

Inverse Symmetry Breaking and the Exact Renormalization Group

M. Pietroni, N. Rius¹ and N. Tetradis

*Theory Division, CERN
CH-1211 Geneva 23, Switzerland*

Abstract

We discuss the question of inverse symmetry breaking at non-zero temperature using the exact renormalization group. We study a two-scalar theory and concentrate on the nature of the phase transition during which the symmetry is broken. We also examine the persistence of symmetry breaking at temperatures higher than the critical one.

CERN-TH/96-341
November 1996

¹Permanent address after 1/11/96: Departament de Física Teòrica, Universitat de València and IFIC, València, Spain.

It has been known for a long time [1, 2, 3], that simple multiscalar models can exhibit an anti-intuitive behaviour associated with more broken symmetry as the temperature is increased. We refer to this behaviour as inverse symmetry breaking. This possibility may have remarkable consequences for cosmology, solving the problem of topological defects, thanks to the fact that the phase transition leading to their formation may never have occurred during the thermal history of the Universe [4]. The existence of the phenomenon for supersymmetric theories has been discussed in refs. [5]. However, doubts have been raised on the validity of these results, which are based on the one-loop approximation to the non-zero-temperature effective potential. This approximation is known to be unreliable for the discussion of many aspects of phase transitions. Recently the effect of next-to-leading-order contributions within perturbation theory has been investigated in ref. [6]. This has been done through the study of gap equations, which are equivalent to a resummation of the super-daisy diagrams of the perturbative series. Large subleading corrections have been identified, which lead to a sizeable reduction of the parameter space where inverse symmetry breaking occurs. The question of inverse symmetry breaking has also been studied through the use of the renormalization group, with similar conclusions [7]. A variational approach has been employed in ref. [8]. Contrary to the results of the above studies, a large- N analysis seems to indicate that symmetry is always restored at high temperature [9]. However, the validity of this claim has recently been questioned in ref. [10]. A finite-lattice calculation also supports the symmetry restoration at sufficiently high temperature [11], even though the relevance of this result for the continuum limit is not clear.

In this letter we study the question of inverse symmetry breaking by employing the exact renormalization group [12]. Our formalism is similar to that of ref. [7]. Our approach, however, is based on the real-time formulation of high-temperature field theories. We investigate a two-scalar theory and we identify the universal behaviour associated with the symmetry-breaking phase transition. Our study is based on an evolution equation for the potential of the non-zero temperature theory. Using a polynomial ansatz for the potential, we solve this equation and verify the conclusion of ref. [7] that inverse symmetry breaking is confirmed by the renormalization-group approach. Moreover, we explore the parameter space that leads to inverse symmetry breaking and compare it with the perturbative predictions. In order to study the phase transition, we go beyond the polynomial ansatz and consider a general dependence of the potential on the field that develops an expectation value when the symmetry is broken. The resulting partial differential equation is solved numerically with the use of appropriate algorithms [13]. The Wilson-Fisher fixed point of the effective three-dimensional theory is shown to govern the dynamics near the second-order phase transition.

We consider the simplest model that exhibits inverse symmetry breaking: a two-scalar model with $Z_2 \times Z_2$ symmetry. The tree-level potential is given by

$$V_{tr}(\phi_1, \phi_2) = \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}m_2^2\phi_2^2 + \frac{1}{4}\lambda_1\phi_1^4 + \frac{1}{4}\lambda_2\phi_2^4 - \frac{1}{2}\lambda_{12}\phi_1^2\phi_2^2. \quad (1)$$

This potential is bounded for $\lambda_{1,2} > 0$ and

$$\lambda_1\lambda_2 > \lambda_{12}^2. \quad (2)$$

The thermal correction to the above potential at the one-loop level is given by the well-

known expression [14]

$$\Delta V_T(\phi_1, \phi_2) = T \int_0^\infty \frac{dk}{2\pi^2} k^2 \text{Tr} \log \left[1 - \exp \left(-\frac{1}{T} \sqrt{k^2 + \mathcal{M}_{tr}^2} \right) \right]. \quad (3)$$

Here \mathcal{M}_{tr}^2 indicates the matrix of second derivatives of the tree-level effective potential

$$\left[\mathcal{M}_{tr}^2(\phi_1, \phi_2) \right]_{i,j} = \frac{\partial^2 V_{tr}(\phi_1, \phi_2)}{\partial \phi_i \partial \phi_j}, \quad i, j = 1, 2. \quad (4)$$

When both eigenvalues of \mathcal{M}_{tr}^2 are much smaller than T^2 (which happens for sufficiently small couplings), the leading field-dependent correction takes the form

$$\Delta V_T(\phi_1, \phi_2) \simeq \frac{T^2}{24} \left[(3\lambda_1 - \lambda_{12})\phi_1^2 + (3\lambda_2 - \lambda_{12})\phi_2^2 \right] + \dots \quad (5)$$

For the parameter range

$$3\lambda_1 - \lambda_{12} < 0, \quad (6)$$

which can be consistent with the stability condition of eq. (2), the thermal correction for the mass term of the ϕ_1 field is negative. If the system is in the symmetric phase at zero temperature with $m_{1,2}^2 > 0$, there will be a critical temperature $T_{cr}^2 = 12m_1^2/(\lambda_{12} - 3\lambda_1)$ above which the symmetry will be broken². If the system is in the broken phase at $T = 0$, the symmetry will never be restored by thermal corrections.

