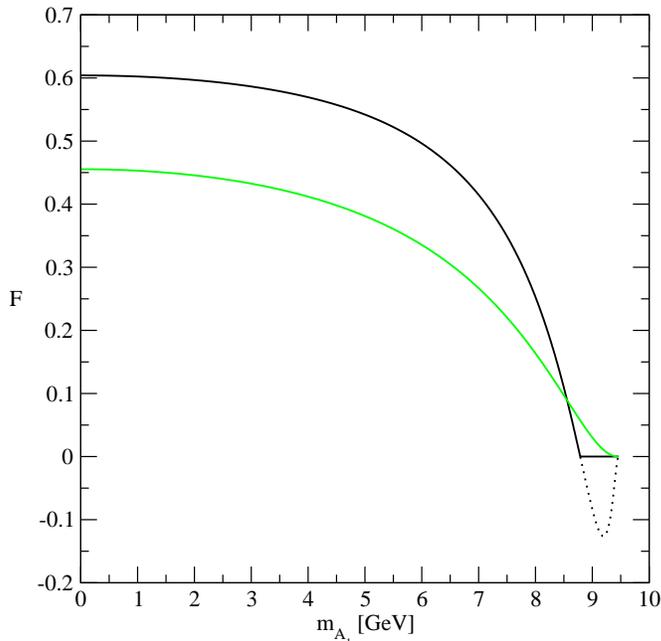


Figure 1: $F(m_{A_1})$ including the bound state and QCD corrections, and a naive extrapolation of the relativistic corrections computed for $m_{A_1} \ll m_\Upsilon$ at larger values of m_{A_1} . Black curve: assuming $m_b = 4.9$ GeV (as used later), green curve: assuming $m_b = 5.3$ GeV.

from CLEO results, we use in this section the smaller values of $F(m_{A_1})$ for larger m_{A_1} , which are obtained by a naive extrapolation of the relativistic corrections [43]. Using the quark model value $m_b = 4.9$ GeV, the resulting behaviour of $F(m_{A_1})$ (including also the bound state and QCD corrections) is shown in Fig. 1, according to which F vanishes (and even becomes negative, in which case we take $F = 0$) for $m_{A_1} \gtrsim 8.8$ GeV. Correspondingly, the CLEO bounds on the NMSSM parameters disappear for $m_{A_1} \gtrsim 8.8$ GeV. (For larger values of m_b as 5.3 GeV, F would vanish only for $m_{A_1} \sim 9.4$ GeV $\sim m_\Upsilon$ as also indicated in Fig. 1.)

Next, in order to translate the CLEO bounds into bounds on $X_d(m_{A_1})$ using eq. (14), the branching ratios $\mathcal{B}(A_1 \rightarrow \tau^+\tau^-/\mu^+\mu^-)$ have to be known, which depend essentially on $\tan\beta$. For m_{A_1} above $2m_\tau$, $\mathcal{B}(A_1 \rightarrow \tau^+\tau^-)$ varies from $\sim 70\%$ for $\tan\beta = 1.5$ to $\sim 95\%$ for $\tan\beta = 50$, whereas $\mathcal{B}(A_1 \rightarrow \mu^+\mu^-)$ is always below 10% even for m_{A_1} below $2m_\tau$ (which implies to reconsider the estimates of the CLEO reach in [44]). Using the code NMSSMTools [33] for the determination of the $\mathcal{B}(A_1 \rightarrow \tau^+\tau^-/\mu^+\mu^-)$ and an interpolation of the CLEO bounds [24], we show our resulting upper limits on X_d as a function of m_{A_1} for two extreme values of $\tan\beta = 1.5$ and 50 in Fig. 2.

Actually, in Ref. [24] the total decay width of A_1 , Γ_{A_1} , is assumed to be below 10 MeV. Although we do not believe that the CLEO bounds disappear completely in the case where Γ_{A_1} (which increases with X_d and m_{A_1}) is larger than 10 MeV, we indicate in Fig. 2 also the region at large X_d and $m_{A_1} \gtrsim 3.5$ GeV where Γ_{A_1} exceeds 10 MeV (depending also slightly on $\tan\beta$). In the updated version 2.1 of the NMSSMTools package [33] these bounds are included.

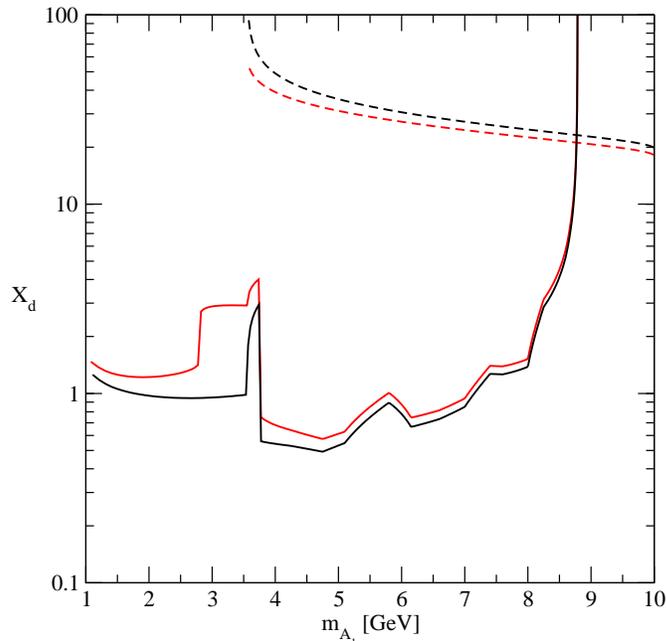


Figure 2: Upper bounds on X_d as a function of m_{A_1} for two extreme values of $\tan\beta = 1.5$ (red curve) and $\tan\beta = 50$ (black curve) using results from CLEO [24]. We also indicate as dashed lines the region at large X_d and $m_{A_1} > 3.5$ GeV where Γ_{A_1} exceeds 10 MeV (same colour code for $\tan\beta = 1.5$ and 50).

4 Mixing of A_1 with the $\eta_b(nS)$ resonances

In the presence of a pseudoscalar Higgs boson with a mass close to one of the different $\eta_b(nS)$ resonances, a significant mixing between these states can occur [12].

The mixing between a CP-odd Higgs and a single $\eta_b(nS)$ ($n = 1, 2$ or 3) resonance can be described by the introduction of off-diagonal elements denoted by δm_n^2 in the mass matrix [12, 17] (here and below we neglect possible induced $\eta_b(nS) - \eta_b(n'S)$ mixings for $n \neq n'$)

$$\mathcal{M}_n^2 = \begin{pmatrix} m_{A_{10}}^2 - im_{A_{10}}\Gamma_{A_{10}} & \delta m_n^2 \\ \delta m_n^2 & m_{\eta_{b0}(nS)}^2 - im_{\eta_{b0}(nS)}\Gamma_{\eta_{b0}(nS)} \end{pmatrix} \quad (15)$$

where the subindex '0' indicates unmixed states: $m_{A_{10}}$ and $m_{\eta_{b0}(nS)}$ ($\Gamma_{A_{10}}$ and $\Gamma_{\eta_{b0}(nS)}$) denote the masses (widths) of the pseudoscalar Higgs boson and $\eta_{b0}(nS)$ states, respectively, before mixing.

