Generalized screened potential model

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Abstract

A new non relativistic quark model to calculate the spectrum of heavy quark mesons is developed. The model is based on an interquark potential interaction that implicitly incorporates screening effects from meson-meson configurations. An analysis of the bottomonium spectrum shows the appearance of extra states as compared to conventional non screened potential models.

1 Introduction

Non Relativistic Quark Models (NRQM) of hadron structure are based on the consideration of effective quark degrees of freedom bound by an interquak interaction potential. For heavy quark mesons $\left(Q\overline{Q}\right)$, in particular bottomonium $(Q = b)$ to which we shall restrict our attention, the form of this potential can be inferred from lattice QCD. More precisely, one can calculate in the lattice $E(r)$, the energy of two static color sources, Q and $\overline{Q}$, in terms of the $Q - \overline{Q}$ distance. By identifying $E(r)$ with the sum of the masses of the Quark (m_Q) and the antiQuark $\left(m_{\overline{Q}}\right)$ plus the $Q\overline{Q}$ potential $V(r)$ one gets $V(r) = E(r) - m_Q - m_{\overline{Q}}$. $E(r)$ is calculated in the lattice from a correlation function.

In the so called quenched approximation (only the bare valence $Q_0\overline{Q}_0$ configuration) a Cornell type of potential, $V_0(r) = \sigma r - \frac{\chi}{r} + E_0$ where σ, χ and E_0 are constants, comes out (see for example [1]). This simple potential form provides a reasonable overall description of the masses of the low lying heavy quarkonia states [2] although some refinements are needed to reach a precise fit (see for example [3]). The form of $E(r)$ is altered when

sea quarks are also taken into account (unquenched lattice calculation). Actually the presence of $q\overline{q}$ pairs where q stands for a light quark $(q = u, d, s)$ gives rise to a screening of the color charges of the bare valence quarks Q_0 and $\overline{Q}_0$. A potential parametrization of this effect was first proposed in the late eighties from a lattice calculation in the two color case including dynamic Kogut-Susskind fermions and with a lattice spacing fixed from the ρ mass [4]. The resulting Quark-antiQuark screened potential form was used, with parameters fixed phenomenologically, to analyze heavy quark mesons as $Q\overline{Q}$ bound states, Q standing for an effective quark [5, 6]. However, more recent lattice data [7] that take into consideration interacting bare valence $Q_0\overline{Q}_0$ and meson $(Q_0\overline{q})$ - meson $\left(\overline{Q}_0 q\right)$ configurations (see next Section), suggest that the form of the potential should be different for energies below and above a meson - meson threshold. In this article we

try to go a step further in the analysis of the heavy quark meson spectrum within a non relativistic quark model framework by proposing the form that a generalized interquark potential incorporating screening may have below and above a meson-meson threshold. This proposal is based on the assumption that screening effects are mainly due to the formation of meson $(Q_0\overline{q})$ - meson $\left(\overline{Q}_0 q\right)$ structures since mesons are color singlets. Then we use the lattice results for the static interquark energy $E(r)$, when the bare valence quark and meson-meson configurations are considered altogether, to build the screened potential. The contents of the article are organized as follows. In Section 2 a brief review

of the lattice results for $E(r)$ is presented. From them a generalized screened potential is defined. The resulting model is applied, in Section 3, to calculate the bottomonium spectrum and to analyze the spectral effect of screening by comparing the masses obtained with the ones calculated from a non screened Cornell potential. Finally in Section 4 our main results and conclusions will be summarized.

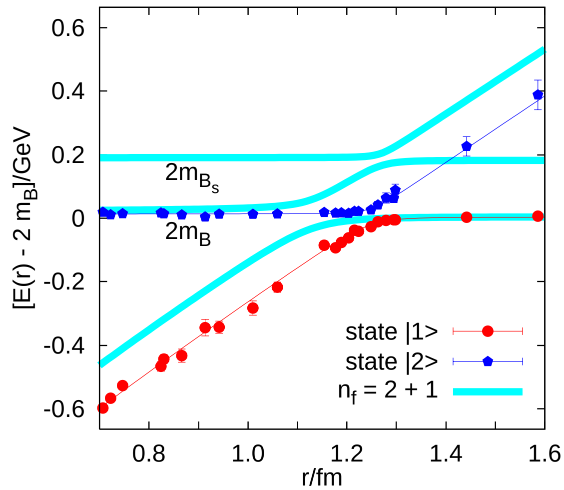

Figure 1: Calculated $b\bar{b}$ energy from lattice QCD when a $B\overline{B}$ configuration with mass $2m_B$ is implemented: circles and pentagons over the thin lines. Educated guess for the case of two meson-meson configurations, $B\overline{B}$ with mass $2m_B$ and $B_s\overline{B}_s$ with mass $2m_{B_s}$: thick lines. From reference [7].

