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π^0-η Mixing and CP Violation*

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Abstract

We discuss π^0-η mixing and its implication for ε'/ε to next-to-leading order in the low-energy expansion. The big effect due to η-η' mixing is shown to be largely cancelled by other contributions occurring at the same order in the chiral expansion.

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1. The recent experimental measurements [1] of direct CP violation in $K^0 \to 2\pi$ decays have led to a new world average [2]

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (21.4 \pm 4.0) \times 10^{-4}\,. \tag{1}$$

The theoretical status of ε'/ε is reviewed in Refs. [3, 4]. Among the ingredients of the theoretical prediction of ε'/ε within the standard model, we concentrate in this note on the quantity Ω_{IB} defined as

$$\Omega_{IB} := \frac{\mathrm{Im}A_{2,IB}}{\omega \mathrm{Im}A_0}\ . \tag{2}$$

We follow the conventional notation: the amplitudes $A_I (I = 0, 2)$ denote the $K \to \pi\pi$ amplitudes with isospin I in the final state, the subscript IB stands for isospin breaking due to $m_u \neq m_d$ (electromagnetic corrections are usually not included in Ω_{IB}) and $\omega := \mathrm{Re}A_2/\mathrm{Re}A_0 \approx 1/22.1$. In the theoretical analyses (e.g., in Refs. [3, 4]), one often takes

$$\Omega_{IB} \simeq \Omega_{\eta+\eta'} \tag{3}$$

arising from π^0-η and η-η' mixing [5].

In chiral perturbation theory (CHPT) [6, 7, 8], the effective field theory of the standard model at low energies, Ω_{IB} occurs already at lowest order, $O(p^2)$:

$$\Omega_{IB} = \frac{2\sqrt{2}\varepsilon^{(2)}_{\pi^0\eta}}{3\sqrt{3}\omega} = 0.13\ , \tag{4}$$

where the lowest-order π^0-η mixing angle $\varepsilon^{(2)}_{\pi^0\eta}$ can be expressed in terms of quark mass ratios as

$$\varepsilon^{(2)}_{\pi^0\eta} = \frac{\sqrt{3}(m_d - m_u)}{4(m_s - \hat{m})} \tag{5}$$

with $\hat{m} = (m_u + m_d)/2$ the average light quark mass. We use the canonical quark mass ratios [9]

$$\frac{m_u}{m_d} = 0.55 \pm 0.04\ , \qquad\qquad \frac{m_s}{m_d} = 18.9 \pm 0.8\ . \tag{6}$$

The η' contribution to Ω_{IB} is known to be large [5]. In the analysis of the Munich group [3, 4, 10], a value $\Omega_{\eta+\eta'} = 0.25 \pm 0.08$ is taken, about twice as big as the lowest-order value (4). This raises the question about the size of possible other contributions competing with η'-exchange. It is the purpose of this letter to answer this question to $O(p^4)$ in CHPT.

To show the sensitivity of ε'/ε to Ω_{IB}, we adopt an approximate formula of the Munich group [3, 4, 10] (not to be used for any "serious" analysis, however)

$$\frac{\varepsilon'}{\varepsilon} \approx 13\,\mathrm{Im}\lambda_t \left[\frac{130\,\mathrm{MeV}}{m_s(m_c)}\right]^2 \left[B_6^{(1/2)}(1-\Omega_{IB}) - 0.4 B_8^{(3/2)}\left(\frac{m_t(m_t)}{165\,\mathrm{GeV}}\right)^{2.5}\right]\left(\frac{\Lambda^{(4)}_{\overline{\mathrm{MS}}}}{340\,\mathrm{MeV}}\right). \tag{7}$$

The so-called B-factors $B_6^{(1/2)}$, $B_8^{(3/2)}$ measure the deviation of the relevant hadronic matrix elements of four-quark operators from the vacuum saturation approximation. Since $\mathrm{Im}\lambda_t\, B_6^{(1/2)} > 0$, a smaller value of Ω_{IB} implies a larger ε'/ε and an increased sensitivity to the precise value of $\mathrm{Im}\lambda_t\, B_6^{(1/2)}$. We come back to this formula at the end of this note.

