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The Longitudinal Muon Polarization in $K_L \to \mu^+\mu^-$ *

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Abstract

From a leading-order calculation of $K_1^0 \to \gamma^*\gamma^* \to \ell^+\ell^-$ in chiral perturbation theory, the longitudinal muon polarization in the decay $K_L \to \mu^+\mu^-$ is found to be approximately $2 \cdot 10^{-3}$ in the standard model. The relative branching ratios $\Gamma(K_S \to \ell^+\ell^-)/\Gamma(K_S \to 2\gamma)$ are predicted as $2 \cdot 10^{-6}$ $(\ell = \mu)$ and $8 \cdot 10^{-9}$ $(\ell = e)$, respectively.

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1 Introduction

The longitudinal polarization P_L of either muon in the decay $K_L \to \mu^+\mu^-$ is a measure of CP violation [1,2]. As for every CP violating observable in the neutral kaon system, there are in general two sources for P_L. In the standard model, the contribution from direct CP violation in the amplitude for $K_2^0 \to \mu^+\mu^-$ is now known to be completely negligible in view of the lower bound $M_H \gtrsim M_Z/2$ for the Higgs mass M_H established by LEP experiments [3]. Over the years, P_L has also been investigated in various extensions of the standard model [4,5]. To summarize the situation in a single sentence, $P_L = O(10^{-1})$ can be obtained at the expense of considerable fine tuning while $P_L = O(10^{-2})$ appears quite naturally in many models (see e.g., the last paper of Ref. [5]). It is worth emphasizing that P_L is especially sensitive to the presence of light scalars with CP violating Yukawa couplings.

The second, model independent source for P_L is indirect CP violation due to K^0-$\bar{K}^0$ mixing. Taking only the absorptive parts of the amplitudes for $K_{1,2}^0 \to \mu^+\mu^-$ into account, Herczeg [4] has estimated $|P_L| \simeq 7 \cdot 10^{-4}$ for the contribution from indirect CP violation and thus at the same time for the complete standard model prediction. However, Herczeg's estimate is based on only one out of four contributions to P_L which could all interfere constructively with unknown magnitudes. The first experiment to measure P_L is expected to have a sensitivity of a few times 10^{-1} [6]. However, in the not too distant future the high kaon fluxes which will be available at kaon factories should allow to reach the interesting region $P_L = O(10^{-2})$. If a polarization of this magnitude will be found, how reliably will we be able to infer the existence of a new mechanism for CP violation from such a measurement?

For this purpose it seems mandatory to attempt a more quantitative calculation of P_L in the standard model to improve Herczeg's estimate $|P_L| \simeq 7 \cdot 10^{-4}$. This has not been possible up to now because of the absence of a trustworthy calculation of the dispersive part of the $K_1^0 \simeq K_S \to \mu^+\mu^-$ amplitude. As a matter of fact, the theoretical situation is by no means better for $K_2^0 \simeq K_L \to \mu^+\mu^-$. However, in this case we can use the accurately known absorptive part together with the measured rate $\Gamma(K_L \to \mu^+\mu^-)$ to get a handle on the dispersive part. Although the absorptive part of $K_1^0 \to \mu^+\mu^-$ is almost as well known [7,8,9], we may have to wait for some time for the actual detection of $K_S \to \mu^+\mu^-$ [10].

The main result of the present paper will be an unambiguous calculation of both the dispersive and the absorptive parts of $K_1^0 \to \ell^+\ell^-$ to lowest order in chiral perturbation theory (CHPT). Not only will the relevant two-loop Feynman amplitude turn out to be finite, but we will demonstrate in addition that there can be no other contributions to $T(K_1^0 \to \ell^+\ell^-)$ to the order considered. Thus, the theoretical quality of the calculated amplitudes corresponds to the successful CHPT predictions for $K_S \to \gamma\gamma$ [11] and $K_L \to \pi^0\gamma\gamma$ [12], both confirmed experimentally [13]. The rates for the rare decays $K_S \to \mu^+\mu^-, e^+e^-$ are an immediate consequence of our calculation. Together with the phenomenologically known amplitude for $K_2^0 \to \mu^+\mu^-$, a reliable estimate of P_L in the standard model becomes possible.

In Sect. 2, the relevant equations for calculating P_L are collected. The essence of the present investigation is contained in Sect. 3 where the transition $K_1^0 \to \ell^+\ell^-$ is considered to lowest order in CHPT. A comparison of our result for the absorptive part Im $T(K_1^0 \to \mu^+\mu^-)$ with previous work is made in Sect. 4. The numerical results for both decay rates and longitudinal muon polarization are presented in Sect. 5. Our conclusions are summarized in Sect. 6.

2 Longitudinal Muon Polarization

The amplitude for the decay of a neutral spin-zero meson φ into a lepton pair has the general form

$$T(\varphi \to \ell^+\ell^-) = \bar{u}(iB + A\gamma_5)v \tag{2.1}$$

with a decay rate

$$\Gamma(\varphi \to \ell^+\ell^-) = \frac{M_\varphi \beta_\ell}{8\pi}(|A|^2 + \beta_\ell^2|B|^2), \qquad \beta_\ell = \left(1 - \frac{4m_\ell^2}{M_\varphi^2}\right)^{1/2} . \tag{2.2}$$

The longitudinal polarization P_L of ℓ^- is defined as [1]

$$P_L = \frac{N_R - N_L}{N_R + N_L} \tag{2.3}$$

where N_R, N_L are the numbers of outgoing ℓ^- with positive or negative helicity, respectively. P_L can be expressed in terms of the amplitudes A, B in (2.1) as [1,2,4]

$$P_L = \frac{M_\varphi \beta_\ell^2}{4\pi\Gamma} \,\mathrm{Im}\,(A^* B). \tag{2.4}$$

Specializing to $\varphi = K_L$ and using the decomposition of the mass eigenstates

$$\begin{aligned} |K_{S,L}\rangle &= (1+|\rho|^2)^{-1/2}(|K_{1,2}^0\rangle + \rho|K_{2,1}^0\rangle) \\ |K_{1,2}^0\rangle &= \frac{1}{\sqrt{2}}(|K^0\rangle \mp |\bar{K}^0\rangle), \end{aligned} \tag{2.5}$$

in terms of CP eigenstates $|K_1^0\rangle$, $|K_2^0\rangle$, one obtains

$$\mathrm{Im}\,(A^* B) = \mathrm{Im}\,\{A_2^*(B_2 + \rho B_1)\} \tag{2.6}$$

to first order in CP violating quantities. The amplitudes A_2, B_1 are CP conserving, B_2 violates CP. We adopt the Wu-Yang phase convention with $\rho = \varepsilon$ where ε is the usual measure of indirect CP violation in $K^0 \to 2\pi$ decays [2].