2 Generalized Screened Potential Model (GSPM)

A lattice calculation of $E(r)$, implying the diagonalization of a correlation matrix involving the bare valence $Q_0\overline{Q}_0$ and meson $(Q_0\overline{q})$ - meson $\left(\overline{Q}_0 q\right)$ interacting configurations, has been carried out in reference [7]. The results when only one meson (B) - meson $\left(\overline{B}\right)$ configuration, with mass $2m_B$, is considered apart from the bare valence quark $\left(Q_0\overline{Q}_0\right)$ are drawn in Fig. 22 of this reference that we reproduce here for completeness as Fig. 1 The two thin curved lines following lattice data (circles and pentagons) represent the

calculated $E(r) - 2m_B$ when only the meson-meson configuration $B\overline{B}$ is implemented whereas the three thick lines correspond to an educated guess for the case of $B\overline{B}$ and $B_s\overline{B_s}$ configurations. We should realize that in both cases $E(r)$ has, when not close to any threshold, a Cornell type form. It is important to emphasize that $E(r)$ from Fig. 1

expresses the energy of two static color sources, Q and $\overline{Q}$, implicitly incorporating screening effects, in terms of the $Q-\overline{Q}$ distance. Notice that Q can be interpreted as a dressed valence quark (different from the bare valence quark $(Q \neq Q_0)$) since the $Q\overline{Q}$ interaction incorporates the effect of meson $(Q_0\overline{q})$ - meson $\left(\overline{Q}_0 q\right)$ configurations. The Generalized

Screened Potential Model (GSPM) is based on the assumption that the dressed valence $Q\overline{Q}$ configuration represents, regarding the spectrum, an effective description of a real meson. Then the meson masses can be calculated from $Q\overline{Q}$ by solving the Schrödinger equation for a $Q-\overline{Q}$ Generalized Screened Potential (GSP) interaction. In order to define the GSP let us start by defining from $E(r)$ an effective quark interaction potential as

$$V(r) \equiv E(r) - m_Q - m_{\overline{Q}}$$

where the masses m_Q and $m_{\overline{Q}}$ are parameters to be fixed phenomenologically. To analyze

the form of $V(r)$ let us consider the more general two meson-meson configuration case in Fig. 1. Let us name the first (second) threshold as T_1 (T_2) (in Fig. 1 $T_1 = B\overline{B}$ $\left(T_2 = B_s\overline{B_s}\right)$. Let us realize that the static approach implies that the two mesons forming the threshold T_i are in a relative $S-$ wave so that the threshold mass M_{T_i} corresponds to the sum of the masses of the mesons. Thus in Fig. 1 $M_{T_1} = 2m_B$ and $M_{T_2} = 2m_{B_s}$. As the forms of $E(r)$ are different below M_{T_1}, in between M_{T_1} and M_{T_2}, and above M_{T_2},

the potential $V(r)$ has different forms in these energy regions. In this sense $V(r)$ is an energy dependent potential. In practice this means that $Q\overline{Q}$ bound states with masses $M_{Q\overline{Q}}$ belonging for example to the energy region $0 < M_{Q\overline{Q}} < M_{T_1}$ should be obtained by solving the Schrödinger equation with the form of the potential corresponding to this energy region and so on. More precisely, in the first energy region defined by

$0 < M_{Q\overline{Q}} < M_{T_1}$, this is for $M_{Q\overline{Q}} \in [M_{T_0}, M_{T_1}]$ where we have defined $M_{T_0} \equiv 0$ in order to unify the notation (let us realize that T_0 does not correspond to any real meson-meson threshold), the form of the potential $V(r)$ will be called $V_{\left[M_{T_0},M_{T_1}\right]}(r)$. This form is given by $V_{\left[M_{T_0},M_{T_1}\right]}(r) = E(r) - m_Q - m_{\overline{Q}}$ with $E(r)$ corresponding to the lower thick line in Fig. 1. According to the form of $E(r)$ when not close to threshold, this potential has at short distances the Cornell type form $\left(\sigma r - \frac{\chi}{r} + V_0\right)$. We shall include the constant V_0 in the definition of the quark and antiquark masses so that we shall write the potential as $\sigma r - \frac{\chi}{r}$. As can be checked from Fig. 1 this form maintains up to a distance close below the crossing distance r_{T_1} defined from $V_{\left[M_{T_0},M_{T_1}\right]}(r_{T_1}) = \sigma r_{T_1} - \frac{\chi}{r_{T_1}} = M_{T_1} - m_Q - m_{\overline{Q}}$. Then $V_{\left[M_{T_0},M_{T_1}\right]}(r)$ starts to flatten approaching its asymptotic value in this energy region $M_{T_1} - m_Q - m_{\overline{Q}}$. In the second energy region defined by $M_{T_1} < M_{Q\overline{Q}} < M_{T_2}$ or