In Ref. [17] only the mixing of the Higgs with the $\eta_{b0}(1S)$ resonance was taken into account. In this paper, we extend the analysis by considering the possible mixing between the Higgs and *any* of the three $\eta_{b0}(nS)$ ($n = 1, 2$ or 3) states. Thus three mixing angles have to be defined; however, only the contribution from the closest $\eta_b(nS)$ state to the hypothetical A_1 mass will be assumed to be significant for the mixing, i.e. only one among the three mixing angles will deviate significantly from zero. The generally complex mixing angle α_n between the pseudoscalar Higgs A_{10} and an $\eta_{b0}(nS)$ state is given by [17]

$$\sin 2\alpha_n = \delta m_n^2 / \Delta_n^2 \quad (16)$$

where

$$\Delta_n^2 = [D_n^2 + (\delta m_n^2)^2]^{1/2} \quad (17)$$

with

$$D_n = (m_{A_{10}}^2 - m_{\eta_{b0}(nS)}^2 - im_{A_{10}}\Gamma_{A_{10}} + im_{\eta_{b0}(nS)}\Gamma_{\eta_{b0}(nS)})/2. \quad (18)$$

The off-diagonal element δm_n^2 can be computed within the framework of a non-relativistic quark potential model as

$$\delta m_n^2 = \left(\frac{3m_{\eta_b(nS)}^3}{8\pi v^2} \right)^{1/2} |R_{\eta_b(nS)}(0)| \times X_d. \quad (19)$$

In a non-relativistic approximation to the bottomonium bound states, the radial wave functions at the origin can be considered as identical for vector and pseudoscalar states, i.e. $R_{\Upsilon(nS)}(0) \simeq R_{\eta_b(nS)}(0)$, and can therefore be determined from the measured $\Upsilon \rightarrow e^+e^-$ decay widths:

$$|R_{\Upsilon(nS)}(0)|^2 \simeq \Gamma[\Upsilon(nS) \rightarrow e^+e^-] \times \frac{9m_{\Upsilon(nS)}^2}{4\alpha^2} \left[1 + \frac{16\alpha_s(m_{\Upsilon}^2)}{3\pi} \right] \quad (20)$$

Substituting recent values for the dielectron widths from [32] we obtain $|R_{\eta_b(1S)}(0)|^2 = 6.60 \text{ GeV}^3$, $|R_{\eta_b(2S)}(0)|^2 = 3.02 \text{ GeV}^3$ and $|R_{\eta_b(3S)}(0)|^2 = 2.18 \text{ GeV}^3$, leading to³

$$\delta m_1^2 = 0.14 \text{ GeV}^2 \times X_d, \quad \delta m_2^2 = 0.11 \text{ GeV}^2 \times X_d, \quad \delta m_3^2 = 0.10 \text{ GeV}^2 \times X_d. \quad (21)$$

The A_1 and $\eta_b(nS)$ physical (mixed) states can be written as

$$\begin{aligned} A_1 &= \cos \alpha_n A_{10} + \sin \alpha_n \eta_{b0}(nS), \\ \eta_b(nS) &= \cos \alpha_n \eta_{b0}(nS) - \sin \alpha_n A_{10} \end{aligned} \quad (22)$$

assuming $\cos^2 \alpha_n + \sin^2 \alpha_n \simeq 1$, i.e. neglecting the imaginary components of α_n . (Here and below we use the notation A_1 and $\eta_b(nS)$ for the mixed states in order to indicate their dominant components for small mixing angles. Clearly, for $\alpha_n \sim 90^\circ$, their dominant components would be reversed.)

The full widths Γ_{A_1} and $\Gamma_{\eta_b(nS)}$ of the A_1 and $\eta_b(nS)$ physical states can be expressed in terms of the widths of the unmixed states according to [17]

$$\begin{aligned} \Gamma_{A_1} &\simeq \cos^2 \alpha_n \Gamma_{A_{10}} + \sin^2 \alpha_n \Gamma_{\eta_{b0}(nS)}, \\ \Gamma_{\eta_b} &\simeq \cos^2 \alpha_n \Gamma_{\eta_{b0}(nS)} + \sin^2 \alpha_n \Gamma_{A_{10}}. \end{aligned} \quad (23)$$

Finally, let us recall that the mixing of the A_{10} with η_{b0} states should lead to mass shifts which can be sizable [12, 17]. These mass shifts might have spectroscopic consequences concerning the hyperfine $\eta_b(nS) - \Upsilon(nS)$ splitting [12, 17, 46] whose predictions within the SM are reviewed in [47], and with respect to which the BaBar result [31] on the $\eta_b(1S) - \Upsilon(1S)$ hyperfine splitting – in the absence of a light CP-odd Higgs – would be somewhat large (see the next section).

5 Upper bounds on X_d from the measured η_b mass, and the mixing-induced η_b mass shift

The observation of an η_b -like state with a mass of $\simeq 9.389 \text{ GeV}$ by BaBar [31], allows to obtain upper limits on the reduced coupling X_d as a function of the lightest CP-odd Higgs mass parameter $m_{A_{10}}$, if $m_{A_{10}}$ is near 9.39 GeV . This follows from the fact that the measured mass squared has

³Similar values can be obtained from a Buchmuller-Tye potential [45].

now to be considered as (the real part of) the eigenvalue of the matrix \mathcal{M}_1^2 (15), corresponding algebraic relations and an estimate of hadronic parameters as $m_{\eta_{b0}(1S)}$.

Subsequently we denote the “ η_b ” mass as measured by BaBar by m_{obs} , and the state $\eta_{b0}(1S)$ by η_{b0} . The observed state has now to be considered as a superposition of A_{10} and η_{b0} . Then the following algebraic identity holds (where δm_1^2 is the off-diagonal element of the matrix \mathcal{M}_1^2 (15)):

$$(\delta m_1^2)^2 = \Delta_A \Delta_\eta \left[1 + \frac{\gamma^2}{(\Delta_A + \Delta_\eta)^2} \right] \quad (24)$$

where

$$\Delta_A = m_{A_{10}}^2 - m_{obs}^2, \quad \Delta_\eta = m_{\eta_{b0}}^2 - m_{obs}^2 \quad (25)$$

and

$$\gamma = m_{A_{10}} \Gamma_{A_{10}} - m_{\eta_{b0}} \Gamma_{\eta_{b0}} . \quad (26)$$

Note that Δ_A and Δ_η must have the same sign, which follows already from properties of eigenvalues of real 2×2 matrices.

Now, if we use estimates for the parameters $m_{\eta_{b0}}$ and γ , eq. (24) allows to obtain an upper bound on X_d as a function of $m_{A_{10}}$. First, for γ we can assume $|\gamma| \lesssim m_{obs} \times 20$ MeV (from $\Gamma_{A_{10}}, \Gamma_{\eta_{b0}} \lesssim 20$ MeV) with the result that the term $\sim \gamma^2$ in (24) is relevant only for $m_{A_{10}}$ very close to m_{obs} .

For $(m_{A_{10}}, m_{\eta_{b0}}) \sim m_{obs}$ (but $(m_{A_{10}}, m_{\eta_{b0}}) - m_{obs}$ larger than a few MeV such that the term $\sim \gamma^2$ can be neglected), eq. (24) can be simplified further with the result

$$(\delta m_1^2)^2 \simeq 4m_{obs}^2 (m_{A_{10}} - m_{obs})(m_{\eta_{b0}} - m_{obs}) \quad (27)$$

and hence, from (21),

$$X_d \simeq 125 \sqrt{(m_{A_{10}} - m_{obs})(m_{\eta_{b0}} - m_{obs})} \text{ GeV}^{-1} . \quad (28)$$

To proceed further, we have to estimate $m_{\eta_{b0}}$. Most of previous estimates for $m_{\eta_{b0}}$ correspond actually to $m_{\eta_{b0}} - m_{obs} > 0$ [47, 48], but subsequently we allow for

$$m_{\eta_{b0}} - m_{obs} = -30 \dots + 40 \text{ MeV} . \quad (29)$$

Next we have to treat the cases $m_{A_{10}} - m_{obs} > 0$ and $m_{A_{10}} - m_{obs} < 0$ separately. Starting with $m_{A_{10}} - m_{obs} < 0$, the maximally possible value for X_d from (28) is assumed for the lowest estimate of $m_{\eta_{b0}}$, with the result

$$X_d^{max}(m_{A_{10}}) \sim 22 \sqrt{m_{obs} - m_{A_{10}}} \text{ GeV}^{-1/2} . \quad (30)$$

For $m_{A_{10}} - m_{obs} > 0$, on the other hand, the maximally possible value for X_d is assumed for the largest estimate of $m_{\eta_{b0}}$, with the result

$$X_d^{max}(m_{A_{10}}) \sim 25 \sqrt{m_{A_{10}} - m_{obs}} \text{ GeV}^{-1/2} . \quad (31)$$

These analytic expressions for $X_d^{max}(m_{A_{10}})$ are fairly good approximations to the numerical upper bounds on $X_d(m_{A_{10}})$ which can be derived from (24) without the approximation (27), apart from the region where $|m_{A_{10}} - m_{obs}|$ is less than about 0.5 MeV (where (30) and (31) would imply $X_d^{max}(m_{A_{10}}) \rightarrow 0$). In fact, with $|\gamma| \sim m_{obs} \times 20$ MeV, one obtains $X_d^{max}(m_{A_{10}}) \sim 0.6$ for $|m_{A_{10}} - m_{obs}| \lesssim 0.5$ MeV (see Fig. 3 below).