In the standard model, the CP violating amplitude B_2 is induced by Higgs exchange with an effective one-loop flavour changing $\bar{s}dH$ coupling [14]. The lower bound [3] on the Higgs mass

$$M_H > 48 \text{ GeV} \qquad (95\% \text{ c.l.}) \tag{2.7}$$

implies [14,15] a conservative upper limit

$$|P_{L,\mathrm{direct}}| < 10^{-4} \tag{2.8}$$

which will turn out to be completely negligible compared to indirect CP violation.

To calculate the dominant contribution to P_L in the standard model, we make use of

$$\varepsilon = |\varepsilon| e^{i\Phi_{SW}} \tag{2.9}$$

$$\Phi_{SW} = \tan^{-1}\frac{2(M_L - M_S)}{\Gamma_S - \Gamma_L} = (43.67 \pm 0.13)^0 \simeq \frac{\pi}{4}$$

to obtain

$$\text{Im }(\varepsilon A_2^* B_1) \simeq \frac{|\varepsilon|}{\sqrt{2}}\{\text{Re } A_2(\text{Re } B_1 + \text{Im } B_1) + \text{Im } A_2(\text{Im } B_1 - \text{Re } B_1)\}. \tag{2.10}$$

The absorptive part Im A_2 is known to be completely dominated by the 2γ-intermediate state [16] with

$$|\text{Im } A_2^{\gamma\gamma}| = \frac{\alpha m_\mu}{4\beta_\mu M_K} \ln\frac{1+\beta_\mu}{1-\beta_\mu}\left[\frac{64\pi\Gamma(K_L \to 2\gamma)}{M_K}\right]^{1/2} \tag{2.11}$$

or

$$B(K_L \to \mu^+\mu^-)_{\text{abs}} \simeq \frac{\alpha^2 m_\mu^2}{2\beta_\mu M_K^2}\left(\ln\frac{1+\beta_\mu}{1-\beta_\mu}\right)^2 B(K_L \to 2\gamma). \tag{2.12}$$

The experimental branching ratio [10] $B(K_L \to 2\gamma) = (5.70 \pm 0.27)\cdot 10^{-4}$ yields

$$|\text{Im } A_2^{\gamma\gamma}| = (2.20 \pm 0.05)\cdot 10^{-12} \tag{2.13}$$

and

$$B(K_L \to \mu^+\mu^-)_{\text{abs}} = (6.81 \pm 0.32)\cdot 10^{-9}. \tag{2.14}$$

Comparing with [10] $B(K_L \to \mu^+\mu^-) = (6.3 \pm 1.1)\cdot 10^{-9}$ one gets an upper bound

$$|\text{Re } A_2| < 0.7(1.1)\cdot 10^{-12} \tag{2.15}$$

at the 1σ (2σ) level.

To compute the longitudinal muon polarization

$$P_L \simeq \frac{\sqrt{2}\beta_\mu|\varepsilon|}{|A_2|^2}\{\text{Re } A_2(\text{Re } B_1 + \text{Im } B_1) + \text{Im } A_2(\text{Im } B_1 - \text{Re } B_1)\} \tag{2.16}$$

we shall calculate the amplitude B_1 responsible for $K_1^0 \simeq K_S \to \mu^+\mu^-$ in the next section. Although Re B_1 and Im B_1 will be completely calculable to lowest order in CHPT, the unknown relative sign between Re A_2 and Im A_2 together with (2.15) will only allow for upper and lower bounds on P_L.

Finally, we note that the estimate of Herczeg [4]

$$|P_L| \simeq 7\cdot 10^{-4} \tag{2.17}$$

is based on the product of absorptive parts Im $A_2\cdot$ Im B_1 in (2.16) only.

3 The Transition $K_1^0 \to \ell^+\ell^-$ in CHPT

It has been known for quite some time [7,8,9] that the amplitude B_1 for $K_1^0 \to \ell^+\ell^-$ is also dominated by the transition $K_1^0 \to \gamma^*\gamma^* \to \ell^+\ell^-$. Although the absorptive part Im B_1 can be computed with some effort [7,8,9], the unknown large-momentum behaviour of $K_1^0 \to \gamma^*\gamma^*$ has so far precluded a reliable calculation of the dispersive part Re B_1.

To lowest order in CHPT, the relevant diagrams are shown in Fig. 1. In principle, the meson loop involves both charged pions and kaons. The crucial observation which allows for an unambiguous calculation of Re B_1 is that chiral symmetry forces the amplitude for the two-loop

diagram of Fig. 1 to be finite. This is analogous to the transitions $K_1^0 \to \gamma\gamma$ and $K_2^0 \to \pi^0\gamma\gamma$ which are both uniquely determined to $O(p^4)$ in CHPT by one-loop Feynman diagrams [11,12].

The proof of finiteness of the two-loop amplitude of Fig. 1 follows the usual procedure of CHPT. The lowest-order local coupling relevant for $K^0 \to \ell^+\ell^-$ with the appropriate transformation property of a $\Delta S = 1$ weak transition is uniquely given by

$$ih \text{ tr }(\lambda_{6-i7}U(\varphi)^\dagger\partial^\mu U(\varphi))\overline{\psi}\gamma_\mu(\gamma_5)\psi + h.c. \tag{3.1}$$

$U(\varphi)$ is the matrix field of pseudoscalar Goldstone fields φ_i ($i = 1,\ldots,8$) transforming as

$$U(\varphi) \to g_R U(\varphi) g_L^\dagger \tag{3.2}$$

under chiral transformations $g_{L,R} \in SU(3)_{L,R}$ and h is a dimensionless coupling constant. It is clear from the structure of (3.1) that only the axial-vector coupling can contribute to $K^0 \to \ell^+\ell^-$. In other words, the two leptons must be in a $^1S_0$ state. Expanding $U(\varphi)$ to first order in φ, one gets from (3.1)

$$\frac{2\sqrt{2}h}{f}\partial^\mu K^0\overline{\psi}\gamma_\mu\gamma_5\psi + h.c. \tag{3.3}$$

with f being equal to the pion decay constant $f_\pi = 93$ MeV to lowest order in CHPT.