$M_{Q\overline{Q}} \in [M_{T_1}, M_{T_2}]$, the form of the potential $V(r)$ will be called $V_{\left[M_{T_1},M_{T_2}\right]}(r)$. This form is given by $V_{\left[M_{T_1},M_{T_2}\right]}(r) = E(r) - m_Q - m_{\overline{Q}}$ with $E(r)$ corresponding to the intermediate thick line in Fig. 1. Therefore it is equal to $M_{T_1} - m_Q - m_{\overline{Q}}$ from $r = 0$ up to a distance close below r_{T_1} , then it rises until getting for a distance close above r_{T_1} the form $\sigma r - \frac{\chi}{r}$. This form is maintained up to a distance close below the crossing distance r_{T_2} defined from $V_{\left[M_{T_1},M_{T_2}\right]}(r_{T_2}) = \sigma r_{T_2} - \frac{\chi}{r_{T_2}} = M_{T_2} - m_Q - m_{\overline{Q}}$ where $V_{\left[M_{T_1},M_{T_2}\right]}(r)$ starts to flatten approaching its asymptotic value $M_{T_2} - m_Q - m_{\overline{Q}}$. This analysis of the two threshold case

can be easily generalized to the general many threshold case by assuming that in between any two thresholds the potential form is similar to $V_{\left[M_{T_1},M_{T_2}\right]}(r)$ through substitution of the corresponding thresholds. For the sake of simplicity we shall reduce the size of the transition regions, from the Cornell to the flat potentials, just to the crossing points r_{T_i}. The Generalized Screened Potential (GSP) $V_{GSP}(r)$ is then defined as:

$$V_{GSP}(r) = V_{\left[M_{T_{i-1}}, M_{T_i}\right]}(r) \quad \text{if} \quad M_{T_{i-1}} < M_{Q\overline{Q}} \leq M_{T_i} \tag{1}$$

with $i \geq 1$, and where the forms of the potential in the different spectral regions are:

$$V_{\left[M_{T_0}, M_{T_1}\right]}(r) = \begin{cases} \sigma r - \frac{\chi}{r} & r \leq r_{T_1} \\ \\ M_{T_1} - m_Q - m_{\overline{Q}} & r \geq r_{T_1} \end{cases} \tag{2}$$

and

$$V_{\left[M_{T_{j-1}}, M_{T_j}\right]}(r) = \begin{cases} M_{T_{j-1}} - m_Q - m_{\overline{Q}} & r \leq r_{T_{j-1}} \\ \\ \sigma r - \frac{\chi}{r} & r_{T_{j-1}} \leq r \leq r_{T_j} \\ \\ M_{T_j} - m_Q - m_{\overline{Q}} & r \geq r_{T_j} \end{cases} \tag{3}$$

for $j > 1$ with the crossing distances $r_{T_{j-1}}$ defined by

$$\sigma r_{T_{j-1}} - \frac{\chi}{r_{T_{j-1}}} = M_{T_{j-1}} - m_Q - m_{\overline{Q}} \tag{4}$$

For instance the generalized screened potential $V_{GSP}(r)$ for $b\overline{b}$ states with $I^G(J^{PC}) = 0^+(0^{++})$ quantum numbers, whose first threshold is $B\overline{B}$, is drawn in Fig. 2 for the first and second energy regions. It is important to emphasize that $V_{GSP}(r)$ defined by (1) is a strictly confining potential (it always rises linearly from any threshold) so that its spectrum only has $Q\overline{Q}$ bound states.

