

First Lattice Study of the Form Factors A_0 and A_3 in the Decay $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$.

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Abstract

We report on a lattice calculation of the form factors A_0 and A_3 for the pseudoscalar to vector meson semileptonic decay $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$. We find that resonant (or pole-type) contributions alone are unable to describe these two form factors simultaneously. For the quantity $A_0(q^2=0)$, which is important phenomenologically for the determination of $|V_{ub}|$, we extract a range of values, $A_0(q^2=0) = (0.16-0.35)_{-6}^{+9}$, where the range is due to systematic uncertainty and the quoted error is statistical. We have also determined $A_2(q^2=0) = 0.28_{-6-5}^{+9+4}$.

Introduction

The semileptonic decay $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ is determined by the matrix element of the $V-A$ weak current between a \bar{B} meson and a ρ meson. The matrix element is,

$$\langle \rho(k, \eta) | \bar{u} \gamma_\mu (1 - \gamma_5) b | \bar{B}(p) \rangle = \eta^{*\beta} T_{\mu\beta}, \quad (1)$$

with form factor decomposition,

$$\begin{aligned} T_{\mu\beta} = & \frac{2V(q^2)}{m_B + m_\rho} \epsilon_{\mu\gamma\delta\beta} p^\gamma k^\delta - i(m_B + m_\rho) A_1(q^2) g_{\mu\beta} \\ & + i \frac{A_2(q^2)}{m_B + m_\rho} (p+k)_\mu q_\beta - i \frac{A(q^2)}{q^2} 2m_\rho q_\mu (p+k)_\beta, \end{aligned} \quad (2)$$

where $q = p - k$ is the four-momentum transfer and η is the ρ polarisation vector. The form factor A can be written as

$$A(q^2) = A_0(q^2) - A_3(q^2), \quad (3)$$

where,

$$A_3(q^2) = \frac{m_B + m_\rho}{2m_\rho} A_1(q^2) - \frac{m_B - m_\rho}{2m_\rho} A_2(q^2), \quad (4)$$

with $A_0(0) = A_3(0)$. In the limit of zero lepton masses, the term proportional to A in equation (2) does not contribute to the total amplitude and hence to the decay rates. Pole dominance models suggest that V , A_i for $i = 1, 2, 3$ and A_0 correspond to 1^- , 1^+ and 0^- exchanges respectively in the t -channel [1].

Neglecting corrections suppressed by inverse powers of the heavy quark mass M , the following relations hold when q^2 is close to the maximum recoil value $q_{\max}^2 = (M - m_\rho)^2$ [2]

$$A_0 \Theta / M^{1/2} = \text{const}, \quad A_3 \Theta / M^{3/2} = \text{const}, \quad (5)$$

where Θ arises from perturbative corrections and is chosen to be 1 at the B mass. In leading order [3],

$$\Theta = \Theta(M, m_B) = \left(\frac{\alpha_s(M)}{\alpha_s(m_B)} \right)^{\frac{2}{\beta_0}}. \quad (6)$$

In the calculations reported below, we will use $\beta_0 = 11$ in the quenched approximation and $\Lambda_{\text{QCD}} = 200 \text{ MeV}$.

The differential decay rate $d\Gamma(\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l) / dq^2$ at $q^2 = 0$ is given by¹,

$$\frac{d\Gamma(\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l)}{dq^2} \Big|_{q^2=0} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} (m_B^2 - m_\rho^2)^3 |A_0(0)|^2, \quad (7)$$

and is determined by the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix element $|V_{ub}|$ and the form factor $A_0(q^2=0)$, which accounts for the nonperturbative QCD contributions. A lattice measurement of $A_0(0)$ can therefore be combined with experimental measurements of the differential decay rate to extract $|V_{ub}|$.

¹The decay rate expression in equation (7) is naturally expressed in terms of $|A_3(0)|$, but since our lattice data for A_0 are of much higher quality, we prefer to use the relation $A_0(0) = A_3(0)$ and express the result in terms of $|A_0(0)|$

It has previously been suggested [4, 5] that hadronic uncertainties can be reduced by combining a measurement of the differential rate with the recent measurement [6] of the rare radiative decay $\bar{B} \rightarrow K^* \gamma$ to determine $|V_{ub}|$, according to the relation:

$$\frac{R(\bar{B} \rightarrow K^* \gamma)}{d\Gamma(\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l)/dq^2|_{q^2=0}} = \frac{192\pi^3}{G_F^2} \frac{1}{|V_{ub}|^2} \frac{(m_B^2 - m_{K^*}^2)^3}{(m_B^2 - m_\rho^2)^3} \frac{1}{m_b^3} \left| \frac{2T_2^{B \rightarrow K^*}(0)}{A_0^{B \rightarrow \rho}(0)} \right|^2. \quad (8)$$

Here, the hadronisation ratio $R(\bar{B} \rightarrow K^* \gamma) = \Gamma(\bar{B} \rightarrow K^* \gamma)/\Gamma(b \rightarrow s \gamma)$ depends only on the form factors $T_1(0) = iT_2(0)$ for the exclusive radiative decay, and is given up to $\mathcal{O}(1/m_b^2)$ and perturbative QCD corrections by,²

$$R(\bar{B} \rightarrow K^* \gamma) = 4 \left(\frac{m_B}{m_b} \right)^3 \left(1 - \frac{m_{K^*}^2}{m_B^2} \right)^3 |T_2(0)|^2. \quad (9)$$

Using equation (8) to determine $|V_{ub}|$ is advantageous if the ratio $2T_2^{B \rightarrow K^*}(0)/A_0^{B \rightarrow \rho}(0)$ is known in advance. There are some predictions that this ratio is close to 1 [4, 9]. Lattice studies can help here by allowing a model-independent calculation.

