

# About the realization of chiral symmetry in $QCD_2^*$

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## Abstract

Two dimensional massless Quantum Chromodynamics presents many features which resemble those of the true theory. In particular the spectrum consists of mesons and baryons arranged in flavor multiplets without parity doubling. We analyze the implications of chiral symmetry, which is not spontaneously broken in two dimensions, in the spectrum and in the quark condensate. We study how parity doubling, an awaited consequence of Coleman's theorem, is avoided due to the dimensionality of space-time and confinement. We prove that a chiral phase transition is not possible in the theory.

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# 1 Introduction

In recent work we have obtained a description of  $QCD_2$  in terms of bosonic fields by establishing through the bosonization rules a correspondence between characteristic Green functions of the bosonized theory and well known fermionic amplitudes [1]. We have discovered the crucial role of the chiral sector, which consists loosely speaking of free zero mass mesons, in attributing quantum numbers. These so-called chiral mesons are intimately related to the realization of chiral symmetry à la Berezinkii-Kosterlitz-Thouless [2] and are therefore no Goldstone bosons. The excited mesons have their origin in the gluonic interactions of the colored meson sector. In the chiral limit, the dynamics of the two sectors is completely decoupled. The only *true* baryons of the theory are also massless [3]. Their genesis requires that the chiral sector be endowed with topologically non-trivial boundary conditions. The remaining baryons arise due to an interplay between the interactions in the colored meson sector and the topological non-trivial boundary conditions of the chiral sector. Therefore they appear as pseudo-mesonic excitations on top of the *true* baryons.

The aim of this paper is to complete this description, discussing those issues which have been a matter of debate over the years. We start by recalling the properties of the spectrum and by constructing explicitly the creation operator for the massless baryons using abelian bosonization. It is a matter of notation to establish a connection with our earlier work. We next review the *BKT* phenomenon and 't Hooft's consistency condition in order to set the ground for our discussion on parity doublets. According to Coleman's theorem [4] in two dimensions chiral symmetry cannot be spontaneously broken and consequently one would expect parity doublets in the spectrum, which do not arise [5]. We shall show that the dimensionality of space-time and confinement are responsible for their absence. Finally arguments based on the large number of color approximation lead to a non-vanishing quark condensate and suggest, if naively interpreted, a phase transition between the weak and strong coupling regimes of the theory, purely on chiral symmetry arguments. We interpret this result and confirm Coleman's theorem for any  $N$ .

We shall hereafter consider only one flavor. This restriction is irrelevant in the chiral limit of the theory, i.e., for massless quarks. Under these circumstances the basis of our argumentation, namely the separation between the flavor and the color degrees of freedom remains. Therefore all our discussion can be easily generalized to the many flavor case.

## 2 The spectrum

By bosonizing  $QCD_2$  we have obtained an equivalent theory with a massless meson [1]. This so-called chiral meson is completely decoupled from the remaining fields of the theory and is not affected by the interactions. It is suggestive of a Goldstone boson, which however cannot be, due to Coleman's theorem. The remaining meson spectrum is described by means of colored mesons, leading to a stable, discrete and infinite spectrum with no parity doublets [1, 5].

The interpretation of the baryonic spectrum in the bosonized theory relies on the existence

of zero mass baryons. In the fermionic description they arise due to the kinematics of the two dimensional world [3]. In our bosonic description they are a consequence of the non-triviality of the boundary conditions of the meson fields [6, 7], in a way which we next describe.

In abelian bosonization [8] the following solitonic operators may be constructed for a non-abelian theory

$$s_{\pm}(x, t) =: \exp \left( -i\sqrt{\frac{\pi}{2}} \left( \int_{-\infty}^x d\xi \dot{\varphi}(\xi, t) \pm \varphi(x, t) \right) \right) : \quad (1)$$

and

$$s_{\pm}^{\alpha}(x, t) =: \exp \left( -i\sqrt{\frac{\pi}{2}} (-)^{\alpha} \left( \int_{-\infty}^x d\xi \dot{\eta}(\xi, t) \pm \eta(x, t) \right) \right) : , \quad \alpha = 1, 2 \quad (2)$$

Here  $\varphi$  represents the chiral meson field, a singlet under the non-abelian group, which we shall take for simplicity to be in this section  $SU(2)_C$ ;  $\eta$  is, loosely speaking, a color meson which defines an extraordinary non-local representation of the color group<sup>1</sup>;  $\pm$  represent the corresponding chiralities [10];  $s^{\alpha}$ ,  $\alpha = 1, 2$  transforms as a doublet under the color group. We shall use negative chirality fields from now on unless specifically labelled.

