

Weak decays of doubly heavy hadrons *

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Abstract

We explore the application and usefulness of the heavy quark symmetry to describe the weak decays of hadrons (mesons and baryons) containing two heavy quarks. Firstly, we address the internal dynamics of a heavy-heavy bound system with the help of estimates based on potential models, showing an approximate spin symmetry in the preasymptotic quark mass region including charmonium, bottonium and B_c meson states. However, no asymptotic spin symmetry should hold in the infinite quark mass limit in contrast to singly heavy hadrons. Predictions on semileptonic and two-body nonleptonic decays of B_c mesons are shown. Furthermore, the stemming flavor and spin symmetries from the interaction between the heavy and light components in hadrons (combining in a “superflavor” symmetry) permit their classification in *meson*-type supermultiplets containing singly heavy mesons together with doubly heavy baryons, and *baryon*-type supermultiplets containing singly heavy baryons together with some exotic doubly heavy multiquark states (diquonia). Exploiting their well-defined transformation properties under the superflavor symmetry group, we get predictions on the widths for some semileptonic and two-body nonleptonic decays of baryons containing both b and c quarks.

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1 Introduction

History of Science teaches us that the discovery of new particles, far from complicating our world's description, opens new trends and offers a previously unknown insight into a more general understanding of the laws of Nature. In fact, a paradigmatic example in chemistry is the elements' periodic table whose full significance was not apparent until the development of the knowledge on the subatomic structure of matter and the birth of quantum mechanics. Analogously in elementary particle physics, the appearance of new (though shortly living) particle generations suggests or confirms new symmetries of matter and its fundamental interactions. Strange, charmed and beautiful hadrons have been good examples of it in the past and certainly still will be in the future.

The existence of doubly heavy mesons and baryons (*i.e.* containing any pair of heavy quarks either c or b) can be viewed as a necessary consequence of the attractive strong interaction among quarks. So far, however, there has been no real possibility to produce them at an observable rate with existing accelerators, leaving aside charmonium and bottomonium resonance families.

Hopefully, this situation should definitely change with the advent of high-energy, high-luminosity hadron colliders like Tevatron and mainly LHC. The two by now accepted experiments at LHC (ATLAS and CMS) include B-physics in their physical objectives and the feasibility of the detection of doubly heavy hadrons is being currently addressed [1] [2] [3]. Moreover, a third high-precision experiment at LHC dedicated to the study of Beauty particles could be finally approved.

Recently the hadroproduction of heavy quarks in general, and of doubly heavy mesons and baryons in particular, has attracted a lot of theoretical work [4]. Conversely, no such attention has been yet devoted to the analysis of the subsequent weak decay of these particles especially in the case of baryons, with the exceptions [5] [6] [7] to our knowledge.

At this stage, the purpose of this work is to make estimates on decay rates of doubly heavy hadrons in order to conduct an experimental search for their observation in a not so far future. We have focused, in particular, on initial state hadrons containing one b quark and one c quark. Indeed, in the case of mesons it represents the only possibility since the top quark would not form a bound state due to its large mass while for baryons the reason is an experimental one [1]. We have tried to base our calculations on model-independent assumptions reducing whenever possible the input parameters within the framework of the approximations used in this work. Our final results, though presumably reasonable, ought to be viewed as a starting point for a more detailed treatment, as well as a checking limit of more exact derivations.

This paper is organized as follows. In Section 2 we discuss the (partial) application of the heavy quark effective theory (HQET), primarily developed for heavy-light (h-l) systems, to the internal dynamics of bound states of two heavy quarks. In particular, the appearance of a spin symmetry at leading order (already pointed out elsewhere [8]-[12]) can be exploited in the analysis of the weak decays of the B_c mesons. In Section 3 we also study some weak decay channels of baryons with b and c quarks by means of a "superflavor" symmetry, obtaining predictions on decay widths and branching fractions whose numerical values are the first so far computed, to our knowledge.

2 Effective Lagrangians for heavy quarks

In recent times, several attempts have appeared to describe the heavy quark strong dynamics from more general grounds than particular models by simplifying the original QCD Lagrangian into a more manageable form. On the one hand, the non-relativistic QCD (NRQCD) is a systematic approach to the dynamics of heavy systems as increasing powers of the quark velocity inside the hadron [13]. Once reformulated on a discrete space-time lattice, it constitutes nowadays an extremely active field of research [14].

On the other hand, the last few years have witnessed a rapid progress in a new low-energy effective theory of QCD appropriate when dealing with hadrons consisting of one heavy component along with light degrees of freedom. Starting from the original Lagrangian $\mathcal{L}_{QCD} = \bar{Q}(i\not{D} - m_Q)Q$, HQET can be formulated [15] according to the following field redefinitions which remove the large part of the heavy quark momentum:

$$h_v(x) = e^{im_Q v \cdot x} P_+ Q(x) \quad (1)$$

$$H_v(x) = e^{im_Q v \cdot x} P_- Q(x) \quad (2)$$

where $P_{\pm} = (1 \pm \not{v})/2$ denote the usual projectors onto positive/negative energy states and h_v and H_v stand for the “large” and “small” components of the spinor field. In order to deal with hadrons containing a heavy antiquark, one must introduce the corresponding fields h_v^- and H_v^- [15].

The degrees of freedom corresponding to H_v can be eliminated by means of the equations of motion of QCD yielding a non-local effective Lagrangian. Then, a $1/m_Q$ power expansion in terms of local operators is achieved leading to

$$\mathcal{L}_{\{\}}^{(i)} = \bar{h}_v^{(i)} i v \cdot D h_v^{(i)} + \frac{1}{2m_{Q_i}} [\mathcal{K}^{(\cdot)} + Z(m_{Q_i}/\mu) \mathcal{S}^{(i)}] + \mathcal{O}(1/m_{Q_i}^2) \quad (3)$$

where the superscript i labels the heavy flavor and those terms including

$$\mathcal{K}^{(\cdot)} = \bar{h}_v^{(i)} (iD_{\perp})^2 h_v^{(i)} \quad (4)$$

$$\mathcal{S}^{(\cdot)} = \frac{g_s}{2} \bar{h}_v^{(i)} \sigma^{\alpha\beta} G_{\alpha\beta} h_v^{(i)} \quad (5)$$

correspond to the (gauged) kinetic energy of the quark i in the hadron’s rest frame, and the analog of the Pauli term describing the chromomagnetic interaction of the quark i with the light degrees of freedom. D_{\perp} denotes the transverse covariant derivative, $D_{\perp} = D - v \cdot D$. The renormalization coefficient Z takes into account the short-distance physics in the chromomagnetic interaction [15].

The leading contribution in the Lagrangian of eq. (3) represents a static quark field, possessing two types of new symmetries not present in the original theory. In fact, the symmetry of this effective theory is larger than merely the product of the flavor and spin ones for the fermionic heavy quark fields, even extending to systems in which the massive colored particles have different spins. This issue will be discussed in more detail in Section 2.2 and used in Section 3.

2.1 Doubly heavy versus singly heavy hadronic systems

In a weakly bound state of two heavy quarks such as charmonium or bottomonium, the residual momentum k (defined as the quark’s momentum measured in the hadron’s rest

frame) neither should be too large on the average, amounting to a relative small off-shellness of the constituents [16].

Indeed, according to a large variety of potential models $\langle k/m_Q \rangle$ (directly related to the average internal quark velocity v' in non-relativistic motion) scales down with some power of the quark mass, *i.e.* $\langle k/m_Q \rangle \sim m_Q^{-\nu}$ where ν takes a value near 0.5 for a logarithmic-like potential, or near 0.34 for a ‘‘Cornell-type’’ potential [17]. This behavior must be understood limited to a certain mass region becoming inappropriate in a dominant short-distance regime.

Motivated by those arguments, we shall insist in applying the inverse heavy quark expansion generated by the field redefinitions (1) and (2) in order to get an effective non-relativistic Lagrangian to describe heavy-heavy (h-h) bound systems. As we shall see, important limitations of the low-energy effective theory will appear with regard to h-l systems, in particular the lack of flavor symmetry even at leading order whereas a spin symmetry will hold in a certain preasymptotic mass domain.

In h-h systems the internal motion of the heavy constituents cannot be neglected even at leading order. This can be viewed as a consequence of the delicate balance between the average potential and kinetic energies in the bound state, as pointed out a long time ago in terms of the virial theorem [18]. Thereby, one must keep the kinetic piece even at lowest order yielding a Schrödinger-type Lagrangian plus the Pauli term

$$\mathcal{L}_{\{\{\}}^{(\prime)}}^{(\prime)} = \mathcal{L}_i^{(\prime)} + \frac{1}{2m_{Q_i}} Z(m_{Q_i}/\mu) \mathcal{S}^{(\prime)} + \mathcal{O}(1/m_{Q_i}^2) \quad (6)$$

where

$$\mathcal{L}_i^{(\prime)} = \bar{h}_v^{(i)} i v \cdot D h_v^{(i)} + \frac{\mathcal{K}^{(i)}}{2m_{Q_i}} \quad (7)$$

The term including the operator (5) describes the chromomagnetic interaction of the quark i with the gluon field generated by itself and its heavy partner.

Let us stress that in the Lagrangian expansion for a h-h system it is the order in the internal velocity v' rather than the dimension of the operators what matters in neglecting further terms ¹. Thus, from arguments based on power-counting rules [19] the expected magnitude of the Pauli term is suppressed by a power v'^2 with respect to \mathcal{L}_i . In fact, observe that for the sake of consistency in the characteristic magnitude, one should have considered some *further terms* of the $m_{Q_i}^{-1}$ expansion contributing at the same order in the internal quark velocity. Such next terms (up to order v'^4) would include relativistic corrections to the kinetic energy as well as spin-dependent effects like spin-orbit forces. The latter, however, vanish for s -wave states so that in this case the only relevant spin effect comes from the operator (5) while the former can be incorporated into a spin-independent Lagrangian beyond $\mathcal{L}_i^{(\prime)}$.

