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Scalar $K\pi$ form factor and light quark masses

Matthias Jamin,* José Antonio Oller,† and Antonio Pich‡
*Institució Catalana de Recerca i Estudis Avançats (ICREA), Theoretical Physics Group, IFAE, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona, Spain,
†Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain, and
‡Departament de Física Teòrica, IFIC, Universitat de València – CSIC, Apt. Correus 22085, E-46071 València, Spain.

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Recent experimental improvements on K-decay data allow for a precise extraction of the strangeness-changing scalar $K\pi$ form factor and the related strange scalar spectral function. On the basis of this scalar as well as the corresponding pseudoscalar spectral function, the strange quark mass is determined to be $m_s(2\,\mathrm{GeV}) = 92 \pm 9\,\mathrm{MeV}$. Further taking into account chiral perturbation theory mass ratios, the light up and down quark masses turn out to be $m_u(2\,\mathrm{GeV}) = 2.7 \pm 0.4\,\mathrm{MeV}$ as well as $m_d(2\,\mathrm{GeV}) = 4.8 \pm 0.5\,\mathrm{MeV}$. As a by-product, we also find a value for the Cabibbo angle $|V_{us}| = 0.2236(29)$ and the ratio of meson decay constants $F_K/F_\pi = 1.203(16)$. Performing a global average of the strange mass by including extractions from other channels as well as lattice QCD results yields $m_s(2\,\mathrm{GeV}) = 94 \pm 6\,\mathrm{MeV}$.

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I. INTRODUCTION

Together with the strong coupling, quark masses are fundamental QCD parameters of the Standard Model (SM). Precise values are required as an input in many phenomenological applications and they can also prove useful in constraining unified theories which go beyond the SM. In the light quark sector, of special interest is the mass of the strange quark, as it plays an important role in theoretical predictions of the parameter for direct CP violation ε'/ε and SU(3) breaking in hadronic B decays.

Present day approaches to determine quark mass values include on a more fundamental level lattice gauge theory, whereas on a more phenomenological level, operator product expansion (OPE) as well as analyticity properties can be exploited to relate the quark-gluon picture of QCD to hadronic observables [1], thereby also allowing for an extraction of quark masses. Further information on the light quark masses can be obtained from chiral perturbation theory (χPT) [2], which permits to make predictions for quark mass ratios. In this work, the latter two approaches will be considered to first determine the mass of the strange quark m_s and from that also the up and down quark masses m_u and m_d.

Good sensitivity to m_s is achieved in the analysis of scalar and pseudoscalar hadronic channels, since the corresponding two-point correlation functions turn out to be proportional to m_s^2. Both channels have already been studied extensively in the literature. In the scalar channel, after the pioneering work of [3], recent analyses have been performed in refs. [4–11], whereas the pseudoscalar channel has been investigated in refs. [12, 13]. A fundamental ingredient for the scalar channel analysis is the scalar strange spectral function, which has been obtained in a series of articles [4, 14, 15] on the basis of dispersion relations and S-wave $K\pi$ scattering data.

Recent experimental improvements on K-decay data, to be discussed in the next section, allow for a precise determination of the ratio $F_K/F_\pi/F_+^{K\pi}(0)$, and consequently for a substantial improvement of the scalar $K\pi$ form factor and the related strange spectral function which will be performed in section 3. In section 4, the scalar strange spectral function is employed to determine the strange mass m_s. For comparison, we also consider the m_s determination from the pseudoscalar channel in section 5. Finally, in the conclusions, our final average for m_s will be presented and compared to recent extractions from lattice QCD, and from our result for m_s and χPT mass ratios, also m_u and m_d are deduced.