2. To investigate π^0-η mixing beyond leading order, we consider the inverse matrix propagator in the space of pseudoscalar octet fields (π_3, π_8):

$$\Delta(q^2)^{-1} = q^2 \mathbf{1} - M_2^2 - \Pi(q^2) \ . \tag{8}$$

The lowest-order mass matrix is given by

$$M_2^2 = B \begin{pmatrix} 2\hat{m} & (m_u - m_d)/\sqrt{3} \\ (m_u - m_d)/\sqrt{3} & 2(2m_s + \hat{m})/3 \end{pmatrix} \tag{9}$$

where B is a low-energy constant of the lowest-order chiral Lagrangian related to the quark condensate [8]. The self-energy matrix $\Pi(q^2)$ of $O(p^4)$ has the simple form

$$\Pi(q^2) = Cq^2 + D \tag{10}$$

with symmetric matrices C, D independent of the momentum q.

The inverse matrix propagator can now be written as

$$\Delta(q^2)^{-1} = (\mathbf{1} - C/2) \left[q^2 \mathbf{1} - M_2^2 - D - \{C, M_2^2\}/2 \right] (\mathbf{1} - C/2) \ . \tag{11}$$

We first diagonalize M_2^2, the mass matrix of $O(p^2)$, with an orthogonal matrix O_2 depending on the π^0-η mixing angle $\varepsilon^{(2)}_{\pi^0\eta}$ given in (5). Neglecting terms of higher order in $m_u - m_d$, we have

$$\begin{aligned} O_2 &= \mathbf{1} + \varepsilon^{(2)}_{\pi^0\eta} \sigma \ , \ \sigma := i\sigma_2 \\ M_{2d}^2 &= O_2 M_2^2 O_2^T = \mathrm{diag}(2B\hat{m}, 2B(2m_s + \hat{m})/3) \ . \end{aligned} \tag{12}$$

It remains to diagonalize the resulting mass matrix in the basis of tree-level eigenstates,

$$M_{2d}^2 + O_2 D O_2^T + \{O_2 C O_2^T, M_{2d}^2\}/2 \ . \tag{13}$$

This is achieved with another orthogonal matrix $O_4 = \mathbf{1} + \varepsilon^{(4)}_{\pi^0\eta}\sigma$ with

$$\begin{aligned} \varepsilon^{(4)}_{\pi^0\eta} &= (M_\pi^2 - M_\eta^2)^{-1} \Big[D_{38} + C_{38}(M_\pi^2 + M_\eta^2)/2 + \varepsilon^{(2)}_{\pi^0\eta}(D_{88} - D_{33}) \\ &\quad + \varepsilon^{(2)}_{\pi^0\eta}(C_{88} - C_{33})(M_\pi^2 + M_\eta^2)/2 \Big] \ , \end{aligned} \tag{14}$$

working as always up to $O(p^4)$ and neglecting terms of higher than first order in $m_u - m_d$. The expression in square brackets in (14) is just the off-diagonal element of the mass matrix (13).

The inverse propagator now assumes its final form

$$\Delta(q^2)^{-1} = (\mathbf{1} - C/2) O_2^T O_4^T \left(q^2 \mathbf{1} - M_d^2 \right) O_4 O_2 (\mathbf{1} - C/2) \tag{15}$$

with $M_d^2 = \mathrm{diag}(M_{\pi^0}^2, M_\eta^2)$. The transformation from the original fields (π_3, π_8) to the mass eigenfields (π^0, η) of $O(p^4)$ is therefore accomplished by a matrix

$$\begin{aligned} V &= (\mathbf{1} + C/2) O_2^T O_4^T \\ &= \mathbf{1} - (\varepsilon_{\pi^0\eta}^{(2)} + \varepsilon_{\pi^0\eta}^{(4)})\sigma + C/2 - \varepsilon_{\pi^0\eta}^{(2)} C\sigma/2 \; . \end{aligned} \tag{16}$$

