

# Unitarity and $K_L \rightarrow \pi^0 \gamma \gamma$

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## ABSTRACT

We discuss the rare decay  $K_L \rightarrow \pi^0 \gamma \gamma$ . Although the recently measured  $2\gamma$ -invariant-mass distribution is in quite good agreement with the lowest-order chiral perturbation theory prediction, there seems to be a discrepancy with the calculated branching ratio. We extend the previous computations to next-to-leading order,  $\mathcal{O}(p^6)$ . The two dominant mechanisms at this order are emphasized: the two-pion intermediate state and vector meson exchange.

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Although effective chiral Lagrangians have proven extremely successful in describing the interactions of the light mesons, there remain several processes that require special care. One of these,  $K_L \rightarrow \pi^0 \gamma \gamma$ , has recently been observed experimentally and the measured branching ratio does not agree as well as one might have hoped with previous predictions. The observed branching ratio is

$$\text{BR}(K_L \rightarrow \pi^0 \gamma \gamma) = \begin{cases} (1.7 \pm 0.3) \times 10^{-6} & \text{NA31 [1],} \\ (1.86 \pm 0.60 \pm 0.60) \times 10^{-6} & \text{E731 [2],} \end{cases} \quad (1)$$

while the prediction of chiral perturbation theory (CHPT) to leading order ( $\mathcal{O}(p^4)$ ) is  $\text{BR} \simeq 0.7 \times 10^{-6}$  [3]. The measured invariant-mass distribution of the final photons is nevertheless in good agreement with the lowest-order CHPT prediction. In this note we review why this particular mode is calculable at all, and attempt to determine the dominant contributions at next-to-leading order ( $\mathcal{O}(p^6)$ ). The results will be consistent with the recent data, although some ambiguities will remain.

CHPT is a technique for implementing all of the symmetries of the standard model in an effective low momentum expansion of the amplitudes involving light mesons. This expansion is characterized by a parameter,  $\Lambda_\chi \equiv 4\pi F_\pi \simeq 1.2$  GeV. Of course, the amplitudes of the light mesons are not polynomial functions of the kinematic variables, but must display the complicated analytic structure associated with the appearance of physical intermediate states, such as two-pion cuts, etc. Since these features are constrained by unitarity, one can in principle reconstruct the amplitude from its polynomial parts given by a power series expansion in momenta. In CHPT, this is most easily done by constructing an effective field theory of mesons, with an effective Lagrangian that has the appropriate power series in derivatives. Since this theory *is* a relativistic field theory, it is guaranteed to satisfy unitarity and have the correct analytic structure; the full amplitudes can then be constructed by a conventional Feynman-Dyson perturbation expansion [4,5].

We can follow this line of reasoning in the case of  $K_L \rightarrow \pi^0 \gamma \gamma$ : we first construct local operators with a definite number of derivatives (or masses, when we include explicit chiral symmetry breaking). For our process, these operators appear first at  $\mathcal{O}(p^6)$  in CHPT (keeping in mind that the electromagnetic field strength,  $F_{\mu\nu} \equiv [D_\mu, D_\nu]$ , counts as two derivatives):

$$\mathcal{L}_6^{\Delta S=1} = \frac{G_8 \alpha_{em}}{(4\pi)^3} \{c_1 F_{\mu\nu} F^{\mu\nu} \text{Tr}(\lambda \partial_\alpha U^\dagger \partial^\alpha U) + \dots\} . \quad (2)$$

Here  $U$  is the non-linearly realized field of mesons,  $\lambda$  is the matrix that projects out  $\Delta S = 1$ , and  $G_8 \simeq 9 \times 10^{-6}$  GeV<sup>-2</sup> is measured in the decays  $K_S \rightarrow 2\pi$ .

Instead of writing down all operators of  $\mathcal{O}(p^6)$  allowed by the symmetries of the standard model, we observe that they all reduce to three different Lorentz structures in terms of meson and photon fields, two derivative and one non-derivative couplings:

$$F_{\mu\nu}F^{\mu\nu}\partial_\lambda K_L\partial^\lambda\pi^0, \quad F_{\mu\nu}F^{\mu\lambda}\partial^\nu K_L\partial_\lambda\pi^0, \quad M_K^2 F_{\mu\nu}F^{\mu\nu}K_L\pi^0. \quad (3)$$

These couplings will give rise to three *a priori* undetermined constants in the amplitudes for  $K_L \rightarrow \pi^0\gamma\gamma$ .

