

Non-perturbative renormalisation of four fermion operators and $B^0 - \bar{B}^0$ mixing with Wilson fermions ^{*} †

SPQcdR Collaboration

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We present new results for the renormalisation and subtraction constants for the four fermion $\Delta F = 2$ operators, computed non-perturbatively in the RI-MOM scheme (in the Landau gauge). From our preliminary analysis of the lattice data at $\beta = 6.45$, for the $B^0 - \bar{B}^0$ mixing bag-parameter we obtain $B_B^{RGI} = 1.46(7)(1)$.

Matrix elements of the four fermion (4f) operators play an important rôle in the studies of CP-violation in the Standard Model and beyond. Of particular interest are the matrix elements describing the $B^0 - \bar{B}^0$ and $K^0 - \bar{K}^0$ mixing amplitudes. Lattice QCD is at present the best suited method to compute such quantities non-perturbatively (NP). To keep the theoretical uncertainties under control, it is of vital interest to renormalise the corresponding 4f-operators non-perturbatively. An additional difficulty arises when working with Wilson fermions on the lattice, namely all the parity even ($\Delta F = 2$) operators mix among themselves. Therefore, besides the overall renormalisation, the spurious mixing should be subtracted.

1. Non-perturbative renormalisation of four fermion operators

To compute the values of the subtraction and renormalisation constants we use the method described in ref. [1]. Following those papers, we compute the Green functions (GF) of all $\Delta F = 2$ operators, sandwiched by the quark

fields, in a specific gauge (in practice we opt for the Landau gauge). After projecting the amputated GF's onto all independent Dirac structures we get $\Gamma_{ij}(p^2)$, on which the following (RI-MOM) renormalisation condition is imposed

$$\Gamma_{ij}(p^2)|_{\mu^2=-p^2} = \Gamma_{ij}^{(0)}(p^2), \quad (1)$$

where $\Gamma_{ij}^{(0)}$ denote the tree level values of the GF's. We employed this prescription in ref. [4] to compute the renormalisation and subtraction constants for all four fermion operators. Here, we focus on the parity even ones, *i.e.*,

$$\begin{aligned} O_1 &= \bar{b}\gamma^\mu q \bar{b}\gamma_\mu q + \bar{b}\gamma^\mu \gamma_5 q \bar{b}\gamma_\mu \gamma_5 q \\ O_2 &= \bar{b}\gamma^\mu q \bar{b}\gamma_\mu q - \bar{b}\gamma^\mu \gamma_5 q \bar{b}\gamma_\mu \gamma_5 q \\ O_3 &= \bar{b}q \bar{b}q - \bar{b}\gamma_5 q \bar{b}\gamma_5 q \\ O_4 &= \bar{b}q \bar{b}q + \bar{b}\gamma_5 q \bar{b}\gamma_5 q \\ O_5 &= \frac{1}{2} \bar{b}\sigma^{\mu\nu} q \bar{b}\sigma_{\mu\nu} q. \end{aligned} \quad (2)$$

As mentioned above, the lattice regularisation with Wilson fermions breaks chirality, thus inducing the spurious mixing among all of the above operators. Therefore to renormalise the operator O_1 , we should subtract the effects of this mixing, namely

$$\hat{O}_1(\mu) = Z(\mu a) \left[O_1(a) + \sum_{i=2}^5 \Delta_i(a) O_i(a) \right]. \quad (3)$$

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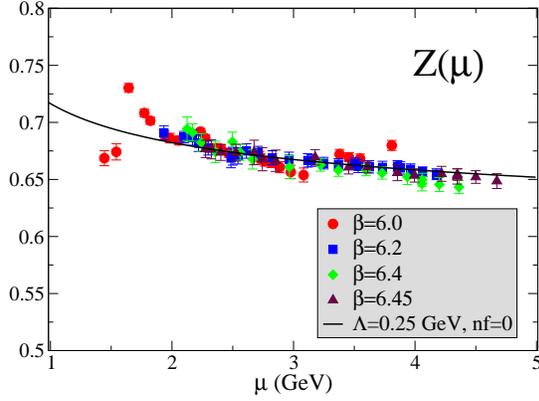