The matrix element of interest that contributes to all amplitudes involving π^0-η mixing to $O(p^4)$ is

$$V_{\pi_8\pi^0} = \varepsilon_{\pi^0\eta}^{(2)} + \varepsilon_{\pi^0\eta}^{(4)} + C_{38}/2 + \varepsilon_{\pi^0\eta}^{(2)} C_{88}/2 \; . \tag{17}$$

Is the expression (17) the generalization of the lowest-order mixing angle $\varepsilon_{\pi^0\eta}^{(2)}$ to $O(p^4)$? The answer is no because (17) is in fact not a measurable quantity. Looking first at the last two terms, C_{38} and C_{88} are actually divergent. Moreover, whereas the η mass shift $\Pi(M_\eta^2) = C_{88} M_\eta^2 + D_{88}$ is invariant under field redefinitions, C_{88} is not. On the other hand, $\varepsilon_{\pi^0\eta}^{(4)}$ is a well-defined and therefore measurable quantity. It was first calculated in Ref. [11] for the analysis of isospin violation in K_{l3} form factors. Without electromagnetic corrections [12], which are by definition not included in Ω_{IB}, the result is

$$\begin{aligned} \varepsilon_{\pi^0\eta}^{(4)} = & \frac{2\varepsilon_{\pi^0\eta}^{(2)}}{3(4\pi F_\pi)^2 (M_\pi^2 - M_\eta^2)} \Big\{ 64 (4\pi)^2 (M_K^2 - M_\pi^2)^2 [3L_7 + L_8^r(\mu)] \\ & - M_\eta^2 (M_K^2 - M_\pi^2) \ln \frac{M_\eta^2}{\mu^2} - 2M_K^2 (M_K^2 - 2M_\pi^2) \ln \frac{M_K^2}{\mu^2} \\ & + M_\pi^2 (M_K^2 - 3M_\pi^2) \ln \frac{M_\pi^2}{\mu^2} - 2M_K^2 (M_K^2 - M_\pi^2) \Big\} \end{aligned} \tag{18}$$

where F_π is the pion decay constant and $L_7, L_8^r(\mu)$ are low-energy constants of the chiral Lagrangian of $O(p^4)$ [8]. The scale dependence of $L_8^r(\mu)$ is of course cancelled by the chiral logarithms in (18).

A possible definition of the π^0-η mixing angle up to $O(p^4)$ is therefore provided by

$$\varepsilon_{\pi^0\eta} := \varepsilon_{\pi^0\eta}^{(2)} + \varepsilon_{\pi^0\eta}^{(4)} \; . \tag{19}$$

In the notation of Ref. [8], the mixing angle (19) corresponds to $(\varepsilon_1 + \varepsilon_2)/2$.

3. After this general treatment of π^0-η mixing to $O(p^4)$, we now turn to the decays $K^0 \to \pi\pi$. As the previous discussion of the expression (17) has shown, the non-measurable part

$$C_{38}/2 + \varepsilon_{\pi^0\eta}^{(2)} C_{88}/2$$

must combine with other contributions of $O(p^4)$ specific to the decay $K^0 \to \pi^0\pi^0$ to produce a measurable S-matrix element. This implies that there are additional contributions to the $K^0 \to \pi\pi$ decay amplitudes of $O[(m_u - m_d)p^2]$ that are not included in the π^0-η mixing angle (19). Some of these additional contributions have recently been considered in Ref. [13]. The complete $K \to \pi\pi$ amplitudes to $O[(m_u - m_d)p^2]$ including electromagnetic corrections up to $O(e^2p^2)$ [14] will be presented and analysed elsewhere [15].