We can now invoke CPS invariance [17] of the $\Delta S = 1$ non-leptonic weak interactions for the Lagrangian (3.1) to show that (3.3) is actually of the form

$$\frac{2\sqrt{2}h}{f}\partial^\mu(K^0 + \bar{K}^0)\overline{\psi}\gamma_\mu\gamma_5\psi = \frac{4h}{f}\partial^\mu K_2^0\overline{\psi}\gamma_\mu\gamma_5\psi. \tag{3.4}$$

For our purposes it is sufficient and more straightforward to consider the limit of CP invariance to derive (3.4) from (3.3).

The final step of the proof consists in power counting. Since we are going to perform the two-loop integration explicitly anyway we only state the result of a straightforward analysis here. If $T(K^0 \to \gamma^*\gamma^*)$ is taken to $O(p^4)$ in CHPT as indicated in Fig. 1, the possible divergences of the amplitude $T(K^0 \to \gamma^*\gamma^* \to \ell^+\ell^-)$ correspond to the local Lagrangian (3.1) with divergent constants h.

Consequently, the final result (3.4) involving only K_2^0 gives rise to two important conclusions:

i) The two-loop amplitude of Fig. 1 for $K_1^0 \to \ell^+\ell^-$ is necessarily finite.

ii) There are no other contributions to this order such as direct short-distance contributions, meson resonance contributions, etc. B_1 is unambiguously determined by the Feynman diagram of Fig. 1 to lowest order in CHPT.

It is instructive to comment on the completely different situation for $K_2^0 \to \ell^+\ell^-$. Although Eq. (3.4) would allow for a divergent amplitude to leading order in CHPT, $T(K_2^0 \to \gamma^*\gamma^*)$ actually vanishes to $O(p^4)$ due to a complete destructive interference between the π^0 and η_8 pole contributions [12,18]. Nevertheless, there is a well-known direct short-distance contribution to the coupling constant h of Eq. (3.4) [19].

The two-loop amplitude corresponding to the diagram of Fig. 1 can be written as

$$iB_1\bar{u}(k_-)v(k_+) = 8\alpha^2 G_8 f(M_K^2 - M_\pi^2)\int \frac{d^4q_1}{(2\pi)^4}\frac{T_{\mu\nu}(q_1,q_2)\bar{u}(k_-)\gamma^\mu(\not{k}_- - \not{q}_1 + m_\ell)\gamma^\nu v(k_+)}{(q_1^2+i\varepsilon)(q_2^2+i\varepsilon)[(k_- - q_1)^2 - m_\ell^2 + i\varepsilon]}$$

$$q = q_1 + q_2 = k_+ + k_-, \qquad q^2 = M_K^2. \tag{3.5}$$

The reduced off-shell amplitude $T_{\mu\nu}(K_1^0 \to \gamma^*\gamma^*)$ is given by

$$T_{\mu\nu} = g_{\mu\nu}H_1(r_1,r_2) + \frac{2q_{2\mu}q_{1\nu}}{M_\pi^2}H_2(r_1,r_2) \tag{3.6}$$

$$H_1 = \int_0^1 du\int_0^{1-u} dv\frac{(r_1+r_2-R)uv + r_2 v(2v-1)}{P(u,v)}, \qquad H_2 = \int_0^1 du\int_0^{1-u} dv\frac{uv}{P(u,v)}$$

$$r_i = \frac{q_i^2}{M_\pi^2}, \qquad R = \frac{M_K^2}{M_\pi^2} = \frac{1}{r_\pi^2}, \qquad P(u,v) = 1 - Ruv - (1-u-v)(r_1 u + r_2 v).$$

The octet coupling constant G_8 is defined by the $O(p^2)$ chiral Lagrangian [20]

$$\mathcal{L}_2^{\Delta S=1} = \frac{1}{2}G_8 f^4 \ \mathrm{tr}\ (\lambda_{6-i7}\partial_\mu U^\dagger\partial^\mu U) + h.c. \tag{3.7}$$

Its numerical value from $\Gamma(K^0 \to 2\pi)$ to lowest order is

$$|G_8| \simeq 9\cdot 10^{-6}\ \mathrm{GeV}^{-2}, \tag{3.8}$$

but this value is rather uncertain due to unknown higher order corrections to $K^0 \to 2\pi$ transitions. To circumvent this uncertainty, we will normalize our results to the decay $K_S \to \gamma\gamma$. In the notation of Ref. [21]

$$\Gamma(K_1^0 \to \gamma\gamma) = \frac{\alpha^2 G_8^2 f^2}{4\pi^3}M_K^3(1-r_\pi^2)^2|H(0)|^2 \tag{3.9}$$

with

$$H(0) = -H_1(r_1 = r_2 = 0), \qquad |H(0)| = 0.67. \tag{3.10}$$

The theoretical prediction [11] on the basis of (3.8)

$$B(K_S \to \gamma\gamma) = 2.0\cdot 10^{-6} \tag{3.11}$$

compares well with the experimental value [13a]

$$B(K_S \to \gamma\gamma) = (2.4 \pm 1.2)\cdot 10^{-6}, \tag{3.12}$$

which still has a rather big error.

Before proceeding with the calculation, we note that G_8 is real in the Wu-Yang phase convention. According to (3.6), the kaon loop contribution to $K_1^0 \to \gamma^*\gamma^*$ vanishes even for off-shell photons. For later use we emphasize that the amplitude B_1 in (3.5) is not only ultraviolet but obviously also infrared convergent because H_1 is non-singular if either r_1 or r_2 (or both) tend to zero.