II. THE RATIO $F_K/F_\pi/F_+^{K\pi}(0)$

Todays knowledge of the pseudoscalar meson decay constants dominantly originates from the leptonic decays $P \to l\nu$ [16]. The ratio of the decay rates $\Gamma[K \to l\nu_l(\gamma)]$ and $\Gamma[\pi \to l\nu_l(\gamma)]$ is directly proportional to the square of F_K/F_π. Including electromagnetic radiative corrections according to Marciano and Sirlin [17], it reads

$$\frac{\Gamma[K \to l\nu_l(\gamma)]}{\Gamma[\pi \to l\nu_l(\gamma)]} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2 M_K}{F_\pi^2 M_\pi}\frac{(1-x_K^2)^2}{(1-x_\pi^2)^2}\frac{[1+\frac{\alpha}{\pi}F(x_K)]}{[1+\frac{\alpha}{\pi}F(x_\pi)]}$$
$$\cdot\left\{1-\frac{\alpha}{\pi}\left[\frac{3}{2}\ln\frac{M_\pi}{M_K}+\Delta C_1+\Delta C_2\frac{m_l^2}{M_\rho^2}\ln\frac{M_\rho^2}{m_l^2}+\Delta C_3\frac{m_l^2}{M_\rho^2}\right]\right\} \quad (1)$$

where $x_P \equiv m_l/M_P$ and $\Delta C_i \equiv C_{iK} - C_{i\pi}$. The constants C_{iP} depend on the hadronic structure, and an explicit expression for the function $F(x_P)$ can be found in ref. [17].

While extracting F_K/F_π from the ratio (1), the dominant uncertainty is due to the CKM matrix element V_{us}.

*Electronic address: jamin@ifae.es
†Electronic address: Oller@um.es
‡Electronic address: Antonio.Pich@uv.es

Thus, it is preferable to present a value for the ratio

$$\frac{|V_{us}|}{|V_{ud}|}\frac{F_K}{F_\pi} = 0.27618\,(39)\,(27)\,(6) = 0.27618\,(48)\,. \quad (2)$$

To arrive at this result, the experimental $K_{\mu 2}$ and $\pi_{\mu 2}$ decay rates have been used. For the $K_{\mu 2}$ decay rate, we have employed the very recent result by the KLOE collaboration $B[K^+ \to \mu^+\nu_\mu(\gamma)] = 0.6366\,(18)$ [18], and all other experimental inputs have been taken from the Review of Particle Physics [16]. ΔC_1 has recently been calculated in the framework of χPT [19] with the result $\Delta C_1 = Z/2\,\ln(M_K/M_\pi)$, where the chiral coupling $Z = 0.8\pm 0.2$ arises from the electromagnetic mass difference of the pion. For the remaining constants, the generous estimates $\Delta C_2 = 0\pm 1$ and $\Delta C_3 = 0\pm 3$ [17] have been employed. In eq. (2), the separated errors correspond to the uncertainty resulting from the $K_{\mu 2}$ branching ratio, the remaining experimental inputs, being dominated by the K lifetime, and the radiative corrections, respectively. The total radiative correction in eq. (1) then amounts to $-\,3.05\,(16)\,\frac{\alpha}{\pi}$, in agreement to the result of ref. [20], obtained in a different framework.

To extract a value for $F_K/F_\pi/F_+^{K\pi}(0)$, we now have to assume inputs for $|V_{us}|\,F_+^{K\pi}(0)$ and $|V_{ud}|$. For both quantities, we employ the results $|V_{us}|\,F_+^{K\pi}(0) = 0.2173\,(8)$ from K_{e3} decays as well as $|V_{ud}| = 0.9738\,(3)$, presented in the very recent review [21]. (For comparison, see however also ref. [22].) One then obtains:

$$\frac{F_K}{F_\pi\,F_+^{K\pi}(0)} = 1.2377\,(22)\,(46)\,(5) = 1.2377\,(51)\,, \quad (3)$$

where the different errors correspond to the uncertainties of eq. (2), $|V_{us}|\,F_+^{K\pi}(0)$, and $|V_{ud}|$, respectively. From eq. (3), we observe that the uncertainty on the considered ratio is dominated by the experimental result for $|V_{us}|\,F_+^{K\pi}(0)$. The influence of $|V_{ud}|$ is rather small, and even increasing its error by a factor of two practically would have no effect.

In all previous expressions the hadronic matrix elements F_K, F_π and $F_+^{K\pi}(0)$ are defined in the framework of pure QCD, i.e. in the limit $\alpha_{\rm QED} = 0$. To the quoted level of precision, electromagnetic corrections have a crucial effect in the measured physical observables and are taken into account through explicit correction factors, as shown in eq. (1) for F_K/F_π [17, 23]. The quoted value for $|V_{us}|\,F_+^{K\pi}(0)$, has been derived from K_{e3} data [21], using the electromagnetic corrections computed in ref. [24].