The coefficients  $c_i$  in (2) (or equivalently the undetermined constants coming from the structures in (3)) are expected to be of order 1 by naïve chiral dimensional analysis. Unfortunately, any choice of these coefficients of  $\mathcal{O}(1)$  leads to a total rate for the process  $K_L \rightarrow \pi^0\gamma\gamma$  which is too small by more than one order of magnitude. The conclusion is either that these coefficients are significantly enhanced compared with the usual size expected by naïve dimensional analysis (which is not unknown—the coefficient conventionally called  $L_9$  [5] is in fact larger than naïve expectations owing to the prominent effect of vector meson exchange [5,6]) or the dominant contribution to this process comes from the cut associated with some physical intermediate state. In this case the relevant intermediate states are expected to be two charged mesons (plus the outgoing  $\pi^0$ ). The latter possibility would actually be good news: the non-trivial analytic behaviour associated with a physical intermediate state is *calculable* without the introduction of new, arbitrary chiral coefficients—we need only know the on-shell  $\Delta S = 1$  amplitude involving four mesons, such as  $K_L \rightarrow 3\pi$ . The remaining ambiguity in this case would be the polynomial parts corresponding to the operators in Eqs. (2) and (3). But as we have already argued, these terms are much too small to produce the observed rate and thus should only make a small correction.

Since the dominant physical intermediate states are two pions, we need only consider the  $K_L$  interacting with a neutral pion plus two charged pions, and the subsequent annihilation of the two charged pions into two photons. The second part of this transition is given to a high degree of approximation by lowest-order scalar electrodynamics. The first part, the  $\Delta S = 1$  interaction of four mesons, can be taken from experimental data on  $K_L$  decays into three pions. The use of experimental data would at first sight seem to be a retreat from an analytic treatment of this process. However, the  $\Delta S = 1$  four-meson interaction is well fit in CHPT if we include four-derivative terms [7], and it is this form of the interaction that we will use. This allows us to continue the analytic treatment of the problem as far as possible, resorting to numerical calculations only for our final plots.

We might also consider truncating the four-meson interaction at two derivatives rather than four, since this fits the  $K_L$  decays reasonably well. In the spirit of CHPT, we expect this approximation to introduce an error of perhaps 20% in amplitude. In fact, this calculation would be precisely equivalent to the standard calculation of Ref. [3]; our purpose here is to use the above analyticity argument to include the most important corrections to this calculation. As we will see, these corrections are significant, although not inconsistent with our CHPT expectations.

We emphasize that our calculation will not be a complete CHPT computation at  $\mathcal{O}(p^6)$ . We argue that the principal contribution to our process should come from the strong (non-polynomial) enhancement associated with the 2-pion intermediate state, and we ignore other  $\mathcal{O}(p^6)$  terms.

The form we use for the  $K_L \rightarrow 3\pi$  amplitude is [7]:

$$\mathcal{A}(K_L(p_K) \rightarrow \pi^+(p_+)\pi^-(p_-)\pi^0(p_0)) = G_8 M_K^2 a_1(z) + \frac{G_8 a_2}{\Lambda_\chi^2} [p_K \cdot (p_+ - p_-)]^2, \quad (4)$$

$$\begin{aligned} a_1(z) &= 0.38 + 0.13Y_0 - 0.0059Y_0^2, \\ a_2 &= 6.5, \end{aligned} \quad (5)$$

with  $z = (p_+ + p_-)^2 / M_K^2$ ,  $r_\pi = M_\pi / M_K$ , and  $Y_0 = (z - r_\pi^2 - \frac{1}{3}) / r_\pi^2$ . For comparison, we recall the lowest-order  $K_L \rightarrow 3\pi$  amplitude

$$\begin{aligned} a_1(z) &= z - r_\pi^2, \\ a_2 &= 0, \end{aligned} \quad (6)$$

corresponding to the calculation of Ref. [3].

Our task at this point is clear—we simply use the above physical amplitude to compute the absorptive part of  $K_L \rightarrow \pi^0 \gamma \gamma$ , and then use a subtracted dispersion relation to obtain the full amplitude. There can be at most three subtraction constants corresponding to the operators in Eqs. (2) and (3).

