IFIC/03-29
UWThPh-2003-2

Isospin violation in ϵ' *

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(Dated: December 1, 2013)

On the basis of a next-to-leading-order calculation in chiral perturbation theory, the first complete analysis of isospin breaking for direct CP violation in $K^0 \to 2\pi$ decays is performed. We find a destructive interference between three different sources of isospin violation in the CP violation parameter ϵ'. Within the uncertainties of large-N_c estimates for the low-energy constants, the isospin violating correction for ϵ' is below 15 %.

Direct CP violation in $K^0 \to 2\pi$ decays is now established [1] to an accuracy of less than 10 %:

$$\text{Re}\ \epsilon'/\epsilon = (1.66 \pm 0.16) \times 10^{-3}\ . \quad (1)$$

The theoretical situation is less satisfactory due to large uncertainties in the hadronic matrix elements of four-quark operators. Although several calculations exist that come close to the experimental result (e.g., Ref. [2]) the accuracy is still rather limited.

K decays involve a delicate interplay between electroweak and strong interactions in the confinement regime. Chiral perturbation theory (CHPT) provides a convenient framework for a systematic low-energy expansion of the relevant amplitudes. In this letter, we concentrate on a quantitative analysis of isospin violation in the CP-violating parameter ϵ'. The results presented here are part of a complete calculation of $K \to 2\pi$ decays [3] to next-to-leading order in the chiral expansion and to first order in isospin violation, including both strong isospin violation ($m_u \neq m_d$) and electromagnetic corrections.

Although isospin violation is in general a small effect it must be included [4, 5, 6, 7] in a precision calculation of ϵ' because it affects the destructive interference between the two main contributions to ϵ' from normal and electromagnetic penguin operators.

The main features of our approach are the following:

1. We include for the first time both strong and electromagnetic isospin violation in a joint analysis.

2. Nonleptonic weak amplitudes in CHPT depend on a number of low-energy constants (LECs): we use leading large-N_c estimates for those constants [2, 3]. Uncertainties arise from (i) input parameters in the leading $1/N_c$ expressions as well as from (ii) potentially large subleading effects in $1/N_c$. We discuss the impact of both (i) and (ii) on the relevant quantities.

We adopt the following parametrization of $K \to \pi\pi$ amplitudes [3, 6]:

$$\begin{aligned} A(K^0 \to \pi^+\pi^-) &= A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}}\, A_2\, e^{i\chi_2} \\ A(K^0 \to \pi^0\pi^0) &= A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} \quad (2) \\ A(K^+ \to \pi^+\pi^0) &= \frac{3}{2}\, A_2^+\, e^{i\chi_2^+}\ . \end{aligned}$$

In the limit of CP conservation, the amplitudes A_0, A_2, and A_2^+ are real and positive. In the isospin limit, $A_2 = A_2^+$, $\chi_2 = \chi_2^+$ in the standard model and the phases χ_i coincide with the corresponding pion-pion phase shifts at $E_{\text{cms}} = M_K$. In terms of these amplitudes, the direct CP violation parameter ϵ' is given by

$$\epsilon' = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \frac{\text{Re}A_2}{\text{Re}A_0} \left[\frac{\text{Im}A_0}{\text{Re}A_0} - \frac{\text{Im}A_2}{\text{Re}A_2}\right]\ . \quad (3)$$

For the systematic low-energy expansion of $K \to \pi\pi$ amplitudes to next-to-leading order, a number of effective chiral Lagrangians is needed. Here, we only write down the relevant nonleptonic weak Lagrangian, referring to Ref. [3] for further details:

$$\begin{aligned} \mathcal{L}_{\text{weak}} &= G_8 F^4 (L_\mu L^\mu)_{23} + G_8 g_{\text{ewk}} e^2 F^6 (U^\dagger Q U)_{23} \\ &+ G_{27} F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3} L_{\mu 21} L^\mu_{13}\right) \quad (4) \\ &+ \sum_i G_8 N_i F^2 O_i^8 + \sum_i G_{27} D_i F^2 O_i^{27} + \text{h.c.} \end{aligned}$$