Introducing two more Feynman parameters, the amplitude B_1 can be expressed as a fourfold integral

$$B_1 = C_\ell I_\ell, \qquad C_\ell = \frac{\alpha^2}{\pi^2}G_8 f m_\ell(1-r_\pi^2)$$

$$I_\ell = \int_0^1 du\int_0^{1-u} dv\int_0^1 dy\int_0^1 dx\left\{(2u+2v-1)[1+x(1-y)] - \frac{u}{1+\gamma x+i\varepsilon}\right\}\Big/(\alpha+\beta x - i\varepsilon) \tag{3.13}$$

$$\begin{aligned}\alpha &= y[r_\pi^2 - v(1-v)]\\ \beta &= yv(1-u-v)\left(1-\frac{yu}{u+v}\right) + \frac{m_\ell^2}{M_K^2}(1-y)^2(u+v)(1-u-v) \geq 0\\ \gamma &= -1 + y\frac{u(1-u)-r_\pi^2}{(u+v)(1-u-v)}.\end{aligned}$$

Carrying out the x-integration in (3.13) gives rise to the decomposition

$$I_\ell = I_{\ell,\mathrm{disp}} + iI_{\ell,\mathrm{abs}}$$

$$I_{\ell,\mathrm{disp}} = \int_0^1 du\int_0^{1-u} dv\int_0^1 dy\left\{\frac{(2u+2v-1)}{\beta}\left[1-y+\ln\left|\frac{\alpha+\beta}{\alpha}\right|(1-\frac{\alpha}{\beta}(1-y))\right] + \right.$$
$$\left. + \frac{u}{\alpha\gamma-\beta}\ln\left|\frac{\alpha+\beta}{\alpha(1+\gamma)}\right|\right\} \tag{3.14}$$

$$I_{\ell,\mathrm{abs}} = \pi\int_0^1 du\int_0^{1-u} dv\int_0^1 dy\left\{\Theta(-\alpha)\Theta(\alpha+\beta)\frac{(2u+2v-1)}{\beta^2}[\beta-\alpha(1-y)] + \right.$$
$$\left. + \lim_{\varepsilon\to 0}\frac{u(\alpha\gamma-\beta)}{(\alpha\gamma-\beta)^2+\varepsilon^2}[\Theta(-\alpha)\Theta(\alpha+\beta) - \Theta(r_\pi^2 - u(1-u))]\right\} \tag{3.15}$$

into dispersive and absorptive parts. The relative branching ratio is given by

$$\frac{\Gamma(K_1^0 \to \ell^+\ell^-)}{\Gamma(K_1^0 \to \gamma\gamma)} = k_\ell|I_\ell|^2 \tag{3.16}$$

$$k_\ell = \frac{\alpha^2\beta_\ell^3 m_\ell^2}{2\pi^2|H(0)|^2M_K^2}, \qquad k_\mu = 2.0\cdot 10^{-7}, \qquad k_e = 6.3\cdot 10^{-12}.$$

Although the representation (3.13) for I_ℓ seems to suggest the presence of a double-absorptive part due to

$$\frac{1}{(\alpha+\beta x - i\varepsilon)(1+\gamma x+i\varepsilon)} \sim \delta(\alpha+\beta x)\delta(1+\gamma x) + \ldots \tag{3.17}$$

the corresponding integral actually vanishes. Since the absence of such a term is not at all transparent in Eq. (3.13), we briefly recall the argument given in Ref. [7]. A double-absorptive part could only arise if both the two pions and the two photons are on-shell in the diagram of Fig. 2. Except for inessential factors, the Cutkosky rules associate the following absorptive parts to the two cuts shown in Fig. 2:

$$\begin{aligned}T_{\mathrm{abs}}^{(1)} &\sim \int d\Omega_{\pi\pi}T(K_1^0 \to \pi\pi)T(\ell^+\ell^- \to \pi\pi)^*\\ T_{\mathrm{abs}}^{(2)} &\sim \int d\Omega_{\gamma\gamma}T(K_1^0 \to \gamma\gamma)T(\ell^+\ell^- \to \gamma\gamma)^*\end{aligned} \tag{3.18}$$

where $d\Omega_{\pi\pi}$, $d\Omega_{\gamma\gamma}$ denote the two-particle phase space integrations. Likewise, we find

$$\begin{aligned} T(\ell^+\ell^- \to \pi\pi)_{\rm abs} &\sim \int d\Omega_{\gamma\gamma} T(\ell^+\ell^- \to \gamma\gamma) T(\pi\pi \to \gamma\gamma)^* \\ T(K_1^0 \to \gamma\gamma)_{\rm abs} &\sim \int d\Omega_{\pi\pi} T(K_1^0 \to \pi\pi) T(\gamma\gamma \to \pi\pi)^* . \end{aligned} \tag{3.19}$$

The double-absorptive part is therefore given by

$$\begin{aligned} iT_{\rm double-abs} &\sim \int d\Omega_{\pi\pi} T(K_1^0 \to \pi\pi)[iT(\ell^+\ell^- \to \pi\pi)_{\rm abs}]^* + \\ &\quad + \int d\Omega_{\gamma\gamma} iT(K_1^0 \to \gamma\gamma)_{\rm abs} T(\ell^+\ell^- \to \gamma\gamma)^* \\ &\sim \int d\Omega_{\pi\pi} d\Omega_{\gamma\gamma} \{T(K_1^0 \to \pi\pi)(-i)T(\ell^+\ell^- \to \gamma\gamma)^* T(\pi\pi \to \gamma\gamma) + \\ &\quad + iT(K_1^0 \to \pi\pi)T(\gamma\gamma \to \pi\pi)^* T(\ell^+\ell^- \to \gamma\gamma)^*\} = 0 \end{aligned} \tag{3.20}$$

because $T(\pi\pi \to \gamma\gamma) = T(\gamma\gamma \to \pi\pi)^*$ for the tree level amplitudes relevant here. Thus, the real part of I_ℓ is due exclusively to the dispersive part (3.14).

4 Absorptive Part

The absorptive part Im B_1 receives contributions from the three cuts displayed in Fig. 3. However, the different contributions to $I_{\ell,\rm abs}$ in (3.15) are not associated to definite intermediate states. In particular, Im $B_1(\pi\pi)$ and Im $B_1(\pi\pi\gamma)$ from Fig. 3 are separately infrared divergent [7,8,9], whereas each part of $I_{\ell,\rm abs}$ in (3.15) is infrared convergent just as the original amplitude B_1 in (3.5).