Once taken into account, the chromomagnetic operator resolves the degeneracy between the spin-singlet and spin-triplet states, both in h-l systems as in h-h systems. An important difference arises, however, in the behavior of the hyperfine splitting in the $m_Q \rightarrow \infty$ limit. In h-l systems the degeneracy becomes asymptotically more and more exact. In sharp contrast, the validity of an approximate spin symmetry in doubly heavy states will be

¹This is the crucial difference with respect to h-l systems where $v' \simeq \Lambda_{QCD}/m_Q$.

justified in a certain intermediate quark mass region, but it should certainly be broken above a more or less defined “threshold”. We examine below this issue with the help of potential models. (Although potential models do not come directly from first principles they describe very well the general features of hadron spectroscopy).

In order to assess the relative importance of the Pauli term in (6), we shall assume a Coulomb-plus-linear potential

$$V(r) = V_0 - \frac{\kappa}{r} + \sigma r$$

For those states above the pure Coulomb well, the wavefunction at the origin is largely determined by the confining potential. In particular for the 1S state, $|\psi(0)|^2 \approx e_1(\hat{m}_{ij}\sigma/2\pi)$ [20], where $e_1 = 1.935$ and $\hat{m}_{ij} = m_{Q_i}m_{Q_j}/(m_{Q_i} + m_{Q_j})$ stands for the reduced mass of the Q_iQ_j system. For definiteness and simplicity, we ascribe the non-confining part of the potential to single-gluon exchange, hence $\kappa = c_F\alpha_s$ where $c_F = 4/3, 2/3$ is a color factor for a quark-antiquark pair, quark-quark pair respectively. We thus write the well-known expression [20] for the hyperfine mass splitting as

$$\Delta M|_{hyp} = \frac{8c_F\pi}{3} \frac{\alpha_s}{m_{Q_i}m_{Q_j}} |\psi(0)|^2 \quad (8)$$

Inserting the $|\psi(0)|^2$ expectation

$$\Delta M|_{hyp} \simeq \frac{8c_F}{3} \frac{\alpha_s\sigma}{m_{Q_i} + m_{Q_j}} \quad (9)$$

and we see that $\Delta M|_{hyp}$ decreases whenever any m_{Q_i} increases. In the particular case of J/ψ or Υ , $m_{Q_i} = m_{Q_j} = m_Q$ and $\Delta M|_{hyp} \simeq 16\alpha_s\sigma/9m_Q$.

One can establish a relationship between the above expressions based on a potential model and an analogous definition of the λ_2 parameter for h-l mesons [15] [21], now for a doubly heavy bound system. To this end, we consider the spin-spin effect in the mass spectrum due to the chromomagnetic interaction of the quark i with a background of color field according to

$$\delta M^{(i)} = - \frac{\langle H(v) | \int d^3x \delta\mathcal{L}_{\{\{\}}^{(i)}}(x) | H(v) \rangle}{\langle H(v) | H(v) \rangle}$$

where the correction term is here in particular

$$\delta\mathcal{L}_{\{\{\}}^{(i)}} = \frac{1}{2m_{Q_i}} Z(m_{Q_i}/\mu_i) \mathcal{S}^{(i)} \quad (10)$$

and the hadron states are normalized non-relativistically. Let us now introduce

$$\frac{\langle H(v) | \mathcal{S}^{(i)}(0) | H(v) \rangle}{\langle H(v) | \bar{h}_v^{(i)} h_v^{(i)} | H(v) \rangle} = d_H \lambda_2^{(i)} \quad (11)$$

where $d_H = 3, -1$ for the spin-singlet, spin-triplet state respectively. As far as we will be interested in s -wave states, the spin-spin interaction vanishes everywhere except in the origin $\vec{x} = \vec{0}$. Thereby, taking into account that with the above normalization the denominator of (11) is unity, one can readily identify $\delta M^{(i)}|_{hyp} = -d_H Z\lambda_2^{(i)}/2m_{Q_i}$.

In reality, the hyperfine effect can be “ascribed” to either heavy quark i or j . Because of the symmetry of the spin interaction between both heavy quarks in a s -wave state (of the dipole-dipole type), the following identity can be formulated by requiring that $\delta M^{(i)}|_{hyp} = \delta M^{(j)}|_{hyp}$:

$$Z(m_{Q_i}/\mu_i) m_{Q_j} \lambda_2^{(i)}(\mu_i) = Z(m_{Q_j}/\mu_j) m_{Q_i} \lambda_2^{(j)}(\mu_j) = Z(\hat{m}_{ij}/\mu) \hat{m}_{ij} \hat{\lambda}_2(\mu) \quad (12)$$

where $\hat{\lambda}_2$ may be interpreted as resulting from the chromomagnetic interaction of a fictitious quark of mass \hat{m}_{ij} moving with the quarks’ relative velocity through the background of the color field generated by both i and j quarks. Now, including a factor 1/2 to avoid double counting, we write in an explicitly symmetric form

$$\begin{aligned} \Delta M|_{hyp} = M_{H^*} - M_H &= \frac{1}{2} [\delta M^{(i)}|_{hyp} + \delta M^{(j)}|_{hyp}] = \\ \left[Z(m_{Q_i}/\mu_i) \frac{1}{m_{Q_i}} \lambda_2^{(i)}(\mu_i) + Z(m_{Q_j}/\mu_j) \frac{1}{m_{Q_j}} \lambda_2^{(j)}(\mu_j) \right] &= Z(\hat{m}_{ij}/\mu) \frac{2}{m_{Q_i} + m_{Q_j}} \hat{\lambda}_2(\mu) \end{aligned} \quad (13)$$

and thus $Z\hat{\lambda} = (M_{H^*}^2 - M_H^2)/4$, in analogy to h-l systems [21]. By comparison with eq. (9) we further get

$$Z(\hat{m}_{ij}/\mu) \hat{\lambda}_2(\mu) \simeq \frac{4c_F}{3} \alpha_s(r_0^{-1}) \sigma \quad (14)$$

where r_0 stands for the typical radius of the hadron, *i.e.* $r_0^{-1} \simeq c_F \alpha_s(r_0^{-1}) \hat{m}_{ij}$. Note that $\hat{\lambda}_2$ is proportional to *the string tension* σ and depends on the renormalization scale μ but $Z(\mu)\lambda_2(\mu)$ does not. Needless to say that eq. (14) must not be taken literally due its naive model dependence but rather as an indication of the close relationship between the $\hat{\lambda}_2$ parameter and the string tension. Thus, the behavior of the hyperfine mass splitting for the J/ψ and Υ , roughly of the type m_Q^{-1} as in h-l systems, amounts to the \hat{m}_{ij} -independence of $\hat{\lambda}_2$.

A numerical estimate using $\alpha_s \simeq 0.4$, $\sigma = 0.18 \text{ GeV}^2$ yields $\hat{\lambda}_2(\mu \simeq 1 \text{ GeV}) \simeq 0.13 \text{ GeV}^2$, in significant accordance with the respective value of $\lambda_2(\mu \simeq 2\bar{\Lambda}) \simeq 0.15 \text{ GeV}^2$ for a h-l meson. Indeed, this is not surprising for it could be directly guessed from the numerical similarity between the mass splittings of the D , D^* states and the η_c , J/ψ states .

In summary, we advocate the validity of the application of the spin symmetry to the analysis of the weak decays of doubly heavy hadrons in analogy to singly heavy hadrons on the basis of the m_Q -independence and smallness of $\hat{\lambda}_2$.

Nevertheless, this emerging symmetry cannot be considered as asymptotically valid in h-h states. Since in the Coulomb regime $v' \sim \alpha_s$, we may then write $\Delta M|_{hyp} \sim m_Q \alpha_s^4$. The “transition” mass domain where the linear behavior takes over the logarithmic fall off can be crudely estimated (from above) to be around $m_0 \simeq e^4 \Lambda_{QCD} \simeq 10 \text{ GeV}$. Only below it (thereby including J/ψ , B_c and Υ states) spin symmetry can be used as an approximation. Indeed, in the language of potential models, a dominant Coulomb regime determines the wavefunction at the origin as $|\psi(0)|^2 = (c_F \alpha_s \hat{m}_{ij})^3 / \pi$ leading to the growth of the hyperfine splitting as follows from eq. (8). Stated in other way, $\hat{\lambda}_2$ would be proportional to the reduced mass squared yielding the growth of $\Delta M|_{hyp}$.

2.2 Superflavor multiplets

In the preceding Section we analyzed the interaction dynamics of a pair of heavy quarks borrowing to some extent the formalism developed for a h-l system. In the following, we shall examine the interaction of a heavy diquark with a light component on the basis of an “additional” effective *diquark* theory.