We emphasize, however, that most previous estimates for $m_{\eta_{b0}}$ correspond to $m_{\eta_{b0}} - m_{obs} > 0$ [47, 48] in contrast to our more conservative assumption (29). These estimates can still be correct within the present framework, if an additional A_{10} state with $m_{A_{10}} - m_{obs} > 0$ exists, which mixes strongly with the η_{b0} , reducing the lower eigenvalue of the mass matrix (15). The induced mass shift $m_{\eta_{b0}} - m_{obs}$ can easily be derived from eq. (28):

$$m_{\eta_{b0}} - m_{obs} \simeq \frac{X_d^2 \times 1 \text{ GeV}^2}{1.56 \cdot 10^4 \cdot (m_{A_{10}} - m_{obs})} \quad (32)$$

which, for $m_{\eta_{b0}} - m_{obs} > 0$, would unwittingly be interpreted as an excess of the ‘‘observed’’ hyperfine splitting $m_{\Upsilon(1S)} - m_{obs}$. For instance, an induced mass shift of $m_{\eta_{b0}} - m_{obs} \sim 20$ MeV would be generated by a CP-odd Higgs with mass $m_{A_{10}}$ and a reduced coupling X_d satisfying $X_d \simeq 17.7 \times \sqrt{m_{A_{10}} - m_{obs}} \text{ GeV}^{-1/2}$ as, e.g., $X_d \simeq 12$ for $m_{A_{10}} \simeq 9.85$ GeV.

Note, however, that this mechanism would imply a heavier mass eigenvalue of the mixing matrix (15) is not too far above m_{obs} . This favours a heavy mass eigenvalue below 10.5 GeV, which not only satisfies LEP constraints but could even, as mentioned in the introduction, explain an observed excess of events at LEP.

6 Comparison of constraints from CLEO, BaBar, B physics and the muon anomalous magnetic moment

In addition to the constraints obtained in section 3 from CLEO and in section 5 from BaBar, the (m_{A_1}, X_d) -plane is already constrained by processes from B physics [26, 27] and the muon anomalous magnetic moment $(g - 2)_\mu$ [29]. (Upper limits on X_d have been derived by OPAL [49] from Yukawa production of a light neutral Higgs Boson at LEP, which seem more restrictive than the constraints from CLEO for $m_{A_1} \gtrsim 9.2$ GeV. We believe, however, that the $\eta_b(nS) - A_1$ mixing, which is relevant here, depends on an additional b - $b\eta_b$ form factor, where the initial b -quark is far off-shell. Since this effect has not been considered in [49], we will not consider the corresponding limits below.)

In the following, we will compare the different constraints in the (X_d, m_{A_1}) - and (X_d, M_A) -planes. (In this section, m_{A_1} is the CP-odd Higgs mass parameter denoted as $m_{A_{10}}$ in the mass matrix (15). However, the difference between $m_{A_{10}}$ and m_{A_1} would hardly be visible in the Figures below.)

For this purpose we have performed a scan over the NMSSM parameter space using the NMHDECAY program from the NMSSMTools package [33]. NMHDECAY allows to verify simultaneously the phenomenological constraints from SUSY searches, Higgs searches, B physics and $(g - 2)_\mu$. We have varied the NMSSM parameters (4) λ , κ , A_λ , A_κ , μ_{eff} , $\tan\beta$ (the latter between 1 and 50), as well as the SUSY breaking gaugino, squark and slepton masses and trilinear couplings, keeping only points where $m_{A_1} < 10.5$ GeV. Then we identified regions in the parameter space which are ruled out by the various phenomenological constraints for *any* choice of parameters. In particular, LEP constraints from Higgs searches require $\tan\beta \gtrsim 1.5$ in the NMSSM, while constraints from $(g - 2)_\mu$ lead to $\tan\beta \gtrsim 2$ for $m_{A_1} < 10.5$ GeV.

The various curves in the (X_d, m_{A_1}) -plane in Fig. 3 indicate lower bounds on X_d from various phenomenological constraints. We found that even for very large X_d there always exist parameter choices such that no region is *always* excluded by either the constraints from $\mathcal{B}(B \rightarrow X_s \gamma)$ or $\mathcal{B}(\bar{B}^+ \rightarrow \tau^+ \nu_\tau)$. However, constraints from $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ and ΔM_q , $q = d, s$ (shown as a green dashed line) always exclude a funnel for $m_{A_1} \sim M_{B_q} \sim 5.3$ GeV, the width of which depends on

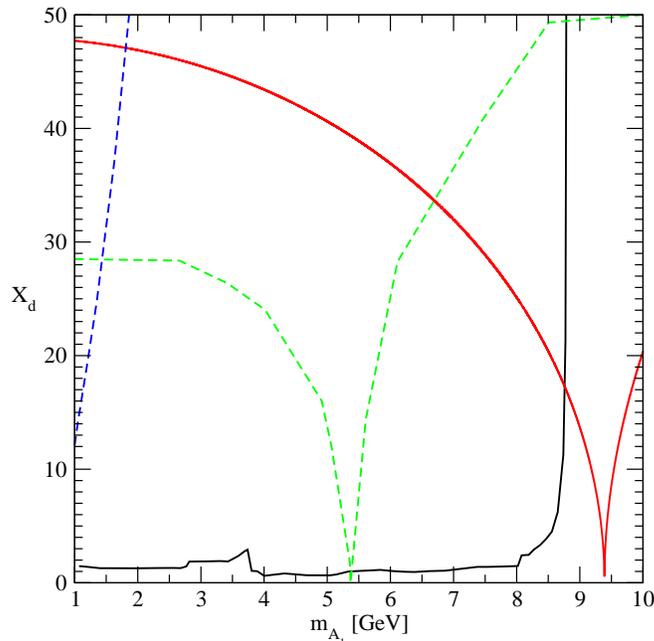


Figure 3: Upper bounds on X_d versus the A_1 mass for all parameters scanned over (see text). Indicated are constraints from $B_s \rightarrow \mu^+\mu^-$ and ΔM_q , $q = d, s$ as a green dashed line, constraints from $(g - 2)_\mu$ as a blue dashed line, the latest bounds from CLEO on $\mathcal{B}(\Upsilon \rightarrow \gamma\tau\tau)$ as a black line and constraints due to the measured $\eta_b(1S)$ mass by Babar as a red line.

the loop-induced $b - s - A_1$ coupling. This coupling being proportional to X_d , the excluded region broadens steadily with X_d as can be observed in Fig. 3, leading to the exclusion of all pseudoscalars with masses below ~ 6 GeV for $X_d \gtrsim 30$. However, the CLEO constraints indicated as a black line are much more restrictive, apart from a narrow window around $m_{A_1} \sim 5.3$ GeV.

