

# The Minimal Supersymmetric Model without a $\mu$ term

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**ABSTRACT:** We propose a supersymmetric extension of the standard model which is a realistic alternative to the MSSM, and which has several advantages. No “ $\mu$ ” supersymmetric Higgs/Higgsino mass parameter is needed for sufficiently heavy charginos. An approximate  $U(1)_R$  symmetry naturally guarantees that  $\tan\beta$  is large, explaining the top/bottom quark mass hierarchy. This symmetry also suppresses supersymmetric contributions to anomalous magnetic moments,  $b \rightarrow s\gamma$ , and proton decay, and these processes place no lower bounds on superpartner masses, even at large  $\tan\beta$ . The soft supersymmetry breaking mass parameters can easily be obtained from either gauge or Planck scale mediation, without the usual  $\mu$  problem. Unlike in the MSSM, there are significant *upper* bounds on the masses of superpartners, including an upper bound of 114 GeV on the mass of the lightest chargino. However the MSSM bound on the lightest Higgs mass does not apply.

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## 1. Introduction

Supersymmetric theories with softly broken supersymmetry have no quadratically divergent contributions to the Higgs mass, and so supersymmetry is hailed as a solution to the gauge hierarchy problem. In supersymmetric theories the  $W$  and  $Z$  masses are technically natural provided the superpartners have mass in the vicinity of the weak scale.

However in the Minimal Supersymmetric Model (MSSM) of particle physics there is a *supersymmetric* Higgs mass parameter, “ $\mu$ ”, which must be of order the superpartner masses for successful phenomenology. It would seem to be a bizarre and unnatural coincidence that a supersymmetric mass should be of nearly the same size as the scale of supersymmetry breaking, unless both mass scales have a common origin. Most solutions to this “ $\mu$  problem” focus on obtaining  $\mu$  as a consequence of supersymmetry breaking, or obtaining both supersymmetry breaking and the  $\mu$  parameter from common inputs.

In this paper we explore the consequences of a different approach to the  $\mu$  problem, namely side-stepping this problem by building a viable model which does not have a  $\mu$  parameter. We show that it is possible to obtain a spectrum of superpartner masses which is experimentally acceptable without  $\mu$ , provided the matter content of the MSSM is extended. In this model, which we call the “ $\mu$ -less Supersymmetric Standard Model” ( $\mu$ SSM), all mass arises directly from supersymmetry breaking or from electroweak symmetry breaking, and since the Higgs potential is determined exclusively from supersymmetry breaking terms, the electroweak scale is directly tied to the scale of superpartner masses. In the MSSM, where both  $\mu$  and supersymmetry breaking terms contribute to the Higgs potential, there is the logical possibility of fine-tuning the  $\mu$  parameter against supersymmetry breaking parameters to make the superpartners much heavier than the weak scale. In the  $\mu$ SSM, such finetuning is not possible and the superpartner masses *must* be at the weak scale.

In the next section we discuss the model and its spectrum. Remarkably, there is an upper bound on the mass of the lightest chargino of 114 GeV, not far beyond the kinematic reach of the recently completed LEP II experiment and within reach of the Tevatron.

In section 3 we show how the supersymmetry breaking terms of the  $\mu$ SSM can arise from gauge mediation, when supersymmetry breaking in the messenger sector is mostly of the  $D$ -term type which respects an  $U(1)_R$  symmetry. In section 4 we show that the approximate  $U(1)_R$  symmetry is also natural in the case of gravity mediated supersymmetry breaking, when the hidden supersymmetry breaking sector contains a gauge  $U(1)$  with a nonvanishing  $D$ -term and no gauge singlets. In section 5.1 we compute the leading one-loop contribution to the electroweak  $T$  parameter. In sections 5.2 and 6 we discuss some other desirable consequences of the approximate  $U(1)_R$  symmetry: naturally large  $\tan\beta$  without the usual enhancement of supersymmetric contributions to  $g-2$ ,  $b \rightarrow s\gamma$ , and proton decay. In section 7 we discuss a messenger sector which renders the gauge mediated model compatible with gauge coupling unification, provided the messenger scale is less than  $\sim 10^7$  GeV.

## 2. The $\mu$ /SSM model and its low energy spectrum

We start with the principle that all mass terms arise directly either from electroweak symmetry breaking or from supersymmetry breaking. We therefore do not allow a supersymmetric  $\mu$  term. The MSSM without a  $\mu$  term would have charginos lighter than the  $W$  boson, which should have been found at LEP II, so we will have to extend the theory. We add the minimal matter content to the MSSM which will allow all charginos and visible neutralinos to obtain mass beyond the current limits. What these current limits are is somewhat ambiguous since experimental limits are model dependent and have not been studied for this model. We will assume that charginos should be heavier than 104 GeV, the kinematic reach of LEP II.

In the MSSM, the charginos arise from the charged spin 1/2 components of the Higgs superfields  $H_2$  and  $H_1$ , and gauge  $W^\pm$  fields. With no  $\mu$  term the mass matrix is

$$\begin{array}{|c|cc|}
 \hline
 & -i\lambda^+ & \Psi_{H_2}^+ \\
 \hline
 -i\lambda^- & \tilde{m}_2 & \sqrt{2} m_W s_\beta \\
 \Psi_{H_1}^- & \sqrt{2} m_W c_\beta & 0 \\
 \hline
 \end{array} \tag{2.1}$$

where  $\tilde{m}_2$  is a supersymmetry breaking Majorana gaugino mass term and the off-diagonal entries break electroweak symmetry. For any value of  $\tan\beta \equiv \langle H_2/H_1 \rangle$  this mass matrix always has an eigenvalue less than the  $W$  mass and so is ruled out by the LEP II chargino mass bounds.

We will remedy this by adding matter which can mix with the MSSM charginos via supersymmetry breaking or electroweak symmetry breaking terms. The minimal such addition is a chiral superfield  $T$  which transforms as a triplet under the  $SU(2)$  gauge group, and is uncharged under the other gauge groups. We add a superpotential coupling

$$\int d^2\theta \ h_T H_1 T H_2 . \tag{2.2}$$

Now the chargino mass matrix is

$$\begin{array}{|c|ccc|}
 \hline
 & \Psi_T^+ & -i\lambda^+ & \Psi_{H_2}^+ \\
 \hline
 \Psi_T^- & 0 & \tilde{M}_2 & -h_T v_1 \\
 -i\lambda^- & \tilde{M}_2 & \tilde{m}_2 & \sqrt{2} m_W s_\beta \\
 \Psi_{H_1}^- & h_T v_2 & \sqrt{2} m_W c_\beta & 0 \\
 \hline
 \end{array} \tag{2.3}$$

where  $\tilde{M}_2$  is a soft supersymmetry breaking Dirac mass term. In the next sections we will show how  $\tilde{M}_2$  can be generated from gauge mediation or from hidden sector supersymmetry breaking. Note that all the charginos can be made heavier than 104 GeV without a  $\mu$  parameter.

A potential problem with the superpotential coupling (2.2) is that the corresponding scalar trilinear can lead to a large electroweak  $T$  parameter. With a scalar trilinear  $A_T H_2 T H_1$ , the Higgs vevs induce a tadpole for the  $T$  scalar, which will then get a vev. An electroweak triplet vev is highly constrained by the electroweak  $T$  parameter. The tadpole for the scalar

may be suppressed in three ways. First, the tadpole requires both the up and down type Higgs vevs and is suppressed at large  $\tan\beta$ . Second, the scalar trilinear is prohibited by  $U(1)_R$  symmetry. Third, trilinear terms are rather suppressed even in conventional gauge mediated models, arising only at 2 loops. All three suppressions are naturally present in a class of models arising from the nearly  $U(1)_R$  symmetric gauge mediation we discuss in section 3, and the first two are present in the Planck scale mediated models discussed in section 4. Another potentially troubling scalar trilinear arises from the coupling of the triplet to the  $SU(2)$   $D$ -term,  $A_D T_a (H_2^\dagger t_a H_2 - H_1^\dagger t_a^* H_1)$ , which is induced in both the models of supersymmetry breaking mediation we discuss. Sufficient suppression of the resulting tadpole will be possible if the mass of the  $T$  scalar is larger than a few TeV. In the gauge mediated models, a very large mass for the scalar, of order several TeV, is automatic.