Let us start by recalling that QCD has, among others, a notable property: two color triplets can bind together to yield another color (anti)triplet. In particular, if both quarks are heavy enough, they will form an almost pointlike source of color behaving as a heavy antiquark with respect to a third light component in a hadron. This property of QCD, combined with the stemming spin-flavor symmetry at the very large quark mass limit should permit to classify hadrons containing one quark and two heavy antiquarks in multiplets for the same velocity with similar properties *as far as the light degrees of freedom are concerned*. Indeed, the fundamental representation of the $SU(6) \otimes U(1)$ spin-flavor symmetry group consists of the two spin states of the fermionic quark field, the scalar di-antiquark field and the three spin states of the axial-vector di-antiquark field. All these objects appear to be the same to any bound light antiquark². In order to explicitly exhibit this symmetry, we shall write the representation of the superflavor group as “quark” supermultiplets (for each fixed v) expressed as *nine*-component column vectors

$$\Psi_v = \begin{pmatrix} h_v \\ S_v \\ A_v^\mu \end{pmatrix}$$

where A_v^μ satisfies the constraint $v_\mu A_v^\mu = 0$. (We have omitted any reference to the flavor of the constituents). The effective Lagrangian then reads

$$\mathcal{L}_{\Psi} = \frac{1}{2} \bar{\Psi}_v \mathcal{M} i v \cdot \overleftrightarrow{D} \Psi_v$$

where \mathcal{M} is a 9×9 mass matrix depending on the normalization of the fields which can be taken from [22] as

$$\mathcal{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2m_S & 0 \\ 0 & 0 & -2m_A \end{pmatrix}$$

and

$$\bar{\Psi}_v = \Psi_v^\dagger \begin{pmatrix} \gamma^0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & g \end{pmatrix}$$

where $g = \text{diag}(1, -1, -1, -1)$.

One can form a superflavor “*meson*” multiplet by the tensor product of Ψ_v and one light antiquark field (there is no need to specify its flavor assuming unbroken light flavor $SU(3)$). Therefore, such hadronic supermultiplet puts together singly heavy mesons and doubly heavy antibaryons. The latter, though with different quantum numbers, really look like “*mesons*” in many respects, as commented more extensively in Section 3. In the

²The underlying superflavor symmetry in the very heavy mass limit of the strong interaction was firstly developed by Georgi, Carone, Wise and Savage [22][23].

infinite quark mass limit, the behavior of the light degrees of freedom becomes independent of the heavy constituent flavors i, j and of the di-antiquark spin. Let us note that such supermultiplet is, however, not completely flavor-independent even at this asymptotic limit because of the mass dependence of the heavy di-antiquark internal dynamics³. On the other hand, singly heavy baryons cannot be included in any *meson* supermultiplet since the groundstate light di-quark (either spin-0 or spin-1) should behave quite differently than a spin-1/2 light antiquark.

Instead, one can construct a *baryon* superflavor multiplet by combining either one heavy quark or one heavy di-antiquark with a light di-quark⁴. Moreover, we sort out two possibilities according to the spin of the light component. If it is a spinless object the supermultiplet contains Λ_Q baryons together with doubly heavy diquonia⁵ ($Q_i Q_j \bar{q} \bar{q}$). Other supermultiplets would include baryon (Σ_Q, Σ_Q^*) states with a spin-1 light component.

3 Weak decays of hadrons with two heavy quarks

Hadrons (mesons or baryons) containing two heavy quarks $Q_i Q_j$ are particularly interesting since their lowest lying states would only decay weakly in contrast to charmonium and bottomonium resonances. In the case of mesons, the only available possibility of gathering together two different heavy quarks is the B_c meson. This is probably the reason why B_c is nowadays object of growing attention both because of its weak decay and in hadron spectroscopy due to its interpolating position between the J/ψ and the Υ [29].

3.1 Semileptonic decay of the B_c meson

As pointed out in the pioneering works by Nussinov and Wetzel [30], Voloshin and Shifman [31] the description of the decay of a h-l hadron like a B meson simplifies dramatically in the infinite quark mass limit. When the heavy quark decays into another still massive quark, the surrounding brown-muck acts as a pure spectator with only one form factor (the Isgur-Wise universal function $\xi(w)$, $w = v_1 \cdot v_2$) governing the transition dynamics of the light degrees of freedom. At zero recoil $\xi(w = 1)$ is unity, reflecting a perfect overlap between the initial and final hadron wave functions [15].

On the other hand, it has been argued in recent literature about the possibility of testing the spectator behavior by experimentally discriminating among specific channels for b or c decays [2] in B_c ($\bar{b}c$) mesons. Let us mention, however, that a simple-minded spectator picture of the B_c decay should differ with regard to the analogous decays of B and D mesons. In h-l mesons the spectator behavior makes sense as an asymptotic

³One may extend the often quoted pedagogical analogy between the atom and a h-l meson. Now, the physics of the compound nucleus (the di-antiquark) is not at all the same for different isotopes although they have similar chemical properties (those of the light degrees of freedom).

⁴Henceforth, the reader himself will kindly realize whether we are referring to quarks or antiquarks so for the sake of brevity we generally suppress any explicit reference to antiquarks.

⁵Historically four-quark systems were in fact referred to as “baryonia” partly due to their expected decay into a baryon-antibaryon pair [24]. In references [25] [26] [27] the similarity between $Q_i Q_j \bar{q} \bar{q}$ states and Λ_Q baryons is established in the context of quark/potential models. Let us also mention that there is an alternative interpretation for the existence of such exotic states as di-mesons, *i.e.* $(Q_i \bar{q}) - (Q_j \bar{q})$ states [28]. Obviously, the latter would not be the supersymmetric partners of singly heavy baryons.

expectation whereas both heavy quarks are necessarily involved in the dynamics of a h-h hadron even for infinite constituent masses.

Let us start by considering the \bar{b} quark undergoing the decay, yielding the exclusive semileptonic channels:

$$B_c \rightarrow \eta_c(J/\psi) + \bar{\ell} \nu \quad (15)$$

Applying the spin symmetry discussed in Section 2 (see the appendix for more details), the expressions for the widths obtained in the non-recoil approximation are

$$\Gamma(B_c \rightarrow \eta_c + \bar{\ell} \nu) \simeq \frac{G_F^2 m_1^5}{192 \pi^3} \varphi(x) f_{12}^2 |V_{cb}|^2 \quad (16)$$

$$\Gamma(B_c \rightarrow J/\psi + \bar{\ell} \nu) \simeq \frac{G_F^2 m_1^5}{192 \pi^3} [\varphi(x) + \phi(x)] f_{12}^2 |V_{cb}|^2 \quad (17)$$

where $x = m_2/m_1$ and φ, ϕ are phase space factors for vanishing leptonic masses whose expressions can be found in the appendix. Notice that we keep both φ, ϕ functions separately to facilitate later comparisons. The form factor is given by $f_{12} = \sqrt{m_2/m_1} \hat{\eta}_{12}(v_1 = v_2)$, where the overlap factor $\hat{\eta}_{12}$ is not fixed at zero recoil by a current conservation due to the lack of an underlying flavor symmetry. Instead, it can be estimated by means of a non-relativistic model of hadrons (see appendix) like the ISGW model [32] according to expression (A.9).

Next we want to apply our simplified approach to the decaying c -quark in the semileptonic channels of B_c mesons:

$$B_c \rightarrow B_s(B_s^*) + \bar{\ell} \nu \quad (18)$$

where the heavy quark spin symmetry and the non-recoil approximation will be employed once again. In [8] the authors made an estimate of the overlap integral between the B_c 's and B_s 's wavefunctions, denoted by $\hat{\Omega}$, by means of an operator product expansion retaining only the leading contribution. We have rederived the expression for $\hat{\Omega}$ at zero recoil resulting, however,

$$\hat{\Omega} \simeq 4 \left(\frac{\pi}{3} \right)^{1/2} f_{B_s} \sqrt{m_{B_s}} r_0^{3/2} \quad (19)$$

where $r_0 = 1/c_F \hat{m}_{bc} \alpha_s(\hat{m}_{bc})$ is the radius of the B_c meson with \hat{m}_{bc} denoting the reduced mass of the b and c quarks. Let us note some discrepancies between “our” formula and the original one presented in [8]. On the one hand, there is a factor $\sqrt{2}$ apparently due to a different normalization of states in (to $2v^0$ in our case instead of v^0). Moreover, there is an extra factor $\sqrt{3}$ in the denominator due to color⁶. In fact, let us note that $f_{B_s} \sqrt{m_{B_s}}/2\sqrt{3}$ can be interpreted as the wave function $\psi_{B_s}(0)$ at the origin according to quark models. Therefore, the “constant” $\psi_{B_s}(0)$ factorizes out in the overlap integral of the initial and final wavefunctions. (This can be accounted for since the variation of ψ_{B_s} takes place over longer distances than the typical size r_0 of the B_c). However, let us observe that the evaluation of $\hat{\Omega}$ in (19) should be somewhat overestimated because of the radial decrease of the B_s wavefunction.

⁶In writing the initial B_c state, a sum over colors must be understood in eq. (4.1) of ref. [8] and a normalization factor $1/\sqrt{3}$ should be included. Keeping color indices throughout, *i.e.* in the wavefunction and the color singlet weak current and taking into account the color Kröneckers delta implicitly contained in eq. (4.4) of [8], the normalization factor $1/\sqrt{3}$ survives all the steps of the calculation.

The decay widths come finally to be

$$\Gamma(B_c \rightarrow B_s + \bar{\ell}\nu) \simeq \frac{G_F^2 m_1^5}{192 \pi^3} \varphi(x) f_{12}^2 |V_{cs}|^2 \quad (20)$$

$$\Gamma(B_c \rightarrow B_s^* + \bar{\ell}\nu) \simeq \frac{G_F^2 m_1^5}{192 \pi^3} [\varphi(x) + \phi(x)] f_{12}^2 |V_{cs}|^2 \quad (21)$$

where $f_{12} = \sqrt{m_2/m_1} \hat{\Omega}$.