Our aim is to discuss the above scenario in the context of the Wilson approach to the renormalization group. The main ingredient in this approach is an exact flow equation that describes how the effective action of the system evolves as the ultraviolet cutoff is lowered. We consider the lowest order in a derivative expansion of the effective action, which contains a general effective potential and a standard kinetic term. At non-zero temperature this approach can be formulated either in the imaginary-time [15] or in the real-time formalism [16]. In the latter formulation, the evolution of the potential lowering the cutoff scale Λ is given by the partial differential equation [16]

$$\Lambda \frac{\partial}{\partial \Lambda} V_\Lambda(\phi_1, \phi_2) = -T \frac{\Lambda^3}{2\pi^2} \text{Tr} \left\{ \log \left[1 - \exp \left(-\frac{1}{T} \sqrt{\Lambda^2 + \mathcal{M}_\Lambda^2} \right) \right] \right\}, \quad (7)$$

where

$$\left[\mathcal{M}_\Lambda^2(\phi_1, \phi_2) \right]_{i,j} = \frac{\partial^2 V_\Lambda(\phi_1, \phi_2)}{\partial \phi_i \partial \phi_j}, \quad i, j = 1, 2. \quad (8)$$

Notice the formal similarity with the Dolan-Jackiw one-loop result of eq. (3). The main difference is that the ‘running’ mass matrix in the exponential of eq. (8) replaces the tree-level one in eq. (3). The initial condition for the above equation, at a scale $\Lambda_0 \gg T$, is the renormalized effective potential at zero temperature. In this work we consider small quartic couplings, so that the logarithmic corrections of the zero-temperature theory can be safely neglected. The initial condition for the evolution is a zero-temperature potential given by eq. (1). Integrating the evolution equation (7), we obtain the non-zero-temperature

² For $3\lambda_2 - \lambda_{12} < 0$ the symmetry is broken in the ϕ_2 direction. The discussion of this case is completely analogous to the one we consider. Notice that it is not possible to break the symmetry in both the ϕ_1 and ϕ_2 directions, because of the stability condition of eq. (2).

effective potential in the limit $\Lambda \rightarrow 0$. In the approximation that the ‘running’ mass matrix on the r.h.s. of the evolution equation is taken to be constant, equal to the tree-level mass matrix, the integration reproduces the perturbative result of eq. (3). The non-trivial behaviour that we describe in the following paragraphs is obtained when the full scale-dependence of the mass matrix is taken into account. An interesting limit corresponds to the evolution at scales $\Lambda \ll T$. Let us denote by $[\tilde{\mathcal{M}}_\Lambda^2]_l$ the eigenvalues of the mass matrix that satisfy $[\tilde{\mathcal{M}}_\Lambda^2]_l \lesssim \Lambda^2$. The remaining eigenvalues correspond to decoupled massive modes that do not contribute to the evolution at the scale Λ [17]. Keeping the leading contribution in the r.h.s. of eq. (7) and omitting the field-independent terms, we obtain

$$\Lambda \frac{\partial}{\partial \Lambda} \left(\frac{V_\Lambda(\phi_1, \phi_2)}{T} \right) = -\frac{\Lambda^3}{4\pi^2} \text{Tr} \left\{ \log \left(\Lambda^2 + [\tilde{\mathcal{M}}_\Lambda^2]_l \right) \right\} . \quad (9)$$

The rescaled potential V_Λ/T has dimensions (mass)³ and its evolution is typical of that of a three-dimensional theory [17, 18]. In the limit $\Lambda \ll T$, dimensional reduction takes place, and a four-dimensional theory at non-zero temperature behaves as an effective three-dimensional one at $T = 0$. We can cast eq. (9) in a form that does not depend explicitly on the scale Λ by defining the dimensionless parameters:

$$\bar{\phi}_{1,2} = \frac{\phi_{1,2}}{\sqrt{\Lambda T}} , \quad \bar{V}_\Lambda(\bar{\phi}_1, \bar{\phi}_2) = \frac{V_\Lambda(\phi_1, \phi_2)}{\Lambda^3 T} , \quad [\bar{\mathcal{M}}_\Lambda^2]_l = \frac{[\tilde{\mathcal{M}}_\Lambda^2]_l}{\Lambda^2} . \quad (10)$$

The dimensionless mass matrix $\bar{\mathcal{M}}_\Lambda^2$ is related to the rescaled potential \bar{V} through an equation analogous to eq. (8). The evolution equation now reads

$$\Lambda \frac{\partial}{\partial \Lambda} \bar{V}_\Lambda(\bar{\phi}_1, \bar{\phi}_2) = -3\bar{V} - \frac{1}{4\pi^2} \text{Tr} \left\{ \log \left(1 + [\bar{\mathcal{M}}_\Lambda^2]_l \right) \right\} , \quad (11)$$

where again we have omitted the field-independent terms. The scale-invariant (fixed-point) solutions of the effective three-dimensional theory can be obtained from the above equation for $\Lambda \partial \bar{V}_\Lambda / \partial \Lambda = 0$.