3 Bottomonium

Bottomonium, made of heavy quarks $\left(b\overline{b}\right)$ is the better framework, due to its non relativistic character, for the application of the GSPM we have developed. One should keep in mind though that even in this case the GSPM may be representing a rather simple approach to a real meson description. On the one hand the model only incorporates screening from meson - meson channels and no threshold widths have been taken into account. Moreover the same effect from thresholds with $s\overline{s}$, $u\overline{u}$ or $d\overline{d}$ content has been considered but it could be different for thresholds with $s\overline{s}$ content. On the other hand the Cornell potential form of $V_{GSP}(r)$ when not close to any threshold, $\sigma r - \frac{\chi}{r}$, does not contain spin dependent terms that, apart from relativistic corrections, we know may give significant contributions to the masses of the lower spectral states. Anyhow, keeping in mind these possible shortcomings, we think it is worthwhile to examine the physical

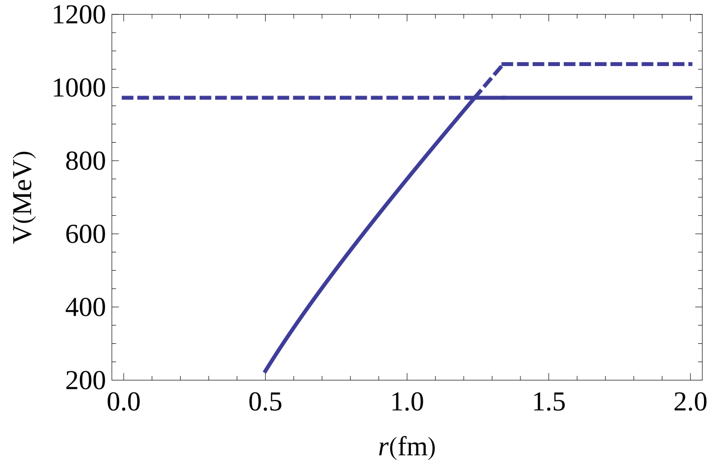

Figure 2: Generalized screened potential $V_{GSP}(r)$. The solid (dashed) line indicates the potential in the first (second) energy region for $0^{+}(0^{++})$ $b\overline{b}$ states with $m_b = 4793$ MeV, $\sigma = 850$ MeV/fm, $\chi = 100$ MeV.fm, $M_{T_1} = 10558$ MeV and $M_{T_2} = 10650$ MeV (values of the parameters and threshold masses from Section 3).

consequences deriving from this simple dynamic model for bottomonium to try to learn from them possible avenues for future progress. In this Section we proceed to the calculation of the bottomonium spectrum. For this purpose we fix first the values of the parameters of the model and we list next the open flavor meson-meson threshold masses to be considered. Then we detail the calculation of the spectrum for a particular case and compile the bulk of results. From them the spectral effect of screening is analyzed.

3.1 Parameters

To establish a criterion to fix the parameters σ, χ and m_Q let us realize that in the first spectral region $[M_{T_0}, M_{T_1}]$, for energies far below the first threshold, we hardly expect any screening effect. In other words the Cornell potential

$$V_{Cor}(r) \equiv \sigma r - \frac{\chi}{r} \qquad (r : 0 \rightarrow \infty) \tag{5}$$

should describe reasonably well this part of the spectrum. Actually this is the case. It turns out that for a value of the Coulomb strength $\chi = 100$ MeV.fm corresponding to a strong quark-gluon coupling $\alpha_s = \frac{3\chi}{4\hbar} \simeq 0.38$ (in agreement with the value derived from QCD from the hyperfine splitting of $1p$ states in bottomonium [8]), one can choose correlated values of σ and m_Q to get such description. In this regard, as we are dealing with a spin independent potential, we may compare as usual the calculated $s-$ wave states with spin-triplets, the $p-$ wave states with the centroids obtained from data and the $d-$ wave states with the only existing experimental candidates. Indeed it would be better a comparison with the centroids for all states but the dearth of spin singlet data makes this unfeasible. Thus, by choosing for example $\sigma = 850$ MeV/fm, a value within the acceptable interval of values for the string tension in QCD, and $m_b = 4793$ MeV, the differences from the calculated Cornell masses to data below the first corresponding thresholds turn out to be less than 30 MeV what constitutes a reasonable overall description. We shall adopt these values so that the set of parameters that will be used henceforth is

$$\begin{aligned} \sigma &= 850 \text{ MeV/fm} \\ \chi &= 100 \text{ MeV.fm} \\ m_b &= 4793 \text{ MeV} \end{aligned} \qquad (6)$$

Let us advance that the degree of arbitrariness in the choice of the parameters has no significant effect on the spectrum when they are required to be correlated for a reasonable description of the lowest spectral states.