We can also follow a slightly more convenient although less obvious path: we can treat the amplitude (4) as the vertex of an effective chiral field theory, include the standard scalar electrodynamic couplings, and use conventional Feynman diagram techniques. This guarantees that we satisfy perturbative unitarity as well as ensuring the correct absorptive parts. Note that gauge invariance requires the Feynman rules for the amplitude in Eq. (4) to also include vertices where the momentum of any charged particle is replaced by

a photon, although these extra graphs make no contribution to the absorptive part. The sum of the relevant Feynman graphs is divergent and requires regularization and renormalization. The divergent part can be removed by adding a polynomial in momenta, but leaves an ambiguous final result. Of course, this ambiguity corresponds precisely to the three undetermined constants in our counterterm Lagrangian—we have anticipated that our answer would be ambiguous up to this polynomial. In the dispersive approach, these constants appear as the subtraction constants needed to obtain a convergent dispersion relation for the amplitude.

The result of this computation is given in terms of two invariant amplitudes  $A, B$  for the three-body decay  $K_L \rightarrow \pi^0 \gamma \gamma$  (assuming CP conservation):

$$\begin{aligned} \mathcal{A}(K_L(p_K) \rightarrow \pi^0(p_0)\gamma(q_1)\gamma(q_2)) &= \frac{G_8 M_K^2 \alpha_{em}}{4\pi} \epsilon_\mu(q_1) \epsilon_\nu(q_2) \left\{ (q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu}) \frac{A}{M_K^2} \right. \\ &\quad \left. + (p_K \cdot q_1 q_2^\mu p_K^\nu + p_K \cdot q_2 q_1^\nu p_K^\mu - p_K^\mu p_K^\nu q_1 \cdot q_2 - p_K \cdot q_1 p_K \cdot q_2 g^{\mu\nu}) \frac{2B}{M_K^4} \right\}. \end{aligned} \quad (7)$$

We have factored out the weak and electromagnetic coupling constants which must be present, and applied powers of  $M_K^2$  to leave  $A$  and  $B$  dimensionless. The result of our computation for these amplitudes is:

$$\begin{aligned} A &= 4F\left(\frac{z}{r_\pi^2}\right) \frac{a_1(z)}{z} + 4\frac{F(z)}{z}(1 + r_\pi^2 - z) \\ &\quad + \frac{a_2 M_K^2}{\Lambda_\chi^2} \left\{ \frac{4r_\pi^2}{z} F\left(\frac{z}{r_\pi^2}\right) + \frac{2}{3} \left(2 + \frac{z}{r_\pi^2}\right) \left[\frac{1}{6} + R\left(\frac{z}{r_\pi^2}\right)\right] - \frac{2}{3} \log \frac{M_\pi^2}{M_\rho^2} \right. \\ &\quad \left. - \frac{8r_\pi^2}{z^2} (x_1 + x_2)^2 \left[\frac{z}{12r_\pi^2} + F\left(\frac{z}{r_\pi^2}\right) + \frac{z}{r_\pi^2} R\left(\frac{z}{r_\pi^2}\right)\right] \right. \\ &\quad \left. + \frac{8r_\pi^2}{z^2} (x_1 - x_2)^2 \left[\frac{z}{12r_\pi^2} + F\left(\frac{z}{r_\pi^2}\right) + \frac{z}{2r_\pi^2} F\left(\frac{z}{r_\pi^2}\right) + \frac{3z}{r_\pi^2} R\left(\frac{z}{r_\pi^2}\right)\right] \right\} \\ &\quad + 4a_V(3 - z + r_\pi^2) + \alpha_1(z - r_\pi^2) + \alpha_2, \end{aligned} \quad (8a)$$

$$\begin{aligned} B &= \frac{a_2 M_K^2}{\Lambda_\chi^2} \left\{ \frac{4r_\pi^2}{z} F\left(\frac{z}{r_\pi^2}\right) + \frac{2}{3} \left(10 - \frac{z}{r_\pi^2}\right) \left[\frac{1}{6} + R\left(\frac{z}{r_\pi^2}\right)\right] + \frac{2}{3} \log \frac{M_\pi^2}{M_\rho^2} \right\} \\ &\quad - 8a_V + \beta. \end{aligned} \quad (8b)$$