Constraints from $(g - 2)_\mu$ originate from the contribution of a light pseudoscalar, which is enhanced by X_d^2 . For m_{A_1} below ~ 3 GeV, the pseudoscalar contribution has the opposite (negative) sign with respect to the deviation of the Standard Model prediction from the measured value of $(g - 2)_\mu$ [29]. This results in the exclusion of very light A_1 below ~ 2 GeV for large values of X_d , as indicated in Fig. 3 – a region which is now also covered by CLEO constraints.

Finally, constraints due to the measured $\eta_b(1S)$ mass by Babar as discussed in section 5 exclude a funnel around $m_{A_1} \sim 9.4$ GeV, which is outside the region covered by CLEO.

In the (X_d, M_A) -plane shown in Fig. 4 one sees that, as discussed qualitatively in section 2, large values of X_d can occur only for not too large values of M_A ; here this statement can be verified quantitatively. No region is generically excluded by the constraints from CLEO, Babar or $(g - 2)_\mu$, since m_{A_1} varies from 1 to 10.5 GeV for each point in this plane. The upper bounds on X_d from $B_s \rightarrow \mu^+\mu^-$ and ΔM_q , $q = d, s$ are indicated as in Fig. 3, and lower bounds on X_d from $\mathcal{B}(B \rightarrow X_s\gamma)$ now appear as well at small M_A . (There, the contribution to $\mathcal{B}(B \rightarrow X_s\gamma)$ from a charged Higgs with a mass $\sim M_A$ has to be compensated by a contribution $\sim X_d$ involving charginos or neutralinos, which requires a sufficiently large value for X_d .)

The most important results of this section are contained in Fig. 3, which shows that the combined present constraints rule out most of the region where $m_{A_1} \lesssim 8.5$ GeV – except if X_d is sufficiently small – whereas $m_{A_1} \sim 8.5 - 10.5$ GeV remains an interesting region in parameter space allowing for large X_d . The corresponding necessary values of M_A can be deduced from Fig. 4.

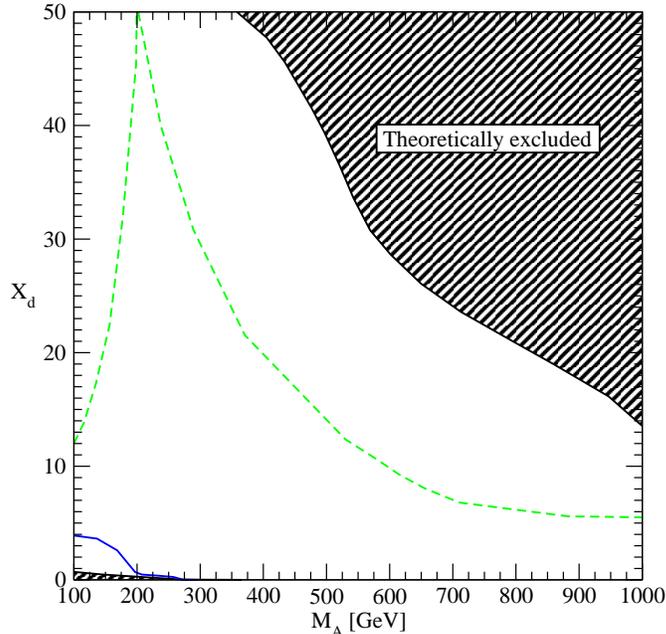


Figure 4: Bounds on X_d versus M_A for A_1 masses below 10.5 GeV. Indicated are upper bounds on X_d from $B_s \rightarrow \mu^+ \mu^-$ and ΔM_q , $q = d, s$ as a green dashed line, and lower bounds on X_d (for $m_A \lesssim 200$ GeV) from $\mathcal{B}(B \rightarrow X_s \gamma)$ as a blue line.

7 Possible lepton universality breaking

As pointed out in [13], one manifestation of the existence of a light CP-odd Higgs boson of mass around ~ 10 GeV could be a breakdown of LU (lepton universality) in Υ decays, if the (not necessarily soft) radiated photon escapes undetected in the experiment, or simply is not specifically searched for in the analysis of events. (The leptonic width is, in fact, an inclusive quantity with a sum over an infinite number of photons.) Higgs-mediated Υ decays would lead to an excess of its tauonic branching ratio (BR), which can be assessed through the ratio

$$\mathcal{R}_{\tau/\ell} = \frac{\mathcal{B}_{\tau\tau} - \mathcal{B}_{\ell\ell}}{\mathcal{B}_{\ell\ell}} = \frac{\mathcal{B}_{\tau\tau}}{\mathcal{B}_{\ell\ell}} - 1 \quad (33)$$

where $\mathcal{B}_{\tau\tau}$ denotes the tauonic, and $\mathcal{B}_{\ell\ell}$ the electronic ($\ell = e$) or muonic ($\ell = \mu$) branching ratios of the Υ resonance, respectively. A statistically significant non-zero value of $\mathcal{R}_{\tau/\ell}$ would be a strong argument in favour of a pseudoscalar Higgs boson mediating the process.

In Table 1 we summarize the current situation of LU obtained from [32]. As already mentioned in the introduction, a $\sim 1\sigma$ effect seems visible in most cases, leading to an overall (positive) $\sim 2\sigma$ effect.

Subsequently we intend to estimate the possible amount of LU breaking in the NMSSM with m_{A_1} in the 9 – 10.5 GeV range and large X_d in order to verify whether it can be assessed experimentally [17], i.e. whether it can be of the order of the few percent.

In principle, also pure SM channels may yield an apparent breaking of LU in Υ decays. In

	$\mathcal{B}(e^+e^-)$	$\mathcal{B}(\mu^+\mu^-)$	$\mathcal{B}(\tau^+\tau^-)$	$R_{\tau/e}(nS)$	$R_{\tau/\mu}(nS)$
$\Upsilon(1S)$	2.38 ± 0.11	2.48 ± 0.05	2.60 ± 0.10	0.09 ± 0.06	0.05 ± 0.04
$\Upsilon(2S)$	1.91 ± 0.16	1.93 ± 0.17	2.00 ± 0.21	0.05 ± 0.14	0.04 ± 0.06
$\Upsilon(3S)$	2.18 ± 0.21	2.18 ± 0.21	2.29 ± 0.30	0.05 ± 0.16	0.05 ± 0.16

Table 1: Measured leptonic branching ratios $\mathcal{B}(\Upsilon(nS) \rightarrow \ell\ell)$ (in %) and error bars (summed in quadrature) of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonances.

fact, all decays through an intermediate pseudoscalar (e.g. η_b) state should break LU due to the leptonic mass dependence of the amplitude as a consequence of helicity conservation. However, such processes (mediated by a two-photon loop or a Z^0 -boson in the SM) provide contributions to the branching ratios well below 1% and can be dropped. On the other hand, phase space should *suppress* the tauonic BR (by about 0.5%) with respect to the electronic and muonic modes.

In the absence of mixing between the Higgs boson A_1 and the η_b resonances, the relevant contribution to $\mathcal{R}_{\tau/\ell}$ would originate exclusively from the first diagram a) in Fig. 5. Assuming a BR($A_1 \rightarrow \tau^+\tau^-$) of 100% (see, however, below), $R_{\tau/\ell}$ would be given by the Wilczek formula (14):

$$\mathcal{R}_{\tau/\ell} = \frac{\mathcal{B}(\Upsilon(nS) \rightarrow \gamma A_1)}{\mathcal{B}(\Upsilon(nS) \rightarrow \mu^+\mu^-)} \equiv R_0 = \frac{G_F m_b^2 X_d^2}{\sqrt{2}\pi\alpha} \left(1 - \frac{m_{A_1}^2}{m_\Upsilon^2}\right) \times F. \quad (34)$$

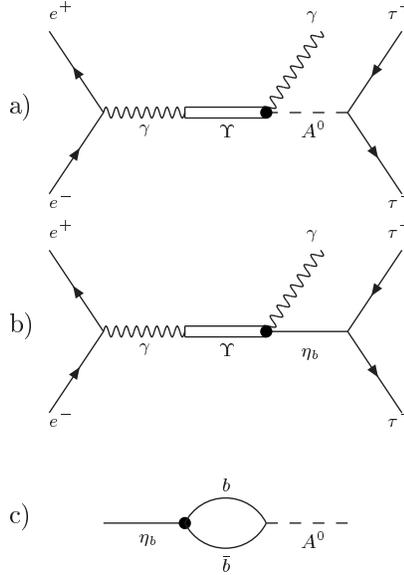


Figure 5: Process $e^+e^- \rightarrow \Upsilon \rightarrow \gamma \tau^+\tau^-$ with a) pseudoscalar Higgs, b) η_b (after mixing) as intermediate states; c) mixing diagram. Diagrams similar to a) and b) could be drawn for the two-gluon decay mode yielding hadrons in the final state instead of taus.