Setting usual numerical values in (19) one gets from the above expressions widths of order 10^{-13} GeV, exceedingly larger than those expected from quark-model calculations, for example [33]. We therefore think that expression (19) actually overestimates the widths in the non-recoil approximation so we prefer to evaluate $\hat{\Omega}$ employing the ISGW model according to eq. (A.9) once again.

In Table I we present our estimates turning out to be in remarkable agreement with the quark model predictions of ref. [33] for the same input parameters. (In fact, our results can be considered as oversimplified quark model calculations). This agreement extends as well over the results by other authors in spite of the simplicity of our approach. There is, however, an important discrepancy (common to all other references quoted in Table I) with the results by Bagan *et al* relative to the widths of those semileptonic channels with a final vector meson (either a J/ψ or a B_s^*).

3.2 Two-body nonleptonic decay of the B_c

In this Section we address some two-body weak decays of the B_c meson. Let us focus on the process induced by the decay of the b -quark,

$$B_c \rightarrow \eta_c(J/\psi) + \pi \quad (22)$$

The exclusive channel corresponding to a final J/ψ is one of the most promising from the point of view of its experimental detectability at a hadron collider [1].

We shall base our theoretical approach on two hypothesis. The assumption of the factorization permits to relate such decay with the semileptonic one at $q^2 = m_\pi^2 \simeq 0$, according to [35]

$$\frac{\Gamma(B_c \rightarrow \eta_c(J/\psi) + \pi)}{d\Gamma/dq^2|_{q^2=0}(B_c \rightarrow \eta_c(J/\psi) + \bar{\ell}\nu)} \simeq 6\pi^2 f_\pi^2 |V_{ud}|^2 \simeq 1 \text{ GeV}^2$$

Besides, we keep the non-recoil approximation in the evaluation of the hadronic matrix element. Thus, making use of the set of equations (A.6) and (A.7) a straightforward derivation yields

$$\Gamma(B_c \rightarrow \eta_c(J/\psi) + \pi) \simeq \frac{G_F^2 (m_1^2 - m_2^2)^3}{192 \pi^3 m_1^3} f_{12}^2 |V_{cb}|^2 \quad (23)$$

where m_2 stands either for the mass of the η_c or the J/ψ . As seen from Table II, the agreement with other quark model predictions is remarkable. Furthermore, the general accordance manifested both in tables I and II suggests the extension of our simplified formalism to the weak decays of baryons with two heavy quarks.

3.3 Semileptonic decay of a doubly heavy baryon

It has been pedagogically introduced in the context of HQET the analogy between an atom and a meson containing one heavy quark along with light degrees of freedom [34]. On the other hand, in baryons with two heavy quarks ($Q_i Q_j q$), the massive quark pair should bind into a $\bar{3}$ source of color interacting with the light degrees of freedom as a static antiquark [23] [36] [37]. Assuming one-gluon exchange, the Bohr radius of the heavy diquark is $\tilde{r}_0 \simeq 1/c_F \alpha_s (r_0^{-1}) \hat{m}_{ij} \simeq 2r_0$, where \hat{m}_{ij} is the reduced mass of the ($Q_i Q_j$) subsystem. Indeed, one expects the size of the heavy diquark be presumably smaller than the typical length of the hadron, so that the analogous picture of a doubly heavy baryon in atomic physics would be a deuterium atom rather than an hydrogen molecule.

If the two heavy quarks have different flavors there are two possible spin configurations of the compact diquark at lowest level. Supposing that the ($Q_i Q_j$) forms a spin-singlet (thus flavor-singlet) state the ground-state hadron will have spin 1/2 which we shall refer to as a Λ_{ij} baryon. If the ($Q_i Q_j$) forms a spin-triplet (thus flavor-triplet) state the ground-state hadron can be either a spin-1/2 baryon (denoted by Σ_{ij}) or a spin-3/2 baryon (denoted by Σ_{ij}^*).

In order to apply the heavy quark spin symmetry to these doubly heavy baryons, we must realize two different scenarios: *i*) the “outer part” formed by the heavy diquark as a whole and the light component surrounding it, and *ii*) the heavy diquark subsystem itself. Ideally, unbroken heavy quark symmetries imply the mass degeneracy among all these hadronic states [23] [5]. Actually, the chromomagnetic hyperfine interaction should resolve the degeneracy leading to the mass level:

$$m(\Sigma_{ij}) < m(\Lambda_{ij}) < m(\Sigma_{ij}^*) \quad (24)$$

according to [36]. Let us remark that these authors apply the hyperfine splitting to doubly heavy baryons mimicking h-l mesons, that is they only consider the spin interaction of the heavy diquark with the light component previously mentioned in *i*), obtaining

$$m(\Lambda_{ij}) = m(\Sigma_{ij}) + \frac{3}{4} \Delta m|_{hyp} = m(\Sigma_{ij}^*) - \frac{1}{4} \Delta m|_{hyp}$$

Note that any discussion about the hyperfine splitting *inside* the heavy diquark has apparently been overlooked in [36] and [23]. In fact, the asymptotic behavior of each chromomagnetic interaction is completely different as stressed in the first part of this work: the former *i*) should fall off as any heavy quark mass increases whereas the latter *ii*) would finally violate the spin symmetry.

Nevertheless, the inequality (24) should probably hold for c , b constituent quarks since the effect of the “inner” hyperfine splitting is suppressed by roughly a factor $(1/2)^4$ with respect to the “outer” hyperfine splitting (a $(1/2)^3$ comes from the the square of the (Coulombic) wavefunction at the origin and an additional 1/2 factor due to $\kappa = c_F \alpha_s$ in equation (9)). Accordingly, observe that the lowest lying state seems likely to be the Σ_{bc} , *i.e.* the spin 1/2 baryon with a diquark which might be considered as a $J^P = 1^+$ quasi-particle.

Finally, let us stress that among all the possible doubly heavy baryons to be produced at the new generation of hadron colliders, only those with one b and one c have a real chance to be observed. (This experimental issue will be discussed in more detail elsewhere

[1]). Hereafter, we shall concentrate on the weakly decaying spin-1/2 Σ_{bc} baryon.

$\Sigma_{bc} \rightarrow N_{cc}$ transitions

According to our notation, if the heavy diquark (bc) in the baryon undergoes a semileptonic weak decay yielding another heavy diquark (cc) (figure 1), the final groundstate baryon can be either a Σ_{cc} or a Σ_{cc}^* . Sometimes we shall design generically both of them as N_{cc} .

The quark-level current $J^\mu = \bar{c}\gamma^\mu(1 - \gamma_5)b$ responsible for the hadronic transition $\Sigma_{bc} \rightarrow N_{cc}$ can be written as [5],

$$J^\mu = \hat{\eta}_{12}(v_1 \cdot v_2) \left[-\frac{1}{2} A_{cc}^{\dagger\beta} A_{bc\beta}(v_1 + v_2)^\mu + \frac{1}{2} i\varepsilon^{\mu\nu\alpha\beta} A_{cc\nu}^\dagger A_{bc\alpha}(v_1 + v_2)_\beta \right] \quad (25)$$

where A_{ij} (A_{ij}^\dagger) annihilates (creates) an axial-vector diquark (ij); $\hat{\eta}_{12}(v_1 \cdot v_2)$ stands for the wavefunction overlap between the parent and daughter heavy diquarks.

Both initial Σ_{bc} and final N_{cc} particles belong to *meson* supermultiplets of the $SU(6) \otimes U(1)$ superflavor symmetry, as explained in Section 2. Thereby, their well-defined transformation properties under this symmetry group permits the use of the trace formalism ensuring a single form factor concerning the overlap of the light degrees of freedom at leading order approximation. We thus shall write the overall form factor η_{12} governing the hadronic transition factorizing into two pieces [5] [7], namely

$$\eta_{12} = \hat{\eta}_{12}(v_1 \cdot v_2) \xi(v_1 \cdot v_2) \quad (26)$$

At low recoil, the light degrees of freedom only play a marginal role in the decay, *i.e.* $\xi(w \simeq 1) \simeq 1$ as in the analogous transitions between h-l mesons. Indeed, as shown in ref. [7], the light component does not change the overall decay rate of the diquark though channels the decay into different spin-states according to some selection rules, as a direct consequence of QCD in the limit of infinite quark masses.

Since no flavor symmetry can be applied to h-h systems, neither $\hat{\eta}_{12}$ nor consequently η_{12} are normalized to unity at zero recoil but can be calculated in a model-dependent way as in [5] using Coulomb wavefunctions,

$$\hat{\eta}_{12}(w = 1) = 8\chi^{3/2} \quad , \quad \left(\chi = \frac{\hat{m}_{bc}\alpha_s(\hat{m}_{bc}) \hat{m}_{cc}\alpha_s(\hat{m}_{cc})}{[\hat{m}_{bc}\alpha_s(\hat{m}_{bc}) + \hat{m}_{cc}\alpha_s(\hat{m}_{cc})]^2} \right)$$

In the numerical evaluation we adopted $m_c = 1.5$ GeV, $m_b = 4.8$ GeV, $\alpha_s(\hat{m}_{bc}) \approx 0.3$ and $\alpha_s(\hat{m}_{cc}) \approx 0.4$. A more exact computation of $\hat{\eta}_{12}$ using wavefunctions from the Coulomb-plus-linear potential of reference [29] provides a very similar numerical value ⁷, close to unity.