At this point the reader might worry that an approximate  $U(1)_R$  symmetry will lead to a light pseudoscalar which is an approximate Goldstone boson. However we will not spontaneously break the symmetry (actually a linear combination of the original  $U(1)_R$  and electroweak hypercharge will remain unbroken). This unbroken symmetry would require  $\tan\beta \rightarrow \infty$ , but we will explicitly break the symmetry by a small amount so that  $H_2$  can get a small vev to give the leptons and down-type quarks mass. Thus we can explain the top/bottom mass hierarchy via an approximate symmetry which gives naturally large  $\tan\beta$  (as was also done in ref. [1]).

We now turn to a discussion of the spectrum of the  $\mu$ SSM, from the bottom up. We start by giving the charge assignments of some of the components of Higgs and electroweak gauge fields under the unbroken  $U(1)_R$ :

$\Psi_{H_1}$	1
$\Psi_{H_2}$	-1
$\Psi_T^\pm$	-1
$H_1$	2
$H_2$	0
$\lambda^\pm$	1

(2.4)

It is also possible to assign  $U(1)_R$  charges to quarks and leptons to allow the usual MSSM superpotential coupling.

With this  $U(1)_R$  unbroken the chargino mass matrix becomes

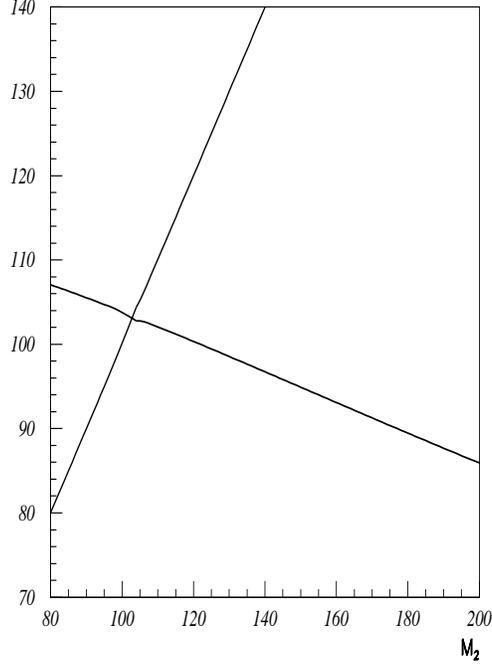
	$\Psi_T^+$	$-i\lambda^+$	$\Psi_{H_2}^+$
$\Psi_T^-$	0	$\tilde{M}_2$	0
$-i\lambda^-$	$\tilde{M}_2$	0	$\sqrt{2}m_W$
$\Psi_{H_1}^-$	$h_T v_2$	0	0

(2.5)

The eigenvalues of this matrix are:

$$m_{\chi_1^\pm} = \tilde{M}_2 \tag{2.6}$$

$$m_{\chi_{2,3}^\pm} = \frac{1}{\sqrt{2}} \sqrt{\tilde{M}_2^2 + h_T^2 v^2 + 2m_W^2 \pm \sqrt{(\tilde{M}_2^2 + h_T^2 v^2 + 2m_W^2)^2 - 8h_T^2 v^2 m_W^2}} \tag{2.7}$$



**Figure 1:** Lighter chargino masses for  $h_T = 1$ ,  $\tan\beta = 60$  and  $\tilde{m}_2 = 5$  GeV.

$$\sim \sqrt{2}m_W \frac{h_T v}{\sqrt{h_T^2 v^2 + \tilde{M}_2^2}}, \sqrt{h_T^2 v^2 + \tilde{M}_2^2 + m_W^2 \frac{2\tilde{M}_2^2}{h_T^2 v^2 + \tilde{M}_2^2}} \quad (2.8)$$

This will get modified slightly by small  $U(1)_R$  breaking effects, which will get us away from the limit  $\tan\beta \rightarrow \infty$  and set  $\tan\beta$  to a moderate value  $\sim 60$ . In this limit there is one chargino with mass  $\tilde{M}_2$  and another chargino whose mass decreases with  $\tilde{M}_2$ . To obtain masses for all charginos heavier than 104 GeV, while assuming  $h_T < 1.2$ ,  $\tilde{M}_2$  must be in the range 104-120 GeV. Moreover, the requirement, that all charginos should be heavier than 104 GeV leads to a lower bound on the Yukawa coupling,  $h_T \gtrsim 1$ . Note that  $\sqrt{2}m_W = 114$  GeV is an upper bound on the mass of the lightest chargino. Thus in the region where all charginos are heavier than 104 GeV we have two charginos with mass between 104 and 120 GeV and one heavier one.

We plot in Fig. 1 the lighter chargino masses as a function of  $\tilde{M}_2$ , for  $h_T = 1$ ,  $\tan\beta = 60$  and  $\tilde{m}_2 = 5$  GeV. In the range of  $\tilde{M}_2$  where all charginos are heavier than 104 GeV the mass of the heavier chargino is  $\sim 270$  GeV.

We now turn to the neutralino sector. In order to give the Bino a Dirac mass, analogous

to the Wino mass term, we will add a singlet chiral superfield  $S$  to the theory, and a coupling  $h_S S H_1 H_2$  to the superpotential. This is reminiscent of the NMSSM, but in the present case the expectation value of the  $S$  scalar will be much smaller than the electroweak scale. Alternatively, the singlet could be omitted, resulting in a nearly massless neutralino. Quantitative exploration of this more economical alternative has led us to the conclusion that it is difficult to simultaneously satisfy the constraints on the invisible width of the  $Z$  and the  $T$  parameter without the singlet, so we will describe the theory with the singlet included.

The neutralino mass matrix is then:

	$\Psi_T^3$	$\Psi_S$	$-i\lambda'$	$-i\lambda^3$	$\Psi_{H_1}^1$	$\Psi_{H_2}^2$
$\Psi_T^3$	0	0	0	$\tilde{M}_2$	$h_T v_2/\sqrt{2}$	$h_T v_1/\sqrt{2}$
$\Psi_S$	0	0	$\tilde{M}_1$	0	$h_S v_2/\sqrt{2}$	$h_S v_1/\sqrt{2}$
$-i\lambda'$	0	$\tilde{M}_1$	$\tilde{m}_1$	0	$-m_Z s_W c_\beta$	$m_Z s_W s_\beta$
$-i\lambda^3$	$\tilde{M}_2$	0	0	$\tilde{m}_2$	$m_Z c_W c_\beta$	$-m_Z c_W s_\beta$
$\Psi_{H_1}^1$	$h_T v_2/\sqrt{2}$	$h_S v_2/\sqrt{2}$	$-m_Z s_W c_\beta$	$m_Z c_W c_\beta$	0	0
$\Psi_{H_2}^2$	$h_T v_1/\sqrt{2}$	$h_S v_1/\sqrt{2}$	$m_Z s_W s_\beta$	$-m_Z c_W s_\beta$	0	0

(2.9)

In the large  $\tan\beta$ ,  $U(1)_R$  symmetric limit the masses become approximately Dirac, with a mass matrix of the form

	$\Psi_T^3$	$\Psi_S$	$\Psi_{H_2}^2$
$-i\lambda'$	0	$\tilde{M}_1$	$m_Z s_W$
$-i\lambda^3$	$\tilde{M}_2$	0	$-m_Z c_W$
$\Psi_{H_1}^1$	$h_T v_2/\sqrt{2}$	$h_S v_2/\sqrt{2}$	0

(2.10)

Note that in this limit there is always a nearly Dirac neutralino with mass lighter than the  $Z$ . In Fig. 2 we show the neutralino masses as a function of the soft mass term  $\tilde{M}_1$ , for  $\tilde{M}_2 = 104$  GeV,  $h_T = 1$  and  $h_S = 0.1$ . In principle the Yukawa coupling  $h_S$  is a free parameter, but large values are disfavored by electroweak precision measurements (see section 5.1). We have taken all Majorana gaugino masses equal to 5 GeV and  $\tan\beta = 60$ .