Here, we are concerned with the contribution of the π^0-η mixing angle to the quantity Ω_{IB}. In fact, we can demonstrate that there are no other contributions of the type $(m_u - m_d)\ L_i$ in Ω_{IB}, where the L_i are the ten low-energy constants in the chiral Lagrangian of $O(p^4)$ [8]. The first observation is that the strong chiral Lagrangian of $O(p^4)$ does not generate vertices with three mesons. Therefore, only the bilinear terms appearing in the two-point functions can contribute to the decays $K \to \pi\pi$ at $O(p^4)$. In addition to $\varepsilon_{\pi^0\eta}$, Ω_{IB} contains the following combination of self-energy matrix elements:

$$C_{38} + \varepsilon^{(2)}_{\pi^0\eta}(C_{88} - C_{33}) \ . \tag{20}$$

The explicit dependence of these matrix elements on the constants L_i is given by

$$\begin{aligned} C_{38}(L_i) &= -\frac{8B}{\sqrt{3}F^2}\ L_5(m_u - m_d) \ , \\ C_{33}(L_i) &= -\frac{16B}{F^2}\{L_4(m_s + 2\hat{m}) + L_5\hat{m}\} \ , \\ C_{88}(L_i) &= -\frac{16B}{F^2}\{L_4(m_s + 2\hat{m}) + L_5(2m_s + \hat{m})/3\} \ . \end{aligned} \tag{21}$$

With the help of (5) one finds that the combination (20) is indeed independent of the L_i.

Therefore, the complete dependence of Ω_{IB} on the strong low-energy constants of $O(p^4)$ is contained in the π^0-η mixing angle (19) and we arrive at our final result

$$\Omega^{\pi^0\eta}_{IB} = \frac{2\sqrt{2}\varepsilon_{\pi^0\eta}}{3\sqrt{3}\omega} \ . \tag{22}$$

The superscript in $\Omega^{\pi^0\eta}_{IB}$ serves as a reminder that there are other isospin-violating contributions to Ω_{IB} in addition to π^0-η mixing.

4. For the numerical discussion, let us first look at the contributions of the low-energy constants L_7, L_8. As is well known, to $O(p^4)$ the effect of the η' is completely contained in L_7 [8]. Taking the standard (mean) value $L_7 = -0.4 \times 10^{-3}$, the contribution of the η' to the π^0-η mixing angle normalized to the lowest-order value is

$$\varepsilon^{(4)}_{\pi^0\eta}(L_7)/\varepsilon^{(2)}_{\pi^0\eta} = 1.10 \ . \tag{23}$$

In agreement with earlier calculations [5], η' exchange more than doubles the lowest-order π^0-η mixing angle. The surprise comes from the second contribution due to $L_8^r(M_\rho)$ for which we take again the standard value 0.9×10^{-3}:

$$\varepsilon^{(4)}_{\pi^0\eta}(L_8^r(M_\rho))/\varepsilon^{(2)}_{\pi^0\eta} = -0.83 \ . \tag{24}$$

The remaining (loop) contributions in (18) almost cancel for $\mu = M_\rho$. Altogether, we obtain the (scale-independent) result

$$\Omega_{IB}^{\pi^0\eta} = 0.16 \tag{25}$$

to be compared with the lowest-order value $\Omega_{IB} = 0.13$ in (4).

Before estimating the theoretical error, we briefly discuss the physical origin of the L_8 contribution that nearly cancels the η' contribution encoded in L_7. For this purpose we recall that the phenomenological values of the $L_i^r(M_\rho)$ can be well understood in terms of meson resonance exchange [16]. In particular, $L_8^r(M_\rho)$ is only sensitive to the octet scalar resonances. In the case at hand, it is the $a_0(983)$ that couples both to $\eta\pi^0$ and to an isospin-violating tadpole proportional to $m_u - m_d$. Therefore, a_0-exchange contributes to π^0-η mixing via L_8 and this is the only low-lying meson resonance contribution. However, we hasten to emphasize that the result (18) is a strict consequence of QCD to $O(p^4)$ in the low-energy expansion and is independent of any specific interpretation of the numerical value of L_8^r.