QCD sum rule analyses [9] – [15], quark models [16, 17] and a combined theoretical and phenomenological analysis [18] give the following picture for the form factors for $\bar{B} \rightarrow \rho$ transitions: V and A_0 have significant q^2 dependence, while A_1 is rather flat in q^2 and A_2 shows intermediate behaviour.

In this note, we present the first lattice study of the form factors $A_0(q^2)$ and $A_3(q^2)$, and give a range of values for $A_0(q^2 = 0)$ which, given experimental measurements of $d\Gamma(\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l)/dq^2|_{q^2=0}$, may be used to determine $|V_{ub}|$, according to equation (7)³. We first study $A_0(q^2)$ and then argue that a combined fit to A_0 and A_3 can give more accurate results for $A_0(0)$ as lattice data improve. We also report on a value for $A_2(q^2=0)$. Other lattice calculations of A_2 for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ can be found in references [20, 21, 22].

Lattice Details

The results described below come from 60 $SU(3)$ gauge configurations generated by the UKQCD collaboration on a $24^3 \times 48$ lattice at $\beta = 6.2$ in the quenched approximation. The $\mathcal{O}(a)$ improved Sheikholeslami–Wohlert (SW) [23] action was used for fermions, with “rotated” fermion fields appearing in all operators used for correlation function calculations [24].

Three-point correlators of the operator in equation (1) with a heavy pseudoscalar meson (the “ \bar{B} ”) and a light vector meson were calculated, for various combinations of heavy (\mathbf{p}) and light (\mathbf{k}) meson three momenta. We refer to each combination with the notation $|\mathbf{p}| \rightarrow |\mathbf{k}|$ where the momenta are in lattice units of $\pi/12a$ (a subscript \perp on \mathbf{k} indicates that \mathbf{p} and \mathbf{k} are perpendicular). Here, the inverse lattice spacing determined from the ρ mass is $a^{-1} = 2.7(1)$ GeV [25]. We use this value for a^{-1} since we are dealing with decays to a ρ meson, but it should be remembered that different physical observables lead to different values for a^{-1} (UKQCD determinations of a^{-1} are summarised in [26]). This uncertainty in a^{-1} should have little effect on the dimensionless form factors evaluated in this paper.

²The theoretical prediction for this ratio may be subject to long-distance effects [7, 8]

³The CLEO collaboration have measured exclusive semileptonic $B \rightarrow \rho$ and $B \rightarrow \pi$ decays, and have preliminary results for q^2 distributions [19].

channel	q^2/GeV^2	$A_0(q^2)$	$A_3(q^2)$	$A_3^{\text{calc}}(q^2)$
$1 \rightarrow \sqrt{2}_\perp$	14.4_{-3}^{+3}	0.98_{-13}^{+12}	-0.51_{-61}^{+45}	-0.50_{-72}^{+54}
$0 \rightarrow \sqrt{2}$	15.3_{-3}^{+3}	0.94_{-8}^{+8}	-0.52_{-31}^{+23}	-0.64_{-42}^{+31}
$1 \rightarrow 1_\perp$	16.7_{-2}^{+2}	1.26_{-12}^{+17}	-0.34_{-65}^{+59}	-0.25_{-68}^{+67}
$0 \rightarrow 1$	17.5_{-2}^{+2}	1.38_{-10}^{+11}	-0.60_{-44}^{+49}	-0.64_{-53}^{+59}
$1 \rightarrow 0$	19.71_{-1}^{+1}	2.24_{-61}^{+55}	—	—

Table 1 Values of the form factors $A_0(q^2)$ and $A_3(q^2)$ for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$. Errors are statistical only. The final column contains A_3 values obtained from previous results for A_1 and A_2 [30].

For relating lattice results to the continuum, we have used the perturbative value for the renormalisation constant of the axial current operator [27]:

$$Z_A = 0.97. \quad (10)$$

Uncorrelated fits were used for extrapolations in the heavy quark mass, although the extraction of the form factors from calculated three-point correlation functions used correlated fits [28]. Statistical errors are 68% confidence limits obtained from 250 bootstrap samples: unless otherwise specified, errors quoted below are statistical only. More details of the lattice calculations can be found in references [29] and [30].

The SW improved action reduces the leading discretisation errors from $O(a)$ in the Wilson fermion action to $O(\alpha_s a)$. For quark masses m around that of the charm quark, $\alpha_s m a$ can be of order 10%. For this first study we have not tried to disentangle these m -dependent discretisation effects from physical mass dependence. We leave them as a component of the systematic error.

Computing $A_0(0)$

We have extracted values for $A_0(q^2)$ for five momentum channels (we cannot extract A_0 for the zero recoil channel, which has $q^2 = q_{\text{max}}^2$), and for four heavy quark masses around the charm mass. We choose channels where the value of ω defined by,

$$\omega = v \cdot v' = \frac{M^2 + m^2 - q^2}{2Mm}, \quad (11)$$

is nearly constant as the heavy quark mass varies, where M , v and m , v' are the masses and four-velocities of the pseudoscalar and vector meson respectively. For these channels, we have extrapolated linearly and quadratically in $1/M$ to the B scale at constant ω , by appealing to heavy quark symmetry. This gives five values for the desired heavy-to-light form factor $A_0(q^2)$ with q^2 values near q_{max}^2 : they are shown in table 1 (the table also gives values for $A_3(q^2)$ for later use).