III. THE SCALAR $K\pi$ FORM FACTOR

As the next step towards the determination of light quark masses, the scalar $K\pi$ form factor $F_0^{K\pi}(t) \equiv F_0(t)$ [50] will be calculated along the lines of the dispersion theoretic approach of refs. [14, 15], employing additional constraints both at long and short distances. For solving the system of dispersion-integral equations, in this approach two external input parameters are required. Like in ref. [15], these inputs will be the value of the form factor at $t = 0$, $F_0(0) = F_+(0)$, as well as its value at the Callan-Treiman point $\Delta_{K\pi} \equiv M_K^2 - M_\pi^2$, $F_0(\Delta_{K\pi})$.

For $F_0(0)$, we employ an average over recent determinations from the lattice and effective field theory approaches [25–30]

$$F_0(0) = 0.972(12)\,, \quad (4)$$

which is also compatible with the original estimate by Leutwyler and Roos [31]. Together with the results of the previous section, this choice corresponds to

$$|V_{us}| = 0.2236(29) \quad \text{and} \quad \frac{F_K}{F_\pi} = 1.203(16)\,. \quad (5)$$

The value for $|V_{us}|$ is compatible with unitarity at the $1.2\,\sigma$ level, while F_K/F_π is about $1\,\sigma$ lower than the result of ref. [31]. The ratio of decay constants is also in nice agreement with the recent lattice results [32].

Because the value at the origin only concerns the global normalisation of $F_0(t)$, from the dispersive approach a more precise determination of the form factor shape can be obtained by first concentrating on the ratio $F_0(t)/F_0(0)$. On the other hand, to a very good approximation, the second required input $F_0(\Delta_{K\pi})$ is given by F_K/F_π, which then turns into the relation

$$\frac{F_0(\Delta_{K\pi})}{F_0(0)} = \frac{F_K}{F_\pi\,F_0(0)} + \frac{\Delta_{\rm CT}}{F_0(0)} = 1.2346(53)\,, \quad (6)$$

where $\Delta_{\rm CT} = -\,3\cdot 10^{-3}$ has been obtained within χPT at order p^4 [33], and half of this value has been added quadratically to the uncertainty of eq. (3).

Using this input parameter and performing an average over the different fits to the S-wave $K\pi$ scattering data, as discussed in detail in [15], for the slope and the curvature of the form factor at the origin, we obtain:

$$\frac{F_0'(0)}{F_0(0)} = 0.773(21)\,{\rm GeV}^{-2}\,, \; \frac{F_0''(0)}{F_0(0)} = 1.599(52)\,{\rm GeV}^{-4}\,. \quad (7)$$

Our result for the slope can also be expressed in terms of the scalar $K\pi$ squared radius, or the parameter λ_0:

$$\langle r^2_{K\pi}\rangle \;=\; 6\,\frac{F_0'(0)}{F_0(0)} = 0.1806(49)\,{\rm fm}^2\,, \quad (8)$$

$$\lambda_0 \;=\; M_\pi^2\,\frac{F_0'(0)}{F_0(0)} = 0.0147(4)\,. \quad (9)$$

The value for λ_0 is compatible with our previous result presented in [25]. It is also in good agreement with the recent experimental result by KTeV [34], but is about 3σ lower then the corresponding ISTRA result [35], where however $F_0''(0)$ was found to be compatible with zero. In figure 1, we display our result for the the scalar $K\pi$ form factor $F_0(t)$ as a function of $\sqrt{t}$, while varying its value at the Callan-Treiman-point according to eq. (6).

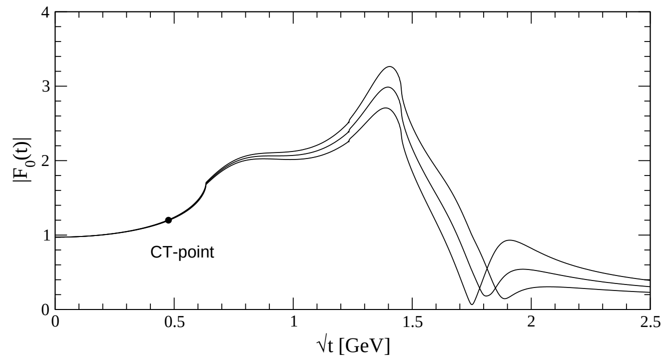

FIG. 1: Result for the scalar $K\pi$ form factor $F_0(t)$ with the range corresponding to a variation of the form factor at the Callan-Treiman-point according to eq. (6).