Finding the solution of eq. (7) is a difficult task. An approximate solution can be obtained [15, 7] by expanding the potential in a power series in the fields. In this way the partial differential equation (7) is transformed into an infinite system of ordinary differential equations for the coefficients of the expansion. This system can be solved approximately by truncation at a finite number of equations. In effect, the potential is approximated by a finite-order polynomial. As a first step, we follow this procedure and define the running masses and couplings at the origin

$$m_{1,2}^2(\Lambda) = \left. \frac{\partial^2 V_\Lambda}{\partial \phi_{1,2}^2} \right|_{\phi_{1,2}=0} , \quad \lambda_{1,2}(\Lambda) = \left. \frac{1}{6} \frac{\partial^4 V_\Lambda}{\partial \phi_{1,2}^4} \right|_{\phi_{1,2}=0} , \quad \lambda_{12}(\Lambda) = - \left. \frac{1}{2} \frac{\partial^4 V_\Lambda}{\partial \phi_1^2 \partial \phi_2^2} \right|_{\phi_{1,2}=0} . \quad (12)$$

The corresponding evolution equations can be obtained by differentiating eq. (7) and neglecting the higher derivatives of the potential. We find

$$\begin{aligned} \Lambda \frac{\partial}{\partial \Lambda} m_{1,2}^2 &= -6C_{1,2}\lambda_{1,2} + 2C_{2,1}\lambda_{12} \\ \Lambda \frac{\partial}{\partial \Lambda} \lambda_{1,2} &= -18D_{1,2}\lambda_{1,2}^2 - 2D_{2,1}\lambda_{12}^2 \\ \Lambda \frac{\partial}{\partial \Lambda} \lambda_{12} &= -6D_1\lambda_1\lambda_{12} - 6D_2\lambda_2\lambda_{12} + 8\frac{C_1 - C_2}{m_1^2 - m_2^2}\lambda_{12}^2 , \end{aligned} \quad (13)$$

with

$$C_{1,2} = \frac{\Lambda^3}{4\pi^2} \frac{N(\omega_{1,2})}{\omega_{1,2}}, \quad D_{1,2} = \frac{\partial C_{1,2}}{\partial m_{1,2}^2}, \quad \omega_{1,2}^2 = \Lambda^2 + m_{1,2}^2, \quad (14)$$

and $N(\omega) = [\exp(\omega/T) - 1]^{-1}$ the Bose-Einstein distribution function. For $\omega_{1,2} \ll T$ we have

$$C_{1,2} \rightarrow \frac{\Lambda^3}{4\pi^2} \frac{T}{\Lambda^2 + m_{1,2}^2}, \quad (15)$$

and the above equations agree with those considered in ref. [7] in the same limit. For $\omega_{1,2} \gg T$ there is no running, because of the exponential suppression in the Bose-Einstein function.

We have solved numerically the system of equations (13) and determined the range of zero-temperature parameters that lead to inverse symmetry breaking. In fig. 1 we present the results for a zero-temperature theory with positive mass terms $m_1^2(\Lambda_0) = m_2^2(\Lambda_0)$ and $\lambda_2(\Lambda_0) = 0.3$. The temperature has been chosen much higher than the critical one ($T = 500m_1(\Lambda_0)$). The system (13) has been integrated from $\Lambda_0 \gg T$ down to $\Lambda = 0$, where the thermally corrected masses and couplings at non-zero temperature have been obtained. A negative value for the mass term m_1^2 at $\Lambda = 0$ has been considered as the signal of inverse symmetry breaking. This has been achieved for the parameter range of $\lambda_1(\Lambda_0)$, $\lambda_{12}(\Lambda_0)$ above the line (a) in fig. 1. In the same figure we plot the stability bound of eq. (2). The allowed range is below the line (b). We also include the perturbative prediction of eq. (6) for the range that leads to inverse symmetry breaking. It lies above the line (c). The phenomenon of inverse symmetry breaking is confirmed by our study, in agreement with ref. [7], where the imaginary-time formulation of the renormalization-group approach has been used. We observe that the renormalization-group treatment eliminates a large part of the parameter space allowed by perturbative theory. This is in agreement with the results of ref. [6], where the gap-equation approach has been followed. Notice that lines (a) and (c) approach each other near the origin, where perturbation theory becomes more reliable.