These values can then be fitted to various q^2 dependences for A_0 and an extrapolation made to determine $A_0(0)$. We will refer to this as the “constant ω ” method below. As a check, we also extrapolate to $q^2 = 0$ for each of our four heavy quark masses around the charm mass, and then extrapolate $A_0(0)$ to the B scale: this will be referred to as the “ $q^2 = 0$ extrapolation” method. Our methods are described in more detail in references [29] and [30].

n_0	$q^2=0$ extrapolation	constant ω		
	$A_0(0)$	$A_0(0)$	m_0/GeV	χ^2/dof
1	0.35_{-6}^{+5}	0.34_{-6}^{+7}	4.8_{-1}^{+2}	0.32
2	0.24_{-3}^{+3}	0.23_{-6}^{+8}	5.4_{-3}^{+4}	0.39
3	0.16_{-2}^{+2}	0.19_{-6}^{+8}	6.0_{-4}^{+5}	0.42
4	0.09_{-1}^{+1}	0.17_{-6}^{+9}	6.5_{-5}^{+6}	0.44
5	0.04_{-1}^{+1}	0.16_{-6}^{+9}	7.0_{-5}^{+7}	0.45

Table 2 Values of the form factor $A_0(0)$ and the mass parameter m_0 for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ for different assumed q^2 dependences given in equation (12). Errors are statistical only.

We have fitted $A_0(q^2)$ to q^2 dependences of the type,

$$A_0(q^2) = \frac{A_0(0)}{(1 - q^2/m_0^2)^{n_0}}, \quad (12)$$

for a range of “polar powers” $n_0 = 1, \dots, 5$. The higher powers, while not physically motivated, allow us to explore different q^2 dependences away from the pole region. The q^2 dependence of A_0 in equation (12) is consistent with the heavy quark symmetry requirement of equation (5), which applies for q^2 close to q_{max}^2 , provided that,

$$A_0(0) \sim M^{-n_0+1/2}. \quad (13)$$

This dependence on the heavy mass has been used when scaling $A_0(0)$ values from the charm mass to the bottom mass in the $q^2 = 0$ extrapolation method (see figure 1). All extrapolations in the heavy pseudoscalar mass M have been performed to $\mathcal{O}(1/M^2)$. The results for both methods are given in table 2.

Table 2 shows that the $q^2 = 0$ extrapolation method gives results compatible with the constant ω method for powers 1, 2 and 3 in the fitted q^2 dependence, but that the agreement gets worse for higher powers. This discrepancy is not significant however, because the $1/M$ extrapolations are not under control for higher polar powers. Indeed, for the lower powers the linear and quadratic $1/M$ extrapolations agree, but for the higher powers they do not, indicating that higher $1/M$ corrections must be included. This point is illustrated in figure 1. For the constant ω method, in contrast, the linear and quadratic $1/M$ extrapolations all agree within errors for each momentum channel studied. Our results for fits to $A_0(q^2)$ using the extrapolated points at the B scale are shown in figure 2.

In summary, we find that there is some uncertainty in the extrapolation to $q^2 = 0$ (in contrast to our finding for A_1 in our earlier studies [30]). The uncertainty is smaller for the constant ω procedure, but nonetheless, a broad range of 0.16–0.34 has to be given for $A_0(0)$. Higher powers of n_0 do not increase this range when $A_0(0)$ is extracted using the constant ω method. We discard the results with $n_0 \geq 4$ obtained from the $q^2 = 0$ extrapolation method.

Constrained Fits to A_0 and A_3

To gain more control over the extrapolations necessary to determine $A_0(0)$, we combine them with a determination of $A_3(0)$ and apply the constraint $A_0(0) = A_3(0)$. We have previously