Here  $x_i = p_K \cdot q_i / M_K^2$ , and the functions  $F$  and  $R$  are given by:

$$F(z) = \begin{cases} 1 - \frac{4}{z} \arcsin^2(\sqrt{z}/2) & z \leq 4 \\ 1 + \frac{1}{z} \left( \ln \frac{1 - \sqrt{1-4/z}}{1 + \sqrt{1-4/z}} + i\pi \right)^2 & z \geq 4 \end{cases}, \quad (9a)$$

$$R(z) = \begin{cases} -\frac{1}{6} + \frac{2}{z} - \frac{2}{z}\sqrt{4/z-1} \arcsin(\sqrt{z}/2) & z \leq 4 \\ -\frac{1}{6} + \frac{2}{z} + \frac{\sqrt{1-4/z}}{z} \left( \ln \frac{1-\sqrt{1-4/z}}{1+\sqrt{1-4/z}} + i\pi \right) & z \geq 4 \end{cases} . \quad (9b)$$

As guaranteed by CHPT, the amplitudes (8) have the right chiral behaviour as any momentum or chiral symmetry breaking mass goes to zero ( $F(z) \sim -z/12$  and  $R(z) \sim z/60$ , for  $z \rightarrow 0$ ). The loop amplitudes in Eqs. (8) agree with a recent calculation by Cappiello, D'Ambrosio and Miragliuolo [8]. The lowest-order amplitudes [3] are recovered for

$$\alpha_1 = \alpha_2 = \beta = a_V = 0 \quad (10)$$

and with  $a_1, a_2$  from Eq. (6).

As announced, the amplitudes (8) depend on three constants  $\alpha_1, \alpha_2$  and  $\beta$ . Actually, we have chosen to pull out the contribution to these constants from vector meson exchange, characterized by the parameter  $a_V$  [9]. To  $\mathcal{O}(p^6)$ , naïve vector meson dominance suggests  $a_V = 0.32$ . However, as first emphasized in Ref. [9], there are also so-called direct weak contributions associated with V exchange which cannot be written as a strong VMD amplitude with an external weak transition. Several model estimates of  $a_V$  have been made in the literature (for a recent discussion and a quite complete list of references, see Ref. [10]). A fair summary of those attempts is that we know neither the sign nor the magnitude of  $a_V$ . Using the CHPT prediction of lowest order [3] with the addition of V exchange the NA31 Collaboration has extracted the following bound on  $a_V$  from the Dalitz-plot distribution of the two photons [1]:

$$-0.32 < a_V < 0.19 \quad (90\% \text{ CL}) . \quad (11)$$

We stress that this bound used only the lowest-order CHPT calculation, and no longer applies to the present calculation as expressed in the amplitudes (8). In fact, as we shall discuss shortly, the NA31 data actually suggest a large negative value for  $a_V$ , outside the bound (11).

The only remaining issue is what values to choose for the counterterms  $\alpha_1, \alpha_2$  and  $\beta$ . By definition, they do not receive any contributions from the usually dominant vector meson exchange (already incorporated in  $a_V$ ) and can therefore be expected to be of order  $M_K^2/\Lambda_\chi^2$  as suggested by naïve chiral dimensional analysis. For definiteness, we fix these

three remaining constants by using minimal subtraction (with the conventions of Ref. [5]) in our loop calculation and choosing the scale  $\mu$  to be  $M_\rho$ . This gives

$$\begin{aligned}\alpha_1 &= 0 \\ \alpha_2 &= -\frac{5}{9} \frac{a_2 M_K^2}{\Lambda_\chi^2} = -0.65 \\ \beta &= -\frac{1}{9} \frac{a_2 M_K^2}{\Lambda_\chi^2} = -0.13 .\end{aligned}\tag{12}$$

Except for the relatively big coefficient  $a_2 = 6.5$ , these values provided by minimal subtraction correspond to our expectations. It is interesting to trace the origin of  $a_2$  in the  $K_L \rightarrow 3\pi$  amplitude. It should not come as a surprise to the reader that the biggest part of  $a_2$  is once again due to V exchange:  $\rho$  exchange in the  $\pi\pi$  scattering amplitude supplemented by an external non-leptonic weak transition contributes the major part of  $a_2$ .

Nevertheless, the final result is rather insensitive to reasonable variations of  $\alpha_1$ ,  $\alpha_2$  and  $\beta$ ; this is what we expected from our initial argument that the operators of  $\mathcal{O}(p^6)$  could not by themselves account for the large rate observed. On the other hand, the effect of V exchange can be sizeable. In computing the amplitudes (8) we have kept only the strongest infra-red behaviour coming from the 2-pion intermediate state. But there is also the possibility of a significant contribution associated with the relatively light vector mesons, just as we saw in our discussion of  $a_2$  in the  $K_L \rightarrow 3\pi$  amplitude.