Notice that in certain regions of parameter space the lightest quasi-Dirac neutralino is lighter than  $m_Z/2$ , thus the decay of the  $Z$  to these neutralinos is kinematically allowed. The resulting increase in the width of the  $Z$  is typically very small, due to the fact that the lightest neutralino has only a small higgsino component.

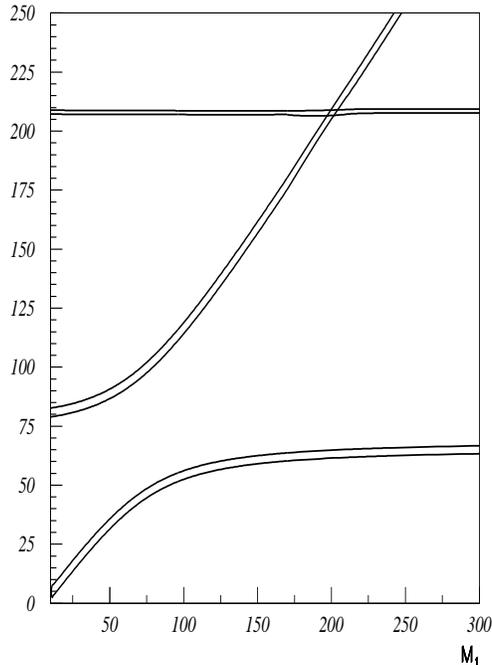
In order to allow a gluino mass without breaking the R-symmetry we also introduce a chiral superfield  $O$ , which is a color octet, and a supersymmetry breaking Dirac mass term

$$\tilde{M}_3 \psi_O \lambda_8 \tag{2.11}$$

where  $\lambda_8$  is the gluino field.

The scalar superpartners receive soft supersymmetry breaking masses as usual. Trilinear scalar couplings are suppressed by  $U(1)_R$  and quite small. A very small scalar  $\mu B$  term of order a few GeV<sup>2</sup>

$$\mu B H_1 H_2 \tag{2.12}$$



**Figure 2:** Neutralino masses as a function of  $\tilde{M}_1$ , for  $\tilde{M}_2 = 104$  GeV,  $h_T = 1$ ,  $h_S = 0.1$ ,  $\tan\beta = 60$  and  $\tilde{m}_1 = \tilde{m}_2 = 5$  GeV.

will be needed in order to induce a small vev for  $H_1$ . It is natural for this term to be small as it breaks the approximate  $U(1)_R$  symmetry. Because the symmetry is explicitly broken rather than spontaneously broken, there is no light pseudoscalar. Similarly, in the MSSM, small  $\mu B$  does not lead to a light pseudoscalar when  $\tan\beta$  is large.

Speculations on the origin of these supersymmetry breaking terms are the subject of the next two sections.

### 3. $U(1)_R$ Symmetric Gauge Mediation

In this section we assume that supersymmetry breaking is transmitted to the  $\mu$ S $\mu$ SSM by Gauge Mediated Supersymmetry Breaking (GMSB). As usual, we assume a messenger sector of heavy supermultiplets in a vector-like representation of the standard gauge group. In conventional gauge mediation, the messengers learn about supersymmetry breaking from coupling to a gauge singlet with an  $F$ -term. This transmits both supersymmetry breaking and  $U(1)_R$  symmetry breaking to the MSSM. Since we want an approximately  $U(1)_R$  symmetric  $\mu$ S $\mu$ SSM, we will assume the messenger sector does not contain any singlet. Instead supersymmetry

breaking in the messenger sector is primarily mediated by some new gauge group also carried by the messengers. Such mediation will primarily induce nonholomorphic scalar supersymmetry breaking masses in the messenger sector. Note that it is simple to construct theories with new gauge interactions carried by the messenger fields which produce such soft masses in the messenger sector [2, 3]. In fact, such theories are even simpler and more natural than most conventional gauge mediated models, which generally require some complicated model building in order to induce the required singlet  $F$ -term and messenger mass scale.

For simplicity, in this section we assume the usual messenger matter content of chiral superfields  $L, \bar{L}, D, \bar{D}$  where  $L, \bar{L}$  transform under  $SU(2) \otimes U(1)$  in conjugate representations and  $D, \bar{D}$  carry color. In section 7 we will give examples of representations which allow for successful coupling constant unification.

In order to obtain Dirac gaugino masses,  $S, T$  and  $O$  must couple to the messengers. The messenger superpotential is

$$\lambda_S S \bar{L} L + \lambda'_S S \bar{D} D + \lambda_T T \bar{L} L + \lambda_O O \bar{D} D + M_L \bar{L} L + M_D \bar{D} D . \quad (3.1)$$

The supersymmetric mass parameters  $M_L$  and  $M_D$  can be much heavier than the weak scale, and we will not discuss their origin here.

The mass matrix for, *e.g.* the  $L, \bar{L}$  scalar fields will have the following form

$$\begin{pmatrix} M_L^2 + \tilde{m}_L^2 & 0 \\ 0 & M_L^2 + \tilde{m}_{\bar{L}}^2 \end{pmatrix} \quad (3.2)$$

where  $\tilde{m}_L^2, \tilde{m}_{\bar{L}}^2$  are soft supersymmetry breaking masses. However, with no messenger singlet, to leading order the messenger sector will accidentally have unbroken  $U(1)_R$  symmetry, and no Majorana gaugino masses will be produced. In the models of refs. [2, 3], the lack of one loop Majorana gaugino masses was a phenomenological problem. In the  $\mu$ SSM, however, at one loop, the gauginos couple to the fermionic components of  $T, O$  and  $S$  and get a Dirac supersymmetry breaking mass.

Note also that provided the  $D$ -type masses are generated by new gauge interactions whose generators are orthogonal to electroweak hypercharge, *i.e.*

$$\text{Tr } T_Y T_{\text{new}} = 0 , \quad (3.3)$$

the disaster of generating a  $D$ -term for hypercharge at one loop is avoided.

There are two diagrams contributing to Dirac gaugino masses, which cancel in the limit that  $\tilde{M}_{L,D}^2 = \tilde{M}_{L,\bar{D}}^2$ .

Defining the mass-squared ratio of scalar particles to fermion as

$$y_{L,D} = \frac{M_{L,D}^2 + \tilde{m}_{L,D}^2}{M_{L,D}^2}$$

$$\bar{y}_{L,D} = \frac{M_{L,D}^2 + \tilde{m}_{L,\bar{D}}^2}{M_{L,D}^2} \quad (3.4)$$

$$(3.5)$$

we find Dirac masses  $\tilde{M}_{2,3}$  of

$$\tilde{M}_{2,3} = S_{L,D} M_{L,D} \frac{g_{2,3} \lambda_{T,O}}{2\pi^2} \left[ \frac{y_{L,D} \log(y_{L,D})}{1 - y_{L,D}} - \frac{\bar{y}_{L,D} \log(\bar{y}_{L,D})}{1 - \bar{y}_{L,D}} \right] \quad (3.6)$$

where  $S_{L,D}$  are the Dynkin indices of the  $L, D$  representations respectively. Similarly,  $\tilde{M}_1$  will receive contributions from both  $L$  and  $D$ .