If  $\varphi$  and  $\eta$ , which we shall represent collectively by  $\phi$ , are endowed with non-trivial boundary conditions

$$\phi(\infty) = \sqrt{\frac{\pi}{2}} ; \quad \phi(-\infty) = 0 \quad (3)$$

the  $s$ ,  $s^{\alpha}$  operators represent the  $U(1)$  and  $SU(2)_C$  solitons respectively. Adscribing their winding numbers properly normalized to baryon number ( $B$ ) and color charge ( $T_3$ ) respectively we have for these solitons and for the quarks fields

$$q^{\alpha}(x) = \sqrt{k\mu} s(x) s^{\alpha}(x), \quad \alpha = 1, 2 \quad (4)$$

where  $k$  is a numerical constant and  $\mu$  a normal ordering mass, the quantum numbers shown in Table 1

|       |               |                |                |                |                |
|-------|---------------|----------------|----------------|----------------|----------------|
|       | $s$           | $s^1$          | $s^2$          | $q^1$          | $q^2$          |
| $B$   | $\frac{1}{2}$ | 0              | 0              | $+\frac{1}{2}$ | $+\frac{1}{2}$ |
| $T^3$ | 0             | $-\frac{1}{2}$ | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{2}$ |

Table 1: Quantum numbers of solitons and quarks

It should be noted that despite the complicated transformation properties of the  $\eta$  field, the quark field transforms as the fundamental representation of the color group. Abelian bosonization leads to a highly non-trivial theory, where the off-diagonal currents are spatially non-local. However the baryon currents for zero mass quarks are free [11].

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<sup>1</sup>We follow in this section the work of Halpern [9] very closely. One can look up in it the properties of the non-local representation associated with the  $\eta$  field. The boundary conditions on this field play a very important role in the definition of its transformation properties under the color group, see also [7] for a discussion on this point.

Let us construct the analog of the four dimensional baryon currents [12, 13]

$$B(x) = \varepsilon^{\alpha\beta} q_\alpha(x) q_\beta(x) \quad (5)$$

Since  $[\varphi, \eta] = 0$  its bosonized equivalent is given by

$$B(x) = k\mu s(x) s(x) s^1(x) s^2(x) \quad (6)$$

Using  $: e^C :: e^D := e^{[C^+, D^-]} : e^{C+D} :$ , where the superindices  $\pm$ , as is customary in field theory, indicate the sign of the energy exponents in the field expansion, and the free commutation relations for the boson fields [8] we obtain

$$B(x, t) \stackrel{\varepsilon \rightarrow 0}{\approx} k\mu e^{[A^+(x+\varepsilon, t), A^-(x, t)]_\varphi} e^{-[A^+(x+\varepsilon, t), A^-(x, t)]_\eta} : e^{2A[\varphi]} : \mathbf{1}_\eta \quad (7)$$

where

$$A^\pm[\phi] = -i\sqrt{\frac{\pi}{2}} \left( \int_{-\infty}^x d\xi \dot{\phi}^\pm(\xi, t) - \phi^\pm(x, t) \right) \quad (8)$$

Since for free fields the  $\varphi$  and  $\eta$  commutators are identical we obtain

$$B(x, t) = k\mu : \exp \left( -i\sqrt{2\pi} \left( \int_{-\infty}^x d\xi \dot{\varphi}(\xi, t) - \varphi(x, t) \right) \right) : \quad (9)$$

which certainly creates a baryon number one state. The mass bosonization equation [1] tells us, both in the abelian and non abelian schemes,

$$q_-^{\alpha+} q_{A+} = k\mu s_-^+ s_+ s_-^{\alpha+} s_{A+} = k\mu e^{i\sqrt{2\pi}\varphi} g_A^\alpha \quad (10)$$