We recall that here we are interested in A_1 masses above 9 GeV, in which case the very conservative (small) estimate in section 3 of the correction factor F in (34) ceases to make sense. The aim of section 3 was to derive conservative upper bounds on X_d ; here, however, we aim at realistic estimates of $\mathcal{R}_{\tau/\ell}$. To this end, the tree-level expression (34) corrected by a constant factor $F = 1/2$

should provide an acceptable approximation [26,37]. Then, the CLEO bounds exclude large values of X_d for m_{A_1} up to roughly 9.2 GeV.

In the presence of mixing between the Higgs boson A_1 and one of the the η_b resonances as in section 4, both eigenstates defined in eqs. (22) would contribute to $\mathcal{R}_{\tau/\ell}$. In order to give the Υ branching ratios into these mixed states, it is convenient to define the ratio

$$S_0(n, n') \equiv \frac{\mathcal{B}(\Upsilon(nS) \rightarrow \gamma\eta_{b0}(n'S))}{\mathcal{B}(\Upsilon(nS) \rightarrow \mu^+\mu^-)}, \quad (35)$$

where the branching ratio for a M1 transition between $\Upsilon(nS)$ and $\eta_{b0}(n')$ states ($n' \leq n$) is given by [48]

$$\mathcal{B}(\Upsilon(nS) \rightarrow \gamma\eta_{b0}(n'S)) = \frac{16\alpha}{3} \left(\frac{Q_b}{2m_b} \right)^2 \frac{I_{n'n}^2 \cdot k^3}{\Gamma_\Upsilon}. \quad (36)$$

k is the photon energy (depending on the mass difference $m_{\Upsilon(nS)} - m_{\eta_b(n'S)}$); $I_{n'n}$ denotes the final and initial wave functions overlap, $I_{n'n} = \langle f_{n'} | j_0(kr/2) | i_n \rangle$, where j_0 is a spherical Bessel function. $I_{n'n}$ is numerically close to unity for favoured transitions ($n = n'$) but much smaller for hindered ($n \neq n'$) transitions. As stressed in [48], however, the considerably larger photon energy k in the latter case could compensate this reduction, leading to competitive transition probabilities. Below we set $I_{12} = 0.057$ [48] and $I_{13} = 0.017$. (The latter value is required in order to reproduce the experimental value $\mathcal{B}(\Upsilon(3S) \rightarrow \gamma \text{ hadrons}) = 4.8 \times 10^{-4}$ found by BaBar [31].)

In terms of R_0 and S_0 defined above, the Υ branching ratios into the mixed states A_1 and $\eta_b(n'S)$ are given by (neglecting interference terms, and normalized w.r.t. the branching ratios into $l^+l^- = \mu^+\mu^-$ or e^+e^-)

$$\begin{aligned} \frac{\mathcal{B}(\Upsilon(nS) \rightarrow \gamma A_1)}{\mathcal{B}(\Upsilon(nS) \rightarrow l^+l^-)} &= \cos^2 \alpha_k R_0 + \sin^2 \alpha_k S_0(n, k) \\ \frac{\mathcal{B}(\Upsilon(nS) \rightarrow \gamma\eta_b(n'S))}{\mathcal{B}(\Upsilon(nS) \rightarrow l^+l^-)} &= \sin^2 \alpha_{n'} R_0 + \cos^2 \alpha_{n'} S_0(n, n') \end{aligned} \quad (37)$$

where, as stated above, we assume that at most one possible mixing angle α_k is nonvanishing. (For a given value for $m_{A_{10}}$, the index k is given by the state $\eta_{b0}(kS)$ whose mass is closest to $m_{A_{10}}$. The Υ decays into the remaining unmixed $\eta_b(n'S)$ states with $n' \neq k$ are still described by eq. (36).)

Next we assume that the mixed states A_1 and $\eta_b(nS)$ decay into $\tau^+\tau^-$ only via their A_{10} component. Then we obtain, using eq. (23) for the full widths of the mixed states,

$$\begin{aligned} \mathcal{B}(A_1 \rightarrow \tau^+\tau^-) &= \mathcal{B}(A_{10} \rightarrow \tau^+\tau^-) \times \frac{\cos^2 \alpha_k \Gamma_{A_{10}}}{\cos^2 \alpha_k \Gamma_{A_{10}} + \sin^2 \alpha_k \Gamma_{\eta_{b0}(kS)}}, \\ \mathcal{B}(\eta_b(nS) \rightarrow \tau^+\tau^-) &= \mathcal{B}(A_{10} \rightarrow \tau^+\tau^-) \times \frac{\sin^2 \alpha_n \Gamma_{A_{10}}}{\cos^2 \alpha_n \Gamma_{\eta_{b0}(nS)} + \sin^2 \alpha_n \Gamma_{A_{10}}}, \end{aligned} \quad (38)$$

i.e. $\mathcal{B}(\eta_b(nS) \rightarrow \tau^+\tau^-)$ vanishes for $n \neq k$. Finally we obtain for $\mathcal{R}_{\tau/\ell}$ (for a given $\Upsilon(nS)$, and assuming $\mathcal{B}(A_{10} \rightarrow \tau^+\tau^-) = 90\%$)

$$\begin{aligned} \mathcal{R}_{\tau/\ell} &= \mathcal{R}_{\tau/\ell}^{A_1} + \mathcal{R}_{\tau/\ell}^{\eta_b} \\ &\equiv \frac{\mathcal{B}(\Upsilon(nS) \rightarrow \gamma A_1)}{\mathcal{B}(\Upsilon(nS) \rightarrow l^+l^-)} \times \mathcal{B}(A_1 \rightarrow \tau^+\tau^-) + \frac{\mathcal{B}(\Upsilon(nS) \rightarrow \gamma\eta_b(kS))}{\mathcal{B}(\Upsilon(nS) \rightarrow l^+l^-)} \times \mathcal{B}(\eta_b(kS) \rightarrow \tau^+\tau^-) \\ &= 0.9 (\cos^2 \alpha_k R_0 + \sin^2 \alpha_k S_0(n, k)) \times \frac{\cos^2 \alpha_k \Gamma_{A_{10}}}{\cos^2 \alpha_k \Gamma_{A_{10}} + \sin^2 \alpha_k \Gamma_{\eta_{b0}(kS)}} \end{aligned}$$

$$+ 0.9 (\sin^2 \alpha_k R_0 + \cos^2 \alpha_k S_0(n, k)) \times \frac{\sin^2 \alpha_k \Gamma_{A_{10}}}{\cos^2 \alpha_k \Gamma_{\eta_{b0}(kS)} + \sin^2 \alpha_k \Gamma_{A_{10}}} \quad (39)$$

where either $\alpha_k = 0$ (if none of the states $\eta_b(kS)$ mixes with A_{10}), or α_k is given by the (supposedly only) mixing angle, whose choice and value depend on $m_{A_{10}}$.