IV. M_S FROM THE SCALAR CHANNEL

The numerical analysis of the scalar channel proceeds in complete analogy to the previous analyses [4, 6, 11] where the detailed theoretical expressions can be found. The central equation for the extraction of the light quark masses from the scalar and pseudoscalar spectral functions takes the form

$$u\,\widehat{\Psi}_{th}(u) = \int_0^{s_0} \rho_{ph}(s)\,e^{-s/u}\,ds + \int_{s_0}^{\infty} \rho_{th}(s)\,e^{-s/u}\,ds\,, \quad (10)$$

in which $\widehat{\Psi}_{th}(u)$ denotes the Borel-transformed theoretical 2-point correlator, providing an exponential suppression of higher-energy contributions where no experimental information is available, and u is the so-called Borel variable. Expressions for $\widehat{\Psi}_{th}(u)$ up to $\mathcal{O}(\alpha_s^3)$ can be found in [4, 11] and we have also included the very recent result on the perturbative $\mathcal{O}(\alpha_s^4)$ contribution [12, 36].

The OPE which is employed for calculating $\widehat{\Psi}_{th}(u)$ is valid only at sufficiently large $u \gg \Lambda_{\rm QCD}^2$. As is well known, for the scalar and pseudoscalar channels a breakdown of the OPE is expected to occur at a relatively large u around $1\,{\rm GeV}^2$, due to the presence of non-perturbative vacuum effects which go beyond the local condensate expansion [37, 38]. Models of the correlation function based on instanton configurations, such as the instanton liquid model (ILM) [39–41], allow to penetrate into the region of smaller u. However, as realised e.g. in [13], at sufficiently large $u \approx 2\,{\rm GeV}^2$, the ILM correction turns out to be rather small, and hence we shall avoid it by choosing $2\,{\rm GeV}^2$ as a lower limit for the Borel variable, which furthermore also reduces the uncertainty from higher order α_s corrections.

The relation between the phenomenological spectral function and the strangeness-changing scalar form factor is given by

$$\rho_{ph}(s) = \frac{3\Delta_{K\pi}^2}{32\pi^2}\left[\sigma_{K\pi}(s)|F_0^{K\pi}(s)|^2 + \sigma_{K\eta'}(s)|F_0^{K\eta'}(s)|^2\right], \quad (11)$$

where $\sigma_{KP}(s)$ is the two-particle phase space factor and like in ref. [4], we have also included the $K\eta'$ contribution $F_0^{K\eta'}(s)$. A possible source of systematic uncertainty may be the neglect of more than 2-particle final states, on which we comment further below. Above the energy s_0, the spectral function is again approximated by the theoretical expression $\rho_{th}(s)$.

Performing the m_s analysis on the basis of eqs. (10) and (11), for the running mass in the $\overline{\rm MS}$ scheme we find

$$m_s(2\,{\rm GeV}) = 87.6\,^{+\,8.8}_{-\,6.8}\;{\rm MeV}\,, \quad (12)$$

at a central value $s_0 = 4.4\,{\rm GeV}^2$, where m_s is most stable with respect to variations of u in the range $2\,{\rm GeV}^2 < u < 4\,{\rm GeV}^2$. This and all other input parameters whose variation induces a shift in m_s of more than 1 MeV have been collected in table I.

Parameter	Value	Δm_s [MeV]
$F_0(\Delta_{K\pi})/F_0(0)$	1.2346(53)	$^{+\,7.0}_{-\,5.3}$
$F_0(0)$	0.972(12)	$^{+\,1.0}_{-\,1.1}$
$\alpha_s(M_Z)$	0.119(2)	$^{-3.1}_{+3.8}$
$\mathcal{O}(\alpha_s^4)$	no $\mathcal{O}(\alpha_s^4)$ $2\times\mathcal{O}(\alpha_s^4)$	$+1.8$ -1.1
s_0	$3.9 - 5.5$ GeV^2	$^{+3.2}_{-2.6}$

TABLE I: Values of the main input parameters and corresponding uncertainties for $m_s(2\,{\rm GeV})$ in the scalar channel.