The reliability of our conclusions crucially depends on whether the solution of the system of truncated equations (13) provides an approximate solution to the full partial differential equation (7). In ref. [7], this has been checked by increasing the level of truncations and verifying the convergence of the results. We follow here a different approach that relies on the numerical integration of eq. (7) through the algorithms discussed in ref. [13]. These algorithms have been used for the integration of the evolution equations for potentials that depend on one field only [13, 19]. The generalization to the two-field case is straightforward. However, limitations in computer time have prevented us from reaching full numerical stability for the results. For this reason, we restrict our discussion of eq. (7) along the ϕ_1 axis, which is the direction of expected symmetry breaking for our choice of couplings. We approximate the potential by the expression

$$V_\Lambda(\phi_1, \phi_2) = V_\Lambda(\phi_1) + \frac{1}{2}m_2^2(\Lambda)\phi_2^2 + \frac{1}{4}\lambda_2(\Lambda)\phi_2^4 - \frac{1}{2}\lambda_{12}(\Lambda)\phi_1^2\phi_2^2. \quad (16)$$

The evolution of $m_2^2(\Lambda)$, $\lambda_2(\Lambda)$ and $\lambda_{12}(\Lambda)$ is determined through the truncated eqs. (13). However, the full ϕ_1 dependence is preserved through the numerical integration of eq. (7),

with the eigenvalues of the mass matrix \mathcal{M}_Λ^2 given by

$$[\mathcal{M}_\Lambda^2]_1 = \frac{\partial^2 V_\Lambda(\phi_1)}{\partial \phi_1^2} \quad \text{and} \quad [\mathcal{M}_\Lambda^2]_2 = m_2^2(\Lambda) - \lambda_{12}(\Lambda)\phi_1^2. \quad (17)$$

This treatment permits a reliable study of the order of the symmetry-breaking phase transition. The appearance of secondary minima of the potential at some point in the evolution can be studied in detail. As a result, we can distinguish between first- and second-order phase transitions. This is not possible when local expansions of the potential, such as the one leading to eqs. (13), are employed.

In fig. 2 we present the evolution of the potential for zero-temperature parameters $m_1^2(\Lambda_0) = m_2^2(\Lambda_0)$, $\lambda_1(\Lambda_0) = 0.01$, $\lambda_2(\Lambda_0) = 0.3$, $\lambda_{12}(\Lambda_0) = 0.05$. The location of this parameter choice on the plot of fig. 1 is denoted by a black square. It is within the region for which inverse symmetry breaking is expected. The temperature is chosen very close to the critical one: $T_{cr}/m_1(\Lambda_0) \simeq 33.3$. This value is in very good agreement with the result of ref. [7], for the same choice of zero-temperature parameters. It deviates significantly from the perturbative prediction $T_{cr}/m_1(\Lambda_0) \simeq 24.5$. We use the rescaled variables defined in eqs. (10), which permit the identification of the fixed points that may be relevant for the phase transition. We plot the derivative $\partial \bar{V}_\Lambda / \partial \bar{\rho}_1$ of the potential as a function of the variable $\bar{\rho}_1 = \bar{\phi}_1^2/2$, for decreasing Λ . The early stages of the evolution of the potential, when its curvature at the origin becomes negative, are not clearly visible in this plot. The reason is that $\partial \bar{V}_\Lambda / \partial \bar{\rho}_1$ is very small for large Λ . The important point in this figure is the flow of the potential towards a scale-invariant solution (marked by WF) during the later stages of the evolution. This solution corresponds to the Wilson-Fisher fixed point of the effective three-dimensional theory. For $\Lambda \rightarrow 0$ the theory leaves the fixed point and flows towards the phase with symmetry breaking. The evolution can also be described in terms of the parameters defined in eqs. (12). Their rescaled versions

$$\bar{m}_{1,2}^2(\Lambda) = \frac{m_{1,2}^2(\Lambda)}{\Lambda^2} \quad \text{and} \quad \bar{\lambda}_{1,2,12}(\Lambda) = \frac{\lambda_{1,2,12}(\Lambda)T}{\Lambda} \quad (18)$$

are obtained from the rescaled potential of eqs. (10) through relations analogous to eqs. (12). (Notice that $\bar{m}_1^2(\Lambda)$ is given by the value of $\partial \bar{V}_\Lambda / \partial \bar{\rho}_1$ for $\bar{\rho}_1 = 0$ in fig. 2.) We present the evolution of all these parameters in figs. 3 and 4. In both these plots we observe that the mass term $m_1^2(\Lambda)$ turns negative at some scale $\Lambda \sim T$. The mass term $m_2^2(\Lambda)$ stays positive and grows as Λ decreases. For $\Lambda \ll T$ we observe that $m_2^2(\Lambda) \gg \Lambda^2$ and the ϕ_2 field decouples. Eventually, for $\Lambda \rightarrow 0$, the renormalized parameters $m_2^2(\Lambda)$ and $\lambda_2(\Lambda)$ take constant values. After the decoupling of the ϕ_2 field, only the fluctuations of the ϕ_1 field contribute to the evolution. In this regime, the evolution equation for the potential can be written in the form of eq. (11), with only the eigenvalue $[\bar{\mathcal{M}}_\Lambda^2]_1 = \partial^2 \bar{V}_\Lambda / \partial \bar{\phi}_1^2$ contributing. The scale-invariant solution of the resulting equation corresponds to the Wilson-Fisher fixed point of the one-scalar three-dimensional theory. Its explicit form can be seen in fig. 2. We conclude that the role of the ϕ_2 field is to trigger the inverse symmetry breaking by inducing a negative mass term $m_1^2(\Lambda)$ at the early stages of the evolution. It subsequently decouples and the dynamics of the phase transition is governed by the Wilson-Fisher fixed point of the one-scalar three-dimensional theory. The resulting phase transition is of second order. Its universal behaviour can be parametrized in terms of critical exponents and amplitudes. For a detailed discussion see ref. [15].