In the set of plots of Fig. 6, $\mathcal{R}_{\tau/\ell}$ is shown for $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$, respectively, as a function of $m_{A_{10}}$, using the formulas of section 4 for the determination of the relevant mixing angle. We assume tentatively⁴ $\Gamma_{\eta_{b0}(1S)} = \Gamma_{\eta_{b0}(2S)} = \Gamma_{\eta_{b0}(3S)} = 5$ MeV and $X_d = 12$. (As argued in section 5, values for $m_{A_1} \sim 9.4 \pm 0.2$ GeV are actually ruled out for this value of X_d . However, for lower values of X_d – leading to correspondingly lower values for $\mathcal{R}_{\tau/\ell}$ – the forbidden window for m_{A_1} becomes smaller. Moreover, phenomenologically interesting values for $m_{A_1} \gtrsim 9.6$ GeV as discussed at the end of section 5, are seen to generate interesting values for $\mathcal{R}_{\tau/\ell}$.)

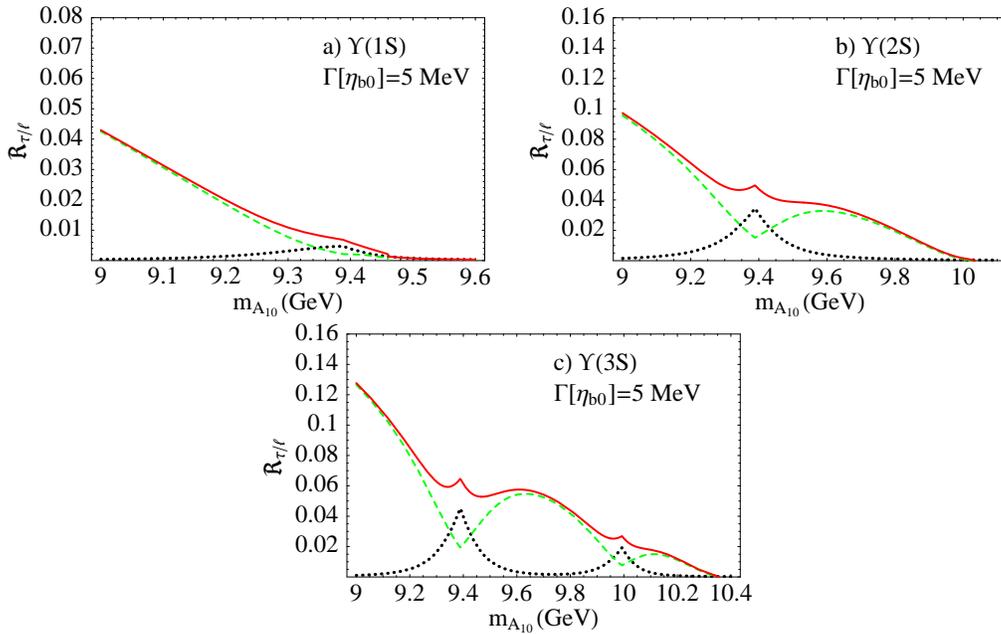


Figure 6: $\mathcal{R}_{\tau/\ell}$ versus the pseudoscalar Higgs mass for a) $\Upsilon(1S)$, b) $\Upsilon(2S)$, and c) $\Upsilon(3S)$ decays using $X_d = 12$, $m_{\eta_{b0}(1S)} = 9.389$ GeV [31], assuming $m_{\eta_{b0}(2S,3S)} = 9.997, 10.32$ GeV respectively, and $\Gamma_{\eta_{b0}(1S,2S,3S)} = 5$ MeV. The contributions from $\mathcal{R}_{\tau/\ell}^{A_1}$ are indicated as dashed green lines, the contributions from $\mathcal{R}_{\tau/\ell}^{\eta_b}$ as dotted black lines, and their sum $\mathcal{R}_{\tau/\ell}$ as solid red lines. Larger (smaller) values of X_d obviously yield higher (lower) values for $R_{\tau/\ell}$.

The contributions from $\mathcal{R}_{\tau/\ell}^{\eta_b}$ (dotted black line) yield the expected bumps around the respective η_b mass values, where the mixing angle becomes large. Conversely, the contributions from $\mathcal{R}_{\tau/\ell}^{A_1}$ (dashed green line) show dips at both $m_{A_1} = m_{\eta_b(1S)} = 9.389$ GeV and $m_{A_1} = m_{\eta_b(2S)} \simeq 10$ GeV, since they become reduced by the mixing. (The expected peak or dip at $m_{A_1} = m_{\eta_b(2S)} \simeq 10.3$ GeV in Fig. 6c) is in fact invisibly small.)

⁴The following expression (asymptotically valid for very heavy quark masses) can be used to estimate the $\eta_b(nS)$ full width from the experimentally known $\eta_c(nS)$ full width: $\Gamma_{\eta_b(nS)}/\Gamma_{\eta_c(nS)} \simeq (m_b/m_c)[\alpha_s(2m_b)/\alpha_s(2m_c)]^5$ [50]. The fifth power of the α_s ratio yields however a large uncertainty to the prediction. On the other hand, theoretical predictions based on the expected ratio of the two-photon and two-gluon widths range from 4 to 20 MeV [51]

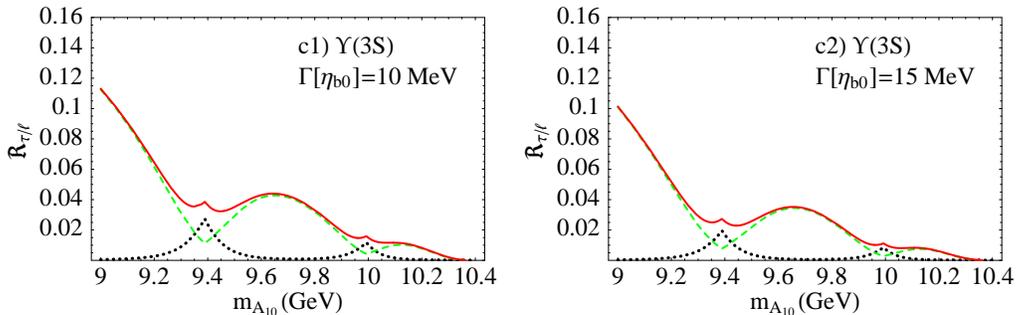


Figure 7: $R_{\tau/\ell}$ versus the pseudoscalar Higgs mass for $\Upsilon(3S)$ decays using $X_d = 12$ and c1) $\Gamma_{\eta_{b0}} = 10$ MeV, c2) $\Gamma_{\eta_{b0}} = 15$ MeV, respectively.

The higher values of $\mathcal{R}_{\tau/\ell}$ and the higher reach in $m_{A_{10}}$ obtained for the $\Upsilon(2S)$ and $\Upsilon(3S)$ (due to the dominant Wilczek mechanism of Fig. 5a)) allow us to conclude that radiative decays of the latter resonances look more promising than the $\Upsilon(1S)$ decays, allowing for the experimental observation of LU breaking (at the few percent level) at a B factory. This result is important for future tests of LU [19].

In order to study the effect of our assumption on the width $\Gamma_{\eta_{b0}}$, we present in the set of Fig. 7 $\mathcal{R}_{\tau/\ell}$ for the $\Upsilon(3S)$ resonance setting $\Gamma_{\eta_{b0}} = 10$ and 15 MeV, respectively. One can observe a slight overall decrease of $\mathcal{R}_{\tau/\ell}$ for larger $\Gamma_{\eta_{b0}}$, as expected from eqs. (38) and (39).

Concerning future measurements, we assume tentatively that a combined statistical and systematic error (summed in quadrature) of 2% for $R_{\tau/\ell}$ is achievable at a (Super) B factory. Fig. 8 shows the foreseen 2σ (95% CL) limits (green region) for testing LU using $\Upsilon(3S)$ decays for a A_1 mass ranging in the interval 9–10.3 GeV.