The matrix $L_\mu = i U^\dagger D_\mu U$ represents the octet of $V - A$ currents to lowest order in derivatives where the $SU(3)$ matrix field U contains the pseudoscalar fields. Q represents the light quark charge matrix. In addition to the lowest-order couplings G_8, G_{27}, and $G_8 g_{\text{ewk}}$, this Lagrangian contains LECs $G_8 N_i$, $G_{27} D_i$ of $\mathcal{O}(G_F p^4)$ [8]. Due to the disparity in size between G_8 and G_{27} ($\Delta I = 1/2$ rule), we consider isospin violating effects in the octet part only. More precisely, we work up to $\mathcal{O}(G_8 p^4, G_8(m_u - m_d)p^2, G_8 e^2 p^2)$ and to $\mathcal{O}(G_{27} p^4)$ for octet and 27-plet amplitudes, respectively. For this purpose, the chiral effective Lagrangian for strong interactions to $\mathcal{O}(p^4)$, the electromagnetic Lagrangian to $\mathcal{O}(e^2 p^2)$ and the electroweak Lagrangian to $\mathcal{O}(G_8 e^2 p^2)$ are also needed [3].

The resulting decay amplitudes depend on a large number of renormalized LECs. In addition to the well-known strong LECs of $\mathcal{O}(p^4)$, we make use of existing estimates for the electromagnetic couplings of $\mathcal{O}(e^2p^2)$ [9]. For the nonleptonic weak LECs of $\mathcal{O}(G_F p^4)$ and $\mathcal{O}(G_8 e^2 p^2)$, we employ leading large-N_c estimates [2, 3]. The starting point is the effective $\Delta S = 1$ Hamiltonian in the three-flavour theory,

$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}\, V_{us}^* \sum_i C_i(\mu_{\rm SD})\, Q_i(\mu_{\rm SD}) \ , \qquad (5)$$

obtained after integrating out all fields with masses larger than $\mu_{\rm SD} \simeq 1$ GeV. The local four-quark operators Q_i contain only the light degrees of freedom whereas the Wilson coefficients C_i are functions of heavy masses and CKM parameters [10, 11]:

$$C_i(\mu_{\rm SD}) \ = \ z_i(\mu_{\rm SD}) \ + \ \tau\, y_i(\mu_{\rm SD}) \qquad (6)$$

$$\tau \ = \ -\frac{V_{td}\, V_{ts}^*}{V_{ud}\, V_{us}^*} \ . \qquad (7)$$

All CP-violating quantities are proportional to Im τ. In our analysis, only ratios of CP-violating amplitudes will appear so we do not need an explicit value for τ.

At leading order in $1/N_c$, the matching between the three-flavour quark theory and CHPT can be done exactly because the T-product of two colour-singlet quark currents factorizes. Since quark currents have well-known realizations in CHPT, the hadronization of the weak operators Q_i is straightforward [2, 3]. The weak LECs are expressed in terms of strong LECs of order p^4, p^6, and e^2p^2. The $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ couplings arising from bosonization of Q_6 are estimated within resonance saturation and are determined by scalar exchange (for details see Refs. [3, 12]). Although admittedly this is not a complete estimate for the $\mathcal{O}(p^6)$ couplings it certainly provides the correct order of magnitude.

Subleading effects in $1/N_c$ are known to be sizable at leading chiral order. We will therefore not use the large-N_c values for ReG_8, ReG_{27} in the numerical analysis but instead determine these couplings from a fit to the $K \to \pi\pi$ branching ratios [3]. The other combination of interest is the ratio Im$(G_8 g_{\rm ewk})$/ImG_8. In this case the size of $1/N_c$ effects is approximately determined by existing calculations going beyond factorization [13]. It turns out that corrections to this ratio are not very large (roughly -30%). The dominant uncertainty comes then from the input parameters in the factorized expressions.