3.2 $I(J^{PC})$ Thresholds

In order to apply the GSPM to a particular set of bottomonium states with definite $I(J^{PC})$ we need the masses M_{T_i} for meson $(Q_0\overline{q})$ - meson $(\overline{Q}_0 q)$ thresholds $(q : u, d, s)$ coupling to these quantum numbers. From these masses the crossing radii r_{T_i} are immediately calculated from (4). Unfortunately not all thresholds are experimentally well known. For example there is a known $0(1^{--})$ threshold from $B^0\overline{B_1(5721)}^0$ where $B_1(5721)^0$ is a $({}^3P_1 -{}^1 P_1)$ mixing state (see for example [9]). However the $({}^3P_1 -{}^1 P_1)$ partner of $B_1(5721)^0$ is not known yet although we expect this missing state to have a mass close to that of $B_1(5721)^0$. We shall call it $B_1(?)$. Therefore the mass of the threshold $B^0\overline{B_1\,(?)}^0$ is not well known. In other cases the situation is reversed since a threshold mass is known but its quantum numbers are not well established. This is for example the situation for thresholds including the meson $B_J^*(5732)$ that we shall tentatively assign to $J^P = 0^+$. The list of thresholds for bottomonium, with their corresponding masses and crossing radii, appear in Tables 1 and 2. The lack of knowledge about further thresholds prevents extending the list to higher energies. It is important to remark that we have used isospin symmetry to construct thresholds with well defined isospin. This means that we are neglecting the mass differences between the electrically neutral and charged members of the same isospin multiplet, for example B^0 and $B^{\pm}$ with PDG quoted masses [9] 5279.53 ± 0.33 MeV and 5279.15 ± 0.31 MeV respectively. Regarding the C parity for a threshold formed by two mesons $\mathfrak{M}_1$ and $\mathfrak{M}_2$ we can construct the combinations $(\mathfrak{M}_1\mathfrak{M}_2 \pm c.c)$ with C parity $+$ and $-$ respectively. Notice though that if $\mathfrak{M}_2 = \overline{\mathfrak{M}_1}$

$I(J^{PC})$	T_i	Bottomonium Thresholds	M_{T_i} (MeV)	r_{T_i} (fm)
$0(0^{++})$				
	T_1	$\left(B^0\overline{B}^0, B^+B^-\right)_{I=0}$	10558	1.24
	T_2	$\left(B^{*0}\overline{B^*}^0, B^{*+}B^{*-}\right)_{I=0}$	10650	1.34
	T_3	$B_s^0\overline{B_s}^0$	10734	1.43
$0(1^{++})$				
	T_1	$(B^0\overline{B^*}^0, B^+\overline{B^*}^-)_{I=0} + c.c.$	10604	1.29
	T_2	$B_s^0\overline{B_s^*} + c.c.$	10782	1.49
$0(2^{++})$				
	T_1	$\left(B^{*0}\overline{B^*}^0, B^{*+}B^{*-}\right)_{I=0}$	10650	1.34
	T_2	$B_s^*\overline{B_s^*}$	10830	1.54

Table 1: Open flavor meson-meson thresholds for $0\,(J^{++})$ $b\overline{b}$ states. Threshold masses (M_{T_i}) obtained from the bottom and bottom strange meson masses quoted in [9]. Crossing distances (r_{T_i}) calculated from (4).

$I(J^{PC})$	T_i	Bottomonium Thresholds	M_{T_i} (MeV)	r_{T_i} (fm)
$0(1^{--})$				
	T_1	$(B^0\overline{B_1(5721)}^0, B^+B_1(5721)^-)_{I=0} - c.c.$	11003	1.73
		$(B^0\overline{B_1(?)}^0, B^+B_1(?)^-)_{I=0} - c.c.$	?	?
	T_2	$(B^{*0}\overline{B_0^*(5732)}^0, B^{*+}B_0^*(5732)^-)_{I=0} - c.c.$	11023	1.76
	T_3	$(B^{*0}\overline{B_1(5721)}^0, B^{*+}B_1(5271)^-)_{I=0} - c.c.$	11049	1.79
		$(B^{*0}\overline{B_1(?)}^0, B^{*+}B_1(?)^-)_{I=0} - c.c.$	?	?
	T_4	$(B^{*0}\overline{B_2^*(5747)}^0, B^{*+}B_2^*(5747)^-)_{I=0} - c.c.$	11072	1.81

Table 2: Open flavor meson-meson thresholds for $0(1^{--})$ $b\bar{b}$ states. Threshold masses (M_{T_i}) calculated from the bottom and bottom strange meson masses quoted in [9]. Crossing distances (r_{T_i}) calculated from (4). For $B_J^*(5732)$ with quoted mass 5691 MeV we have assumed $J = 0$. A question mark has been used for the mass of an unknown meson and the mass of the corresponding threshold.