An observation of lepton universality breaking in Υ decays should lead to a careful search for the quasi-monochromatic photons shown in Figs. 5 in a sample of events firstly selected and enriched using 1-prong tauonic decays and requiring missing energy (neutrinos). Let us recall that there could be two nearby peaks corresponding to two physical eigenstates. However, large A_1 or η_b widths might invalidate this search method: if the m_{A_1} and m_{η_b} masses were not too different (i.e. less than 50 MeV), the two peaks might not be resolved experimentally but yield a broader peak than expected. This conventional search has been unsuccessful so far in $\Upsilon(1S)$ decays, but can (should) be extended to the yet unexplored radiative decays of the $\Upsilon(2S, 3S)$ resonance into τ 's, according to the proposal in [15, 17].

Finally, a light CP-odd Higgs which mixes with one of the η_b states would also decay hadronically and eventually be visible by requiring four or more charged tracks together with a photon in radiative $\Upsilon(2S, 3S)$ decays, a criterium used by Babar [31] for their discovery of the $\eta_b(1S)$. Whereas the Babar data can possibly be used to put bounds on such an additional state for certain ranges of its mass, we believe that a serious search for such an additional state in the inclusive photon spectrum from radiative Υ decays would require a detailed treatment of the various background contributions, which is beyond the scope of the present paper. Corresponding investigations are clearly another interesting task in the future.

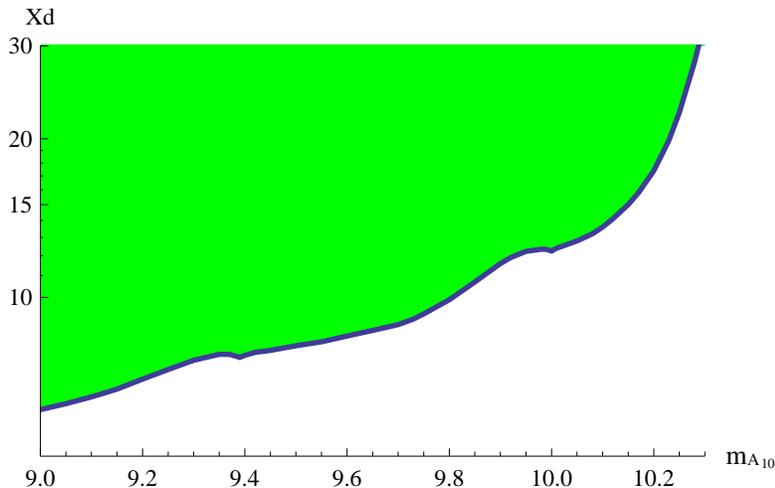


Figure 8: Expected 2σ signal (green area) in the mass range 9 – 10.35 GeV of the CP-odd Higgs A_1 , assuming a total error of 2% for $R_{\tau/\ell}$.

8 Conclusions and outlook

In this paper we have summarized constraints from and perspectives of various processes related to a CP-odd Higgs boson with a mass m_{A_1} below the $B\bar{B}$ threshold of 10.5 GeV. Apart from m_{A_1} , these phenomena depend essentially on its reduced coupling X_d to b -quarks. Within the parameter space of the NMSSM, relatively large values of X_d are possible (for sufficiently large values of $\tan\beta$).

We have compared present constraints on the $m_{A_1} - X_d$ plane from B meson physics, the anomalous magnetic moment of the muon, LEP, and recent results from CLEO and Babar. In spite of the conservative approach towards the bound state corrections to the Wilczek formula the most stringent constraints originate – not astonishingly – from the dedicated (negative) searches by CLEO for $m_{A_1} \lesssim 8.8$ GeV allowing, however, for substantial values of X_d provided that $8.8 \text{ GeV} \lesssim m_{A_1} \lesssim 10.5 \text{ GeV}$.

Given the possible explanation of the 2.3σ excess in searches for a CP-even Higgs boson at LEP [6], this allowed mass range is of particular interest. We emphasize again that the interval $9.4 \text{ GeV} \lesssim m_{A_1} \lesssim 10.5 \text{ GeV}$ can also have an effect on the $\eta_b(1S)$ mass as measured by Babar via mixing, and explain the possibly excessive $\Upsilon(1S) - \eta_b(1S)$ hyperfine splitting.

Such a scenario can and should be tested at presently running B factories, and/or a future Super B factory. Obvious search strategies consist – as already performed – in radiative $\Upsilon(nS)$ decays into both tauonic and hadronic final states, keeping an eye on possible close (but separate) peaks in the photon spectrum. In addition, violation of lepton universality in inclusive radiative Υ decays can be a signal for an additional CP-odd Higgs. We have clarified that corresponding visible signals are well within the reach of future precision experiments.

These searches are complementary to Higgs boson searches at colliders (like the LHC) where it is quite doubtful at present whether a light CP-odd Higgs decaying dominantly into $\tau^+\tau^-$ could be seen. On the contrary, such a CP-odd Higgs can render searches for the lightest CP-even Higgs boson h very difficult, if it decays dominantly into $h \rightarrow A_1 A_1 \rightarrow \tau^+\tau^-\tau^+\tau^-$. Hence, searches at B factories are possibly our only windows into the light Higgs sector, if such a scenario is realized.

Acknowledgments

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References

- [1] P. Fayet, Nucl. Phys. B **90** (1975) 104; Phys. Lett. B **64** (1976) 159; Phys. Lett. B **69** (1977) 489 and Phys. Lett. B **84** (1979) 416; H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120** (1983) 346; J.M. Frere, D.R. Jones and S. Raby, Nucl. Phys. B **222** (1983) 11; J.P. Derendinger and C.A. Savoy, Nucl. Phys. B **237** (1984) 307; A.I. Veselov, M.I. Vysotsky and K.A. Ter-Martirosian, Sov. Phys. JETP **63** (1986) 489; J.R. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D **39** (1989) 844; M. Drees, Int. J. Mod. Phys. A **4** (1989) 3635.
- [2] J.E. Kim and H.P. Nilles, Phys. Lett. B **138** (1984) 150.
- [3] B. A. Dobrescu, G. Landsberg and K. T. Matchev, Phys. Rev. D **63** (2001) 075003 [arXiv:hep-ph/0005308];
B. A. Dobrescu and K. T. Matchev, JHEP **0009** (2000) 031 [arXiv:hep-ph/0008192].
- [4] U. Ellwanger, J. F. Gunion, C. Hugonie and S. Moretti, arXiv:hep-ph/0305109 (in “Physics interplay of the LHC and the ILC”, G. Weiglein *et al.* [LHC/LC Study Group], Phys. Rept. **426** (2006) 47);
U. Ellwanger, J. F. Gunion, C. Hugonie and S. Moretti, arXiv:hep-ph/0401228 (in “The Higgs working group: Summary report 2003”, K. A. Assamagan *et al.* [Higgs Working Group Collaboration], arXiv:hep-ph/0406152).
- [5] R. Dermisek and J. F. Gunion, Phys. Rev. Lett. **95** (2005) 041801 [arXiv:hep-ph/0502105].
- [6] R. Dermisek and J. F. Gunion, Phys. Rev. D **73** (2006) 111701 [arXiv:hep-ph/0510322].
- [7] R. Dermisek and J. F. Gunion, Phys. Rev. D **75** (2007) 075019 [arXiv:hep-ph/0611142].
- [8] U. Ellwanger, J. F. Gunion and C. Hugonie, JHEP **0507** (2005) 041 [arXiv:hep-ph/0503203];
S. Chang, P. J. Fox and N. Weiner, JHEP **0608** (2006) 068 [arXiv:hep-ph/0511250];
P. W. Graham, A. Pierce and J. G. Wacker, “Four taus at the Tevatron,” arXiv:hep-ph/0605162;
S. Moretti, S. Munir and P. Poulose, Phys. Lett. B **644** (2007) 241 [arXiv:hep-ph/0608233];
S. Chang, P. J. Fox and N. Weiner, Phys. Rev. Lett. **98** (2007) 111802 [arXiv:hep-ph/0608310];
T. Stelzer, S. Wiesenfeldt and S. Willenbrock, Phys. Rev. D **75** (2007) 077701 [arXiv:hep-ph/0611242];
U. Aglietti *et al.*, “Tevatron-for-LHC report: Higgs,” arXiv:hep-ph/0612172;
K. Cheung, J. Song and Q. S. Yan, Phys. Rev. Lett. **99** (2007) 031801 [arXiv:hep-ph/0703149].
- [9] R. Dermisek and J. F. Gunion, Phys. Rev. D **76** (2007) 095006 [arXiv:0705.4387 [hep-ph]].