Instead of performing a detailed fit to the experimental rate and distribution, which is left to our experimental colleagues, we choose  $a_V = 0$  and  $a_V = -0.9$  as two instructive examples for our calculation and compare with the result of  $\mathcal{O}(p^4)$  [3]. We find for the branching ratio

$$\text{BR}(K_L \rightarrow \pi^0 \gamma\gamma) = \begin{cases} 0.67 \times 10^{-6}, & \mathcal{O}(p^4), \\ 0.83 \times 10^{-6}, & \mathcal{O}(p^6), a_V = 0, \\ 1.60 \times 10^{-6}, & \mathcal{O}(p^6), a_V = -0.9 . \end{cases}\tag{13}$$

The corresponding spectra in the  $2\gamma$ -invariant mass are shown in Fig. 1, normalized to the 50 unambiguous events of NA31 [1] (neglecting acceptance corrections).

The lessons from this exercise are straightforward. The unitarity corrections by themselves raise the rate only moderately. Moreover, they produce an even more pronounced peaking of the spectrum at large  $m_{\gamma\gamma}$  (dashed curve in Fig. 1). Although the  $\mathcal{O}(p^4)$  distribution (dotted curve) describes the experimental distribution rather well, Fig. 7a of Ref.

[1] already suggests a slight excess of the theoretical curve at high  $m_{\gamma\gamma}$ . Ironically, the unitarity corrections tend to ruin the seeming agreement between theory and experiment for the spectrum. The predicted distribution of  $\mathcal{O}(p^6)$  for  $a_V = 0$  is definitely in disagreement with experiment.

As shown by the full curve in Fig. 1, addition of the V exchange component restores the agreement. Not only does the amplitude of  $\mathcal{O}(p^6)$  reproduce the observed spectrum with  $a_V = -0.9$ , but it also raises the branching ratio to  $1.6 \times 10^{-6}$ , in perfect agreement with the experimental values in Eq. (1). A careful fit taking the experimental acceptance into account is expected to yield a somewhat smaller  $|a_V|$ , but the negative sign of  $a_V$  seems clearly established. In their recent analysis [10], Heiliger and Sehgal have also investigated the contribution from vector meson exchange. Their favoured case of constructive interference corresponds to  $a_V < 0$ . A direct comparison is not possible because unitarity corrections of  $\mathcal{O}(p^6)$  are not included in the amplitudes of Ref. [10]. As we have shown, these corrections have an important impact on the spectrum, although less so for the rate.

In Fig. 2, we show the corresponding distributions in the second Dalitz variable  $y \equiv x_1 - x_2$ . In comparison with the  $\mathcal{O}(p^4)$  distribution, the unitarity corrections and V exchange exhibit again opposite trends. The spectrum for  $a_V = -0.9$  (full curve) is flatter than the  $\mathcal{O}(p^4)$  spectrum (dotted curve), in qualitative agreement with the experimental result of NA31 [1].

Finally, we comment on the implications of our results for the CP-conserving  $2\gamma$ -exchange contribution to the decay  $K_L \rightarrow \pi^0 e^+ e^-$  [9–12]. Neglecting the helicity-suppressed contribution from the amplitude  $A$  and including only the model-independent absorptive part from the  $2\gamma$  cut, the amplitude  $B$  in (8b) gives rise to

$$\text{BR}(K_L \rightarrow \pi^0 e^+ e^-)|_{abs} = \begin{cases} 0.3 \times 10^{-12}, & a_V = 0, \\ 1.8 \times 10^{-12}, & a_V = -0.9. \end{cases} \quad (14)$$

Although the rate increases of course with  $|a_V|$ , there is some destructive interference between the unitarity corrections of  $\mathcal{O}(p^6)$  and the V exchange contribution (for  $a_V = -0.9$ ). For a recent review of the CP-violating one-photon exchange contribution, the reader is referred to Ref. [10].