then, as the two mesons are in a relative $S-$ wave, we have $\overline{\mathfrak{M}_1}\mathfrak{M}_1 = (-)^{j_1+j_1-j}\,\mathfrak{M}_1\overline{\mathfrak{M}_1}$ where j_1 stands for the spin of $\mathfrak{M}_1$ and j for the total spin of the threshold. Therefore only one combination in $\mathfrak{M}_1\overline{\mathfrak{M}_1} \pm c.c$ is allowed for a given value of j (the other vanishes). For example the $I=0$ threshold $B^*\overline{B^*}$ with $j_1 = 1$ has positive C parity when coupled to $j = 0, 2$.

3.3 Spectrum

Bottomonium states are obtained by solving the Schrödinger equation for the GSP potential $V_{GSP}(r)$. In the energy region $\left[M_{T_{i-1}}, M_{T_i}\right]$ they satisfy

$$\left(\mathcal{T} + V_{\left[M_{T_{i-1}},M_{T_i}\right]}\right)\left|(Q\overline{Q})_{k_{[T_{i-1},T_i]}}\right\rangle = M_{k_{[T_{i-1},T_i]}}\left|(Q\overline{Q})_{k_{[T_{i-1},T_i]}}\right\rangle \tag{7}$$

where $\mathcal{T}$ stands for the kinetic energy operator, $\left|(Q\overline{Q})_{k_{[T_{i-1},T_i]}}\right\rangle$ for the bound state and $M_{k_{[T_{i-1},T_i]}}$ for its mass. As we have a radial potential we use the spectroscopic notation $k \equiv nl$, in terms of the radial, n, and orbital angular momentum, l, quantum numbers of the $Q\overline{Q}$ system. To fix the ideas let us consider for example the spectral states for $0^+(0^{++})$ $b\overline{b}$. In the first energy region the potential $V_{\left[M_{T_0},M_{T_1}\right]}(r)$, given by (2), reads (solid line in Fig. 2)

$$V_{[0,10558]}(r) = \begin{cases} \sigma r - \frac{\chi}{r} & r \leq 1.24 \text{ fm} \\ 972 \text{ MeV} & r \geq 1.24 \text{ fm} \end{cases}$$

where M_{T_1} and r_{T_1} have been taken from Table 1 and the values of the parameters (σ, χ, m_b) are given by (6). By solving the Schrödinger equation for $V_{[0,10558]}(r)$ we get the GSPM spectrum in $[M_{T_0}, M_{T_1}]$. It has only three bound states states, $1p_{[T_0,T_1]}$, $2p_{[T_0,T_1]}$ and $3p_{[T_0,T_1]}$, whose masses $M_{k_{[T_0,T_1]}}$ generically denoted by M_{GSP} are listed in Table 3. In the second energy region the potential, $V_{\left[M_{T_1},M_{T_2}\right]}(r)$, reads (dashed line in Fig. 2)

$$V_{[10558,10650]}(r) = \begin{cases} 972 \text{ MeV} & r \leq 1.24 \text{ fm} \\ \sigma r - \frac{\chi}{r} & 1.24 \text{ fm} \leq r \leq 1.34 \text{ fm} \\ 1064 \text{ MeV} & r \geq 1.34 \text{ fm} \end{cases}$$

where the threshold masses and crossing radii are taken from Table 1. The spectrum has only one bound state $1p_{[T_1,T_2]}$ whose mass $M_{1p_{[T_1,T_2]}}$ generically denoted by M_{GSP}

$b\overline{b}$ $0^+(0^{++})$	$[T_{i-1}, T_i]$	$\left[M_{T_{i-1}}, M_{T_i}\right]$ MeV	GSPM States $k_{[T_{i-1},T_i]}$	M_{GSP} MeV
	$[T_0, T_1]$	$[0, 10558]$	$1p_{[T_0,T_1]}$	9920
			$2p_{[T_0,T_1]}$	10259
			$3p_{[T_0,T_1]}$	10521
	$[T_1, T_2]$	$[10558, 10650]$	$1p_{[T_1,T_2]}$	10620

Table 3: Calculated $0^+(0^{++})$ $b\overline{b}$ masses from $V_{GSP}(r)$, generically denoted by M_{GSP}, in the first two energy regions indicated by the thresholds $[T_{i-1}, T_i]$ and their masses $\left[M_{T_{i-1}}, M_{T_i}\right]$.