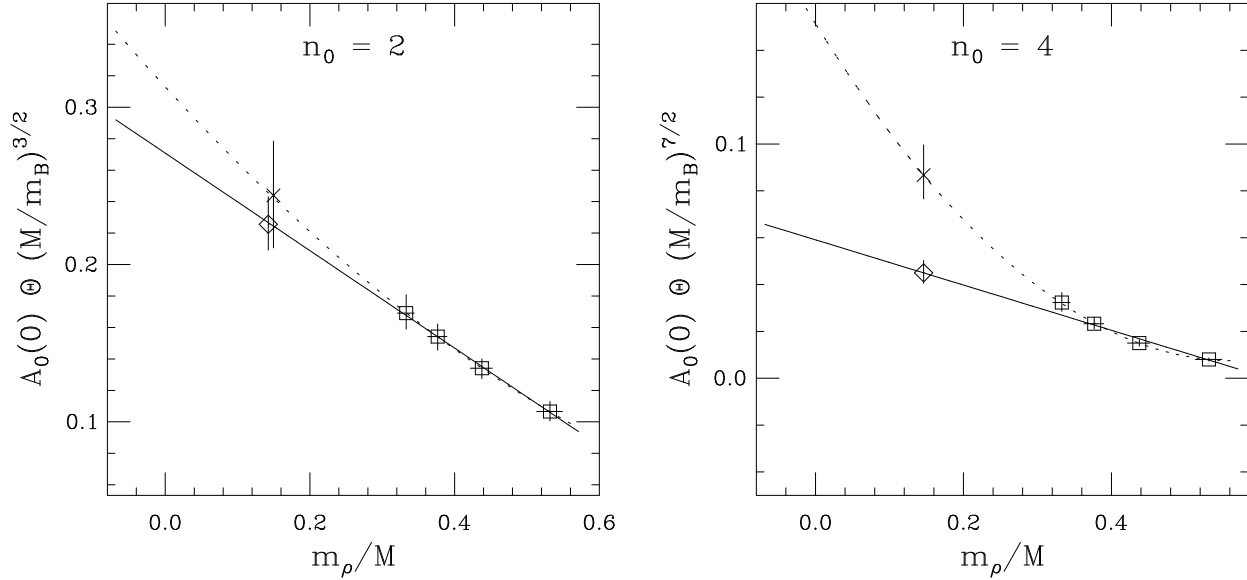


Figure 1 Comparison of linear and quadratic in $1/M$ extrapolations of $A_0(0)$ (“ $q^2 = 0$ extrapolation” method). $A_0(q^2)$ has been fitted at each heavy mass value. The values of $A_0(0)$ from the individual fits (squares) are then extrapolated linearly (solid curve) or quadratically (dashed curve) in $1/M$ to the B scale (diamond or cross). In the left hand plot, the extrapolated points have been displaced slightly for clarity. On the left a dipole form [$n_0 = 2$ in equation (12)] has been used at each heavy mass, and on the right a quadrupole ($n_0 = 4$) form.

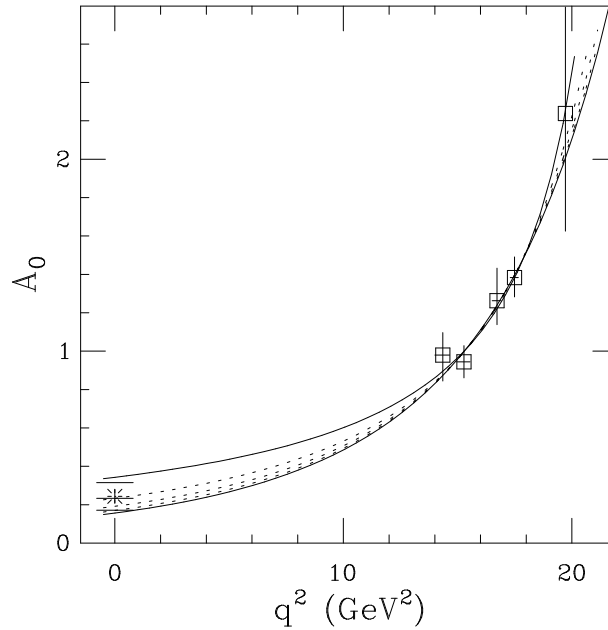


Figure 2 $A_0(q^2)$ for the decay $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ obtained by the constant ω method. Squares denote points extrapolated from measurements around the charm scale in channels where ω is constant (or nearly constant). The curves shown are fits to equation (12) with values $n_0 = 1, 5$ (solid) and $n_0 = 2, 3, 4$ (dotted) for the polar power. The three horizontal bars at $q^2 = 0$ show the value and errors for $A_0(0)$ when $n_0 = 2$, and the burst point shows the value obtained from the $q^2 = 0$ extrapolation method, also with $n_0 = 2$. For both methods, the value of $A_0(0)$ decreases as n_0 increases.