The dominant phenomenological uncertainty on m_s is due to the shape of the form factor $F_0(t)$ while the value $F_0(0)$ only plays a minor role. On the theoretical side the main uncertainty arises from the variation of α_s and to a smaller extent from unknown higher order corrections which are estimated by either removing or doubling the $\mathcal{O}(\alpha_s^4)$ correction. Finally, we have varied the parameter s_0 in a rather generous range. Higher order corrections in the OPE have been included according to [4, 11]. However, they have only small influence and a variation of the corresponding parameters induces errors on m_s of less than 1 MeV in all possible cases.

V. M_S FROM THE PSEUDOSCALAR CHANNEL

In complete analogy to the scalar channel the strange mass m_s can also be extracted from the pseudoscalar channel. The phenomenological spectral function has been modelled along the lines of ref. [13], while our analysis parallels the one of the recent work [12].

From the pseudoscalar channel, at an $s_0 = 4.2\,{\rm GeV}^2$ for which m_s in the region $2\,{\rm GeV}^2 < u < 4\,{\rm GeV}^2$ is most stable against a variation of the Borel variable u, the strange quark mass is found to be

$$m_s(2\,{\rm GeV}) = 97.2\,^{+\,11.3}_{-\,8.0}\;{\rm MeV}\,. \quad (13)$$

Again, in table II the input parameters and their variations which produce a shift of m_s larger than 1 MeV are compiled. The most important parameters are the decay constants of the first two excited K resonances, the $K(1460)$ and $K(1830)$, which have been estimated in ref. [13]. Since especially the decay constant of the second $K(1830)$ resonance is not very well determined, to be conservative a generous range has been assumed. We should note, however, that employing the central input parameters of ref. [12], their results are exactly reproduced. Furthermore, the dependence on the QCD coupling, higher order corrections and the parameter s_0 has been analysed as in the previous section.

Parameter	Value	Δm_s [MeV]
$F_{K(1460)}$	$22.0 \pm 2.6\,\text{MeV}$	$^{+5.2}_{-4.4}$
$F_{K(1830)}$	10^{+6}_{-10}	$^{+6.9}_{-4.6}$
$\alpha_s(M_Z)$	$0.119(2)$	$^{-4.5}_{+6.6}$
$\mathcal{O}(\alpha_s^4)$	no $\mathcal{O}(\alpha_s^4)$ $2\times\mathcal{O}(\alpha_s^4)$	$+1.4$ -0.8
s_0	$3.7 - 5.5\ \text{GeV}^2$	$^{+2.8}_{-1.5}$

TABLE II: Values of the main input parameters and corresponding uncertainties for $m_s(2\,\text{GeV})$ in the pseudoscalar channel.

VI. CONCLUSIONS

In the previous two sections, the strange quark mass has been determined on the basis of improved results for the strangeness-changing scalar spectral function presented in section 3, and the resonance model [13] in the case of the pseudoscalar spectral function. As can be observed from eqs. (12) and (13) within the uncertainties, both results are in reasonable agreement, and thus, we are in a position to average them. Since it is difficult to assign a precise meaning to the theoretical uncertainties, we have decided to take the arithmetic mean, and assigned the larger error of (12) as the total uncertainty, which yields our final result

$$m_s(2\,\text{GeV}) = 92.4 \pm 8.8\ \text{MeV} = 92 \pm 9\ \text{MeV}\,, \qquad (14)$$

providing the first determination of m_s at the 10% level from non-lattice approaches. The fact that m_s from the scalar channel turns out smaller than for the pseudoscalar channel might be attributed to the fact that only 2-particle intermediate states have been included. Nevertheless, on the basis of large-N_C arguments, the contribution from higher-multiplicity states is expected to be suppressed, and also the uncertainties in the pseudoscalar channel are so large that at present no significance can be attributed to this difference. Still, we plan to investigate this question further in the future.