In the limit that the supersymmetry breaking terms are much smaller than  $M_L$ , the result is

$$\tilde{M}_{2,3} = S_{L,D} \frac{g_{2,3} \lambda_{T,O}}{4\pi^2} \frac{\tilde{m}_{L,\bar{D}}^2 - \tilde{m}_{L,D}^2}{M_{L,D}}. \quad (3.7)$$

If the mass squared differences are regarded as arising from a  $D$  component of a  $U(1)$  gauge field  $\mathcal{W}'$ , then these graphs may be regarded as generating a supersymmetric operator

$$\int d^2\theta \frac{\xi_1}{M} \mathcal{W}_1^\alpha \mathcal{W}'_\alpha S + \frac{\xi_2}{M} \mathcal{W}_2^\alpha \mathcal{W}'_\alpha T + \frac{\xi_3}{M} \mathcal{W}_3^\alpha \mathcal{W}'_\alpha O, \quad (3.8)$$

where  $M$  is the messenger mass scale,  $\mathcal{W}_{1,2,3}$  are the standard model  $U(1)$ ,  $SU(2)$  and  $SU(3)$  gauge field strengths and  $\xi_{1,2,3}$  are dimensionless numbers. Note that supersymmetry breaking scalar trilinear couplings of the  $T, O, S$  scalars to the  $D$  components of the MSSM gauge fields, which are contained in the operator (3.8), are also generated by gauge mediation, as are the operators

$$\int d^2\theta \frac{\mathcal{W}'^\alpha \mathcal{W}'_\alpha}{M^2} (\xi'_1 S^2 + \xi'_2 T^2 + \xi'_3 O^2). \quad (3.9)$$

The operators (3.9) will give masses squared of opposite sign to the scalar and pseudoscalar components of  $S, T, O$ . Because of the large positive gauge mediated mass for the spinless components of  $S, T, O$ , an additional negative contribution to one of the masses squared is not troublesome.

The masses of scalar  $\mu$ S SM particles may be found as a special case of the general expressions computed in [4, 5]. Note that obtaining positive squark and slepton masses will require negative supertrace in the messenger sector, i.e.<sup>2</sup>

$$\tilde{m}_{\bar{L}}^2 + \tilde{m}_L^2 < 0 \quad (3.10)$$

$$\tilde{m}_{\bar{D}}^2 + \tilde{m}_D^2 < 0. \quad (3.11)$$

With a negative supertrace of messenger sector masses squared, the scalar components of  $T, S$ , and  $O$  will receive a large positive mass squared at one loop and will therefore be significantly heavier than the other superpartners. This mass is of order a loop factor times the soft masses in the messenger sector, and is not suppressed by the messenger mass scale. The  $T$  and  $O$  scalar masses should not be much larger than  $10^4$  GeV, or they will give

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<sup>2</sup>An alternative possibility for squark and slepton masses is to generate them from finite loops involving the Dirac gaugino mass [6]. However this would require ultra heavy gauginos of order several TeV, which is incompatible with the  $\mu$ S SM.

excessive two loop contributions to squark and slepton masses. The supersymmetry breaking terms in the messenger sector should therefore not be larger than of order  $M_S \sim 10^5$  GeV. Since squark and slepton masses will be of order  $(\alpha/\pi)(M_S^2/M)$ , the messenger mass scale  $M$  should be below  $10^6$  GeV.

Gauge mediated models have a generic  $\mu$  problem. It is quite difficult in gauge mediation to induce a  $\mu$  parameter which is naturally related to supersymmetry breaking, without inducing an excessively large  $B\mu$  parameter [7]. The  $\mu$ SSM avoids the gauge mediated  $\mu$  problem. A  $\mu B$  parameter can be induced which is proportional to a small coupling, and it is not a problem that the resulting  $\mu$  parameter will be much smaller than the weak scale.

It is a simple matter to induce the very small  $B\mu$  term needed for  $\tan\beta < \infty$ . For instance the messenger sector could contain a very heavy gauge singlet field  $S'$  with a coupling  $S'H_1H_2$ . If multi loop effects induce a small vev and  $F$ -term for  $S'$  from the supersymmetry breaking in the messenger sector the requisite  $B\mu$  term can be induced. The  $\mu$  term from  $\langle S' \rangle$  will be quite small, and of no phenomenological importance.

#### 4. $\mu$ SSM with Hidden Supersymmetry Breaking

The  $\mu$ SSM model with the requisite approximate accidental  $U(1)_R$  symmetry may also arise naturally in hidden sector models where the dominant mediation mechanism is from Planck scale physics. In this section we will show that the approximate  $U(1)_R$  is automatic when the hidden supersymmetry breaking sector does not contain any gauge singlets, but does have a gauged  $U(1)$  with a nonvanishing  $D$ -term. It is quite simple to build such models with dynamical supersymmetry breaking. Gravity mediated  $U(1)_R$  symmetric supersymmetry breaking has been considered before [8–11]. Refs. [8,9] introduced the  $O$  but not the  $T$  and  $S$  fields, and always predicted a chargino lighter than the  $W$ , hence are now ruled out. Ref. [10] introduced also a pair of  $T$  fields, and mentioned a possible supergravity origin for Dirac mass term in theories with hidden dynamical supersymmetry breaking.

Following ref. [10], we assume that a hidden supersymmetry breaking sector contains some gauge interactions, including a  $U(1)$  with a gauge field strength  $\mathcal{W}_\alpha$  which has nonvanishing  $D$ -term, and chiral superfield(s)  $X$  which are charged under the hidden sector gauge symmetry, which obtain  $F$ -terms. The ‘4-1’ [12,13] theory of dynamical supersymmetry breaking is a simple example of such a model.

We allow the most general gauge invariant nonrenormalizable interactions between the hidden and the  $\mu$ SSM visible sectors (consistent with enough approximate flavor symmetry to adequately suppress flavor changing neutral currents). Since there is no  $X$  gauge singlet, there are no gauge invariant terms linear in  $X$ , hence any Majorana gaugino masses or trilinear scalar couplings will be suppressed. The Dirac gaugino masses arise from

$$\int d^2\theta \frac{\xi_1}{M_P} \mathcal{W}_1^\alpha \mathcal{W}'_\alpha S + \frac{\xi_2}{M_P} \mathcal{W}_2^\alpha \mathcal{W}'_\alpha T + \frac{\xi_3}{M_P} \mathcal{W}_3^\alpha \mathcal{W}'_\alpha O . \quad (4.1)$$

The D component of  $\mathcal{W}'$ , which we call  $D'$ , will give

$$\tilde{M}_1 = \xi_1 \frac{D'}{M_P} \quad (4.2)$$

$$\tilde{M}_2 = \xi_2 \frac{D'}{M_P} \quad (4.3)$$

$$\tilde{M}_3 = \xi_3 \frac{D'}{M_P} \quad (4.4)$$

Scalar masses squared can arise in the usual way from the operators

$$\int d^4\theta \frac{\xi_0}{M_P^2} X^\dagger X Q^\dagger Q \quad (4.5)$$

where  $Q$  is an  $\mu$ SSM chiral superfield. For  $F_X \sim D'$  these should be the same approximate size as the gaugino masses.

There will also be an anomaly mediated [14, 15] contribution to Majorana gaugino masses and scalar trilinears. These effects are down by a loop factor but will lead to a small amount of  $U(1)_R$  symmetry breaking.

The  $B\mu$  term can arise from the coupling

$$\int d^4\theta \frac{\xi_b}{M_P^2} X^\dagger X H_1 H_2 . \quad (4.6)$$

There is no necessary reason why this term should be smaller than the electroweak scale, but as it breaks the R-symmetry it is quite natural in the sense of 'tHooft that it should be small. Note that a  $\mu$  and a  $B\mu$  term with  $B = m_{3/2}$  will also arise if  $H_1 H_2$  appears in the Kahler potential [14] but again an approximate R symmetry makes it natural for this term to be small.

Kinetic mixing between hypercharge and the hidden  $U(1)$  must be suppressed, as it would lead to a large hypercharge  $D$ -term. Also potentially dangerous are terms linear in the singlet  $S$  such as

$$\int d^2\theta \mathcal{W}'^\alpha \mathcal{W}'_\alpha S + \dots \quad (4.7)$$

which could give the singlet scalar a large tadpole. The necessary suppressions can be guaranteed by symmetries and nonrenormalization theorems, provided hypercharge is unified into a nonabelian group, and provided  $S$ , along with  $T$  and  $O$ , is part of an adjoint of the unified group.