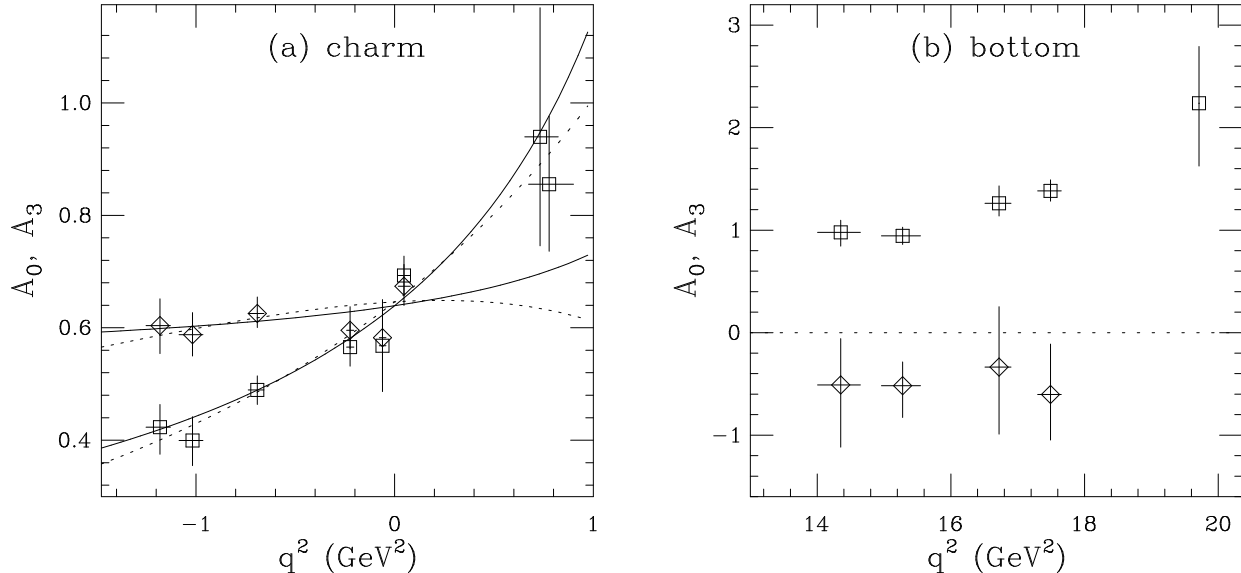


Figure 3 Comparison of q^2 dependences of A_0 and A_3 at (a) the charm (left) and (b) the bottom (right) scales. The squares are values for A_0 and the diamonds are values for A_3 . The charm plot in (a) uses our data with $\kappa = 0.129$ for the heavy quark and with the light quarks extrapolated to $\kappa_{\text{crit}} = 0.14315(1)$ [25]. The curves on the left hand plot are fits using the model of equations (16) and (17), with $n_0 = 1$ (solid) and $n_0 = 5$ (dotted), as explained in the text.

used this idea to perform constrained fits to f^+ and f^0 in $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ as well as T_1 and T_2 for $\bar{B} \rightarrow K^* \gamma$. To fit both A_0 and A_3 with “resonant” (pole or multipole) behaviour in q^2 , whilst maintaining the constraint $A_0(0) = A_3(0)$ and agreement with the heavy quark symmetry requirements of equation (5) at q_{max}^2 , would require a model of the form,

$$A_0(q^2) = \frac{A_{0,3}(0)}{(1 - q^2/m_0^2)^{n_0}}, \quad (14)$$

$$A_3(q^2) = \frac{A_{0,3}(0)}{(1 - q^2/m_3^2)^{n_0+1}}. \quad (15)$$

We find that such a model cannot describe our data.

We were able to extract $A_3(q^2)$ at the B scale for four momentum channels in our lattice simulation. The channel with the largest value of q^2 , $1 \rightarrow 0$ (the zero recoil channel, $0 \rightarrow 0$ cannot be extracted directly for A_3), was too noisy to be extracted reliably: as we shall see, better quality lattice data allowing its extraction would be the easiest way to improve our results. Our results for $A_3(q^2)$ after extrapolation to the B scale in the constant ω method are included in table 1 (fourth column of table).

We find that A_3 is positive for q^2 close to q_{max}^2 in our measured data, with heavy quark masses around the charm scale. However, once extrapolated to the bottom mass, $A_3(q^2)$ is *negative* for q^2 close to q_{max}^2 . A_3 is a linear combination of A_1 and A_2 , with positive and negative coefficients respectively, where the A_1 contribution is most important at the charm scale, but A_1 and A_2 contribute more equally at the bottom scale. Moreover, heavy quark symmetry predicts that A_1 decreases with increasing heavy quark mass, while A_2 increases. In figure 3 we show the q^2 dependence of A_0 and A_3 at the two scales (the meaning of the curves in the charm-scale plot is explained later).

The negative values of A_3 found from our simulation prompt us to make two observations:

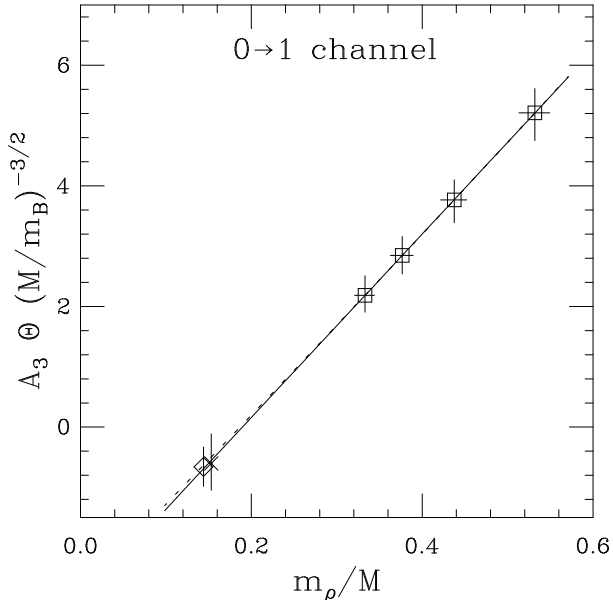


Figure 4 $1/M$ extrapolation for A_3 for the $0 \rightarrow 1$ channel, from the charm scale to the bottom scale. The squares are measured points for our four heavy mass values, which are extrapolated linearly (solid curve) or quadratically (dashed curve) in $1/M$ to the B scale (diamond or cross). The extrapolated points have been displaced slightly for clarity.

- $1/M$ corrections to the heavy quark limit are large for A_3 , as shown in figure 4. A_3 has the behaviour $A_3 \sim MA_2 \sim M^{3/2}$ in the limit $M \rightarrow \infty$, but in extrapolating from scales around the charm mass, we start from a region where the A_1 contribution, with behaviour $MA_1 \sim M^{1/2}$, is non-negligible. A_3 exhibits the largest $1/M$ corrections we have found so far in several studies of processes involving heavy-to-heavy [31, 32, 33] or heavy-to-light [29, 30] quark transitions. This is confirmed by using A_1 and A_2 values extracted previously by us [30] to calculate A_3 from the definition of equation (4): these calculated values are given in the last column of table 1 as A_3^{calc} . We can also reproduce the coefficients found for the $1/M$ expansion of $A_3\Theta(m_B/M)^{3/2}$ by using the definition of equation (4) to express them in terms of the $1/M$ expansion coefficients of $A_1\Theta(m_B/M)^{-1/2}$ and $A_2\Theta(m_B/M)^{1/2}$ found in reference [30].
- Resonant contributions alone, meaning q^2 dependences of the form given in equations (14) and (15), which satisfy the constraint $A_0(0) = A_3(0)$, are incompatible with the measured difference in sign of A_0 and A_3 . The fact that around q_{max}^2 , A_0 is positive and A_3 is negative, is a clear failure of pole dominance models to explain the q^2 behaviour of the form factors in a region away from the physical pole. Sum rule [9] – [15] and quark model [16, 17] calculations, and the analysis of reference [18], also indicate that simple single pole fits cannot be made to all form factors.