The evolution close to the fixed point is apparent in fig. 3, in the range where the rescaled parameters $\bar{m}_1^2(\Lambda)$, $\bar{\lambda}_1(\Lambda)$ become constant. Their fixed-point values are $\bar{m}_{1,fp}^2 \simeq -0.44$, $\bar{\lambda}_{1,fp} \simeq 3.2$. In this regime, the parameters $m_1^2(\Lambda)$, $\lambda_1(\Lambda)$ evolve towards zero according to $m_1^2(\Lambda) = \bar{m}_{1,fp}^2 \Lambda^2$, $\lambda_1(\Lambda) = \bar{\lambda}_{1,fp} \Lambda/T$ (see fig. 4). This explains why the fixed point is relevant very close to the phase transition. If the temperature is chosen such that the evolution stays close to the fixed point for a long ‘time’ $t = \log(\Lambda/T)$, the curvature of the potential at the origin is very small at the end of the evolution. Through sufficient fine-tuning of the temperature, the curvature can be made arbitrarily small. This is the criterion for the occurrence of a second-order phase transition. Notice that the renormalized quartic coupling $\lambda_1(\Lambda = 0)$ is also expected to be zero at the critical temperature [17]. According to eqs. (13) the coupling $\lambda_{12}(\Lambda)$ evolves as $\lambda_{12} \sim \Lambda^a$, with $a = 3\bar{\lambda}_{1,fp}/2\pi^2(1 + \bar{m}_{1,fp}^2)^2 \simeq 1.6$. For $\lambda_2(\Lambda)$ we obtain $\lambda_2(\Lambda) = \lambda_2(\Lambda_i) + c(\Lambda^{2a-1} - \Lambda_i^{2a-1})/(2a - 1)$, where Λ_i is the scale at which the fixed-point solution is initially approached, and c a calculable constant. The predicted evolution of $\lambda_{12}(\Lambda)$ and $\lambda_2(\Lambda)$ is confirmed by fig. 4. If we had used the truncated equations (13) for the discussion of the evolution of $m_1^2(\Lambda)$ and $\lambda_1(\Lambda)$, as in ref. [7], we would have obtained $a = 1/3$. This would have predicted an unphysical, singular behaviour $\lambda_2(\Lambda) \sim -\Lambda^{-1/3}$ for $\Lambda \rightarrow 0$. Our solution of the evolution equation for the full potential along the ϕ_1 axis resolves this problem, and describes the decoupling of the ϕ_2 field properly.

In conclusion, we have used the real-time formulation of the exact renormalization group in order to study the question of inverse symmetry breaking in the context of the two-scalar theory of eq. (1). We have verified the conclusion of ref. [7] that inverse symmetry breaking is confirmed by the renormalization-group approach. We have also determined the parameter range that leads to inverse symmetry breaking, through appropriate truncations of the partial differential equation that describes the evolution of the potential. This parameter range is significantly smaller than the perturbative prediction, in agreement with ref. [6]. We have also obtained a numerical solution of the evolution equation for the full potential along the ϕ_1 axis, without relying on truncations in that direction. In this way we have obtained a detailed picture of the symmetry-breaking phase transition. The ϕ_2 field triggers the symmetry breaking along the ϕ_1 direction and subsequently decouples. The phase transition is of second-order, governed by the Wilson-Fisher fixed point of the effective three-dimensional theory. Our improved treatment gives no indication of symmetry restoration for the range of temperatures above the critical one, in which we obtain stable numerical solutions.

Acknowledgements: We would like to thank M. D’Attanasio, B. Gavela, J. Orloff and G. Senjanović for helpful discussions. M.P. has been supported by an E.C. Fellowship under the H.C.M. program. N.R. has been supported in part by CICYT under grant AEN-96/1718 and by DGICYT under grant PB95-1077 (Spain).