Next, we apply the non-recoil approximation to the evaluation of rates for the semileptonic decays as explained in the appendix. One arrives at

$$\Gamma[\Sigma_{bc} \rightarrow \Sigma_{cc} \text{ or } \Sigma_{cc}^* + \ell\bar{\nu}] \simeq \frac{G_F^2 m_1^5}{576 \pi^3} [5\varphi(x) + 2\phi(x)] f_{12}^2 |V_{cb}|^2 \quad (27)$$

Discriminating Σ_{cc} from Σ_{cc}^* in the final state, one gets readily (see equations (17) and (18) in reference [7]).

$$\Gamma[\Sigma_{bc} \rightarrow \Sigma_{cc} + \ell\bar{\nu}] \simeq \frac{G_F^2 m_1^5}{576 \pi^3} \left[\frac{13}{3}\varphi(x) + \frac{4}{3}\phi(x) \right] f_{12}^2 |V_{cb}|^2 \quad (28)$$

⁷I am indebted to P. González for technical advice in this calculation.

$$\Gamma[\Sigma_{bc} \rightarrow \Sigma_{cc}^* + \ell\bar{\nu}] \simeq \frac{G_F^2 m_1^5}{576 \pi^3} \left[\frac{2}{3} \varphi(x) + \frac{2}{3} \phi(x) \right] f_{12}^2 |V_{cb}|^2 \quad (29)$$

In Table III we show the numerical results using the last expressions. It is worth to mention that the relative branching fraction of the semileptonic decays $\Sigma_{bc} \rightarrow \Sigma_{cc}(\Sigma_{cc}^*) + \ell\bar{\nu}$ is completely inverted with regard to the decay $B_c \rightarrow \eta_c(J/\psi) + \bar{\ell}\nu$. (The underlying diquark transition is different in each case, $1 \rightarrow 1$ in the former and $0 \rightarrow 0(1)$ in the latter. The groundstate Λ_{cc} is forbidden by the Pauli exclusion principle).

$\Sigma_{bc} \rightarrow N_b$ transitions

Let us assume now that the decay of the Σ_{bc} proceeds through the quark decay $c \rightarrow s, d$ yielding a final b -flavored baryon. It might be conceivable that the diquark keeps up and the final baryon could be viewed as made up of a h-l diquark and a light component⁸. Another interpretation assumes that the initial diquark actually breaks up: the b quark becomes a nearly static source of color whereas one light diquark is formed. We adopt this second (more standard) possibility henceforth supposing in addition that the N_b lies at the groundstate. Still, the light dressing can be either a spinless or a spin-1 object. In the following we examine the former case, *i.e.* when the final baryon is of the Λ_b -type.

We are addressing here a transition connecting doubly heavy to singly heavy baryons. In order to take benefit from the compact trace formalism developed in [22] for the evaluation of hadronic matrix elements, we need to recall the fact that both initial and final hadrons belong to (different) superflavor multiplets under the $SU(6) \otimes U(1)$ spin-flavor symmetry group. Therefore, we shall use the following 9×4 matrix representations for the Σ_{bc} and Λ_b (anti)baryons,

$$\tilde{\Psi}_{\Sigma_{bc}}(v) = \frac{1}{\sqrt{6M_\Sigma}} \begin{pmatrix} 0 \\ 0 \\ u_\Sigma^T(v) C \sigma^{\mu\alpha} v_\alpha \gamma_5 \end{pmatrix}, \quad \tilde{\Psi}_{\Lambda_b}(v) = \frac{1}{\sqrt{2M_\Lambda}} \begin{pmatrix} u_\Lambda^T(v) C \\ 0 \\ 0 \end{pmatrix}$$

where C is the conjugation matrix and the spinors u_i are normalized to $\bar{u}_i(v)u_i(v) = 2M_i$ satisfying $\not{v}u_i(v) = u_i(v)$. The corresponding bra matrices are defined as

$$\bar{\tilde{\Psi}}(v) = \gamma^0 \tilde{\Psi}^\dagger(v) \begin{pmatrix} \gamma^0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & g \end{pmatrix} \quad (30)$$

The trace formalism can be employed to compute the hadronic matrix elements for the $\Sigma_{bc} \rightarrow N_b$ transition, as before. There are, however, some crucial differences with respect to the $\Sigma_{bc} \rightarrow N_{cc} + \ell\bar{\nu}$ decay as we shall see.

The initial (bc) diquark (with a very large mass $\simeq m_b + m_c$) should stand very close to the geometrical center of an spherically symmetric baryon. Furthermore, the b -quark itself would tend to occupy a nearly central position inside the heavy diquark because of its relative larger mass.

The mean distance of the b with respect to the center of mass of the (bc) system (and to the center of the baryon as a whole too) is of order $(\hat{m}_{bc}/m_b)\tilde{r}_0$ where $\tilde{r}_0 = 3/2\alpha_s\hat{m}_{bc}$

⁸This may make some sense in particular for an emerging s -quark which could be considered as semi-heavy to some extent; in fact, such a picture has been recently proposed as an alternative interpretation for *any* baryon containing a single heavy quark [38].

denotes the characteristic size of the (bc) bound state, about twice the Bohr radius of the B_c meson. Thereby, the typical distance of the b -quark to the baryon center is $\simeq 3/2\alpha_s m_b$. On the other hand, the final heavy quark (*i.e.* the b) will stay near the central position of the final baryon as well, oscillating with typical “zitterbewegung” frequency $2m_b$ and thus spreading over the corresponding length scale. Long-distance processes due to soft gluons give rise to residual momentum exchanges and intermediate quantum states with energy fluctuations of order k , leading also to a typical distance $\sim 1/k \times k/m_b = 1/m_b$ about the average position of the b quark. Observe that both of its initial and final oscillation amplitudes are of the same order (indeed they are $1/m_b$ effects) vanishing in the limit $m_b \rightarrow \infty$.

Therefore, the decay of the c -quark at low-recoil occurs under the influence of a color field generated by a center-positioned, almost immovable b -quark. (This contrasts with the decay $\Sigma_{bc} \rightarrow N_{cc}$ where a reassembly of the final *heavy* diquark is necessary). Hence, no form factor relative to the heavy component will be considered in the hadronic transition amplitude, at least to leading order, or stated in other words, the wavefunction overlap corresponding to the b -quark is assumed equal to unity.

From the above discussion, one can heuristically deduce that solely those form factors related to the light degrees of freedom will come into play in the weak current. Accordingly the latter, $J^\mu = \bar{s}\gamma^\mu(1 - \gamma_5)c$, acting on a Σ -type baryon can be written as

$$J^\mu = F^{\mu\nu}(v_1, v_2) h_b^\dagger A_{bc\nu}$$

$F^{\mu\nu}$ is a tensor function depending on v_1 and v_2 , which can be parametrized in general as

$$F^{\mu\nu}(v_1, v_2) = a(w) g^{\mu\nu} + b(w) v_1^\mu v_2^\nu + c(w) v_2^\mu v_2^\nu + d(w) i\varepsilon^{\mu\nu\alpha\beta} v_{1\alpha} v_{2\beta}$$

where a , b , c , and d are universal functions concerning the light degrees of freedom. In particular at zero recoil (recall from the appendix the small range $v_1 \cdot v_2 \leq 1.02$ allowed in this process!) only the a -term gives contribution, all others vanishing since $v_1^\nu A_{bc\nu}(v_1) = 0$. The $a(w \sim 1)$ function absorbs the complexity of the non-perturbative decay dynamics of the light degrees of freedom near this kinematic point. Let us remark that it does not coincide with the Isgur-Wise function for transitions among members of *meson*-type supermultiplets. Nevertheless, $a(w)$ is a true universal function no matters the flavor of the constituent heavy quarks.

The hadronic matrix elements near zero recoil can be written as

$$\langle \Lambda_b(v) | J^\mu | \Sigma_{bc}(v) \rangle \simeq \frac{i a(1)}{2\sqrt{3} m_1 m_2} \bar{u}_\Lambda(v) \gamma^\mu \gamma_5 u_\Sigma(v) \quad (31)$$

Finally, we get in the non-recoil approximation $v_1 = v_2$,

$$\Gamma[\Sigma_{bc} \rightarrow \Lambda_b + \bar{\ell}\nu] \simeq \frac{G_F^2 m_1^5}{576 \pi^3} [\varphi(x) + \phi(x)] f_{12}^2 |V_{cs}|^2 \quad (32)$$

where $f_{12} = \sqrt{m_2/m_1} a(1)$.

In order to have an estimate of $a(1)$ one may conjecture that its numerical value should be intermediate between unity (*i.e.* the Isgur-Wise function at zero recoil) and the overlap of the light degrees of freedom corresponding to the $D \rightarrow K$ transition (see figure 2). We

tentatively adopt the last wavefunction overlap factor obtained from the ISGW model with the parameters of Table A.I as an acceptable estimate for $a(1)$. Indeed, note that the presence of the almost immovable b -quark acting as an attractive center of color force should somehow shrink the size of the light degrees of freedom (especially compared to the Kaon system where such a static force center does not exist at all) thereby increasing their wavefunction overlap. However, this effect should be somewhat cancelled by the fact that the quark-quark interaction is about half as attractive as in the quark-antiquark case. Our predictions (with a large uncertainty) are shown in Table III.

3.4 Absolute branching fractions

In order to present our results in a more suitable way from the experimental viewpoint, we tentatively assume that the full width of the Σ_{bc} baryon is similar to the full width of the B_c baryon. This assumption can be somehow justified by the fact that the Λ_b baryon has similar lifetime as the B meson [40].