The negative values found for A_3 near q_{max}^2 are, in fact, still compatible with heavy quark symmetry constraints as $M \rightarrow \infty$. To make a combined fit for A_0 and A_3 , we use the

following q^2 dependences⁴:

$$A_0(q^2) = \frac{A_{0,3}(0)}{(1 - q^2/m_0^2)^{n_0}}, \quad (16)$$

$$A_3(q^2) = A_{0,3}(0) \frac{(1 - cq^2/m_B m_\rho)}{(1 - q^2/m_3^2)^{n_0}}. \quad (17)$$

These forms satisfy the constraint $A_0(0) = A_3(0)$ and are compatible with heavy quark symmetry provided that $A_{0,3}(0) \sim M^{-n_0+1/2}$ and c is independent of M .

Because we had only four points for $A_3(q^2)$, covering a limited range of q^2 , we were unable to determine simultaneously the parameters $A_{0,3}(0)$, c , m_0 and m_3 in equations (16) and (17). We therefore set $m_3 = m_0$, as predicted by heavy quark symmetry in the infinite mass limit. In fact we believe that the 1^+ B resonance should be heavier than the 0^- resonance and therefore that A_3 should be flatter in q^2 than A_0 . By constraining both masses to be equal we make an error apparently at quadratic order in the expansion of the form factor A_3 . However, the fit parameter c can absorb quadratic differences and our errors in fact begin at $\mathcal{O}(q^4)$.

This parameterisation fits our data well at the charm scale, as shown in figure 3a: the figure shows fitted curves for A_0 and A_3 obtained for $n_0 = 1$ and $n_0 = 5$. Over a limited range of q^2 , changes in n_0 can be compensated by variations in the fitted mass parameter, m_0 .

In figure 5 we show the results of the combined fit at the B scale for several choices of the polar power n_0 in the model of equations (16) and (17). The values obtained for $A_{0,3}(0)$ are listed in table 3 as one of the columns in the set labelled “constant ω ”. The table also gives $A_{0,3}(0)$ obtained from constrained fits for each of our heavy quark mass values, followed by an extrapolation of $A_{0,3}(0)$ to the B scale ($q^2 = 0$ extrapolation method), including terms up to quadratic in the $1/M$ extrapolation. From the results of table 3 we can estimate the error due to the approximation $m_3 = m_0$. The largest error corresponds to the momentum channel $0 \rightarrow 1$, which has the largest value, $q^2 = 17.5(2)$ GeV, used in the analysis for A_3 . For this channel, taking $m_3 - m_0 \approx \Lambda_{\text{QCD}} \approx 0.5$ GeV (in the D meson system the analogous mass difference is $m_{D_1(1^+)} - m_{D^0(0^-)} = 2.423$ GeV $-$ 1.865 GeV $=$ 0.558 GeV), the error is 7% for $n_0 = 1, \dots, 5$.

In figure 5, the A_3 curves drop steeply as q^2 approaches the pole at $m_3^2 = m_0^2$. This looks like an artefact of using the same mass parameter for both A_0 and A_3 , when we expect m_3 to be heavier. Higher powers of n_0 , while artificial, effectively increase the pole mass and make the q^2 behaviour of A_3 flatter away from the pole region.

We find a range $(0.16-0.35)_{-6}^{+9}$ for $A_0(0)$. We do not include a 10% systematic error due to the chiral extrapolation and discretisation effects, as discussed in reference [30], because this error is much smaller than both the statistical and systematic (owing to the different q^2 -dependences assumed for A_0 and A_3) errors noted above. Our range for $A_0(0)$, $(0.16-0.35)_{-6}^{+9}$, is compatible with the value obtained in reference [9]: $A_0(0) = 0.24 \pm 0.02$.

The constrained fits give the same result as fitting A_0 alone above, but having data for A_3 close to q_{max}^2 can help choose between different pole behaviours. Improved lattice simulations in the near future with smaller statistical and systematic errors will provide much better quality data for A_3 close to q_{max}^2 . This will allow us to remove the restriction

⁴These are motivated by Stech’s form factor model [16] and by similar forms used recently by L Lellouch [34] for the form factors f^+ and f^0 in semileptonic $B \rightarrow \pi$ decay.