Since

$$s_-^+ s_+ = e^{i\sqrt{2\pi}\varphi} \quad (11)$$

then

$$s_-^{\alpha+} s_{A+} = g_A^\alpha \quad (12)$$

Thus an exact relation can be found between the matrix elements of the non-abelian color fields of ref.([1]) and those of the soliton operator. Using the exact solutions of the free and Wess-Zumino-Witten theory [14] we obtain

$$\langle e^{i\sqrt{2\pi}\varphi(x, t)} e^{-i\sqrt{2\pi}\varphi(0)} \rangle = \langle g_1^{1+}(x, t) g_1^1(0) \rangle = \frac{1}{2\pi k\mu\sqrt{x_+ x_-}} \quad (13)$$

From these expressions one can calculate the correlators for the solitonic operators which are given by

$$\langle s_\pm^+(x, t) s_\pm(0) \rangle = \langle s_\pm^{\alpha+}(x, t) s_\pm^\alpha(0) \rangle = \left( \frac{-i}{2\pi k\mu x_\mp} \right)^{\frac{1}{2}} \quad (14)$$

leading to the baryon correlator

$$\langle B^+(x, t) B(0) \rangle = \frac{-1}{4\pi^2 x_+^2} \quad (15)$$

which corresponds to a zero mass particle.  $B(x, t)$  projects onto a zero mass baryon number one state.

We have throughout assumed free  $\varphi$  and  $\eta$  fields. This is consistent with the fact that the interaction is asymptotically free [11]. Looking back at the derivation we see in Eq.(7), that because the fields are free the color dependence banishes, establishing that the baryon number one soliton current associated with the massless baryon, depends solely on the chiral field.

Let us recover our previous result [1] in this more transparent language. The baryon correlation function in terms of the  $\varphi$  and  $\eta$  meson fields is given by

$$G^B(x_a, x_b; y_a, y_b) = k^2 \mu^2 \langle B^{\varphi+}(x_a, x_b) B^\varphi(y_a, y_b) \rangle \langle B^{\eta+}(x_a, x_b) B^\eta(y_a, y_b) \rangle \quad (16)$$

If one uses a specific gauge [15] such that

$$B^\varphi = s(x_a) s(x_b) \quad \text{and} \quad B^\eta = \varepsilon_{\alpha\beta} s^\alpha(x_a) s^\beta(x_b) \quad (17)$$

it is easy to show, that in the long distance regime ( $l_a = x_a - y_a; l_1 \rightarrow \infty$ ) the zero mass particle dominates the  $\varphi$  piece of the correlator leading to

$$\langle B^{\varphi+}(x_a, x_b) B^\varphi(y_a, y_b) \rangle \sim \frac{1}{l_1^2} \quad (18)$$

Since  $G^B$  is the true propagator, its long distance behavior is a sum of Yukawas. This together with the last equation implies that the  $B^\eta$  correlator cannot have a physical long distance behavior and the intermediate states that saturate it must be unphysical mesons [1].

### 3 Coleman's theorem and parity doubling

It is well known that in two dimensions chiral symmetry cannot be spontaneously broken [4]. In  $QCD_2$  it is realized however not in a conventional way but in an *almost* spontaneously broken fashion [17]. This so-called BKT phenomenon is characterized by a correlation function that falls off as a power law [1]

$$\langle q\bar{q}(x)q\bar{q}(0) \rangle \stackrel{x^2 \rightarrow \infty}{\approx} \langle e^{i\sqrt{\frac{4\pi}{N}}\varphi(x)} e^{-i\sqrt{\frac{4\pi}{N}}\varphi(0)} \rangle \approx \frac{1}{(2\pi k^2 \mu^2 x^2)^{\frac{1}{N}}} \quad (19)$$