The theoretical uncertainty of the result (25) for $\Omega_{IB}^{\pi^0\eta}$ is dominated by the uncertainties of the low-energy constants. The combination $3L_7 + L_8^r(M_\rho)$ can be determined from two observables [8]: the deviation from the Gell-Mann–Okubo mass formula, which is well under control, and a quantity Δ_M related to the $O(p^4)$ corrections for the ratio M_K^2/M_π^2. With a generous upper limit $|\Delta_M| \leq 0.2$ (compared to $|\Delta_M| \leq 0.09$ in [8] and $\Delta_M = 0.065 \pm 0.065$ in [9]) to allow also for higher-order corrections, one finds

$$3L_7 + L_8^r(M_\rho) = (-0.25 \pm 0.25) \times 10^{-3} \ . \tag{26}$$

In comparison, the errors of both the Gell-Mann–Okubo discrepancy and the quark mass ratios (6) entering $\varepsilon_{\pi^0\eta}^{(2)}$ can be neglected. The final result for the contribution of π^0-η mixing to Ω_{IB} is

$$\Omega_{IB}^{\pi^0\eta} = 0.16 \pm 0.03 \ . \tag{27}$$

An independent estimate can be obtained from the analysis of K_{l3} form factors [11]. From the experimentally measured ratio of the $K^+\pi^0$ to $K^0\pi^-$ form factors at $q^2 = 0$ one can directly extract the π^0-η mixing angle $\varepsilon_{\pi^0\eta}$, leading to $\Omega_{IB}^{\pi^0\eta} = 0.19 \pm 0.06$. The two values are consistent with each other. Since the latter value is obtained under the assumption that electromagnetic corrections [12] can be neglected we consider (27) as our final result.

5. The contribution of π^0-η mixing to Ω_{IB} does not include all isospin-violating corrections in $K^0 \to \pi\pi$ decays. As defined here to $O(p^4)$ in the low-energy expansion of QCD, it gives rise to

$$\Omega_{IB}^{\pi^0\eta} = 0.16 \pm 0.03 \ . \tag{28}$$

This value is smaller than the one used previously where only η-η' mixing was included at $O(p^4)$ [5]. To assess the impact on ε'/ε, we adopt the approximate formula (7) with the central values for the B-factors taken by the Munich group [3, 4, 10], $B_6^{(1/2)} = 1$,

$B_8^{(3/2)} = 0.8$. Lowering Ω_{IB} from 0.25 to 0.16 corresponds to an increase of ε'/ε by 21%, bringing the theoretical prediction [3, 4, 10] closer to the experimental value (1). In addition, a smaller value for Ω_{IB} increases the sensitivity of ε'/ε to the B-factor $B_6^{(1/2)}$.

It has been shown recently [17] that final state interactions induce a strong enhancement of the theoretical ε'/ε prediction, correcting the $I = 0$ and $I = 2$ $K \to \pi\pi$ amplitudes with the multiplicative dispersive factors $\Re_0 = 1.41 \pm 0.06$ and $\Re_2 = 0.92 \pm 0.02$, respectively. The bag factors used before, which do not include final state interactions, get then modified to $B_6^{(1/2)}|_{FSI} = 1.4$ and $B_8^{(3/2)}|_{FSI} = 0.7$. The isospin-violating contribution $B_6^{(1/2)}\Omega_{IB}$ corresponds to two final pions with $I = 2$ and, therefore, should be multiplied by $\Re_2$; thus, $B_6^{(1/2)}\Omega_{IB}|_{FSI} = 0.15$. The overall effect is to enhance ε'/ε by a factor 2.3. The so-called "central" value in Refs. [3, 4, 10], $\varepsilon'/\varepsilon = 7.0 \times 10^{-4}$, gets then increased to 16×10^{-4}, in better agreement with the experimental measurement.

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