What have we learned from this calculation? Ideally, we would have found that the  $\mathcal{O}(p^6)$  counterterms make a negligible contribution to this process, and only the terms induced by unitarity and analyticity are relevant. This would have meant that the process is totally free of ambiguities and could be calculated in terms of the experimentally measured



$K_L \rightarrow 3\pi$  amplitude. Unfortunately, nature has not been so kind—the branching ratio with  $a_V = 0$  can only account for about half of the total observed rate and the corresponding  $m_{\gamma\gamma}$  distribution disagrees with experiment. On the other hand, the counterterms by themselves cannot account for this large rate either, without uncomfortably large values for the coefficients and without destroying the approximate agreement in the spectrum between experiment and CHPT at  $\mathcal{O}(p^4)$ . However, the two effects together can give good agreement with experiment with values of the counterterms that are comparable to other operators induced by vector exchange.

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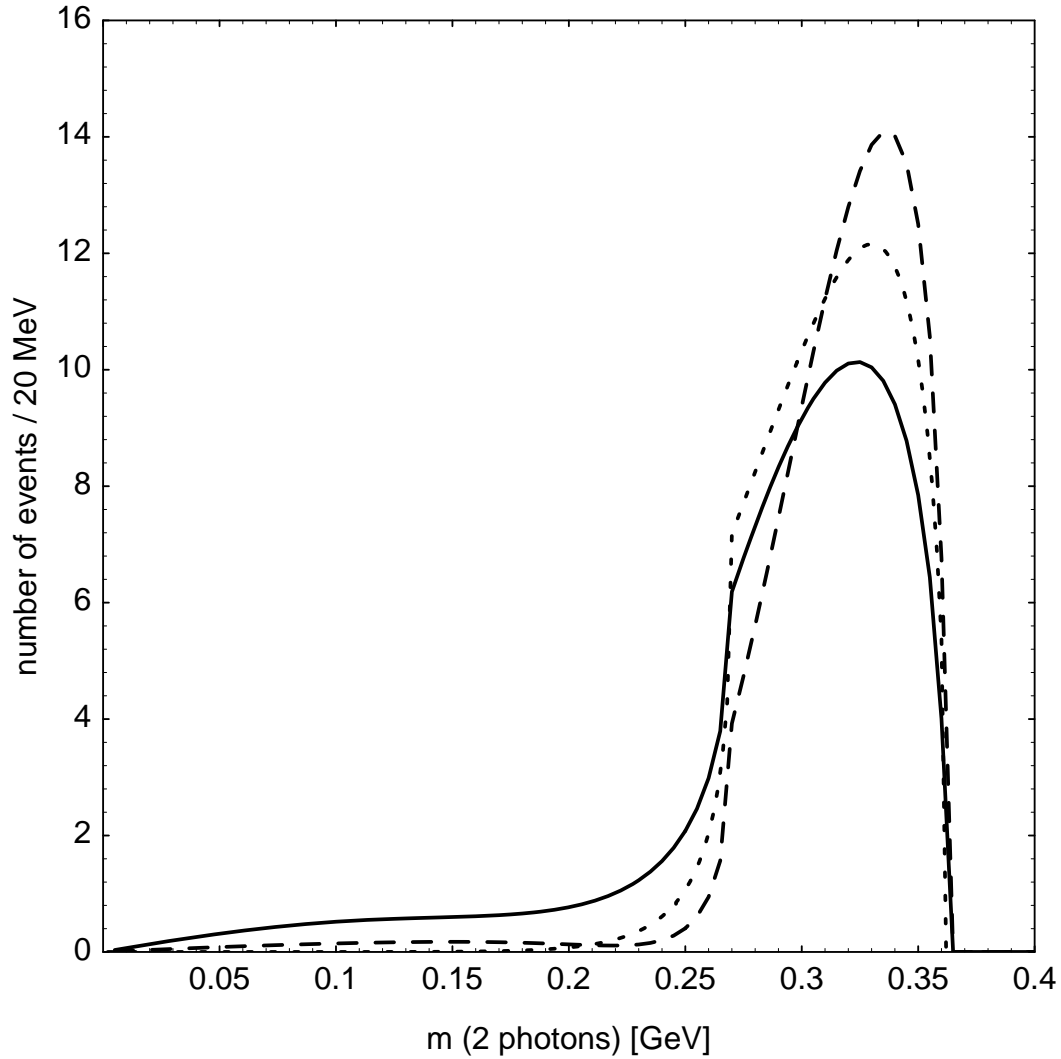


Fig. 1.  $2\gamma$ -invariant-mass distributions for  $K_L \rightarrow \pi^0 \gamma \gamma$ :  $\mathcal{O}(p^4)$  (dotted curve),  $\mathcal{O}(p^6)$  with  $a_V = 0$  (dashed curve),  $\mathcal{O}(p^6)$  with  $a_V = -0.9$  (full curve). The spectrum is normalized to the 50 unambiguous events of NA31 (without acceptance corrections).