is listed in Table 3. By proceeding in the same way for higher energy regions and for different quantum numbers we get the complete GSP bound state spectrum. The spectrum for $0^+(J^{++})$ $b\overline{b}$ states from the generalized screened potential $V_{GSP}(r)$ given by (1) is shown in Table 4. The spectrum from the Cornell potential $V_{Cor}(r)$ given by (5) with the same values of the parameters σ, χ and m_Q given by (6) is also listed for comparison. For 0^- (1^{--}) $b\overline{b}$ states there is some uncertainty in the calculation of the spectrum from the unknown threshold masses. Moreover, the possible accumulative effect of almost degenerate thresholds is out of the scope of the GSP such as has been defined. From this uncertainty we can not reasonably determine the spectrum around and above 11000 MeV. Hence we limit our calculation to the first energy region having taken the first threshold mass at 11003 MeV. In Table 5 we list these results as well as the ones from the Cornell potential, with the same values of the parameters σ, χ and m_Q, for comparison.

3.4 Screening Effects

A look at Table 4 makes clear that the more significant spectral effect from the generalized screened potential $V_{GSP}(r)$ is the bigger number of spectral states above the first meson-meson threshold as compared to the non screened Cornell potential $V_{Cor}(r)$ case. Thus, for example there are three $0^+(0^{++})$ $b\overline{b}$ GSPM bound states with masses $(10620, 10704, 10784)$ MeV between 10558 MeV, the mass of the first threshold, and 10830 MeV, the mass of the last known threshold, for only one Cornell state with mass 10768 MeV in this energy interval. Regarding the spectrum in the first energy region

J^{PC}	GSP States $k_{[T_{i-1},T_i]}$	M_{EQM} MeV	M_{PDG} MeV	$M_{Cor}\,(k)$ MeV
0^{++}	$1p_{[T_0,T_1]}$	9920	$9859.44 \pm 0.42 \pm 0.31$	9920 $(1p)$
1^{++}	$1p_{[T_0,T_1]}$	9920	$9892.78 \pm 0.26 \pm 0.31$	9920 $(1p)$
2^{++}	$1p_{[T_0,T_1]}$	9920	$9912.21 \pm 0.26 \pm 0.31$	9920 $(1p)$
0^{++}	$2p_{[T_0,T_1]}$	10259	$10232.5 \pm 0.4 \pm 0.5$	10259 $(2p)$
1^{++}	$2p_{[T_0,T_1]}$	10259	$10255.46 \pm 0.22 \pm 0.50$	10259 $(2p)$
2^{++}	$2p_{[T_0,T_1]}$	10259	$10268.65 \pm 0.22 \pm 0.50$	10259 $(2p)$
0^{++}	$3p_{[T_0,T_1]}$	10521		10531 $(3p)$
1^{++}	$3p_{[T_0,T_1]}$	10526		10531 $(3p)$
			$10530 \pm 5 \pm 9$	
2^{++}	$3p_{[T_0,T_1]}$	10528		10531 $(3p)$
0^{++}	$1p_{[T_1,T_2]}$	10620		
1^{++}	$1p_{[T_1,T_2]}$	10668		
0^{++}	$1p_{[T_2,T_3]}$	10704		
2^{++}	$1p_{[T_1,T_2]}$	10710		
				10768 $(4p)$
1^{++}	$2p_{[T_1,T_2]}$	10776		
0^{++}	$1p_{[T_3,T_4]}$	10784		
2^{++}	$2p_{[T_1,T_2]}$	10815		

Table 4: Calculated J^{++} bottomonium masses from $V_{GSP}(r) : M_{GSP}$. Masses for experimental resonances, M_{PDG}, have been taken from [9]. For p waves we quote separately the np_0, np_1 and np_2 states. Masses and states from the Cornell potential $V_{Cor}(r)$, denoted by $M_{Cor}\ (k)$ are also shown for comparison.

J^{PC}	EQM States $k_{[T_{i-1},T_i]}$	M_{EQM} MeV	M_{PDG} MeV	$M_{Cor}\,(k)$ MeV
1^{--}	$1s_{[T_0,T_1]}$	9459	9460.30 ± 0.26	9459 $(1s)$
	$2s_{[T_0,T_1]}$	10012	10023.026 ± 0.31	10012 $(2s)$
	$1d_{[T_0,T_1]}$	10157	10163.7 ± 1.4	10157 $(1d)$
	$3s_{[T_0,T_1]}$	10342	10355.2 ± 0.5	10342 $(3s)$
	$2d_{[T_0,T_1]}$	10438		10438 $(2d)$
	$4s_{[T_0,T_1]}$	10608	10579.4 ± 1.2	10608 $(4s)$
	$3d_{[T_0,T_1]}$	10682		10682 $(3d)$
	$5s_{[T_0,T_1]}$	10840		10841 $(5s)$
			10876 ± 11	
	$4d_{[T_0,T_1]}$	10899		10902 $(4d)$