References

- [1] S. Weinberg, Phys. Rev. D 9 (1974) 3357.
- [2] R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 42 (1979) 1651; Phys. Rev. D 20 (1979) 3390.
- [3] P. Langacker and S.-Y. Pi, Phys. Rev. Lett. 45 (1980) 1.
- [4] G. Dvali, A. Melfo and G. Senjanović, Phys. Rev. Lett. 75 (1995) 4559; preprint SISSA-18-96-A, hep-ph/9601376.
- [5] H. Haber, Phys. Rev. D 46 (1982) 1317; M. Mangano, Phys. Lett. B 147 (1984) 307; G. Dvali and K. Tamvakis, Phys. Lett. B 378 (1996) 141; B. Bajc, A. Melfo and G. Senjanović, Phys. Lett. B 387 (1996) 796.
- [6] G. Bimonte and G. Lozano, Phys. Lett. B 366 (1996) 248; Nucl. Phys. B 460 (1996) 155.
- [7] T.G. Roos, Phys. Rev. D 54 (1996) 2944.
- [8] G. Amelino-Camelia, preprint OUTP-96-44P, hep-ph/9610262.
- [9] Y. Fujimoto and S. Sakakibara, Phys. Lett. B 151 (1985) 260; E. Manesis and S. Sakakibara, Phys. Lett. B 157 (1985) 287; K.G. Klimenko, Theor. Math. Phys. 80 (1989) 929.
- [10] J. Orloff, preprint ENSLAPP-AL-615/96, hep-ph/9611398.
- [11] G. Bimonte and G. Lozano, preprint DFTUZ-96-11, hep-th/9603201.
- [12] K.G. Wilson and I.G. Kogut, Phys. Rep. 12 (1974) 75.
- [13] J. Adams, J. Berges, S. Bornholdt, F. Freire, N. Tetradis and C. Wetterich, Mod. Phys. Lett. A 10 (1995) 2367.
- [14] L. Dolan and R. Jackiw, Phys. Rev. D 9 (1974) 3320; D.A. Kirzhnits and A.D. Linde, JETP 40 (1974) 628; Ann. Phys. 101 (1976) 195.
- [15] N. Tetradis and C. Wetterich, Nucl. Phys. 398 (1993) 659.
- [16] M. D'Attanasio and M. Pietroni, Nucl. Phys. B 472 (1996) 711.
- [17] N. Tetradis and C. Wetterich, Nucl. Phys. B 422 (1994) 541; N. Tetradis and D. Litim, Nucl. Phys. B 464 (1996) 492.
- [18] T.R. Morris, Phys. Lett. B 334 (1994) 355.
- [19] N. Tetradis, preprint CERN-TH/96-190, hep-ph/9608272.

Figures

Fig. 1 The parameter space that leads to inverse symmetry breaking ($\lambda_2 = 0.3$).

Fig. 2 The derivative $\bar{V}'_\Lambda = \partial\bar{V}_\Lambda/\partial\bar{\rho}_1$ of the potential as a function of $\bar{\rho}_1 = \bar{\phi}_1^2/2$ for decreasing Λ , for a theory very close to the phase transition.

Fig. 3 The evolution of the rescaled masses and couplings for a theory very close to the phase transition.

Fig. 4 The evolution of the masses and couplings for a theory very close to the phase transition.