A meaningful comparison with previous work [7,8,9] is therefore only possible for Im $B_1(\pi\pi + \pi\pi\gamma)$ and for Im $B_1(\gamma\gamma)$ which can be expressed in terms of the amplitude $H(0)$ defined in Eq. (3.10) as

$$\text{Im } B_1(\gamma\gamma) = C_\mu \text{ Im } I_\mu(\gamma\gamma) = \frac{\alpha^2 G_8}{\pi\beta_\mu} f m_\mu (1 - r_\pi^2) \text{ Re } H(0) \ln \frac{1+\beta_\mu}{1-\beta_\mu}. \tag{4.1}$$

A more principal problem for the comparison seems to be that previous calculations did not take into account the chiral structure of mesonic vertices. However, to the required order in CHPT the absorptive part is in fact insensitive to the chiral vertex structure. This is immediately obvious for Im $B_1(\pi\pi)$ since $T(K_1^0 \to \pi\pi)$ is only needed for on-shell mesons and $T(\pi^+\pi^- \to \ell^+\ell^-)$ involves photonic couplings only. The situation is similar for Im $B_1(\gamma\gamma)$ because the absorptive part of $T(K_1^0 \to \gamma\gamma)$ is once again independent of the derivative structure of the $K\pi\pi$ vertex. Moreover, since there is neither a kaon loop contribution to $K_1^0 \to \gamma\gamma$ in CHPT [11] nor a local counterterm of $O(p^4)$, Re $T(K_1^0 \to \gamma\gamma)$ is given by a convergent dispersion integral [7,22] over Im $T(K_1^0 \to \gamma\gamma)$ and is therefore also insensitive to the chiral vertex structure.

The remaining case of Im $B_1(\pi\pi\gamma)$ requires some discussion because the derivative nature of mesonic couplings in CHPT requires by gauge invariance the presence of a direct $K\pi\pi\gamma$ amplitude contributing to $T(K_1^0 \to \pi^+\pi^-\gamma)$ which is of course absent in the old calculations [7,8,9]. Moreover, in the Bremsstrahlung diagrams for $K_1^0 \to \pi^+\pi^-\gamma$ one pion goes off-shell so the amplitude seems to depend once again on the chiral structure of the $K\pi\pi$ vertex.

However, also in this case gauge invariance (Low's theorem [23]) forces the direct $K\pi\pi\gamma$ amplitude to compensate exactly the chiral off-shell behaviour of the Bremsstrahlung amplitudes

Table 1

Comparison with previous calculations of Im B_1 (total), Im $B_1(\gamma\gamma)$ and Im $B_1(\pi\pi + \pi\pi\gamma)$ for $K_1^0 \to \mu^+\mu^-$ in units of 10^{-12}. $B_1 = \sqrt{2}F_1$ in the notation of Refs. [8,9].

	Im B_1 (total)	Im $B_1(\gamma\gamma)$	Im $B_1(\pi\pi + \pi\pi\gamma)$
This calculation	0.54	−1.53	2.07
Smith and Uy [8]	0.76	−1.54	2.30
Bogomolny et al. [9]	0.56	−1.58	2.14

to reproduce the result corresponding to a non-derivative $K\pi\pi$ vertex [24]. More explicitly, let us consider a general off-shell $K_1^0\pi^+\pi^-$ amplitude of the form

$$A[K_1^0(p) \to \pi^+(p_+) + \pi^-(p_-)] = c_0 + c_1 \frac{p^2}{M_K^2} + c_2 \frac{p_+^2}{M_K^2} + c_3 \frac{p_-^2}{M_K^2} \tag{4.2}$$

with constants c_i. The constant c_0 corresponds to a non-derivative $K\pi\pi$ vertex (forbidden by chiral symmetry), while c_1, c_2, c_3 represent typical chiral couplings of $O(p^2)$. From (4.2) we obtain

$$\Gamma(K_1^0 \to \pi^+\pi^-) \sim |c_0 + c_1 + (c_2 + c_3) r_\pi^2|^2 \tag{4.3}$$

for the rate. Although the amplitude for $K_1^0 \to \pi^+\pi^-\gamma$ consists of both Bremsstrahlung contributions and a direct photon emission amplitude depending on c_1, c_2, c_3, gauge invariance dictates the form

$$T(K_1^0(p) \to \pi^+(p_+) + \pi^-(p_-) + \gamma(k)) = \varepsilon^\mu(k) A_\mu(p_+, p_-, k) \tag{4.4}$$

with

$$A_\mu \sim [c_0 + c_1 + (c_2 + c_3) r_\pi^2] \left[\frac{p_{+\mu}}{M_K^2 - 2pp_-} - \frac{p_{-\mu}}{M_K^2 - 2pp_+} \right].$$

Therefore, to the considered order in the chiral expansion $T(K_1^0 \to \pi^+\pi^-\gamma)$ depends only on the on-shell amplitude for $K_1^0 \to \pi^+\pi^-$ and is therefore insensitive to the chiral vertex structure.

In Table 1 we compare our results for Im B_1 ($\ell = \mu$) with previous calculations. For our numbers given in the Table and also for later results the precision of the numerical integrations is better than the digits given. The agreement with the calculation of Bogomolny et al. [9] is excellent and satisfactory for the results of Smith and Uy [8]. Finally, we note the strong destructive interference between Im $B_1(\gamma\gamma)$ and Im $B_1(\pi\pi + \pi\pi\gamma)$ for the transition $K_1^0 \to \mu^+\mu^-$.