At next-to-leading chiral order we estimate subleading effects in $1/N_c$ by varying the chiral renormalization scale at which the large-N_c results are supposed to apply. We consider two options for the higher-order couplings $G_8 N_i$, ... in the Lagrangian (4): one may either adopt the large-N_c predictions for the ratios $(G_8 N_i)/G_8$ or directly for the couplings $G_8 N_i$. The uncertainties related to these two choices will be taken into account. We also include in the errors the effect of changing the short-distance renormalization scale $\mu_{\rm SD}$ between M_ρ and 1.3 GeV, with the central value at 1 GeV.

Whenever the errors are sizable, they are however largely dominated by the intrinsic uncertainty (an effect of higher order in $1/N_c$) at which chiral scale $\nu_{\rm CH}$ the large-N_c estimates for the renormalized LECs $G_8 N_i^r(\nu_{\rm CH})$, ... apply. We vary this scale between 0.6 GeV and 1 GeV, with the central values at M_ρ.

We now turn to the different sources of isospin violation in ϵ'. The expression (3) is valid to first order in CP violation. Since ImA_I is CP odd the quantities ReA_I and χ_I are only needed in the CP limit $(I = 0, 2)$. We disregard the phase that can be obtained from the $K \to \pi\pi$ branching ratios. The same branching ratios are usually employed to extract the ratio $\omega_S = {\rm Re}A_2/{\rm Re}A_0$ assuming isospin conservation. Accounting for isospin violation via the general parametrization (2), one is then really evaluating $\omega_+ = {\rm Re}A_2^+/{\rm Re}A_0$ rather than ω_S. The two differ by a pure $\Delta I = 5/2$ effect:

$$\omega_S \ = \ \omega_+ \left(1 + f_{5/2}\right) \qquad (8)$$

$$f_{5/2} \ = \ \frac{{\rm Re}A_2}{{\rm Re}A_2^+} - 1 \ . \qquad (9)$$

Because ω_+ is directly related to branching ratios it proves useful to keep ω_+ in the normalization of ϵ', introducing the $\Delta I = 5/2$ correction $f_{5/2}$ [6].

Since ImA_2 is already first order in isospin violation the formula for ϵ' takes the following form, with all first-order isospin breaking corrections made explicit:

$$\epsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2 - \chi_0)} \omega_+ \left[\frac{{\rm Im}A_0^{(0)}}{{\rm Re}A_0^{(0)}} (1 + \Delta_0 + f_{5/2}) - \frac{{\rm Im}A_2}{{\rm Re}A_2^{(0)}} \right] \ , \qquad (10)$$

where

$$\Delta_0 = \frac{{\rm Im}A_0}{{\rm Im}A_0^{(0)}} \cdot \frac{{\rm Re}A_0^{(0)}}{{\rm Re}A_0} - 1 \qquad (11)$$

and the superscript (0) denotes the isospin limit.

To isolate the isospin breaking corrections in ϵ', we write the amplitudes A_0, A_2 more explicitly as

$$A_0\, e^{i\chi_0} = \mathcal{A}_{1/2} \ = \ \mathcal{A}_{1/2}^{(0)} + \delta\mathcal{A}_{1/2}$$

$$A_2\, e^{i\chi_2} = \mathcal{A}_{3/2} \ = \ \mathcal{A}_{3/2}^{(0)} + \delta\mathcal{A}_{3/2} + \mathcal{A}_{5/2} \ , \qquad (12)$$

where $\delta\mathcal{A}_{1/2,3/2}$, $\mathcal{A}_{5/2}$ are first order in isospin violation. To the order we are considering, the amplitudes $\mathcal{A}_{\Delta I}$ have both absorptive and dispersive parts. To disentangle the (CP conserving) phases generated by the loop amplitudes from the CP-violating phases of the various LECs, we express our results explicitly in terms of $\mathcal{D}isp\ \mathcal{A}_{\Delta I}$ and $\mathcal{A}bs\ \mathcal{A}_{\Delta I}$.