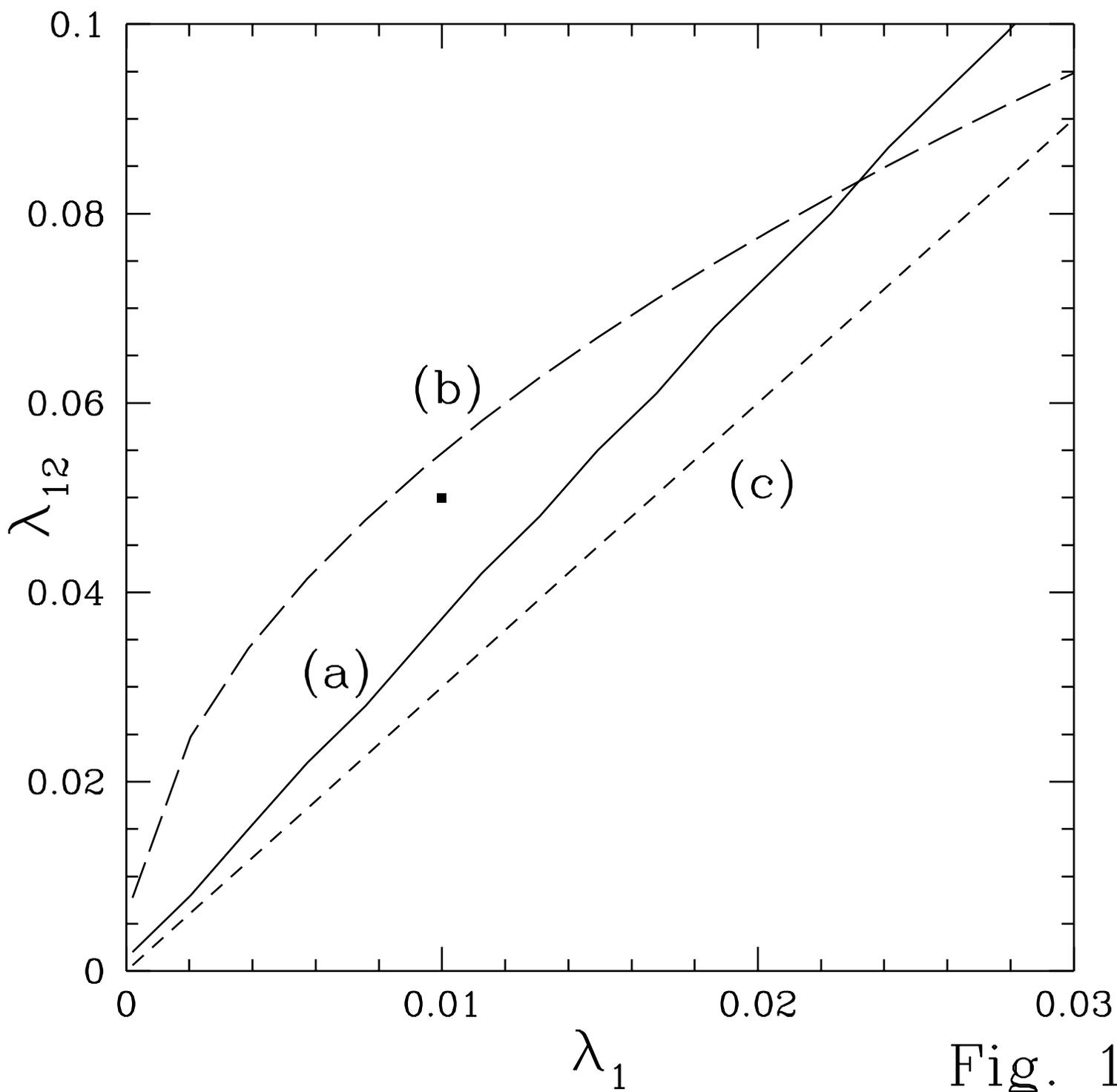


Fig. 1

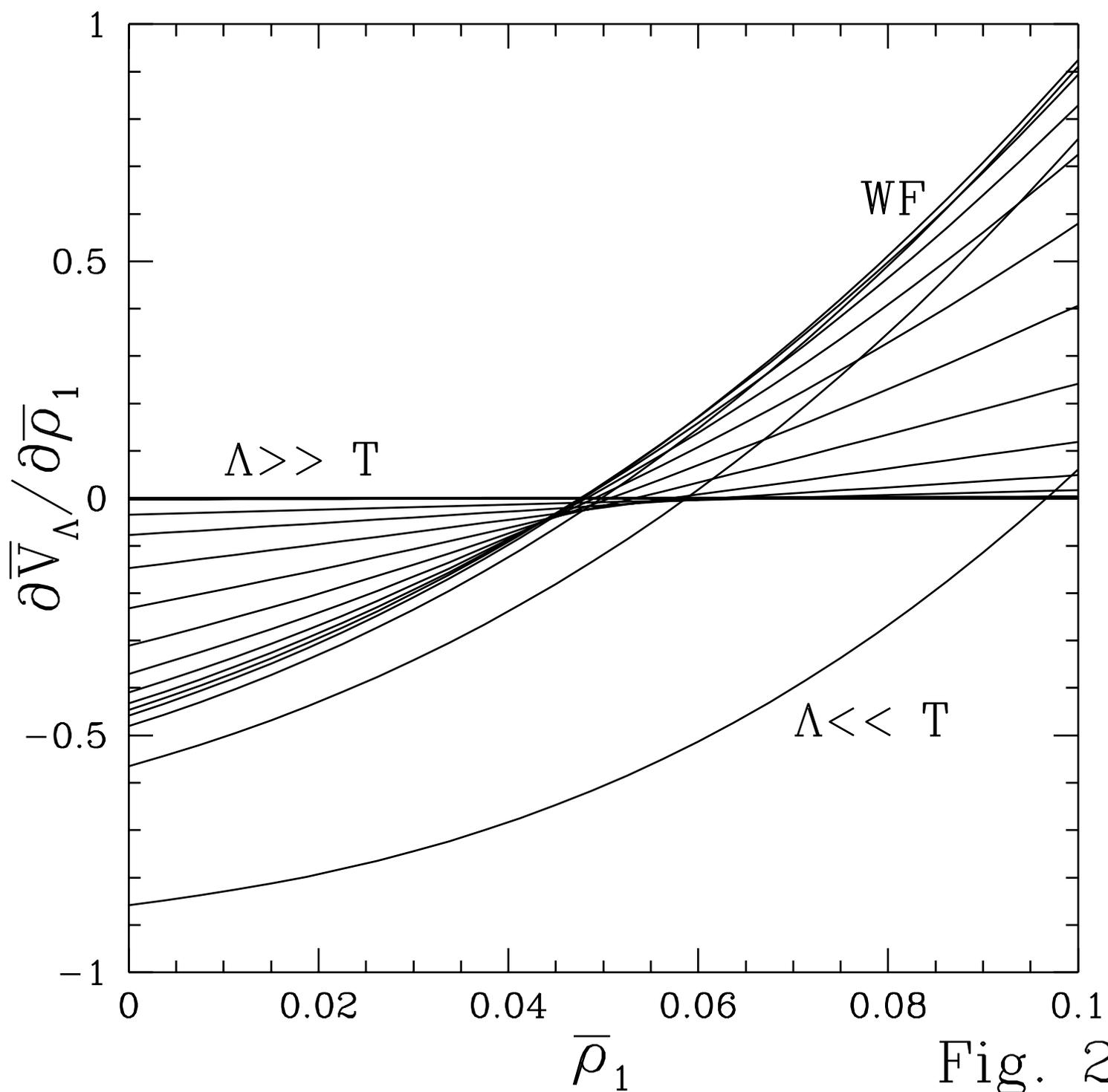