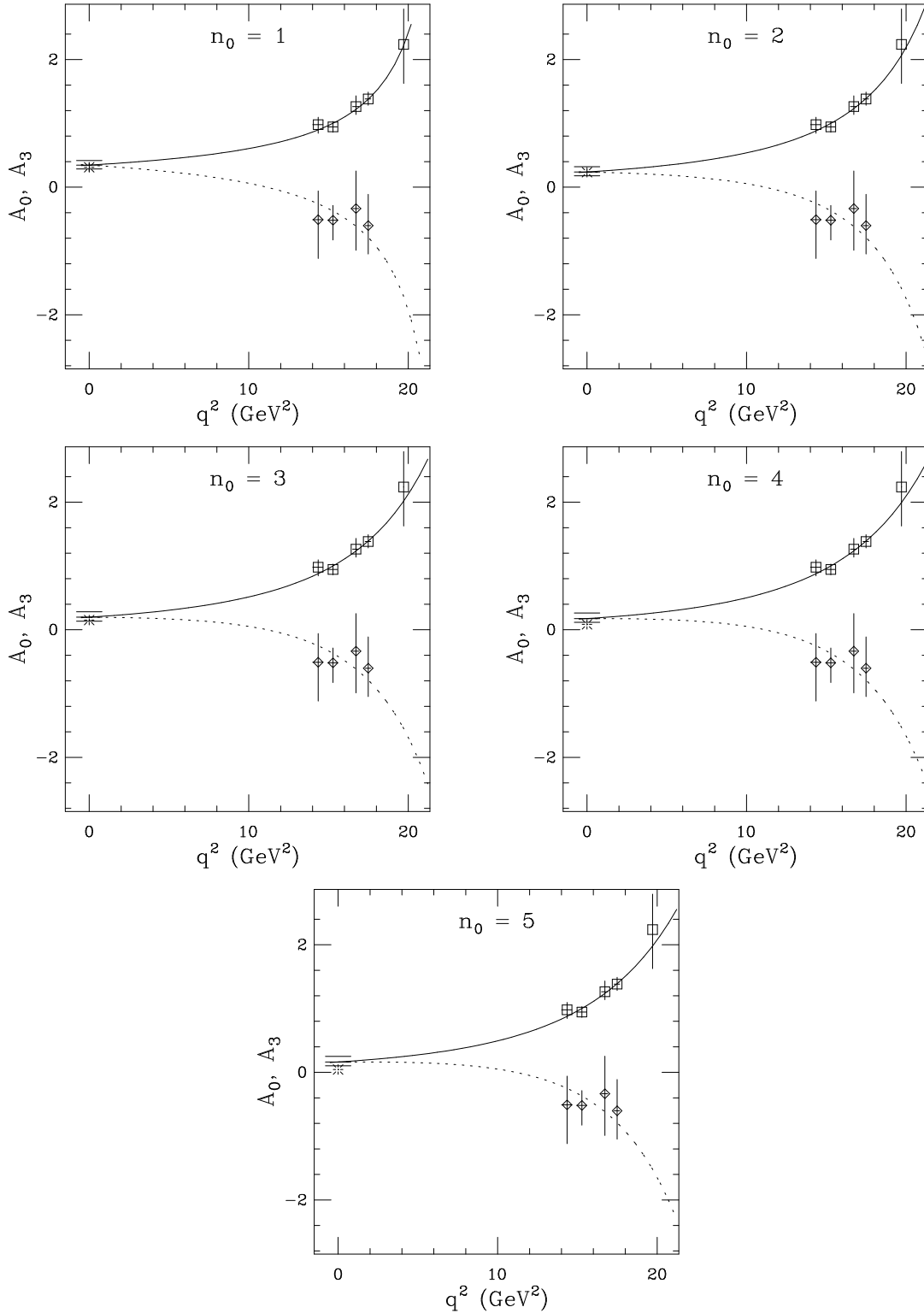


Figure 5 Combined fit to A_0 and A_3 at the B scale using the model of equations (16) and (17) for the solid and dotted curves respectively, with different values of the polar power n_0 as labelled on each plot. Squares are values for $A_0(q^2)$ and diamonds are values for $A_3(q^2)$, the same in all five plots. The three horizontal bars at $q^2 = 0$ show the value and errors for $A_{0,3}(0) = A_0(0) = A_3(0)$ in the combined fit (constant ω method), and the burst point shows $A_{0,3}(0)$ determined from the $q^2 = 0$ extrapolation method.

n_0	$q^2=0$ extrapolation	constant ω			
	$A_{0,3}(0)$	$A_{0,3}(0)$	c	m_0/GeV	χ^2/dof
1	0.31_{-7}^{+7}	0.35_{-6}^{+7}	0.37_{-5}^{+6}	4.8_{-1}^{+2}	0.31
2	0.24_{-4}^{+4}	0.24_{-6}^{+8}	0.36_{-5}^{+6}	5.5_{-3}^{+4}	0.34
3	0.15_{-2}^{+2}	0.20_{-6}^{+8}	0.36_{-5}^{+6}	6.0_{-3}^{+6}	0.36
4	0.09_{-1}^{+1}	0.18_{-6}^{+9}	0.36_{-5}^{+6}	6.6_{-4}^{+7}	0.37
5	0.04_{-1}^{+1}	0.16_{-6}^{+9}	0.36_{-5}^{+6}	7.1_{-5}^{+8}	0.38

Table 3 Values of the form factor $A_0(0) = A_3(0)$, together with the fitted parameters c and m_0 , for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ from a combined fit of A_0 and A_3 using the model given in equations (16) and (17). Errors are statistical only.

$A_2(0)$	Reference
0.28_{-6-5}^{+9+4}	this work
0.24 ± 0.56	APE “a” [20]
0.27 ± 0.80	APE “b” [20]
$0.38 \pm 0.18 \pm 0.04$	ELC “a” [21]
$0.49 \pm 0.21 \pm 0.05$	ELC “b” [21]
0.72_{-35-7}^{+35+10}	[22]

Table 4 Lattice determinations of $A_2(0)$. The letters “a” and “b” denote results from two alternative methods of $1/M$ extrapolation (see [20] and [21] for details).