## 5. SUSY contributions to precision electroweak parameters

### 5.1 $T$ parameter

The approximate  $U(1)_R$  symmetry of the  $\mu$ SSM model and/or a heavy mass for the triplet scalar provides sufficient suppression of the tree level contribution to the electroweak  $T$  parameter. However the superpotential couplings  $h_T T H_1 H_2$  and  $h_S S H_1 H_2$  break custodial  $SU(2)$  symmetry and thus can lead to potentially large one-loop effects in the  $T$  parameter. One

should keep in mind that the oblique approximation is not appropriate for light superpartners, and the complete supersymmetric one-loop corrections in this model should be considered. Such a computation is beyond the scope of the present work, so we shall interpret our results for the  $T$  parameter as an order of magnitude estimate of the radiative corrections expected in the  $\mu$ SSM model. As we will see, the effects can be sizable.

We work in the large  $\tan\beta$  limit. The main contributions to the  $T$  parameter come from chargino and neutralino loops in the  $Z$  and  $W$  self-energies. The complete expression is rather cumbersome and can be evaluated only numerically, however we have found that the approximation of keeping just the entries proportional to  $h_T, h_S$  in the chargino and neutralino mass matrices is quite good and leads to simple analytic results. In this limit we obtain

$$T = \left( \frac{h_T^2 v^2}{32\pi m_Z^2 s_W^2 c_W^2 (1-x)} \right) \times \left[ -5 + 6x - x^2 - 2(x^2 + 2x + 5) \log \left( \frac{h_T^2 (1+x)v^2}{2\mu^2} \right) + 4(x+3) \log \left( \frac{h_T^2 v^2}{\mu^2} \right) \right] \quad (5.1)$$

where  $x = h_S^2/h_T^2$  and the renormalization scale  $\mu$  should be taken to be  $m_Z$ .

Eq. (5.1) shows that the leading contribution to the  $T$  parameter grows as  $h_T^2 \log(h_T^2 v^2/\mu^2)$  and it is therefore very sensitive to the exact value of the coupling  $h_T$ . Recall that there is a lower limit on this coupling from chargino masses. Although the singlet coupling  $h_S$  also contributes to the  $T$  parameter, its contribution is negligible provided  $h_S \lesssim 0.1$  and we will ignore it in the following.

There is also a  $T$  parameter contribution from the scalar sector, due to the mass splitting between the different  $SU(2)$  components of the Higgs doublet  $H_1$  and the triplet  $T$ . This contribution is not enhanced by the  $\log \mu^2$  term though, and can be made very small by the soft supersymmetry breaking scalar masses.

From a global fit of the electroweak precision data one obtains  $T = -0.02 \pm 0.13(+0.09)$ , where the central value assumes  $M_H = 115$  GeV and the parentheses shows the change for  $M_H = 300$  GeV [16]. This bound can be relaxed for larger  $M_H$ , leading to  $T \lesssim 0.6$  at 95% CL [17].

If we impose the kinematic limit from LEP II that charginos should be heavier than 104 GeV,  $h_T \sim 1$  and the contribution to the  $T$  parameter is huge,  $\sim 2.7$ . However if the actual bound on chargino masses in this model were somewhat lower, say 90 GeV, we would obtain  $h_T \gtrsim 0.6$  which leads to  $T \sim 0.6$ . Therefore, given the large sensitivity of the  $T$  parameter to the value of  $h_T$ , a careful calculation of the chargino mass bounds is crucial to determine the viability of the  $\mu$ SSM model.

Moreover, the determination of the  $T$  parameter comes mainly from the  $Z$  width. As we have mentioned in section 2, in certain regions of parameter space the  $Z$  can decay to the lightest nearly Dirac neutralinos, and the above bounds on the  $T$  parameter will not directly apply.

## 5.2 Muon anomalous magnetic moment

In the  $\mu$ SSM model, the approximate  $U(1)_R$  symmetry suppresses the supersymmetric contributions to anomalous magnetic moments, electric dipole moments,  $b \rightarrow s\gamma$ , and proton decay. As an example, we have explicitly computed the contribution to the anomalous magnetic moment of the muon,  $a_\mu$ . The measurement of  $a_\mu$  by the  $g-2$  collaboration [18] is consistent with but slightly higher than the Standard Model prediction,  $a_\mu^{exp} - a_\mu^{SM} = (21 \pm 18)10^{-10}$ , if one uses  $a_\mu^{had} = (697 \pm 10)10^{-10}$  for the hadronic polarization contribution [19,20] and  $(8 \pm 3)10^{-10}$  for the hadronic contribution to light by light scattering. The hadronic contribution to light by light scattering has been calculated in [21–27], with satisfactory agreement between the calculations. However there is intrinsic theoretical uncertainty and hadronic model dependence in these calculations, due to nonperturbative hadronic physics, as emphasized in [28]. A model independent effective chiral Lagrangian treatment includes a low energy constant which is not constrained by other data [29]. The size of this term may be estimated from the model calculations. Assuming the size of the term is of the order indicated by modeling the hadronic physics gives an intrinsic theoretical uncertainty of order  $3 \times 10^{-10}$  in the hadronic contribution to the light by light. However, as emphasized in [29], errors in estimating the parameters which account for nonperturbative effects are not gaussian and a factor of three or more deviation from the estimate of the low energy constant would not be unusual.

The supersymmetric contributions to  $a_\mu$  [30,31] include loops with a chargino and a muon sneutrino and loops with a neutralino and a smuon. Besides the chargino and neutralino mass matrices given in section 2, we also need the smuon mass matrix. Note that the trilinear  $A$  is expected to be small in this model. We will take it to be real, as would occur if it is generated by anomaly or gauge mediation.

$$M_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 + (s_W^2 - \frac{1}{2})m_Z^2 c_{2\beta} & A v_1 \\ A v_1 & m_R^2 - s_W^2 m_Z^2 c_{2\beta} \end{pmatrix}, \quad (5.2)$$

and the sneutrino mass

$$m_{\tilde{\nu}}^2 = m_L^2 + \frac{1}{2}m_Z^2 c_{2\beta}, \quad (5.3)$$

where we have used the notation  $c_{2\beta} \equiv \cos 2\beta$ .

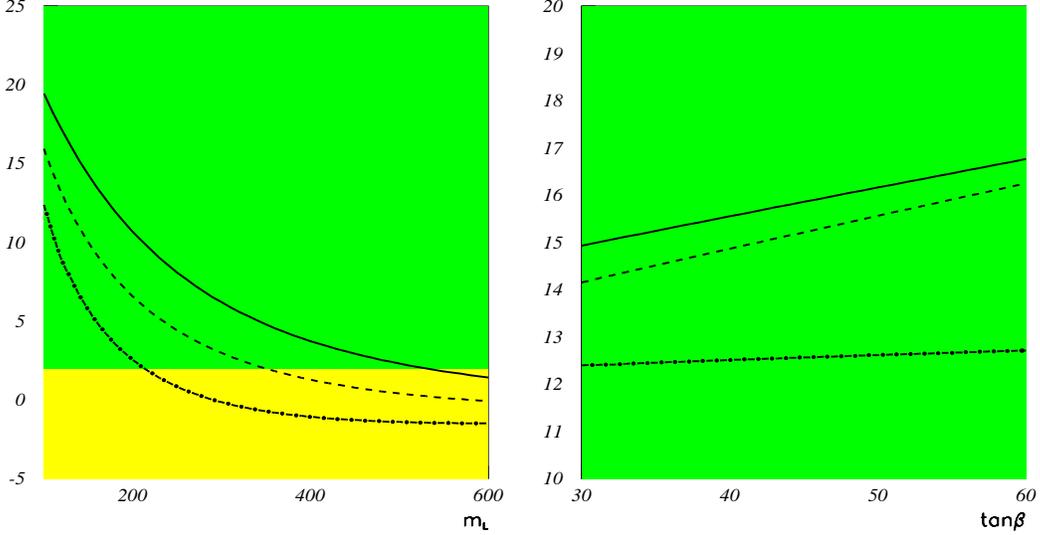
Performing the summation over all chargino, neutralino and smuon mass eigenstates, the result for  $a_\mu$  reads

$$\delta a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2} \sum_{i,m} \left\{ -\frac{m_\mu}{12m_{\tilde{\mu}_m}^2} (|n_{im}^L|^2 + |n_{im}^R|^2) F_1^N(x_{im}) + \frac{m_{\chi_i^0}}{3m_{\tilde{\mu}_m}^2} \text{Re}[n_{im}^L n_{im}^R] F_2^N(x_{im}) \right\} \quad (5.4)$$