where  $N$  is the number of colors. Thus the chiral meson, although not a Goldstone boson, governs the long distance behavior of this correlator, a property shared with its four dimensional analog. This similarity of behavior goes even further. G. 't Hooft analyzed the restrictions imposed by the axial anomaly equation on the spectrum of confining theories with zero mass quarks [16]. In two dimensions the anomaly appears by considering the vector current (axial current) two point function (recall  $j_5^\mu = \varepsilon^{\mu\nu} j_\nu$ ). The discontinuity of this amplitude, a signature of the anomaly [19], can also be calculated by saturating over

all possible intermediate states. In the chiral limit, only massless boundstates contribute at zero momentum. In  $QCD_2$  we have both zero mass mesons and baryons. The massless mesons saturate the anomaly as occurs in the spontaneously broken theories [16, 20]. The zero mass baryons do not contribute [3].

Since chiral symmetry is not spontaneously broken the spectrum should contain parity doublets. However they do not appear, again in direct correspondence with the four dimensional situation. However the mechanism which eliminates the doubling must be very peculiar of two dimensions.

In the bosonized form of the theory the chiral and color sectors are completely decoupled. The chiral sector is generated by the zero mass meson field. Gauge interactions are absent. Since chiral symmetry is not broken one should expect, besides the  $\varphi$  meson, a chiral partner, i.e., a  $\sigma$ . The conservation of the  $U(1)_{axial}$  current leads to

$$\partial_\mu j_5^\mu = 0 \quad (20)$$

while the dimensionality of space-time implies

$$j_5^\mu = \varepsilon^{\mu\nu} j_\nu \quad (21)$$

Since  $j_5^\mu \sim \partial^\mu \varphi$ , Eq.(20) leads to the existence of a pseudoscalar zero mass boson

$$\square \varphi = \partial_\mu j_5^\mu = 0 \quad (22)$$

Moreover the current associated with  $U(1)_{vector}$

$$j_\mu \sim \varepsilon_{\mu\nu} \partial^\nu \varphi \quad (23)$$

is trivially conserved and therefore no  $\sigma$  meson is required. Eq.(23) is not a Noether current. It is conserved irrespective of the dynamics and has therefore a topological character. Thus no zero mass doublets exist!

The color sector is determined by the minimally coupled Wess-Zumino-Witten model, which realizes by construction  $SU(N)_{vector}$  locally and therefore also globally. The vacuum of the theory is also invariant under this group since the symmetry cannot be broken spontaneously. On the other hand  $SU(N) \otimes SU(N)$  is explicitly broken. The breaking is caused by the color anomaly, which in the bosonized language appears explicitly in the action [1, 21]

$$S_{WZW}(g, A_\mu) = S_{WZW}(g) + \int d^2x \text{tr}(A_+ J_- + A_- J_+ + \frac{1}{4} A_+ A_- - \frac{1}{4\pi} A_+ g A_- g^+) \quad (24)$$

where  $S_{WZW}$  is the WZW action and

$$J_\mp^a = -\frac{i}{2\pi} \text{tr}(\partial_\mp g^{\mp 1}) g^{\pm 1} T^a \quad (25)$$

The vacuum of the theory is therefore not invariant under  $SU(N) \otimes SU(N)$ . Finally parity is a good symmetry of the WZW model, because this is exactly what happens in  $QCD$  [22].

The spectrum of the resonant mesons arises from this action. The symmetries of the hamiltonian and the properties of the vacuum impose severe restrictions on the possible allowed quantum numbers and degeneracies of these states. In two dimensions the representations of Poincare's group do not require angular momentum and therefore only a principal quantum number  $n$  associated with the energy of the boundstate will characterize the space-time component of the wave function. Parity,  $P$ , will also be a good quantum number of the states.