Fig. 2

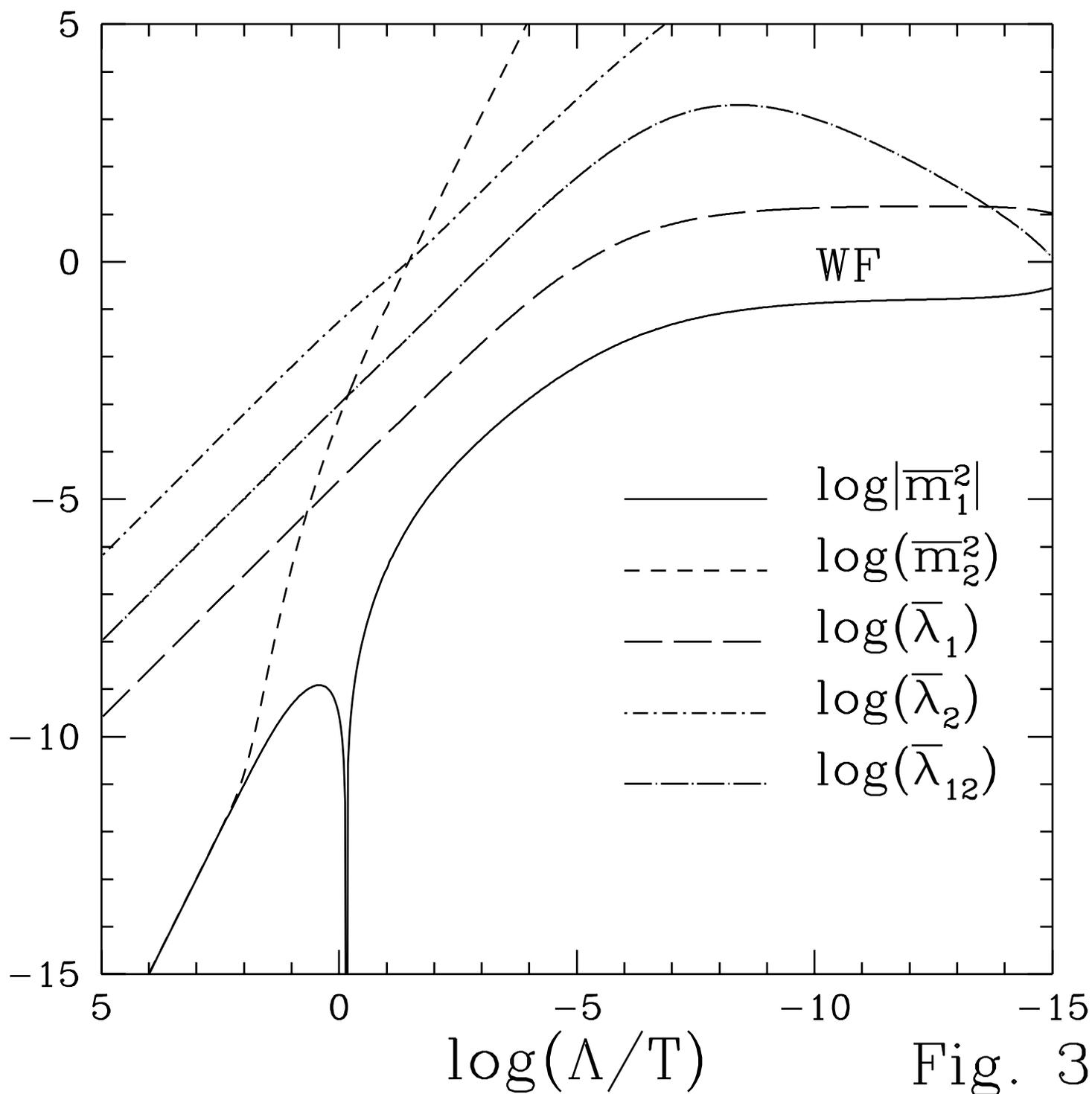


Fig. 3