To first order both in CP violation and in isospin breaking, we obtain

$$
\begin{aligned}
\Delta_0 &= -2\left|\mathcal{A}^{(0)}_{1/2}\right|^{-2}\left(\mathrm{Re}[\mathcal{D}isp\,\mathcal{A}^{(0)}_{1/2}]\ \mathrm{Re}[\mathcal{D}isp\,\delta\mathcal{A}_{1/2}] + \mathrm{Re}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{1/2}]\ \mathrm{Re}[\mathcal{A}bs\,\delta\mathcal{A}_{1/2}]\right) \\
&+ \left[\mathrm{Im}[\mathcal{D}isp\,\mathcal{A}^{(0)}_{1/2}]\,\mathrm{Re}[\mathcal{D}isp\,\mathcal{A}^{(0)}_{1/2}] + \mathrm{Im}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{1/2}]\,\mathrm{Re}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{1/2}]\right]^{-1}\Big\{\mathrm{Im}[\mathcal{D}isp\,\delta\mathcal{A}_{1/2}]\,\mathrm{Re}[\mathcal{D}isp\,\mathcal{A}^{(0)}_{1/2}] \\
&\quad + \mathrm{Im}[\mathcal{D}isp\,\mathcal{A}^{(0)}_{1/2}]\,\mathrm{Re}[\mathcal{D}isp\,\delta\mathcal{A}_{1/2}] + \mathrm{Im}[\mathcal{A}bs\,\delta\mathcal{A}_{1/2}]\,\mathrm{Re}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{1/2}] + \mathrm{Im}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{1/2}]\,\mathrm{Re}[\mathcal{A}bs\,\delta\mathcal{A}_{1/2}]\Big\} \qquad (13) \\
f_{5/2} &= \frac{5}{3}\left|\mathcal{A}^{(0)}_{3/2}\right|^{-2}\Big\{\mathrm{Re}[\mathcal{D}isp\,\mathcal{A}^{(0)}_{3/2}]\,\mathrm{Re}[\mathcal{D}isp\,\mathcal{A}_{5/2}] + \mathrm{Re}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{3/2}]\,\mathrm{Re}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{5/2}]\Big\} \qquad (14) \\
\mathrm{Im}A_2 &= \left|\mathcal{A}^{(0)}_{3/2}\right|^{-1}\Big\{\mathrm{Im}[\mathcal{D}isp\ (\delta\mathcal{A}_{3/2}+\mathcal{A}_{5/2})]\,\mathrm{Re}[\mathcal{D}isp\,\mathcal{A}^{(0)}_{3/2}] + \mathrm{Im}[\mathcal{A}bs\,(\delta\mathcal{A}_{3/2}+\mathcal{A}_{5/2})]\,\mathrm{Re}[\mathcal{A}bs\,\mathcal{A}^{(0)}_{3/2}]\Big\}\,, \qquad (15)
\end{aligned}
$$

where $|\mathcal{A}^{(0)}_n| = \sqrt{(\mathrm{Re}[\mathcal{D}isp\,\mathcal{A}^{(0)}_n])^2 + (\mathrm{Re}[\mathcal{A}bs\,\mathcal{A}^{(0)}_n])^2}$.

These expressions are general results to first order in CP and isospin violation but they are independent of the chiral expansion. Working strictly to a specific chiral order, these formulas simplify. We prefer to keep them in their general form but we will discuss later the numerical differences between the complete and the systematic chiral expressions. The differences are one indication for the importance of higher-order chiral corrections.