5 Numerical Results

Upon numerical integration of the integrals (3.14) and (3.15) we obtain

$$\begin{aligned} I_{\mu,\rm disp} &= -2.82 \\ I_{\mu,\rm abs} &= 1.21 \end{aligned} \tag{5.1}$$

for $K_1^0 \to \mu^+\mu^-$ where $I_{\mu,\mathrm{abs}}$ corresponds, of course, to Im B_1 (total) in Table 1. For $K_1^0 \to e^+e^-$ the results are

$$\begin{aligned} I_{e,\mathrm{disp}} &= 1.4 \pm 0.3 \\ I_{e,\mathrm{abs}} &= -35 \end{aligned} \tag{5.2}$$

For $\ell = e$, $I_{\ell,\mathrm{abs}}$ is enhanced compared to $\ell = \mu$. This is partly due to the constructive interference between the $\gamma\gamma$ and the $\pi\pi + \pi\pi\gamma$ intermediate states in this case. Moreover, the $\gamma\gamma$ contribution is itself enhanced for $\ell = e$:

$$\mathrm{Im}\ I_e(\gamma\gamma) = \frac{\pi\ \mathrm{Re}\ H(0)}{\beta_e} \ln \frac{1+\beta_e}{1-\beta_e} = -14. \tag{5.3}$$

In contrast, there is a strong destructive interference in the dispersive part $I_{e,\mathrm{disp}}$. The amplitude for the electronic decay mode is completely dominated by the absorptive part. Of course, the overall amplitude B_1 in Eq. (3.13) is suppressed for $\ell = e$ by a factor m_e due to helicity.

From Eq. (3.16) we compute the relative branching ratios

$$\begin{aligned} \frac{\Gamma(K_S \to \mu^+\mu^-)}{\Gamma(K_S \to \gamma\gamma)} &= 1.9 \cdot 10^{-6}, \\ \frac{\Gamma(K_S \to e^+e^-)}{\Gamma(K_S \to \gamma\gamma)} &= 7.9 \cdot 10^{-9} \end{aligned} \tag{5.4}$$

well below the present experimental upper limits [10]. The results (5.4) differ from those of a recent calculation employing a scalar resonance dominated $K_1^0 \to \gamma^*\gamma^*$ amplitude [25] which finds larger branching ratios.

To calculate the longitudinal muon polarization we recall Eq. (2.16)

$$P_L = \frac{\sqrt{2}\beta_\mu|\varepsilon|}{|A_2|^2}\{\mathrm{Re}\ A_2(\mathrm{Re}\ B_1 + \mathrm{Im}\ B_1) + \mathrm{Im}\ A_2(\mathrm{Im}\ B_1 - \mathrm{Re}\ B_1)\}. \tag{5.5}$$

Keeping only absorptive parts and inserting the experimental values for $|\mathrm{Im}\ A_2|$, $|A_2|^2$ discussed in Sect. 2 as well as Im B_1 from Table 1, we recover the estimate of Herczeg [4]

$$|P_{L,\mathrm{abs}}| = \frac{\sqrt{2}\beta_\mu|\varepsilon|}{|A_2|^2} \cdot |\mathrm{Im}\ A_2\ \mathrm{Im}\ B_1| \simeq 7 \cdot 10^{-4}. \tag{5.6}$$

Comparison of Eqs. (5.1) and (5.5) shows that Re B_1 and Im B_1 interfere constructively in the dominant contribution to P_L proportional to Im A_2. To find the allowed range for $|P_L|$ we write it in the form

$$\begin{aligned} |P_L| &= \frac{\sqrt{2}\beta_\mu|\varepsilon|C_\mu}{|\mathrm{Im}\ A_2|} g(x) \\ g(x) &= \frac{|b - ax|}{1 + x^2}, \qquad x = \frac{\mathrm{Re}\ A_2}{\mathrm{Im}\ A_2} \\ a &= -I_{\mu,\mathrm{abs}} - I_{\mu,\mathrm{disp}} = 1.6, \qquad b = I_{\mu,\mathrm{abs}} - I_{\mu,\mathrm{disp}} = 4.0 \end{aligned} \tag{5.7}$$

with C_μ defined in Eq. (3.13). The extrema of $g(x)$ in the experimentally permitted range (2.15) corresponding to

$$\begin{aligned} |x| < \ & 0.3 \quad (0.5) \\ & 1\sigma \quad\ 2\sigma \end{aligned} \tag{5.8}$$

are

$$\begin{aligned} g_{\mathrm{max}} &= g(x_{\mathrm{max}}) = 4.2, \qquad x_{\mathrm{max}} = \frac{b - \sqrt{a^2+b^2}}{a} = -0.2 \\ g_{\mathrm{min}} &= g(x = 0.3\ (0.5)) = 3.3\ (2.5). \end{aligned} \tag{5.9}$$

Normalizing the octet constant G_8 appearing in C_μ to $B(K_S \to 2\gamma)$, we obtain the following allowed range for the longitudinal muon polarization

$$1.9\ (1.5) < |P_L| \cdot 10^3 \left(\frac{2 \cdot 10^{-6}}{B(K_S \to \gamma\gamma)}\right)^{1/2} < 2.5\ (2.6). \tag{5.10}$$

The values outside (inside) brackets correspond to 1σ (2σ) limits. The error on the upper bound depends practically only on the error of Im A_2 in Eq. (2.13) since $x_{\mathrm{max}} = -0.2$ is well inside the 1σ interval for Re A_2/Im A_2. The error on the lower bound, on the other hand, is completely dominated by the error of $|\mathrm{Re}\ A_2|$ given in (2.15) or (5.8).

The constructive interference of Re B_1 and Im B_1 has led to a significant increase of $|P_L|$ compared to the estimate (5.6). Nevertheless, the allowed range (5.10) for P_L still shows the magnitude expected for a CP violating observable entirely due to indirect CP violation via K^0-$\bar{K}^0$ mixing. The interval (5.10) is relatively narrow because of the destructive interference of Re B_1 and Im B_1 in the term proportional to the small dispersive amplitude Re A_2.

6 Conclusions

Prior to our calculation, a reliable computation of the longitudinal muon polarization P_L in $K_L \to \mu^+\mu^-$ going beyond an order of magnitude estimate was not possible. To lowest order in CHPT, the transition $K_1^0 \to \ell^+\ell^-$ is completely determined by the two-loop Feynman diagram of Fig. 1. The absence of local counterterms with the appropriate transformation property of a $\Delta S = 1$ non-leptonic weak transition implies both the finiteness of the Feynman amplitude of Fig. 1 and the absence of additional contributions to this order such as direct weak or meson resonance contributions. This is in contrast to $K_2^0 \to \ell^+\ell^-$ where a direct weak amplitude (sensitive to the top quark [26]) appears already to lowest order in CHPT.