In two  $(1+1)$  dimensions degenerate opposite parity states can only arise due to internal degrees of freedom. In massless  $QCD_2$  the internal degrees of freedom are flavor and color. In the bosonized theory flavor only appears in the chiral sector, which on the other hand is color blind. As we have seen, the chiral sector avoids parity doubling through the relation between the axial and vector currents, which arises as a consequence of the dimensionality of space-time. Color describes the dynamics of the WZW model and therefore of the other sector of the bosonized theory. The fields describing this action are flavor singlets, and therefore this theory is flavorless (insipid). A degeneracy between states of different parity can only occur if the color structure of the model allows it. This is not the case due to confinement. Let us be more precise. Imagine for a moment that  $SU(N) \otimes SU(N)$  were not broken. In this scenario we would have many degenerate multiplets of opposite parity. For example in the case of two colors, the only mesonic representations which satisfy confinement are of the form

$$\left(\frac{m}{2}, \frac{m}{2}\right), m \in N \quad (26)$$

Their decomposition in terms of the vector color symmetry is given by

$$\left(\frac{m}{2}, \frac{m}{2}\right) \Rightarrow 0 \oplus 1 \oplus 2 \oplus \dots \oplus m \quad (27)$$

This multiplets contain in general states of different parity. Recall for example the  $(\sigma, \vec{\pi})$  representation of  $SU(2) \otimes SU(2)$ . However there is only one singlet in each representation. So, once we impose confinement, the degeneracy will disappear. The argument generalizes to  $N$  colors. <sup>2</sup>

Thus the dimensionality of space-time and the realization of the color symmetry in the Wess-Zumino-Witten lagrangian conspire to eliminate parity doubling, explaining why the boundstates of the meson equation depend exclusively of  $n$ , which also determines the parity  $P = (-)^{n+1}$  [5]. Similar arguments can be developed for baryons.

## 4 The chiral phase transition

As we have seen in the previous section, Eq.(19), in the large  $N$  limit the quark condensate does not vanish. This has been considered by many people as a signature of spontaneously

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<sup>2</sup>A caveat:  $SU(N) \otimes SU(N)$  is explicitly broken in the confining theory and one could argue, that the breaking mechanism could make opposite parity singlets coalesce to the same multiplet. This can not be, since it would imply, that the symmetry is broken to a larger group than  $SU(N)_{vector}$ .

broken chiral symmetry and therefore as a loophole of Coleman's theorem [18, 23]. We next show that this interpretation is incorrect.<sup>3</sup>

The cornerstone of the suspicion of a chiral phase transition is the singular behavior of the quark condensate  $\langle q\bar{q} \rangle$ , since this expectation value is an order parameter for the breaking of axial flavor symmetry. Taking the behavior of the condensate in the large  $N$  limit one can envisage a phase change from a situation in which axial symmetry is conserved ( $\langle q\bar{q} \rangle = 0$ ), to one in which it is broken ( $\langle q\bar{q} \rangle \neq 0$ ). Since 't Hooft's calculation [5] was performed in this latter case, it could seem that his results would correspond to one of the phases of the theory and therefore not extendible to the other regime. The authors which have taken this position consider 't Hooft's calculation as the weak coupling limit of the theory, since the transition region occurs for  $e' \sim M \rightarrow 0$ ,  $M$  being the quark mass and  $e'$  is the effective coupling,  $e' = \frac{e}{\sqrt{N}}$ . If one accepts this argumentation the physics for finite  $N$  must be substantially different from that of 't Hooft's model and corresponds to a phase where chiral symmetry is restored [24]. Note that this way of thinking requires an  $N$  dependence of the quark mass, which we consider academic, since quark masses are external parameters in  $QCD$  and moreover are associated with flavor, not with color.

Our study of bosonized  $QCD_2$  allows us to clarify the above statements. The exact calculation of the quark correlator shows an anomalous behavior only if  $M = 0, N \approx \infty$ . This behavior is controlled (see Eq.(19)) by the chiral sector of the theory. Let us repeat the argumentation, leading to this formula for the chiral condensate. Using the bosonization formula Eq.(10) we obtain for it

$$\langle q\bar{q} \rangle \sim \langle 0 | : e^{i\sqrt{\frac{4\pi}{N}}\varphi} : | 0 \rangle \langle 0 | tr(g) | 0 \rangle \quad (28)$$