$$\delta a_\mu^{\chi^\pm} = \frac{m_\mu}{16\pi^2} \sum_k \left\{ \frac{m_\mu}{12m_{\tilde{\nu}_\mu}^2} (|c_k^L|^2 + |c_k^R|^2) F_1^C(x_k) + \frac{2m_{\chi_k^\pm}}{3m_{\tilde{\nu}_\mu}^2} \text{Re}[c_k^L c_k^R] F_2^C(x_k) \right\} \quad (5.5)$$

where  $i = 1, \dots, 6$ ,  $k = 1, 2, 3$  and  $m = 1, 2$  are neutralino, chargino and smuon mass eigenstate labels respectively,  $x_{im} = m_{\chi_i^0}^2/m_{\tilde{\mu}_m}^2$  and  $x_k = m_{\chi_k^\pm}^2/m_{\tilde{\nu}_\mu}^2$ . The interaction vertices  $n_{im}^{L,R}$ ,  $c_k^{L,R}$  and the loop functions  $F_i^{N,C}(x)$  can be found in the appendix.

In the  $U(1)_R$  symmetric limit, the contributions to  $a_\mu$  proportional to the neutralino and chargino masses exactly vanish, and there is only a tiny effect proportional to  $m_\mu$ . However, once we take into account the small  $U(1)_R$  symmetry breaking effects, the leading contribution comes from the terms with the neutralino and chargino masses, much as in the MSSM. There are two kinds of corrections, approximately of the same order: terms proportional to the gaugino Majorana masses,  $\sim$  few GeV and terms proportional to  $v_1$  ( $\sim 4$  GeV for  $\tan\beta = 60$ ).



**Figure 3:** Maximum value of  $\delta a_\mu \times 10^{10}$  as a function of  $m_L$  and  $\tan\beta$ , for  $\tilde{m}_1 = \tilde{m}_2 = 0$  (dashed-dotted), 5 GeV (dashed) and 10 GeV (solid). We have taken  $A = 0$ ,  $m_R = 100$  GeV,  $\tilde{M}_1 = 100$  GeV,  $\tilde{M}_2 = 110$  GeV,  $h_T = 0.8$ ,  $h_S = 0.1$ ,  $\tan\beta = 60$  (left) and  $m_L = 100$  GeV (right). The shadowed areas correspond to  $1\sigma$  (dark-green) and  $2\sigma$  (light-yellow) allowed regions from the  $g-2$  collaboration result.

In Fig. 3 (left) we show the maximum possible value of  $\delta a_\mu$  in the  $\mu$ SSM model as a function of the soft supersymmetry breaking mass term  $m_L$ , for several values of the gaugino Majorana masses. We have taken all of them equal, but the results are not very sensitive to this particular choice. We have fixed  $\tilde{M}_1 = 100$  GeV,  $\tilde{M}_2 = 110$  GeV,  $A = 0$ ,  $m_R = 100$  GeV and  $\tan\beta = 60$ . These values of the soft supersymmetry breaking parameters are consistent with the constraint that all charged superpartners are heavier than 100 GeV. In Fig. 3 (right) we have plotted the maximum possible value of  $\delta a_\mu$  as a function of  $\tan\beta$ , for  $m_L = 100$  GeV and the same values of the remaining parameters.

As we discussed in sec. 2, the allowed range of  $\tilde{M}_2$ ,  $h_T$  and  $h_S$  for successful phenomenology is quite constrained, so there is not significant dependence of  $\delta a_\mu$  on these parameters. We also observe that the neutralino contribution is approximately independent of  $\tilde{M}_1$ .

Regarding the dependence of  $\delta a_\mu$  on the soft masses  $m_L, m_R$ , we find that for  $m_L \approx m_R$  the contribution from chargino loops is typically larger than the neutralino one by about one order of magnitude. If  $m_R \gg m_L$ , chargino-sneutrino loops are still dominant, since the sneutrino mass depends only on  $m_L$ , while if  $m_L \gg m_R$ , the chargino-sneutrino loops rapidly decrease, and both contributions become comparable<sup>3</sup>. For instance, if  $m_R = 100$  GeV and  $m_L \geq 500$  GeV, then  $\delta a_\mu^0 \approx \delta a_\mu^\pm$  and as we see in Fig. 3, the total  $\delta a_\mu$  is about one order of magnitude smaller than for  $m_L = m_R = 100$  GeV.

We have performed an analytic approximation, in order to better understand the numerical results. Since the chargino contribution is typically dominant we focus on this one. We have diagonalized the chargino mass matrix in the large  $\tan\beta$  limit. Then we consider the  $U(1)_R$  symmetry breaking entries as perturbations, and diagonalize the complete matrix to first order in the small parameters  $\tilde{m}_2, v_1$ . We obtain

$$\delta a_\mu^{\chi^\pm} = -\frac{\sqrt{2}g_2}{24\pi^2} \frac{m_\mu^2}{m_{\tilde{\nu}_\mu}^2} \{A \tan\beta + B\}, \quad (5.6)$$

where in the limit  $m_W \ll h_T v, \tilde{M}_2$  the coefficients  $A, B$  are given by

$$A = \tilde{m}_2 h_T \frac{\tilde{M}_2}{m_{\chi_3^\pm}^2} [G(m_{\chi_1^\pm}, m_{\chi_3^\pm}) - G(m_{\chi_1^\pm}, m_{\chi_2^\pm})] \quad (5.7)$$

and

$$B = \frac{g_2}{\sqrt{2}} \left( \frac{\tilde{M}_2^2}{m_{\chi_3^\pm}^2} G(m_{\chi_1^\pm}, m_{\chi_2^\pm}) + \frac{h_T^2 v_2^2}{m_{\chi_3^\pm}^2} G(m_{\chi_1^\pm}, m_{\chi_3^\pm}) \right) + h_T \frac{\tilde{M}_2}{m_{\chi_3^\pm}} F(m_{\chi_1^\pm}, m_{\chi_2^\pm}) - \frac{g_2 h_T^2}{\sqrt{2}} \frac{\tilde{M}_2 v_2^2}{m_{\chi_3^\pm}^3} F(m_{\chi_1^\pm}, m_{\chi_3^\pm}) \quad (5.8)$$

where we have defined

$$F(m_{\chi_1^\pm}, m_{\chi_i^\pm}) = \frac{m_{\chi_1^\pm} m_{\chi_i^\pm}}{m_{\chi_i^\pm}^2 - m_{\chi_1^\pm}^2} [F_2^C(x_i) - F_2^C(x_1)], \quad (5.9)$$

$$G(m_{\chi_1^\pm}, m_{\chi_i^\pm}) = \frac{m_{\chi_i^\pm}^2 F_2^C(x_i) - m_{\chi_1^\pm}^2 F_2^C(x_1)}{m_{\chi_i^\pm}^2 - m_{\chi_1^\pm}^2}. \quad (5.10)$$

From the approximate expression (5.6) we see that although the contribution to  $a_\mu$  in the  $\mu$ SSM model is also enhanced for large  $\tan\beta$ , due to the approximate  $U(1)_R$  it is suppressed by the small gaugino Majorana masses and therefore much smaller than in the MSSM.

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<sup>3</sup>Notice that in the  $\mu$ SSM model the trilinear terms are very small, and moreover they are multiplied by  $v_1$  in the smuon mass matrix (5.2), so the mixing in the smuon sector is negligible.

## 6. Proton Decay

One advantage of a  $U(1)_R$  symmetry is that a large ratio of Higgs vevs  $H_2/H_1$  becomes natural, explaining the ratio of top to bottom quark masses in terms of an approximate symmetry. Thus a large number of supersymmetric processes which are normally dangerously large in the large  $\tan\beta$  limit become suppressed. One of the most significant suppressions is of proton decay from dimension 5 operators.