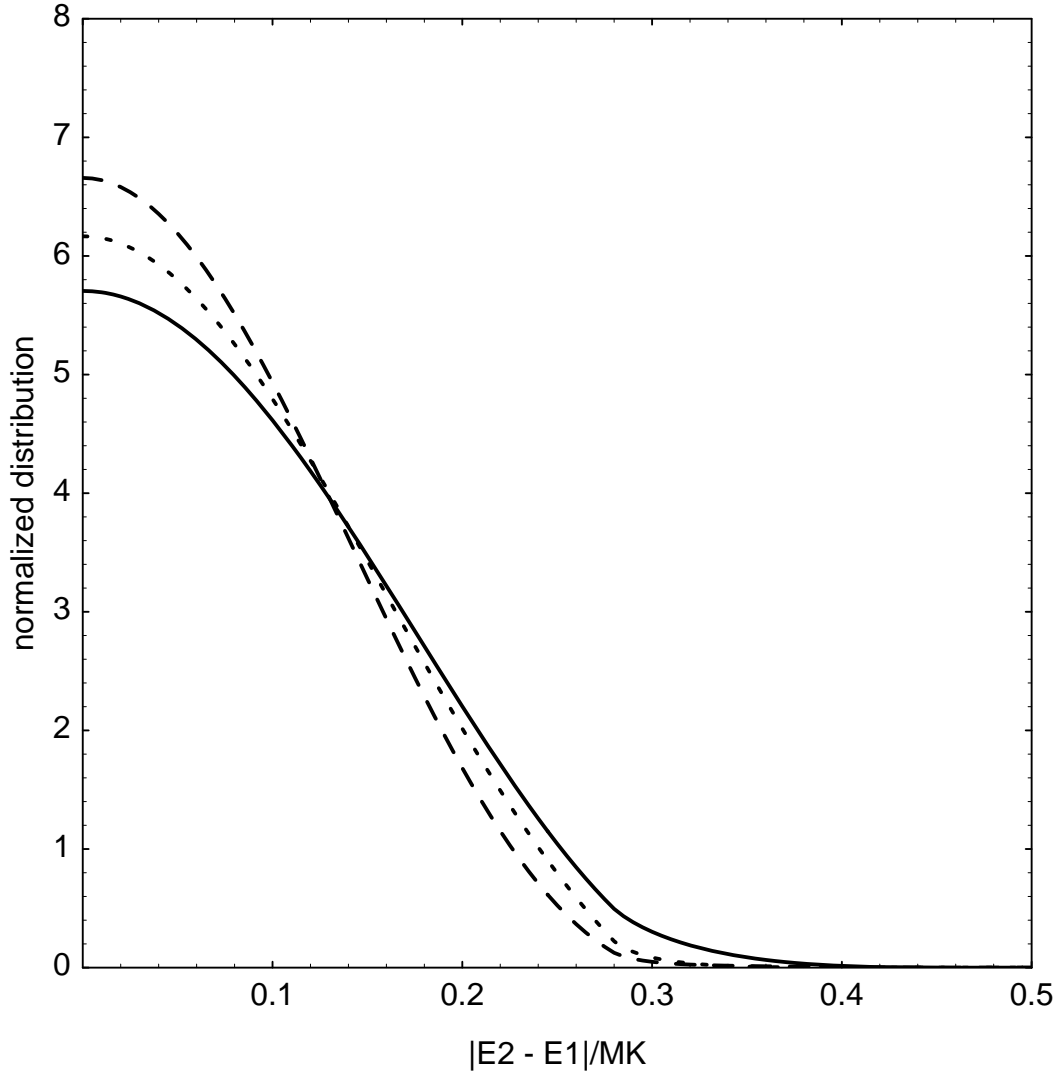


Fig. 2.  $y$ -distributions ( $y \equiv x_1 - x_2$ ) for  $K_L \rightarrow \pi^0 \gamma \gamma$ :  $\mathcal{O}(p^4)$  (dotted curve),  $\mathcal{O}(p^6)$  with  $a_V = 0$  (dashed curve),  $\mathcal{O}(p^6)$  with  $a_V = -0.9$  (full curve). The normalization is arbitrary.