$m_3 = m_0$ and will help to discriminate between the results for the different q^2 dependences reported in table 3 and figure 5. For example, if $A_3(q^2)$ does not decrease as $q^2 \rightarrow q_{\text{max}}^2$, then lower values of $A_0(0)$ will be preferred.

In a previous paper [30], we showed that $A_1(0)$ could be extracted using an assumed pole form for its q^2 dependence. Here we have extracted $A_0(0) = A_3(0)$, which, by the definition of A_3 in equation (4), makes possible a determination of $A_2(0)$. This can help constrain q^2 fits to A_2 . Using a value $A_1(0) = 0.27_{-4-3}^{+7+3}$ taken from our earlier results in reference [30], we find⁵ $A_2(0) = 0.28_{-6-5}^{+9+4}$, where in both cases the first errors are statistical and the second ones are systematic, due to discretisation effects and uncertainties in the chiral extrapolation together with the variation from changing n_0 . Since $A_2(q_{\text{max}}^2) > A_1(q_{\text{max}}^2)$, we find that A_2 has a more pronounced q^2 dependence than A_1 : this is consistent with sum rule [11] and quark model [16, 17] calculations, and with the analysis presented in [18]. Our result for $A_2(0)$ is the most precise obtained from the lattice to date: it is compared with previous lattice determinations in table 4.

Unfortunately, the quality of the numerical results for $A_0(0)$, obtained here, and for T_2 in our previous work [29], does not allow us to calculate the ratio $2T_2^{B \rightarrow K^*}(0)/A_0^{B \rightarrow \rho}(0)$ of hadronic form factors in equation (8). In reference [29], we found two sets of values for $T_2(0)$, 0.15_{-6}^{+7} and 0.26_{-1}^{+2} depending on the q^2 behaviour (pole or constant, respectively) assumed for the form factor $T_2(q^2)$. Some authors [4, 9], have suggested that the ratio I is close to one. This, together with the results for A_0 presented here, would support low values for $T_2(0)$

⁵We quote $A_2(0)$ obtained from $A_3(0)$ with $n_0 = 2$. The central value of $A_2(0)$ ranges from 0.24 for $n_0 = 1$ to 0.30 for $n_0 = 5$. This range of values is smaller than the size of the statistical errors quoted above.

and therefore a pole-type behaviour for its q^2 dependence. In reference [30], by studying the ratio $A_1/2iT_2$ (predicted to be 1 in the heavy-quark limit) and the form factor A_1 , we also found arguments supporting a pole-type behaviour for T_2 .

This study, combined with our previous studies of the form-factors A_1 , A_2 and V in $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ decays [30], $T1$ and $T2$ in $\bar{B} \rightarrow K^* \gamma$ decays and f^+ and f^0 in $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ decays [29], will enable us in the immediate future to perform a global analysis of the heavy-to-light decays $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$, $\bar{B} \rightarrow K^* \gamma$ and $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$. Such a study will serve to check the validity of different theoretical models, for example the one recently suggested by Stech [16]. In conjunction with theoretical input, such as the constraints on f^+ and f^0 in $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ derived from a dispersion relation analysis in [34], it could also help lattice calculations reduce the systematic uncertainties related to the q^2 dependence of the form factors.

Conclusions

We have determined a range of values for $A_0(0)$ from the lattice for the first time. The range we find is: $(0.16-0.35)_{-6}^{+9}$. As data on both A_0 and A_3 improves, it could become an important ingredient in the determination of $|V_{ub}|$.

From fits to A_0 alone, it is hard to dismiss high values for $A_0(0)$ arising from an assumed single pole behaviour of the form factor. Information from A_3 , although not of high quality, can help determine $A_0(0)$ by disfavouring such high values. With our present data we cannot rule out the higher values, but with better quality data, in particular $A_3(q^2)$ values with q^2 closer to q_{\max}^2 , a constrained fit to A_0 and A_3 could give more reliable answers (as was found for the form factors f^+ and f^0 for $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ in [29]). This would allow us to reduce the range of values quoted for $A_0(0)$.

A_3 itself, for q^2 near q_{\max}^2 , has a large dependence on the heavy quark mass, changing from positive to negative as one scales from the charm mass to the bottom mass. Because $A_3(q^2)$ at the B scale is negative when $A_0(q^2)$ is positive at the same q^2 , close to q_{\max}^2 , it cannot be fitted solely by resonant terms. Hence, our lattice measurements bear out the predictions of quark models.

Combining the results for $A_0(0)$ with earlier results for A_1 allows us to determine $A_2(0) = 0.28_{-6-5}^{+9+4}$. Combining this with improved lattice measurements of $A_2(q^2)$ for q^2 near q_{\max}^2 could allow the determination of the full q^2 dependence of A_2 .

Future lattice simulations, with improved statistical and systematic errors, would allow us to take $m_3 \neq m_0$ in the fits to equations (16) and (17). We believe that in these circumstances the most physically motivated polar power, $n_0 = 1$, would be preferred. We expect that the results obtained would coincide with those presented here, where we have set $m_3 = m_0$ (as predicted by heavy quark symmetry) and have limited ourselves to a q^2 -region away from the pole where higher polar powers effectively increase the pole mass and make the q^2 behaviour of A_3 flatter.

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