- [10] M. Carena, T. Han, G. Y. Huang and C. E. M. Wagner, *JHEP* **0804** (2008) 092 [arXiv:0712.2466 [hep-ph]];
 J.R. Forshaw, J.F. Gunion, L. Hodgkinson, A. Papaefstathiou and A.D. Pilkington, *JHEP* **0804** (2008) 090 [arXiv:0712.3510 [hep-ph]];
 A. Djouadi *et al.*, *JHEP* **0807** (2008) 002 [arXiv:0801.4321 [hep-ph]];
 S. Chang, R. Dermisek, J. F. Gunion and N. Weiner, “Nonstandard Higgs Boson Decays,” arXiv:0801.4554 [hep-ph];
 A. Belyaev *et al.*, “The Scope of the 4 tau Channel in Higgs-strahlung and Vector Boson Fusion for the NMSSM No-Lose Theorem at the LHC,” arXiv:0805.3505 [hep-ph].
- [11] S. Schael *et al.* [ALEPH Collaboration], *Eur. Phys. J. C* **47** (2006) 547.
- [12] M. Drees and K.-I. Hikasa, *Phys. Rev. D* **41** (1990) 1547.
- [13] M. A. Sanchis-Lozano, *Mod. Phys. Lett. A* **17** (2002) 2265 [arXiv:hep-ph/0206156];
 M. A. Sanchis-Lozano, *Int. J. Mod. Phys. A* **19** (2004) 2183 [arXiv:hep-ph/0307313];
 M. A. Sanchis-Lozano, *PoS HEP2005* (2006) 334 [arXiv:hep-ph/0510374].
- [14] B. McElrath, *Phys. Rev. D* **72**, 103508 (2005) [arXiv:hep-ph/0506151].
- [15] M. A. Sanchis-Lozano, *J. Phys. Soc. Jap.* **76** (2007) 044101 [arXiv:hep-ph/0610046].
- [16] R. Dermisek, J. F. Gunion and B. McElrath, *Phys. Rev. D* **76** (2007) 051105 [arXiv:hep-ph/0612031].
- [17] E. Fullana and M. A. Sanchis-Lozano, *Phys. Lett. B* **653** (2007) 67 [arXiv:hep-ph/0702190].
- [18] R. N. Hodgkinson, *Phys. Lett. B* **665** (2008) 219 [arXiv:0802.3197 [hep-ph]].
- [19] M. Bona *et al.*, “SuperB: A High-Luminosity Asymmetric e^+e^- Super Flavor Factory. Conceptual Design Report,” arXiv:0709.0451 [hep-ex].
- [20] M. Carena, J. R. Ellis, S. Mrenna, A. Pilaftsis and C. E. M. Wagner, *Nucl. Phys. B* **659** (2003) 145 [arXiv:hep-ph/0211467].
- [21] J. S. Lee and S. Scopel, *Phys. Rev. D* **75** (2007) 075001 [arXiv:hep-ph/0701221].
- [22] T. Han, P. Langacker and B. McElrath, *Phys. Rev. D* **70** (2004) 115006 [arXiv:hep-ph/0405244].
- [23] S. Kraml *et al.*, “Workshop on CP studies and non-standard Higgs physics,” arXiv:hep-ph/0608079.
- [24] W. Love *et al.* [CLEO Collaboration], “Search for Light CP-odd Higgs in Radiative Decays of Upsilon(1S),” arXiv:0807.1427 [hep-ex].
- [25] J. F. Gunion, “A light CP-odd Higgs boson and the muon anomalous magnetic moment,” arXiv:0808.2509 [hep-ph].
- [26] G. Hiller, *Phys. Rev. D* **70**, 034018 (2004) [arXiv:hep-ph/0404220].
- [27] F. Domingo and U. Ellwanger, *JHEP* **0712** (2007) 090 [arXiv:0710.3714 [hep-ph]].

- [28] Z. Heng, R. J. Oakes, W. Wang, Z. Xiong and J. M. Yang, Phys. Rev. D **77** (2008) 095012 [arXiv:0801.1169 [hep-ph]].
- [29] M. Krawczyk, Acta Phys. Polon. B **33** (2002) 2621 [arXiv:hep-ph/0208076];
F. Domingo and U. Ellwanger, JHEP **0807** (2008) 079 [arXiv:0806.0733 [hep-ph]].
- [30] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Perseus Publishing, Cambridge (US), MA, 1990).
- [31] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **101** (2008) 071801 [arXiv:0807.1086 [hep-ex]].
- [32] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.
- [33] U. Ellwanger, J. F. Gunion and C. Hugonie, JHEP **0502** (2005) 066 [arXiv:hep-ph/0406215];
U. Ellwanger and C. Hugonie, Comput. Phys. Commun. **177** (2007) 399,
<http://www.th.u-psud.fr/NMHDECAY/nmssmtools.html>
- [34] R. N. Hodgkinson and A. Pilaftsis, Phys. Rev. D **76** (2007) 015007 [arXiv:hep-ph/0612188].
- [35] B. Ananthanarayan and P. N. Pandita, Int. J. Mod. Phys. A **12** (1997) 2321 [arXiv:hep-ph/9601372].
- [36] F. Wilczek, Phys. Rev. Lett. **40** (1978) 279.
- [37] H. E. Haber, A. S. Schwarz and A. E. Snyder, Nucl. Phys. B **294** (1987) 301.
- [38] J. Polchinski, S. R. Sharpe and T. Barnes, Phys. Lett. B **148** (1984) 493.
- [39] J. T. Pantaleone, M. E. Peskin and S. H. H. Tye, Phys. Lett. B **149** (1984) 225.
- [40] W. Bernreuther and W. Wetzel, Z. Phys. C **30** (1986) 421.
- [41] M. I. Vysotsky, Phys. Lett. B **97** (1980) 159.
- [42] P. Nason, Phys. Lett. B **175** (1986) 223.
- [43] I. G. Aznaurian, S. G. Grigorian and S. G. Matinyan, JETP Lett. **43** (1986) 646.
- [44] D. McKeen, "Constraining Light Bosons with Radiative $\Upsilon(1S)$ Decays," arXiv:0809.4787 [hep-ph].
- [45] E. J. Eichten and C. Quigg, Phys. Rev. D **66** (2002) 010001 [arXiv:hep-ph/9503356].
- [46] M. A. Sanchis-Lozano, Nucl. Phys. Proc. Suppl. **142** (2005) 163 [arXiv:hep-ph/0407320].
- [47] N. Brambilla *et al.* [Quarkonium Working Group], "Heavy quarkonium physics," arXiv:hep-ph/0412158.
- [48] S. Godfrey and J. L. Rosner, Phys. Rev. D **64** (2001) 074011 [Erratum-ibid. D **65** (2002) 039901].
- [49] G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. C **23** (2002) 397 [arXiv:hep-ex/0111010].

- [50] A. Le Yaouanc et al., *Hadron transitions in the quark model*, (Gordon and Breach Science Publishers, 1988).
- [51] C. S. Kim, T. Lee and G. L. Wang, Phys. Lett. B **606** (2005) 323 [arXiv:hep-ph/0411075].