Although $\mathrm{Im}A_2$ is itself first order in isospin breaking we now make the usual (but scheme dependent) separation of the electroweak penguin contribution to $\mathrm{Im}A_2$ from the isospin breaking effects generated by other four-quark operators:

$$\mathrm{Im}A_2 = \mathrm{Im}A_2^{\mathrm{emp}} \ + \ \mathrm{Im}A_2^{\mathrm{non-emp}}\ . \qquad (16)$$

In order to perform such a separation within the CHPT approach, we need to identify the electroweak penguin contribution to a given low-energy coupling. In other words, we need a matching procedure between CHPT and the underlying theory of electroweak and strong interactions. Such a matching procedure is given here by working at leading order in $1/N_c$. Then, the electroweak LECs of $\mathcal{O}(G_8 e^2 p^n)$ $(n = 0, 2)$ in $\mathrm{Im}A_2^{\mathrm{non-emp}}$ must be calculated by setting to zero the Wilson coefficients C_7, C_8, C_9, C_{10} of electroweak penguin operators.

Splitting off the electromagnetic penguin contribution to $\mathrm{Im}A_2$ in this way, we can now write ϵ' in a more familiar form as

$$\epsilon' = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\,\omega_+\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}}\right] \qquad (17)$$

where

$$\Omega_{\mathrm{eff}} = \Omega_{\mathrm{IB}} - \Delta_0 - f_{5/2} \qquad (18)$$

$$\Omega_{\mathrm{IB}} = \frac{\mathrm{Re}A_0^{(0)}}{\mathrm{Re}A_2^{(0)}}\cdot\frac{\mathrm{Im}A_2^{\mathrm{non-emp}}}{\mathrm{Im}A_0^{(0)}}\ . \qquad (19)$$

The quantity Ω_{eff} includes all effects to leading order in isospin breaking and it generalizes the more traditional parameter Ω_{IB}. Although Ω_{IB} is in principle enhanced by the large ratio $\mathrm{Re}A_0^{(0)}/\mathrm{Re}A_2^{(0)}$ the actual numerical analysis shows all three terms in (18) to be relevant when both strong and electromagnetic isospin violation are included.

We present numerical results for the following two cases:

i. We calculate Ω_{eff} and its components for $\alpha = 0$ (purely strong isospin violation). In this case, there is a clean separation of isospin violating effects in $\mathrm{Im}A_2$. We compare the lowest-order result of $\mathcal{O}(m_u - m_d)$ with the full result of $\mathcal{O}[(m_u - m_d)p^2]$.

ii. Here we include electromagnetic corrections explicitly, comparing again $\mathcal{O}(m_u - m_d, e^2p^0)$ with $\mathcal{O}[(m_u - m_d)p^2, e^2p^2]$. In this case, the splitting between $\mathrm{Im}A_2^{\mathrm{emp}}$ and $\mathrm{Im}A_2^{\mathrm{non-emp}}$ is performed at leading order in $1/N_c$.

In Table I the uncertainties in the "LO" entries are dominated by error propagation from the input parameters (these ranges account however for known $1/N_c$ corrections). Apart from loops, new effects in "NLO" entries depend on ratios of next-to-leading to leading-order LECs. In Table I, we use the leading $1/N_c$ estimates for the ratios $(G_8N_i)/G_8, \ldots$, and estimate the uncertainty as discussed above. The final error for each of the quantities Ω_{IB}, Δ_0, $f_{5/2}$, and Ω_{eff} is obtained by adding in quadrature the LO error and the one associated to weak LECs at NLO. Moreover, only $f_{5/2}$ and $\mathrm{Re}A_0^{(0)}/\mathrm{Re}A_2^{(0)}$ depend on the ratio G_8/G_{27}. In these cases we rely on the phenomenological value implied by our fit [3]. Some of the errors in Table I are manifestly correlated, e.g., in the LO column for $\alpha \neq 0$.

The NLO results are obtained with the full expressions (13), ..., (15). Using instead the simplified expressions corresponding to a fixed chiral order, the results are found to be well within the quoted error bars. The same is true for the alternative procedure of applying large-N_c directly to the LECs G_8N_i, ... The isospin violating ratios in Table I are hardly changed at all in this case.