The main objection to this motivated analogy may arise from the fact the annihilation channel of the meson $B_c \rightarrow \bar{\ell}\nu$ has no analogous counterpart in the Λ_{bc} baryon decay. However, noting that the expected branching ratio of the B_c into all leptonic channels is $\approx 18\%$ [33] while the main part of the decay (82%) corresponds to “spectator” diagrams, it remains conceivable that the Σ_{bc} ’s lifetime is not very different from the B_c ’s [33]. Therefore, adopting $\tau(\Sigma_{bc}) \simeq 0.5$ ps we get the set of branching fractions shown in Table III (since one would expect $\tau(B_c) < \tau(\Sigma_{bc})$ we are underestimating these branching fractions in some sort).

3.5 Two-body nonleptonic decay of the Σ_{bc}

In this Section we shall examine the two-body weak decay generated by the quark-level process $b \rightarrow c\bar{c}s$

$$\Sigma_{bc} \rightarrow \Xi_c + J/\psi$$

where Ξ_c denotes a ground-state Λ_Q -type baryon. We have focused on this exclusive channel yielding a J/ψ since its subsequent decay into a lepton pair could be useful to provide a tagging signal among the huge hadronic background at LHC [1]. Proceeding throughout in a very naive way, the hadronic matrix element

$$\langle \Xi_c J/\psi | J^\mu j_\mu | \Sigma_{bc} \rangle \quad (33)$$

where $J^\mu = \bar{c}\gamma^\mu(1 - \gamma_5)b$, $j^\mu = \bar{s}\gamma^\mu(1 - \gamma_5)c$ can be Fierz-reordered. Thus assuming in addition the factorization hypothesis, it can be written as

$$\langle \Xi_c | \bar{s}\gamma^\mu(1 - \gamma_5)b | \Sigma_{bc} \rangle \langle J/\psi | \bar{c}\gamma^\mu(1 - \gamma_5)c | 0 \rangle \quad (34)$$

We next relate this amplitude to the semileptonic decay at $q^2 = m_{J/\psi}^2$ according to [35] as we did for the B_c ,

$$\frac{\Gamma(\Sigma_{bc} \rightarrow \Xi_c + J/\psi)}{d\Gamma/dq^2|_{q^2=m_\psi^2}(\Sigma_{bc} \rightarrow N_c + \ell\bar{\nu})} \simeq 6\pi^2 f_{J/\psi}^2 s_F |V_{cs}|^2$$

where s_F stands for a *suppression* factor that we assume to be 4/9 (the factor 9 comes from color mismatch whereas the enhancing factor 4 is due to the existence of two final

c quarks to combine with the emerging \bar{c} to yield the J/ψ). Note that the denominator represents an “unreal” semileptonic process as an artefact with similar initial and final baryon states as the real physical process in the numerator. Then we use the following expression (see eq. (A.7)) in the non-recoil approximation

$$\frac{d\Gamma(1\rightarrow 0)}{dq^2} = \frac{G_F^2 [\lambda^{3/2} + 12q^2 m_1^2 \lambda^{1/2}]}{576 \pi^3 m_1^3} f_{12}^2 |V_{cb}|^2$$

With regard to the overlap factors contained in f_{12} we shall stretch the arguments used for the semileptonic decay $\Sigma_{bc} \rightarrow N_b + \ell\nu$, whereas this time the c -quark (always remaining within a region of dimension $\simeq 1/m_c$) playing the spectator role instead of the b . Therefore, we (tentatively) disregard any form factor due to the rearrangement of the heavy component, whereas we evaluate the light component overlap according to the ISGW model like a “ $B \rightarrow K$ ” transition.

In table IV we present our predictions for the partial width and branching fraction of the process. It is interesting to note that the BF turns out to be of order 1%. Of course, such estimate suffers from many theoretical uncertainties but, however, is in remarkable accordance with an equivalent calculation by Mannel and Roberts [41] for the analogous $\Lambda_b \rightarrow \Lambda J/\psi$ decay once taken into account the enhancing factor 4.

4 Summary

We have analyzed the dynamics of the weak decay of hadrons containing two heavy quarks employing the formalism of the Heavy Quark Effective Theory to some extent. Firstly we focused on the internal dynamics of heavy-heavy bound systems. Notice that contrary to heavy-light systems, there is no flavor symmetry arising from the leading-order Schrödinger-type Lagrangian.

We showed on the basis of estimates from potential models that in the range of masses limited to the preasymptotic domain including J/ψ , B_c and Υ states, the Pauli chromomagnetic term in the effective Lagrangian behaves as a genuine subleading term in a $1/m_Q$ expansion. For heavy quark masses larger than $\simeq 10$ GeV, the Coulomb regime will prevail and the spin symmetry will be definitely lost.

On the other hand, the interaction of a heavy diquark with a light component in a hadron exhibits a “superflavor” symmetry arising in the low-energy limit of the strong interactions, meaning that in the infinite mass limit both spin and flavor of the color source become irrelevant. Thereby, the massive quark and diquark fields can be joined into a common “quark” supermultiplet as far as their interactions with light degrees of freedom are concerned. Furthermore, such “quark” supermultiplet can be combined with either a light quark or a light diquark field to yield a classification scheme of hadrons according to the behavior of the light component. In particular, one can form a *meson*-type multiplet including singly heavy mesons and doubly heavy (anti)baryons, all of them with well-defined transformations properties under the superflavor group. All members of a hadron supermultiplet, though with different quantum numbers, would show similar spectroscopy concerning the excitations of the light component. In a parallel way, one can form two kinds of *baryon*-type multiplets by combination of the “quark” supermultiplet with either a spinless or a spin-1 light component. Thus, singly heavy baryons and exotic multiquark states (doubly heavy diquonia $Q_i Q_j \bar{q} \bar{q}$) would belong to the same supermultiplet.

In this work we made a phenomenological application of all the above ideas to the decays of hadrons containing b and c quarks in particular. Besides, we focused on those exclusive channels leading (at least) to a final charged lepton. In fact, to be realistic, the presence of a J/ψ resonance decaying into a charged lepton pair will likely be an almost necessary condition to disentangle the decay from the huge hadronic background in hadron colliders (for more details see [1]).

Firstly, use was made of spin symmetry in the analysis of some weak decays of the B_c meson. Predictions on partial widths and branching fractions by means of the non-recoil approximation are shown in Tables I, II. In general our evaluations are in agreement with other quark model calculations clearly favoring the final spin-triplet state versus the spin-singlet state. (However, a significant discrepancy to this general rule appears in the work of Bagan *et al* using QCD sum rules).

Motivated by this general agreement, we have extended our formalism to the analysis of some weak decays of the Σ_{bc} baryon. One ought to sort out two possible scenarios depending on whether the decaying quark is either the bottom or the charm quark.

Let us suppose that the b quark undergoes the weak decay yielding a doubly heavy final baryon (N_{cc}) and we look upon it as made of one heavy diquark along with light degrees of freedom. Then, both parent and daughter particles belong to *meson*-type multiplets and the trace formalism based on the superflavor symmetry developed by Georgi *et al* [22] can be applied to straightforwardly.

In this case, two form factors are required to describe the hadronic transition at leading order in the effective theory. One of them measures the overlap of the wavefunctions of the initial and final heavy diquarks, while the other is simply the Isgur-Wise function, common to transitions among members of this kind of supermultiplet. Moreover, near zero recoil the light component should not change the overall decay of the heavy diquark.

Predictions on decay widths and branching fractions for semileptonic decays adopting the non-recoil approximation and assuming similar lifetimes for the Σ_{bc} baryon and the B_c meson can be found in Table III.

Instead, if the transition proceeds through the c -quark decay and the final baryon consists of the surviving b -quark and one light diquark, we are actually confronted to a distinct physical situation. Indeed, the final hadron belongs to a *baryon*-type supermultiplet whereas the initial one is of the *meson*-type. We are thus dealing with a transition between members of *different* supermultiplets.

Notice that although the heavy diquark would break up, the b -quark would remain within a small central region of the baryon before and after the disintegration, in this particular channel. Therefore no form factor measuring the rearrangement of the heavy component in the baryon comes into play now at leading order. Instead, we introduced a form factor related to the light component in analogy to $D \rightarrow K$ transitions. Table III shows our predictions for the partial width and branching fraction.

Lastly, we have evaluated the expected branching fraction for the two-body nonleptonic decay of the Σ_{bc} baryon into Ξ_c and J/ψ relying on the factorization hypothesis. Our results are presented in Table IV.

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Appendix

A

In this paper, the hadronic matrix elements for weak transitions of doubly heavy hadrons are evaluated employing the non-recoil approximation (the parent and daughter hadrons are at rest in their common rest frame). We will retain, nevertheless, relativistic phase space factors in the calculation of decay widths but leptonic masses will be neglected.

Throughout, subscripts 1 and 2 will design initial and final quantities. In particular m_1, v_1, p_1 and m_2, v_2, p_2 stand for the mass, four-velocity and four-momentum of the initial and final hadron respectively.

The validity of the non-recoil approximation as an estimate is strongly supported by the fact that the kinematic variable $w = v_1 \cdot v_2$ is restricted to values close to unity. To appreciate this, let us consider a hadron containing two heavy quarks denoted by Q_i and Q_j . Let quark Q_j , for example, undergo a weak decay into Q_k . At $q^2 = (p_1 - p_2)^2 = 0$,

$$(v_1 \cdot v_2)_{max} = 1 + \frac{(m_1 - m_2)^2}{2m_1 m_2} \simeq 1 + \frac{(m_{Q_j} - m_{Q_k})^2}{2(m_{Q_i} + m_{Q_j})(m_{Q_i} + m_{Q_k})} \quad (\text{A.1})$$

Considering for definiteness the decay $c \rightarrow s$ in an initial hadron containing one bottom quark/antiquark and one charm quark/antiquark,

$$(bc)_{v_1} \rightarrow (bs)_{v_2} : \quad (v_1 \cdot v_2)_{max} \simeq 1.02$$

for the mass values $m_b = 4.8$ GeV, $m_c = 1.5$ GeV and $m_s = 0.3$ GeV.