In this expression the vacuum expectation value (vev) of  $tr(g)$  is always different from zero, since the  $SU(N) \otimes SU(N)$  symmetry is explicitly broken. Thus only the vev of the chiral operator  $e^{i\sqrt{\frac{4\pi}{N}}\varphi}$  can be the cause of the vanishing of the condensate. This is precisely what happens. The chiral operator changes under transformations of the  $U(1) \otimes U(1)$  group, defined by  $\varphi \rightarrow \varphi + \alpha$  as

$$e^{i\sqrt{\frac{4\pi}{N}}\varphi} \rightarrow e^{i\sqrt{\frac{4\pi}{N}}(\varphi+\alpha)} \quad (29)$$

If the vacuum is invariant under  $U(1) \otimes U(1)$  (Coleman's theorem) the vev must vanish. This vev is therefore the order parameter in the bosonized theory for any phase transition associated with chiral symmetry.

In the large  $N$  limit the vev of the quark condensate does not vanish. Naively this would imply that our order parameter does not vanish and that the vacuum would break spontaneously the symmetry. This argument is incorrect. In the large  $N$  limit the fluctuations of the  $\varphi$  field disappear and  $e^{i\sqrt{\frac{4\pi}{N}}\varphi}$  becomes the unit operator, so that the quark condensate becomes in this limit purely an  $SU(N)$  object ( $\langle tr(g) \rangle$ ), which is different from zero due to the gauge interaction. Thus our order parameter, which was such because it transformed

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<sup>3</sup>A beautiful discussion about the long distance behaviour of the chiral condensate and its relation to the  $\frac{1}{N}$  expansion for the  $SU(N)$  Thirring model can be found in ref.[17].

non trivially under the group, ceases to be an order parameter in the large  $N$  limit because it becomes the unit operator which is trivially invariant under the group. In this way, the fact that the condensate becomes different from zero in the large  $N$  limit is independent of chiral dynamics and simply reflects the fact that  $SU(N) \otimes SU(N)$  is explicitly broken.

## 5 Conclusion

We have used abelian and non-abelian bosonization techniques to study the properties of two-dimensional  $QCD$ . The bosonization rules have provided a well defined connection between the Green functions of the bosonized models and those of their fermionic counterparts. The crucial importance of the apparently naive chiral sector has been unveiled, both for mesons and baryons, and the role of the gauge dynamics in the formation of the spectrum has been clarified.

The zero mass baryon is essential to describe the excited baryons. Its existence is guaranteed by the presence of a topological chiral sector. We have constructed its current operator and described the properties of the baryonic spectrum as obtained from the four point correlators and its absence from the chiral anomaly.

The chiral meson is instrumental in describing the flavor symmetry à la BKT. This mechanism converts the impossibility of manifesting itself as the Goldstone boson of a spontaneously broken symmetry, into a dominance of the long range behavior of the correlators.

The resonant mesonic spectrum has its origin in the dynamics of color. The non-existence of parity multiplets is due to the dimensionality of space-time and confinement. The latter mechanism is a consequence of the gauge coupling, which also leads to an explicit breaking of the axial color symmetry in such a way as to preserve parity and  $SU(N)_{vector}$ . In bosonized  $QCD_2$  this breaking appears as an explicit term in the lagrangian, while in its fermionic equivalent version it is due to the anomaly, and therefore a consequence of the non-invariance of the fermionic measure under the axial color symmetry.

We have analyzed the possibility of a phase transition in  $QCD_2$ . The cornerstone of this proposal is the behavior of the chiral condensate in the large  $N$  limit. We have proven that only one phase exists and that the apparent anomalous behavior of the chiral order parameter is a consequence of the trivialization of the chiral sector in that limit. We have shown that the physics becomes independent of the chiral dynamics. We therefore must conclude that all argumentation aiming at justifying the existence of a phase transition in  $QCD_2$ , based on the spontaneous symmetry breaking of the  $U(1) \otimes U(1)$  symmetry is spurious and interprets incorrectly the mechanisms regarding the symmetries of  $QCD_2$ .

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