In the MSSM, dimension 5 operators in the superpotential of the form  $qqq\ell$  and  $\bar{u}\bar{u}\bar{d}\bar{e}$  can lead to rapid proton decay and must be quite suppressed. These are dangerous even if associated with an inverse power of the Planck scale and with the same small parameters which suppress the light quark and lepton masses. However in the  $U(1)_R$  symmetric limit, such supersymmetric operators do not lead to proton decay, since a linear combination of baryon number and  $U(1)_R$  charge remains unbroken. The proton decay rate is thus suppressed by a welcome factor of the square of ratio of the R symmetry breaking terms to the R symmetric supersymmetry breaking terms, a factor of approximately  $1/\tan^2\beta \sim 4 \times 10^{-4}$ .

## 7. Unification of couplings

One rationale for supersymmetry is coupling constant unification. If we add matter to the MSSM in incomplete multiplets under the unifying group the usual successful prediction of  $s_W^2 \approx .23$  may be lost. In the  $\mu$ SSM we have added matter in the adjoint representation of  $U(1) \otimes SU(2) \otimes SU(3)$ , which will not preserve the usual prediction. It is, however a simple matter to embed the  $T, S$  and  $O$  fields into a complete adjoint multiplet of a GUT such as  $SU(3)^3$  [32] or  $SU(5)$  [33].

It would be economical, although not necessary, to have the other fields of the multiplet serve as the messenger fields of a gauge mediated model. Thus we could take the messenger multiplets to transform under the standard  $SU(3) \otimes SU(2) \otimes U(1)$  gauge group as any number of complete unified multiplets plus in addition a multiplet transforming either as

$$(1, 2, \pm 1/2) + (1, 1, \pm 1) + (1, 1, \pm 1) \quad (7.1)$$

or as

$$(3, 2, -5/6) + (\bar{3}, 2, 5/6) . \quad (7.2)$$

There is an upper bound on the mass scale of the new fields for unification will be preserved, with the usual one loop result being obtained in the limit where all fields are at the electroweak scale. However the resulting constraint is very mild.

If one assumes all the  $\mu$ SSM superpartners are at the weak scale, and computes the one-loop running neglecting threshold effects, one can fit the scales of the new matter multiplet and GUT to the low energy gauge coupling constants. The result is

$$M_{\text{new}} = M_{\text{weak}} e^{\frac{2\pi}{3} \left( \frac{12}{\alpha_2} - \frac{5}{\alpha_1} - \frac{7}{\alpha_3} \right)}$$

(7.3)

$$M_{\text{GUT}} = M_{\text{weak}} e^{\frac{5\pi}{6} \left( \frac{3}{\alpha_2} - \frac{1}{\alpha_1} - \frac{2}{\alpha_3} \right)}. \quad (7.4)$$

By taking values for the coupling constants at the edge of their allowed ranges, *e.g.*  $\alpha(M_Z) = 1/127.7$ ,  $\alpha_s(M_Z) = 0.122$ , and  $s_W^2 = 0.233$  the additional matter fields can be as heavy as  $3 \times 10^7$  GeV and the GUT scale as high as  $10^{18}$  GeV. Threshold effects at the GUT, messenger and  $\mu$ SSM scales and higher loop corrections make order one changes in these predictions. This constraint is less stringent than the upper bound on the messenger scale found in section 3, and so if the additional fields are part of the messenger sector the gauge couplings will unify, to a good approximation.

It is tempting to speculate on an extra dimensional origin for such split adjoint matter multiplets. After all, extra dimensional theories in which gauge bosons live in the bulk and chiral matter fields live on a three brane typically have additional matter fields in the adjoint representation when described four dimensionally, unless the extra dimension is orbifolded. The adjoint fields might be  $N = 2$  superpartners of the gauge fields. We will leave aside such model building issues here, but these will be explored in ref. [6].

## 8. Summary

We have proposed a viable supersymmetric model without a  $\mu$  parameter, by extending the matter content of the MSSM. Charginos, neutralinos and gluinos get supersymmetry breaking Dirac mass terms by mixing with, respectively, the fermionic components of a  $SU(2)$  triplet chiral superfield, a singlet chiral superfield, and a color octet. The  $\mu$ SSM can naturally arise from either gauge or gravity mediation, if the supersymmetry breaking sector respects an approximate  $U(1)_R$  symmetry. Such an approximate symmetry can easily arise by accident, as a consequence of the absence of gauge singlet chiral superfields with  $F$ -terms in the supersymmetry breaking or mediation sector.

We have studied aspects of the phenomenology of the  $\mu$ SSM, which contains two light charginos with masses within the reach of the TeVatron, and a quasi-Dirac neutralino lighter than the  $Z$  boson. Such a light quasi-Dirac particle might have some unusual features as a dark matter candidate, along the lines of [34].

A very strong constraint on the  $\mu$ SSM model comes from the contribution to the electroweak  $T$  parameter. In the absence of a  $\mu$  term, the superpotential coupling  $h_T$  in (2.2) should be large for the charginos to be heavier than the current experimental limit from LEP II. This coupling breaks custodial  $SU(2)$  symmetry and gives the main contribution to the  $T$  parameter. The size of this correction is directly correlated with the chargino mass bounds, so it would be very interesting to find the experimental bounds on the chargino masses in this particular model, since the existing ones for the MSSM are not applicable.

We have also computed the supersymmetric contribution to the muon anomalous magnetic moment within the  $\mu$ SSM. We find a strong suppression due to the approximate  $U(1)_R$

symmetry; as a consequence, the effect is small even for large values of  $\tan \beta$  and light superpartners.

The MSSM bound on the lightest Higgs mass does not apply, though, since the scalar sector is also enlarged by the scalar components of the  $SU(2)$  triplet and scalar chiral superfields, and there are new,  $F$ -component contributions to the Higgs quartic coupling. There will still be some upper bounds from triviality of the superpotential couplings, as computed in general models with Higgs triplets in refs. [35,36].

There are many aspects of this model deserving further study such as the most effective search strategies for the superpartners, a more complete, predictive and explicit gauge mediated origin for the supersymmetry breaking parameters, the origin of the approximate  $U(1)_R$  symmetry and of the small explicit breaking terms, and whether the required fields and symmetries could be obtained from extra dimensions.

## 9. Appendix

The left- and right- muon-smuon-neutralino and muon-sneutrino-chargino vertices which appear in  $\delta a_\mu$ , eqs. (5.4), (5.5), are given by

$$n_{im}^R = \sqrt{2}N_{i3}X_{m2} + y_\mu N_{i5}X_{m1} \quad (\text{A. 1})$$

$$n_{im}^L = \frac{1}{\sqrt{2}}(g_2 N_{i4} + g_1 N_{i3})X_{m1}^* - y_\mu N_{i5}X_{m2}^* \quad (\text{A. 2})$$

$$c_k^R = y_\mu U_{k3} \quad (\text{A. 3})$$

$$c_k^L = -g_2 V_{k2} \quad (\text{A. 4})$$

where  $N, U, V, X$  are the unitary transformations that diagonalize the neutralino, chargino and smuon mass matrices, i.e. they satisfy

$$N^* M_{\chi^0} N^\dagger = \text{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}, m_{\chi_5^0}, m_{\chi_6^0}) \quad (\text{A. 5})$$

$$U^* M_{\chi^\pm} V^\dagger = \text{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}, m_{\chi_3^\pm}) \quad (\text{A. 6})$$

$$X M_{\tilde{\mu}}^2 X^\dagger = \text{diag}(m_{\tilde{\mu}_1}^2, m_{\tilde{\mu}_2}^2) \quad (\text{A. 7})$$

The loop functions have the form

$$F_1^N(x) = \frac{2}{(1-x)^4}(1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x) \quad (\text{A. 8})$$

$$F_2^N(x) = \frac{3}{(1-x)^3}(1 - x^2 + 2x \log x) \quad (\text{A. 9})$$

$$F_1^C(x) = \frac{2}{(1-x)^4}(2 + 3x - 6x^2 + x^3 + 6x \log x) \quad (\text{A. 10})$$

$$F_2^C(x) = \frac{-3}{2(1-x)^3}(3 - 4x + x^2 + 2 \log x) \quad (\text{A. 11})$$

and are normalized so that  $F_i^N(1) = F_i^C(1) = 1$  ( $i = 1, 2$ ), corresponding to degenerate particles.