If the b undergoes the weak decay,

$$(bc)_{v_1} \rightarrow (cc)_{v_2} : \quad (v_1 \cdot v_2)_{max} \simeq 1.29$$

Analogous values should also hold for mesons and baryons with charm and bottom. Such small ranges of the velocities product should render the estimates of widths from hadronic amplitudes evaluated at $w = 1$ as an acceptable first approximation.

On the other hand, the spin symmetry exhibited by eq. (7) permits to represent the covariant spin wavefunctions of doubly heavy bound states in a similar way as mesons with a single heavy quark, that is [15] [42]

$$J^P = 0^- : \mathcal{H}(\underline{\square}) = -\sqrt{\mathcal{M}} \mathcal{P}_{+\gamma_{\nabla}} \quad ; \quad \mathcal{J}^P = \infty^- : \mathcal{H}(\underline{\square}) = \sqrt{\mathcal{M}} \mathcal{P}_{+\not{\epsilon}} \quad (\text{A.2})$$

for a s -wave pseudoscalar meson. In the case of a s -wave bound state of two quarks,

$$J^P = 0^+ : \mathcal{H}(\underline{\square}) = \sqrt{\mathcal{M}} \mathcal{P}_+ \quad ; \quad \mathcal{J}^P = \infty^+ : \mathcal{H}(\underline{\square}) = \sqrt{\mathcal{M}} \mathcal{P}_{+\gamma_{\nabla}\not{\epsilon}} \quad (\text{A.3})$$

where $P_{\pm} = (1 \pm \not{v})/2$ are energy projectors.

In this way, the hadronic matrix elements corresponding to transitions among bound states containing two heavy quarks can be evaluated according to the well-known trace formalism: [39]

$$\langle H_2(v_2) | \overline{Q}_k \Gamma Q_j | H_1(v_1) \rangle = -\hat{\eta}_{12}(v_1 \cdot v_2) \text{Tr}[\overline{\mathcal{H}}_2(v_2) \Gamma \mathcal{H}_1(v_1)] \quad (\text{A.4})$$

where the initial and final hadronic systems can be scalar, pseudoscalar, vector or pseudovector states; Γ is a combination of γ matrices and $\hat{\eta}_{12}$ plays the role of the dimensionless Isgur-Wise function for transitions between h-l mesons [15], measuring as well the overlap of the initial and final hadronic wavefunctions. Yet, $\hat{\eta}_{12}(w)$ depends on the type of hadrons involved in the process and it is not a universal function of the velocity difference only. On the other hand, unbroken spin symmetry ensures a single $\hat{\eta}_{12}$ (reduced) form factor for hadronic transitions, irrespective of Γ , between different spin-states. Finally, let us stress that $\hat{\eta}_{12}$ is not absolutely normalized to unity at zero recoil via the vector current since h-h systems do not belong to a multiplet of a heavy quark flavour symmetry group (there should be a mismatch between the parent and daughter hadron wavefunctions even at zero recoil).

A summary of the relevant hadronic matrix elements in the non-recoil approximation ($v_1 = v_2 = v$) for the weak process $Q_j \rightarrow Q_k W^*$ is given by:

$$\begin{aligned}
\langle 0^P, \epsilon_2 | V^\mu | 0^P \rangle &\simeq \pm 2 \hat{\eta}_{12} \sqrt{m_1 m_2} v^\mu \\
\langle 1^P, \epsilon_2 | A^\mu | 0^P \rangle &\simeq \pm 2 \hat{\eta}_{12} \sqrt{m_1 m_2} \epsilon_2^{*\mu} \\
\langle 0^P | A^\mu | 1^P, \epsilon_1 \rangle &\simeq \pm 2 \hat{\eta}_{12} \sqrt{m_1 m_2} \epsilon_1^\mu \\
\langle 1^P, \epsilon_2 | V^\mu | 1^P, \epsilon_1 \rangle &\simeq \mp 2 \hat{\eta}_{12} \sqrt{m_1 m_2} (\epsilon_1 \cdot \epsilon_2^*) v^\mu \\
\langle 1^P, \epsilon_2 | A^\mu | 1^P, \epsilon_1 \rangle &\simeq \mp 2 \hat{\eta}_{12} \sqrt{m_1 m_2} i \varepsilon^{\mu\nu\alpha\beta} v_{1\nu} \epsilon_{1\alpha} \epsilon_{2\beta}^*
\end{aligned} \tag{A.5}$$

where the upper(lower) sign applies when $P = \mp$ respectively; the vector and axial-vector currents are $V^\mu = \bar{Q}_k \gamma^\mu Q_j$ and $A^\mu = \bar{Q}_k \gamma^\mu \gamma_5 Q_j$. Upon contraction with the leptonic current $\bar{\ell} \gamma_\mu (1 - \gamma_5) \nu$, squaring, summing and averaging over polarization states whenever needed, the resulting differential widths for each massless lepton species and charge mode are

$$\frac{d\Gamma}{dq^2}(0^P \rightarrow 0^P)|_{vector} \simeq \frac{d\Gamma}{dq^2}(1^P \rightarrow 1^P)|_{vector} \simeq \frac{G_F^2 \lambda^{3/2}}{192 \pi^3 m_1^3} f_{12}^2 |V_{jk}|^2 \tag{A.6}$$

$$\frac{d\Gamma}{dq^2}(1^P \rightarrow 0^P)|_{axial} \simeq \frac{1}{3} \frac{d\Gamma}{dq^2}(0^P \rightarrow 1^P)|_{axial} \simeq \frac{1}{2} \frac{d\Gamma}{dq^2}(1^P \rightarrow 1^P)|_{axial} \simeq \frac{G_F^2 (\lambda^{3/2} + 12q^2 m_1^2 \lambda^{1/2})}{576 \pi^3 m_1^3} f_{12}^2 |V_{jk}|^2 \tag{A.7}$$

where $\lambda \equiv \lambda(m_1^2, m_2^2, q^2)$ denotes the Källén function, V_{jk} is the KM mixing matrix element involved in the quark's decay. The (single) hadronic form factor f_{12} at the non-recoil approximation is related to $\hat{\eta}_{12}$ by:

$$f_{12} = \sqrt{\frac{m_2}{m_1}} \hat{\eta}_{12}(w = 1) \tag{A.8}$$

assumed nearly constant over the whole available q^2 range. This introduces an uncertainty lying within the framework of the approximations made in this work. Using the ISGW model [32] at q_{max}^2 we may write

$$\hat{\eta}_{12} = \left(\frac{2\beta_1 \beta_2}{\beta_1^2 + \beta_2^2} \right)^{3/2} \tag{A.9}$$

Those values of the β_i parameter of interest in our work are collected in table A.I.

Integrating over q^2 between 0 and $q_{max}^2 = (m_1 - m_2)^2$,

$$\Gamma(0^P \rightarrow 0^P)|_{vector} \simeq \Gamma(1^P \rightarrow 1^P)|_{vector} \simeq \frac{G_F^2 m_1^5}{192 \pi^3} \varphi(x) f_{12}^2 |V_{jk}|^2 \tag{A.10}$$

$$\frac{1}{3} \Gamma(0^P \rightarrow 1^P)|_{axial} \simeq \Gamma(1^P \rightarrow 0^P)|_{axial} \simeq \frac{1}{2} \Gamma(1^P \rightarrow 1^P)|_{axial} \simeq \frac{G_F^2 m_1^5}{576 \pi^3} [\varphi(x) + \phi(x)] f_{12}^2 |V_{jk}|^2 \quad (\text{A.11})$$

where φ and ϕ are phase space factors depending on the mass ratio $x = m_2/m_1$ (neglecting lepton masses). Their expressions are: [10]

$$\begin{aligned} \varphi(x) &= \frac{1}{4} [(1 - x^4)(1 - 8x^2 + x^4) - 24 x^4 \ln x] \\ \phi(x) &= 2 (1 - x^2) [(1 + x^2)^2 + 8x^2] + 24 (1 + x^2) x^2 \ln x \end{aligned}$$

In the limit $x \rightarrow 1$, corresponding *e.g.* to the Shifman-Voloshin condition $(m_1 - m_2)^2 \ll (m_1 + m_2)^2$, *i.e.* small four-momentum transfer compared to the hadronic masses, phase space factors behave as:

$$\frac{1}{2} \phi(x) \simeq \varphi(x) \rightarrow \frac{16}{5} (1 - x)^5 \quad (\text{A.12})$$

In this extreme non-relativistic limit,

$$\begin{aligned} \Gamma(0^P \rightarrow 0^P)|_{vector} \simeq \Gamma(1^P \rightarrow 1^P)|_{vector} &\simeq \frac{1}{3} \Gamma(0^P \rightarrow 1^P)|_{axial} \simeq \Gamma(1^P \rightarrow 0^P)|_{axial} \simeq \frac{1}{2} \Gamma(1^P \rightarrow 1^P)|_{axial} \\ &\simeq \frac{G_F^2 (m_1 - m_2)^5}{60 \pi^3} f_{12}^2 |V_{jk}|^2 \end{aligned} \quad (\text{A.13})$$

Assuming in addition that the spin-singlet and spin-triplet states are perfectly mass degenerate, the spin-rule yielding a factor 3 in favor of final vector states arises from the above expressions at once. In reality, however, broken spin symmetry may spoil such simple expectations mainly because of the different available phase space in each case.