The present calculation yields the essentially parameter free predictions

$$\begin{aligned} \frac{\Gamma(K_S \to \mu^+\mu^-)}{\Gamma(K_S \to \gamma\gamma)} &= 2 \cdot 10^{-6} \\ \frac{\Gamma(K_S \to e^+e^-)}{\Gamma(K_S \to \gamma\gamma)} &= 8 \cdot 10^{-9} \end{aligned} \tag{6.1}$$

for the rare decays $K_S \to \ell^+\ell^-$ within the standard model. As usual, such small rates can be viewed in two different ways. The pessimistic attitude stresses the experimental difficulties to detect such rare decays which for optimists may serve as sensitive measures of physics beyond the standard model (e.g., light scalars). The theoretical quality of the predictions (6.1) matches the successful CHPT predictions for $K_S \to \gamma\gamma$ [11] and $K_L \to \pi^0\gamma\gamma$ [12].

The bounds (5.10) for the longitudinal muon polarization can be summarized by the approximate equality

$$|P_L| \sim 2 \cdot 10^{-3}. \tag{6.2}$$

Although bigger by about a factor 3 than the estimate of Herczeg [4] based on absorptive parts only, the standard model prediction (6.2) for P_L is typical for a CP violating quantity sensitive to indirect CP violation only.

Taking the inherent uncertainties of CHPT and the imprecisely known octet coupling G_8 into account, we arrive at the final conclusion that experimental indications for $|P_L| > 5 \cdot 10^{-3}$ would constitute clear evidence for physics and more precisely for additional mechanisms of CP violation beyond the standard model.

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References

[1] A. Pais and S.B. Treiman, Phys. Rev. *176* (1968) 1974;
L.M. Sehgal, Phys. Rev. *181* (1969) 2151.

[2] For reviews of CP violation phenomenology see
J.F. Donoghue, B.R. Holstein and G. Valencia, Int. J. Mod. Phys. *A2* (1987) 319;
W. Grimus, Forts. d. Physik *36* (1988) 201.

[3] D. Decamp et al. (ALEPH-Coll.), Searches for the standard Higgs boson produced in the reaction $e^+e^- \to H^0Z^*$, preprint CERN-PPE/91-19 and references therein.

[4] P. Herczeg, Phys. Rev. *D27* (1983) 1512.

[5] D. Chang and R.N. Mohapatra, Phys. Rev. *D30* (1984) 2005;
T. Kurimoto, Phys. Lett. *B172* (1986) 357;
J. Liu and L. Wolfenstein, Nucl. Phys. *B289* (1987) 1;
J. Liu, Phys. Rev. *D36* (1987) 2178;
J. Liu, Z. Phys. *C37* (1987) 139;
J.-M. Frère and J. Liu, Nucl. Phys. *B324* (1989) 333;
C.Q. Geng and J.N. Ng, Phys. Rev. *D39* (1989) 3330; ibid. *D42* (1990) 1509.

[6] T. Inagaki, Preliminary design for the muon longitudinal polarization in $K_L \to \mu^+\mu^-$, KEK-preprint 89-11, in Proc. of the Workshop on CP Violation at KAON Factory, Ed. J.N. Ng, Dec. 1988, TRIUMF, Vancouver.

[7] B.R. Martin, E. de Rafael and J. Smith, Phys. Rev. *D2* (1970) 179;
ibid. *D4* (1971) 272 (E).

[8] J. Smith and Z.E.S. Uy, Phys. Rev. *D7* (1973) 2738.

[9] E.B. Bogomolny, M.A. Shifman and M.Z. Shmatikov, Phys. Rev. *D8* (1973) 3237.

[10] Review of Particle Properties, Phys. Lett. *B239* (1990).

[11] G. D'Ambrosio and D. Espriu, Phys. Lett. B175 (1986) 237;
J.L. Goity, Z. Phys. *C34* (1987) 341.

[12] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. *B189* (1987) 363;
L. Cappiello and G. D'Ambrosio, Nuovo Cim. *99A* (1988) 153.

[13] a) H. Burkhardt et al. (CERN-NA31), Phys. Lett. *B199* (1987) 139;
b) G.D. Barr et al. (CERN-NA31), Phys. Lett. *B242* (1990) 523;
V. Papadimitriou et al. (FNAL-E731), A measurement of the branching ratio of the decay $K_L \to \pi^0\gamma\gamma$, preprint EFI 91-01.

[14] F.J. Botella and C.S. Lim, Phys. Rev. Lett. *56* (1986) 1651.

[15] C.Q. Geng and J.N. Ng, Phys. Rev. *D39* (1989) 3330.

[16] M.K. Gaillard and H. Stern, Ann. Phys. (N.Y.) *76* (1973) 580 and references therein.

[17] C. Bernard et al., Phys. Rev. *D32* (1985) 2343.

[18] J.F. Donoghue, B.R. Holstein and Y.C.R. Lin, Phys. Rev. Lett. *55* (1985) 2766; Nucl. Phys. *B277* (1986) 651.

[19] M.K. Gaillard and B.W. Lee, Phys. Rev. *D10* (1974) 897;
R.E. Shrock and M.B. Voloshin, Phys. Lett. *87B* (1979) 375.

[20] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. *B291* (1987) 692.

[21] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. *B303* (1988) 665.

[22] L.M. Sehgal, Phys. Rev. *D7* (1973) 3303.

[23] F.E. Low, Phys. Rev. *110* (1958) 974.

[24] E. de Rafael, Nucl. Phys. B (Proc. Suppl.) *7A* (1989) 1.

[25] A.N. Ivanov, M. Nagy, M.K. Volkov and N.I. Troitskaya, Czech. J. Phys. *40* (1990) 990.

[26] L. Bergström, E. Massó and P. Singer, Phys. Lett. *B249* (1990) 141 and references therein.

Figure Captions

Fig. 1: Two-loop Feynman diagram for $K_1^0 \to \ell^+\ell^-$ involving the one-loop amplitude $K_1^0 \to \gamma^*\gamma^*$ with charged mesons in the loop. The actual computation shows that only pions in the loop contribute to the amplitude.