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## References

- [1] A. E. Nelson and L. Randall, *Naturally large  $\tan\beta$* , *Phys. Lett.* **B316** (1993) 516–520, [<http://arXiv.org/abs/hep-ph/9308277>].
- [2] L. Randall, *New mechanisms of gauge-mediated supersymmetry breaking*, *Nucl. Phys.* **B495** (1997) 37–56, [<http://arXiv.org/abs/hep-ph/9612426>].
- [3] C. Csaki, L. Randall, and W. Skiba, *Composite intermediary and mediator models of gauge-mediated supersymmetry breaking*, *Phys. Rev.* **D57** (1998) 383–390, [<http://arXiv.org/abs/hep-ph/9707386>].
- [4] E. Poppitz and S. P. Trivedi, *Some remarks on gauge-mediated supersymmetry breaking*, *Phys. Lett.* **B401** (1997) 38–46, [<http://arXiv.org/abs/hep-ph/9703246>].
- [5] N. Arkani-Hamed, G. F. Giudice, M. A. Luty, and R. Rattazzi, *Supersymmetry-breaking loops from analytic continuation into superspace*, *Phys. Rev.* **D58** (1998) 115005, [<http://arXiv.org/abs/hep-ph/9803290>].
- [6] P. Fox, A. Nelson, and N. Weiner, *Dirac gaugino masses and supersoft supersymmetry breaking*, <http://arXiv.org/abs/hep-ph/0206096>.
- [7] M. Dine, A. E. Nelson, and Y. Shirman, *Low-energy dynamical supersymmetry breaking simplified*, *Phys. Rev.* **D51** (1995) 1362–1370, [<http://arXiv.org/abs/hep-ph/9408384>].
- [8] L. J. Hall and L. Randall,  *$U(1)$ - $r$  symmetric supersymmetry*, *Nucl. Phys.* **B352** (1991) 289–308.
- [9] L. Randall and N. Rius, *The minimal  $U(1)$ - $r$  symmetric model revisited*, *Phys. Lett.* **B286** (1992) 299–306.
- [10] M. Dine and D. MacIntire, *Supersymmetry, naturalness, and dynamical supersymmetry breaking*, *Phys. Rev.* **D46** (1992) 2594–2601, [<http://arXiv.org/abs/hep-ph/9205227>].
- [11] J. L. Feng, N. Polonsky, and S. Thomas, *The light higgsino-gaugino window*, *Phys. Lett.* **B370** (1996) 95–105, [<http://arXiv.org/abs/hep-ph/9511324>].
- [12] M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, *New tools for low-energy dynamical supersymmetry breaking*, *Phys. Rev.* **D53** (1996) 2658–2669, [<http://arXiv.org/abs/hep-ph/9507378>].
- [13] E. Poppitz and S. P. Trivedi, *Some examples of chiral moduli spaces and dynamical supersymmetry breaking*, *Phys. Lett.* **B365** (1996) 125–131, [<http://arXiv.org/abs/hep-th/9507169>].
- [14] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B557** (1999) 79–118, [<http://arXiv.org/abs/hep-th/9810155>].

- [15] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **12** (1998) 027, [<http://arXiv.org/abs/hep-ph/9810442>].
- [16] **Particle Data Group** Collaboration, D. E. Groom *et. al.*, *Review of particle physics*, *Eur. Phys. J.* **C15** (2000) 1–878.
- [17] R. S. Chivukula, *Limits on the mass of a composite higgs boson*, <http://arXiv.org/abs/hep-ph/0005168>.
- [18] **Muon g-2** Collaboration, H. N. Brown *et. al.*, *Precise measurement of the positive muon anomalous magnetic moment*, *Phys. Rev. Lett.* **86** (2001) 2227–2231, [<http://arXiv.org/abs/hep-ex/0102017>].
- [19] M. Davier and A. Hocker, *New results on the hadronic contributions to  $\alpha(m(z)^{**2})$  and to  $(g-2)(\mu)$* , *Phys. Lett.* **B435** (1998) 427–440, [<http://arXiv.org/abs/hep-ph/9805470>].
- [20] F. Jegerlehner, *The effective fine structure constant at Tesla energies*, <http://arXiv.org/abs/hep-ph/0105283>.
- [21] M. Hayakawa and T. Kinoshita, *Comment on the sign of the pseudoscalar pole contribution to the muon g-2*, <http://arXiv.org/abs/hep-ph/0112102>.
- [22] M. Knecht, A. Nyffeler, M. Perrottet, and E. De Rafael, *Hadronic light-by-light scattering contribution to the muon g-2: An effective field theory approach*, *Phys. Rev. Lett.* **88** (2002) 071802, [<http://arXiv.org/abs/hep-ph/0111059>].
- [23] M. Knecht and A. Nyffeler, *Hadronic light-by-light corrections to the muon g-2: The pion-pole contribution*, *Phys. Rev.* **D65** (2002) 073034, [<http://arXiv.org/abs/hep-ph/0111058>].
- [24] W. J. Marciano and B. L. Roberts, *Status of the hadronic contribution to the muon (g-2) value*, <http://arXiv.org/abs/hep-ph/0105056>.
- [25] E. Bartos, A. Z. Dubnickova, S. Dubnicka, E. A. Kuraev, and E. Zemlyanaya, *Scalar and pseudoscalar meson pole terms in the hadronic light-by-light contributions to muon  $a(\mu)(had)$* , <http://arXiv.org/abs/hep-ph/0106084>.
- [26] J. Bijnens, E. Pallante, and J. Prades, *Comment on the pion pole part of the light-by-light contribution to the muon g-2*, *Nucl. Phys.* **B626** (2002) 410–411, [<http://arXiv.org/abs/hep-ph/0112255>].
- [27] I. Blokland, A. Czarnecki, and K. Melnikov, *Pion pole contribution to hadronic light-by-light scattering and muon anomalous magnetic moment*, *Phys. Rev. Lett.* **88** (2002) 071803, [<http://arXiv.org/abs/hep-ph/0112117>].
- [28] K. Melnikov, *On the theoretical uncertainties in the muon anomalous magnetic moment*, *Int. J. Mod. Phys.* **A16** (2001) 4591–4612, [<http://arXiv.org/abs/hep-ph/0105267>].
- [29] M. Ramsey-Musolf and M. B. Wise, *Hadronic light-by-light contribution to muon g-2 in chiral perturbation theory*, <http://arXiv.org/abs/hep-ph/0201297>.
- [30] T. Moroi, *The muon anomalous magnetic dipole moment in the minimal supersymmetric standard model*, *Phys. Rev.* **D53** (1996) 6565–6575, [<http://arXiv.org/abs/hep-ph/9512396>].
- [31] S. P. Martin and J. D. Wells, *Muon anomalous magnetic dipole moment in supersymmetric theories*, *Phys. Rev.* **D64** (2001) 035003, [<http://arXiv.org/abs/hep-ph/0103067>].

- [32] V. A. Rizov, *A gauge model of the electroweak and strong interactions based on the group  $su(3)_l \times su(3)_r \times su(3)_c$* , *Bulg. J. Phys.* **8** (1981) 461–477.
- [33] H. Georgi and S. L. Glashow, *Unity of all elementary particle forces*, *Phys. Rev. Lett.* **32** (1974) 438–441.
- [34] D. R. Smith and N. Weiner, *Inelastic dark matter*, *Phys. Rev.* **D64** (2001) 043502, [<http://arXiv.org/abs/hep-ph/0101138>].
- [35] J. R. Espinosa and M. Quiros, *Higgs triplets in the supersymmetric standard model*, *Nucl. Phys.* **B384** (1992) 113–146.
- [36] J. R. Espinosa and M. Quiros, *Gauge unification and the supersymmetric light higgs mass*, *Phys. Rev. Lett.* **81** (1998) 516–519, [<http://arXiv.org/abs/hep-ph/9804235>].