References

- [1] ATLAS internal note, Phys-NO-041 (1994); ATLAS internal note Phys-NO-058 (1994).
- [2] A. Fridman and B. Margolis, CERN preprint CERN-TH 6878/93 (1993).
- [3] P. Lebrun and R.J. Oakes, FERMILAB preprint FERMILAB-Conf-93/303 (1993).
- [4] P. Nason et al., CERN preprint CERN-TH.7134/94 (1994); A.F. Falk, M. Luke, M.J. Savage and M.B. Wise, Phys. Rev. **D 49** (1994) 555; E. Braaten, K. Cheung and T.C. Yuan, Phys. Rev. **D 48** (1993) 5049; C-H Chang and Y-Q Chen, Phys. Rev. **D 48** (1993) 4086; M. Lusignoli, M. Masetti and S. Petrarca, Phys. Lett. **B 266** (1991) 142; S. S. Gershtein, A.K. Likhoded and S.R. Slabospitsky, Int. J. Mod. Phys. **A 13** (1991) 2309.
- [5] M.J. White and M.J. Savage, Phys. Lett. **B 271** (1991) 410.
- [6] M.J. Savage and R.P. Springer, Int. J. Mod. Phys. **A 6** (1991) 1701.
- [7] M.A. Sanchis-Lozano, Phys. Lett. **B 321** (1994) 407.
- [8] E. Jenkins, M. Luke, A.V. Manohar and M.J. Savage, Nuc. Phys. **B 390** (1993) 463.
- [9] R. Casalbuoni et al, Phys. Lett. **B 302** (1993) 95; Phys. Lett. **B 309** (1993) 163.
- [10] M.A. Sanchis, Phys. Lett. **B 312** (1993) 333; Z. Phys. **C 62** (1994) 271.
- [11] A. Pineda and J. Soto, Universitat de Barcelona preprint UB-ECM-PF-94/19 (1994).
- [12] T. Mannel and G.A. Schuler, CERN preprint CERN-TH.7468/94 (1994).
- [13] E. Eichten, Nuc. Phys. B (Proc. Suppl.) **4** (1988) 179; G.P. Lepage and B.A. Thacker *ibid* 199.
- [14] Lattice 94, Nucl. Phys. (Proc. Suppl.) **B 34** (1994).
- [15] M. Neubert, Phys. Rep. **245** (1994).
- [16] F. Hussain, J.G. Körner and G. Thompson, Ann. Phys. **206** (1991) 334.
- [17] W. Kwong, J.L. Rosner and C. Quigg, Ann. Rev. Nucl. Part. Sci. **37** (1987) 325; A. Le Yaouanc, Ll. Oliver, O. Pène and J-C Raynal, Hadron Transitions in the Quark Model (Gordon and Breach, 1988).
- [18] L.S. Brown and W.I. Weisberger, Phys. Rev. **D 20** (1979) 3239.
- [19] G.P. Lepage et al, Phys. Rev. **D 46** (1992) 4052.
- [20] W. Lucha, F.F. Schöberl and D. Gromes, Phys. Rep. **200** (1991) 127.
- [21] A.F. Falk and M. Neubert, Phys. Rev. **D 47** (1993) 2965; M. Neubert, Phys. Lett. **B 322** (1994) 419.
- [22] H. Georgi and M.B. Wise, Phys. Lett. **B 243** (1990) 279; C.D. Carone, Phys. Lett. **B 253** (1991) 408.
- [23] M.J. Savage and M.B. Wise, Phys. Lett. **B 248** (1990) 177.
- [24] T. Barnes, RAL preprint RAL-94-056 (1994).

- [25] H.J. Lipkin, Phys. Lett. **B 172** (1986) 242.
- [26] S. Zouzou et al., Z. Phys. **C 30** (1986) 457.
- [27] C. Semay and B. Silvestre-Brac, Z. Phys. **C 61** (1994) 271.
- [28] A.V. Manohar and M.B. Wise, Nuc. Phys. **B 399** (1993) 17.
- [29] E.J. Eichten and C. Quigg, Phys. Rev. **D 49** (1994) 5845.
- [30] S. Nussinov and W. Wetzel, Phys. Rev. **D 36** (1987) 130.
- [31] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. **47** (1988) 511.
- [32] N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. **D 39** (1989) 799.
- [33] M. Lusignoli and M. Masetti, Z. Phys. **C 51** (1991) 549.
- [34] B. Grinstein, Ann. Rev. Nuc. Part. Sci. **42** (1992) 101.
- [35] J.D. Bjorken, Nuc. Phys. B (Proc. Suppl.) **11** (1989) 325.
- [36] T. Ito, T. Morii and M. Tanimoto, Z. Phys. **C 59** (1993) 57.
- [37] M. Anselmino et al., Rev. Mod. Phys. **65** (1993) 1199.
- [38] N.A. Viet, Syracuse University preprint SU-4240-561 (1993).
- [39] A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nuc. Phys. **B 343** (1990) 1.
- [40] Particle Data Group, Review of Particle Properties, Phys. Rev. **D 50** (1994).
- [41] T. Mannel and W. Roberts, Z. Phys. **C 59** (1993) 179.
- [42] A.F. Falk, Nuc. Phys. **B 378** (1992) 79.
- [43] V.V. Kiselev and A. Tkabladze, Phys. Rev. **D 48** (1993) 5208.
- [44] C-H Chang and Y-Q Chen, Phys. Rev. **D 49** (1994) 3399.
- [45] E. Bagan et al., Z. Phys. **C 64** (1994) 57.

Figure Captions:

Figure 1) Graphs representing the weak decay of doubly heavy hadrons. For the weak interaction, the graphs may be interpreted as Feynman diagrams. In the case of the baryon the light quark should act as spectator at low recoil, whereas the diquark can be considered as a decaying quasi-particle.

Figure 2) A visualization of the hadronic transitions: a) $\Sigma_{bc} \rightarrow N_{cc}$, b) $\Sigma_{bc} \rightarrow N_b$, c) $D \rightarrow K$. At zero recoil, the overlap factor of the light degrees of freedom in the case b) should be somehow intermediate between a) and c).

Table I.a). Partial widths of the semileptonic decays of B_c ($\bar{b}c$) in units 10^{-15} GeV. The relative branching fraction refers to the final states with a vector meson with respect to those with a pseudoscalar meson. The same input values of the parameters of the ISGW model, $V_{cb} = 0.046$ etc as in ref. [33] have been used to facilitate the comparison.

Mode	this work	[33]	[43]	[44]	[45]
$B_c \rightarrow \eta_c + \bar{\ell}\nu$	10	10.6	15	14.2	9.4
$B_c \rightarrow J/\psi + \bar{\ell}\nu$	42	38.5	44	34.4	12.2
relative BF	4.2	3.6	3	2.4	1.3
$B_c \rightarrow B_s + \bar{\ell}\nu$	18	16.4	—	26.6	14.2
$B_c \rightarrow B_s^* + \bar{\ell}\nu$	43	40.9	—	44	14.2
relative BF	2.4	2.5	—	1.7	1

Table I.b). Branching fractions from our results in Table I.a) using a lifetime of the B_c equal to 0.5 ps [33].

Mode	$BF(\%)$
$B_c \rightarrow \eta_c + \bar{\ell}\nu$	0.75
$B_c \rightarrow J/\psi + \bar{\ell}\nu$	3.15
$B_c \rightarrow B_s + \bar{\ell}\nu$	1.35
$B_c \rightarrow B_s^* + \bar{\ell}\nu$	3.22

Table II.a). Partial widths of nonleptonic decays of the B_c in units 10^{-15} GeV, where we have used the same input values as in reference [33].

Mode	this work	[33]	[44]
$B_c \rightarrow \eta_c + \pi$	2.4	2.1	3.29
$B_c \rightarrow J/\psi + \pi$	2.3	2.2	3.14
relative BF	~ 1	~ 1	~ 1

Table II.b). Branching fractions from our results in Table II.a) using a lifetime of the B_c equal to 0.5 ps [33].

Mode	$BF(\%)$
$B_c \rightarrow \eta_c + \pi$	0.18
$B_c \rightarrow J/\psi + \pi$	0.17

Table III. Partial widths of semileptonic decays of the Σ_{bc} baryon (bcq) in units 10^{-15} GeV. We have used $M_{\Sigma_{bc}} = 6.93$ GeV, $M_{\Sigma_{cc}} = 3.63$ GeV, $M_{\Sigma_{cc}^*} = 3.73$ GeV, $M_{\Lambda_b} = 6.0$ GeV [45]. For the estimates of the absolute BF we set the Σ_{bc} lifetime equal to 0.5 ps.

Mode	width	BF(%)
$\Sigma_{bc} \rightarrow \Sigma_{cc} + \ell \bar{\nu}$	40	3.0
$\Sigma_{bc} \rightarrow \Sigma_{cc}^* + \ell \bar{\nu}$	12	0.9
$\Sigma_{bc} \rightarrow \Lambda_b + \ell \nu$	34	2.6

Table IV). Width (in units of 10^{-15} GeV) and branching fraction of a two-body nonleptonic decay of the Σ_{bc} assuming its lifetime equal to 0.5 ps, $M_{\Xi_c} = 2.47$ [40] GeV and $f_{J/\psi} = 0.385$ GeV [33].

Mode	width	BF(%)
$\Sigma_{bc} \rightarrow \Xi_c + J/\psi$	10	0.8

Table A.I. Values in GeV of the β_i parameter in the ISGW model [32] needed for the evaluation of several wavefunction overlaps at zero recoil used in this work.

$i =$	K	D	B	B_s	J/ψ	B_c
β_i	0.31	0.39	0.41	0.51	0.66	0.82