	$\alpha = 0$		$\alpha \neq 0$	
	LO	NLO	LO	NLO
$\Omega_{\rm IB}$	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
Δ_0	-0.004	-0.41 ± 0.05	8.7 ± 3.0	8.3 ± 3.6
$f_{5/2}$	0	0	0	8.3 ± 2.4
$\Omega_{\rm eff}$	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 7.7

TABLE I: Isospin violating corrections for ϵ' in units of 10^{-2}. The first two columns refer to strong isospin violation only ($m_u \neq m_d$), the last two contain the complete results including electromagnetic corrections. LO and NLO denote leading and next-to-leading orders in CHPT.

Altogether, we expect our errors to be realistic estimates of higher-order effects in the chiral expansion.

As first observed by Gardner and Valencia [14], there is a source of potentially large isospin violation through terms of the type $(m_u - m_d) G_8 N_i$, which get contributions from strong LECs of $\mathcal{O}(p^6)$ at leading order in large N_c. The crucial LEC of $\mathcal{O}(p^6)$ can be related [15] to the mass splitting in the lightest scalar nonet that survives in the large-N_c limit. The results in Table I are based on the evidence discussed in Ref. [12] that the isotriplet and isodoublet states in the lightest scalar nonet are $a_0(1450)$ and $K_0^*(1430)$, respectively (Scenario A). In the large-N_c limit, the results are independent of the assignment of isosinglet scalar resonances. If instead the $a_0(980)$ is the isotriplet member of the lightest nonet surviving in the large-N_c limit the couplings $(m_u - m_d) G_8 N_i$ would be strongly enhanced [14], implying large (of the order of 100 %) and negative values of $\Omega_{\rm IB}$. In this case, ϵ'/ϵ would be more than twice as big as in the isospin limit. We consider this an additional argument against such a scenario for the lightest scalar nonet (Scenario B in Ref. [12]).

Finally, we have investigated the impact of some subleading effects along the lines of Ref. [12]. Although by no means a systematic expansion in $1/N_c$, those nonet breaking terms may furnish yet another indication for the intrinsic uncertainties of some of the LECs. The size of those terms depends on the assignment of isosinglet scalar resonances. Since nonet breaking effects are large in the scalar sector they affect most of the entries in Table I in a non-negligible way, although always within the quoted uncertainties. Employing again scenario A for the lightest scalar nonet [12], $\Omega_{\rm eff}$ in (17) decreases from $6.0 \cdot 10^{-2}$ to $-1.4 \cdot 10^{-2}$.

We have performed a complete analysis of isospin violation in ϵ', due to both the light quark mass difference and electromagnetism. In particular, we have included for the first time isospin violation in the ratio ${\rm Im}A_0/{\rm Re}A_0$, parametrized by the quantity Δ_0. This ratio gets only a small contribution from strong isospin violation but the electromagnetic part is important, being dominated by electromagnetic penguin contributions. Both Δ_0 and the purely electromagnetic $\Delta I = 5/2$ contribution $f_{5/2}$ interfere destructively with $\Omega_{\rm IB}$ to yield a final value $\Omega_{\rm eff} = (6.0 \pm 7.7) \cdot 10^{-2}$ for the overall measure of isospin violation in ϵ'. If electromagnetic penguin contributions are included in theoretical calculations of ${\rm Im}A_0/{\rm Re}A_0$, Δ_0 can be dropped in $\Omega_{\rm eff}$ to a very good approximation. Finally, if all electromagnetic corrections are included in ${\rm Im}A_0/{\rm Re}A_0$, ${\rm Im}A_2/{\rm Re}A_2$, and ${\rm Re}A_2/{\rm Re}A_0$, $\Omega_{\rm eff}$ is essentially determined by $\Omega_{\rm IB}$.

We thank J. Bijnens for useful information. The work of V.C. and A.P. has been supported in part by MCYT, Spain (Grant No. FPA-2001-3031) and by ERDF funds from the European Commission.

* Work supported in part by HPRN-CT2002-00311 (EURIDICE) and by Acciones Integradas, Project No. 19/2003 (Austria), HU2002-0044 (MCYT, Spain).

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