Making use of our final result on m_s of eq. (14), and two particular quark mass ratios obtained from χPT [42], $R \equiv m_s/\hat{m} = 24.4 \pm 1.5$ as well as $Q^2 \equiv (m_s^2 - \hat{m}^2)/(m_d^2 - m_u^2) = (22.7 \pm 0.8)^2$, where $\hat{m} \equiv (m_u + m_d)/2$, we are also in a position to calculate the light up and down quark masses with the result:

$$m_u(2\,\text{GeV}) \;=\; 2.7 \pm 0.4\ \text{MeV}\,, \qquad (15)$$

$$m_d(2\,\text{GeV}) \;=\; 4.8 \pm 0.5\ \text{MeV}\,. \qquad (16)$$

To conclude, let us compare our determinations of m_s presented in eqs. (12) and (13) with other recent extractions of this quantity from sum rules, and lattice QCD. To this end, in figure 2 the m_s values obtained in this work, as well as the recent determinations from e^+e^--scattering [43] and hadronic τ decays [44], are displayed as the full circles. The most recent results from lattice simulations at $N_f = 2+1$ [45, 46] and $N_f = 2$ [47–49] quark flavours are shown as the full squares.

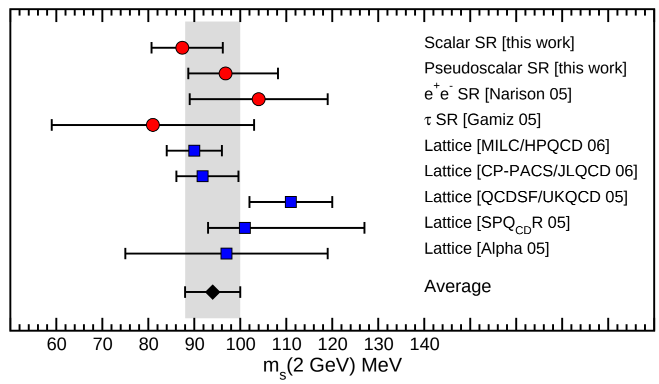

FIG. 2: Compilation of recent determinations of m_s from sum rules (circles) and lattice QCD (squares).

All results for m_s are in good agreement – perhaps with the exception of the QCDSF/UKQCD value [47] which lies a bit high – and thus we are in a position to present a global average. To do this, we have calculated a weighted average of all numbers, taking the larger uncertainty in the case of unsymmetric errors, which leads to

$$m_s(2\,\text{GeV}) = 94 \pm 6\ \text{MeV}\,, \qquad (17)$$

where as the total uncertainty we have chosen the error of the single best determination of m_s from the lattice.