Fig. 2: Double-absorptive part for $K_1^0 \to \ell^+\ell^-$ where pions and photons would be simultaneously on-shell.

Fig. 3: Unitarity cuts yielding the absorptive parts Im $B_1(\gamma\gamma)$, Im $B_1(\pi\pi)$ and Im $B_1(\pi\pi\gamma)$.

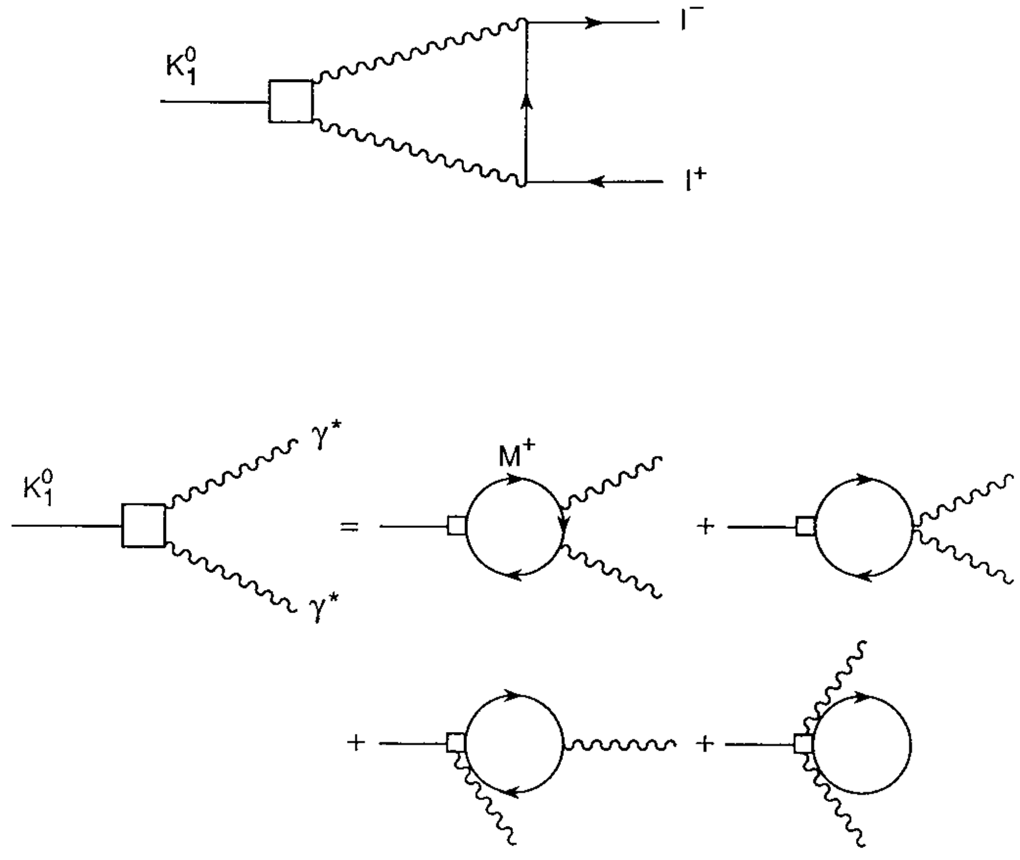


Figure 1

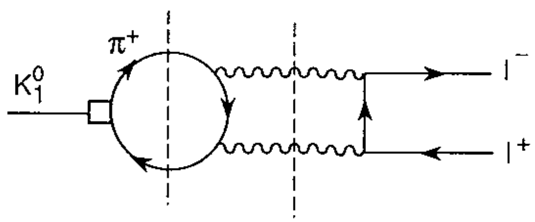
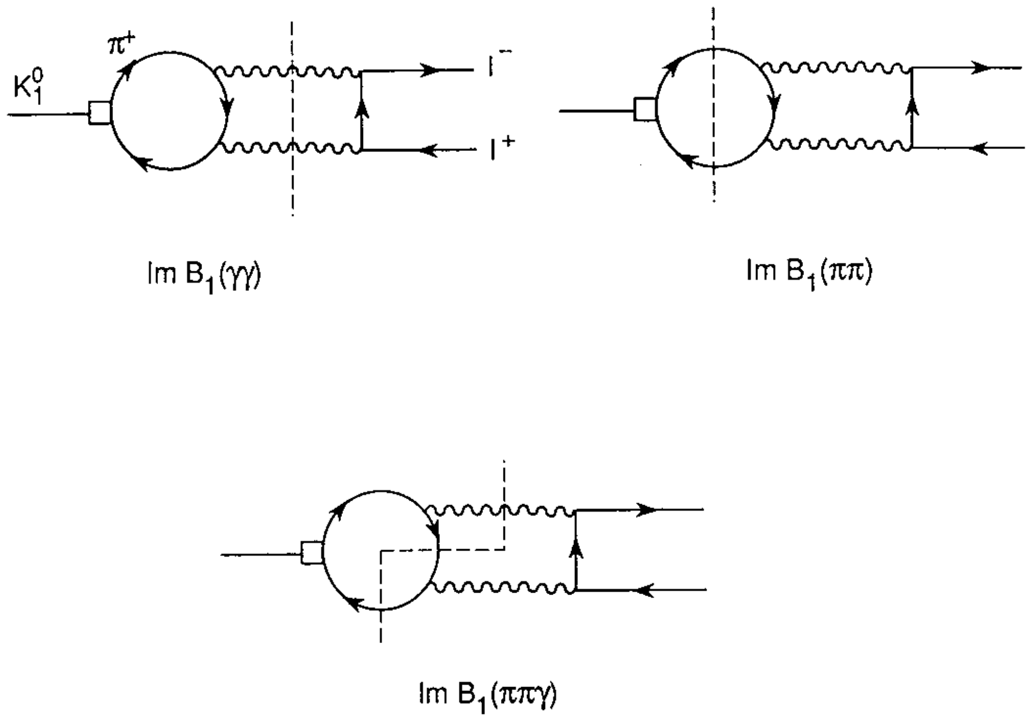


Figure 2



- Figure 3 -

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