Table 5: Calculated 1^{--} bottomonium masses from $V_{GSP}(r) : M_{GSP}$. Masses for experimental resonances, M_{PDG}, have been taken from [9]. Masses and states from the Cornell potential $V_{Cor}(r)$, denoted by $M_{Cor}\,(k)$ are also shown for comparison.

is almost identical for both potentials, the only difference being a slightly bigger attraction for $V_{GSP}(r)$ which makes the states close below threshold to be lower in mass than the corresponding Cornell ones. Notice that this extra attraction could make in some particular case that a state that is close above threshold for the Cornell potential lies close below threshold for the screened potential. An additional effect from the screened

potential is the breaking of the $J^{++} = (0,1,2)^{++}$ degeneracy implied by the Cornell potential. This is due to the different values of the threshold masses in each case. However, at the level of precision of our calculation, we obtain for the masses of the $1p\,(0,1,2)^{++}$ the same value (9919.6 MeV). This has to do with the fact that these states lying quite below the first threshold are very little affected by it. A similar argument applies to the $2p\,(0,1,2)^{++}$ states with calculated masses (10258.5 MeV, 10258.6 MeV, 10258.6 MeV), rounded off to 10259 MeV in Table 4. One should not forget though that a a more important contribution to this breaking may come from the non considered spin-dependent terms in the potential. Therefore we may conclude that a denser spectral pattern than

conventionally considered is the main feature resulting from the application of the GSPM. In other words screening effects in the way we have implemented them give rise to the appearance of new spectral states (not present in the non screened potential case). This can be understood if we think of an alternative (but much more complicated technically) equivalent method for calculating the spectrum based on the consideration of interacting bare valence $Q_0\overline{Q}_0$ and meson $(Q_0\overline{q})$ - meson $\left(\overline{Q}_0 q\right)$ configurations. Then it is clear that through configuration mixing more spectral states than the pure Cornell (non screened) states corresponding to $Q_0\overline{Q}_0$ are present. Unfortunately we have not yet enough data to validate or refute this conclusion. A scan for J^{++} states in the mentioned energetic region (from 10558 MeV to 10830 MeV) could shed definite light about this prediction. With respect to this it should be mentioned that most of the new spectral states are related to at least one threshold with $s\overline{s}$ content what could imply a reduction of the formation probability for them. The only exception is the $0^+(0^{++})(10620)$ which is a priory the ideal candidate to check the GSPM.

4 Summary

A new nonrelativistic quark model to study the spectrum of heavy quark mesons has been developed. The model is built in terms of effective quark degrees of freedom interacting through a potential that incorporates screening effects from meson-meson configurations. The form of this interaction potential that we call Generalized Screened Potential, or abbreviate GSP, has been proposed from lattice results and exhibits a characteristic dependence on the energy interval of application. The model, called Generalized Screened Potential Model, or abbreviate GSPM, has been applied to calculate the bottomonium spectrum (the only non relativistic meson system). A richer spectrum (bigger number of bound states) than the one resulting from the non-screened Cornell potential is obtained. In particular extra J^{++} bottomonium states above the first meson-meson threshold appear. Certainly the masses of these new states may be shifted when dynamic corrections are implemented. However as the form of the potential in between two thresholds is determined to a large extent by the threshold masses (indeed it can be approximated by a spherical well) we hardly expect any change in the number of calculated states when these corrections are incorporated. Therefore we consider the presence of these extra states a quite robust distinctive prediction of the model. It should be emphasized that by construction the model is suited for the calculation of spectral masses. In this regard it represents a very simple and efficient alternative to a couple channel calculation of the spectrum involving bare valence and meson - meson configurations. As a counterpart a quantitative treatment of decay processes (for example strong decays to open flavor mesons) may require the explicit consideration of meson - meson configurations as in a couple channel scheme. The generalization of the Generalized Screened Potential Model to other meson sectors is feasible but some dynamic implementations may be required. In particular the analysis of charmonium with a richer spectrum than conventionally expected deserves special attention and will be the subject of future work. In summary we have proposed a spectral Generalized Screened Potential Model which can be considered as a first attempt to incorporate lattice screening effects from meson-meson thresholds

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