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[1] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448.
[2] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465.
[3] S. Narison, N. Paver, E. de Rafael and D. Treleani, Nucl. Phys. **B212** (1983) 365.
[4] M. Jamin, J.A. Oller and A. Pich, Eur. Phys. J. **C 24** (2002) 237, hep-ph/0110194.
[5] K. Maltman, Phys. Lett. **B462** (1999) 195, hep-ph/9904370.
[6] M. Jamin, Nucl. Phys. Proc. Suppl. **64** (1998) 250, hep-ph/9709484. Proc. of *QCD 97*, Montpellier, July 1997.
[7] P. Colangelo, F. de Fazio, G. Nardulli and N. Paver, Phys. Lett. **B408** (1997) 340, hep-ph/9704249.
[8] T. Bhattacharya, R. Gupta and K. Maltman, Phys. Rev. **D57** (1998) 5455, hep-ph/9703455.
[9] K.G. Chetyrkin, D. Pirjol and K. Schilcher, Phys. Lett. **B404** (1997) 337, hep-ph/9612394.
[10] K.G. Chetyrkin, C.A. Dominguez, D. Pirjol and K. Schilcher, Phys. Rev. **D51** (1995) 5090, hep-ph/9409371.
[11] M. Jamin and M. Münz, Zeit. Phys. **C66** (1995) 633, hep-ph/9409335.
[12] K.G. Chetyrkin and A. Khodjamirian, Eur. Phys. J. C **C46** (2006) 721, hep-ph/0512295.
[13] K. Maltman and J. Kambor, Phys. Rev. **D65** (2002) 074013, hep-ph/0108227.
[14] M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. **B587** (2000) 331, hep-ph/0006045.
[15] M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. **B622** (2002) 279, hep-ph/0110193.
[16] S. Eidelman et al., Phys. Lett. **B592** (2004) 1, `http://pdg.lbl.gov`.
[17] W.J. Marciano and A. Sirlin, Phys. Rev. Lett. **71** (1993) 3629.
[18] KLOE Collaboration, F. Ambrosino et al., Phys. Lett. **B632** (2006) 76, hep-ex/0509045.
[19] M. Knecht, H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. **C12** (2000) 469, hep-ph/9909284.
[20] M. Finkemeier, Phys. Lett. **B387** (1996) 391, hep-ph/9505434.
[21] E. Blucher et al. (2005), hep-ph/0512039.
[22] M. Battaglia et al., CERN Yellow Report 2003-002 (2003), hep-ph/0304132.
[23] W.J. Marciano, Phys. Rev. Lett. **93** (2004) 231803, hep-ph/0402299.
[24] V. Cirigliano, H. Neufeld and H. Pichl, Eur. Phys. J. **C35** (2004) 53, hep-ph/0401173.
[25] M. Jamin, J.A. Oller and A. Pich, J. High Energy Phys. **02** (2004) 047, hep-ph/0401080.
[26] D. Becirevic et al., Nucl. Phys. **B705** (2005) 339, hep-ph/0403217.
[27] V. Cirigliano et al., JHEP **04** (2005) 006, hep-ph/0503108.
[28] C. Dawson et al., PoS **LAT2005** (2005) 337, hep-lat/0510018.
[29] JLQCD Collaboration, N. Tsutsui et al., PoS **LAT2005** (2005) 357, hep-lat/0510068.
[30] Fermilab Lattice, MILC and HPQCD Collaboration, M. Okamoto, PoS **LAT2005** (2005) 013, hep-lat/0510113.
[31] H. Leutwyler and M. Roos, Zeit. Phys. **C25** (1984) 91.
[32] MILC Collaboration, C. Aubin et al., Phys. Rev. **D70** (2004) 114501, hep-lat/0407028. Poster presented by R. Sugar at Lattice 06, Tucson, Arizona.
[33] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 517.
[34] KTeV Collaboration, T. Alexopoulos et al., Phys. Rev. **D70** (2004) 092007, hep-ex/0406003.
[35] O.P. Yushchenko et al., Phys. Lett. **B589** (2004) 111, hep-ex/0404030.
[36] P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, Phys. Rev. Lett. **96** (2006) 012003, hep-ph/0511063.
[37] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Phys. Lett. **B86** (1979) 347.
[38] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B191** (1981) 301.
[39] E.V. Shuryak and J.J.M. Verbaarschot, Nucl. Phys. **B410** (1993) 55, hep-ph/9302239.
[40] A.E. Dorokhov, S.V. Esaibegian, N.I. Kochelev and N.G. Stefanis, J. Phys. **G23** (1997) 643, hep-th/9601086.
[41] V. Elias, F. Shi and T.G. Steele, J. Phys. **G24** (1998) 267.
[42] H. Leutwyler, Phys. Lett. **B378** (1996) 313, hep-ph/9602366.
[43] S. Narison (2005), hep-ph/0510108.
[44] E. Gámiz et al., Phys. Rev. Lett. **94** (2005) 011803, hep-ph/0408044. Nucl. Phys. Proc. Suppl. **144** (2005) 59-64, hep-ph/0411278.
[45] MILC/HPQCD Collaboration, Q. Mason et al., Phys. Rev. **D73** (2006) 114501, hep-ph/0511160. Poster presented by R. Sugar at Lattice 06, Tucson, Arizona.
[46] CP-PACS/JLQCD Collaboration, T. Ishikawa et al., PoS **LAT2005** (2006) 057, hep-lat/0509142. Talk presented by T. Ishikawa at Lattice 06, Tucson, Arizona.
[47] QCDSF/UKQCD Collaboration, M. Göckeler et al., Phys. Rev. **D73** (2006) 054508, hep-lat/0601004.
[48] SPQCDR Collaboration, D. Becirevic et al., Nucl. Phys. **B734** (2006) 138, hep-lat/0510014.
[49] ALPHA Collaboration, M. Della Morte et al., Nucl. Phys. **B729** (2005) 117, hep-lat/0507035.
[50] To simplify the notation, from now on the superscript "$K\pi$" on the form factors will be dropped.