Figure 1. $Z(\mu)$ as computed at four values of β and rescaled to $\beta = 6.45$ as indicated in eq. (4). The full line correspond to the perturbative running of the operator $O_1(\mu)$ at NLO in the RI-MOM scheme (see eq. (5) in which we use $\Lambda_{n_f=0}^{\overline{\text{MS}}} = 250$ MeV).

The condition (1) allows us to compute all subtraction $\Delta_{2-5}(a)$ and the renormalisation constant $Z(\mu a)$. We perform such a calculation for 4 different values of the lattice spacing, corresponding to $\beta = 6.0, 6.2, 6.4, 6.45$. At each β , we work with 4 values of the light quark mass, which allows us to extrapolate to the chiral limit (in which the RI-MOM scheme is defined). In this extrapolation a special attention is given to the subtractions of the pseudo-Goldstone boson (PGB) pole. These artefacts are the consequence of the fact that the operators in our Green functions are inserted at zero momentum. To subtract the effects of the PGB pole, we extend the method of ref. [2], in a way described in ref. [3].

In fig. 1 we plot $Z(\mu a)$, obtained for all of our values of β and then rescaled to $\beta = 6.45$ according to,

$$Z(\mu a') = R(a', a) Z(\mu a), \quad (4)$$

where $R(a', a)$ is μ -independent if the discretisation effects of $\mathcal{O}(\mu a)$ are negligible. In fig. 1 we also show the renormalisation group running at NLO in perturbation theory, *i.e.*,

$$Z(\mu) = Z(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{\gamma_0}{2\beta_0}} \times \left(1 + \frac{\alpha_s(\mu_0) - \alpha_s(\mu)}{4\pi} J_{RI} \right), \quad (5)$$

where $\beta_0^{(n_f=0)} = 11$, $\gamma_0 = 4$ and $J_{RI}^{(n_f=0)} = 8 \log 2 - 1933/726$ [5]. We have chosen $\mu_0 \approx$

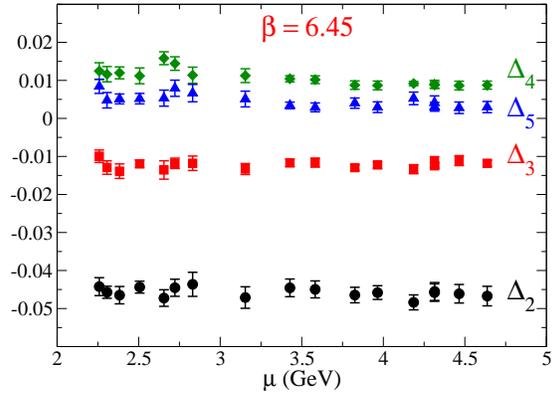


Figure 2. Subtraction constants Δ_{2-5} computed at $\beta = 6.45$ for several values of the renormalisation scale μ . We observe that, to a good accuracy, $\partial\Delta_{2-5}/\partial\mu \simeq 0$, as it should.

3 GeV. We see that eq. (5) describes our data quite well. Moreover, the $\mathcal{O}(\mu a)$ effects appear to be small, a feature already observed in the case of the renormalisation constants of the bilinear quark operators, computed with this same method [6]. In fig. 2 we show Δ_{2-5} , computed at $\beta = 6.45$. We note that they are very weakly dependent on the renormalisation scale μ , as one expects in presence of small lattice artefacts. We also mention that their values decrease with the lattice spacing a (*i.e.* with larger value of β). After fitting each one of them to a constant, we obtain

$$\Delta_{2-5} = \{-4.6(2), -1.2(1), 1.0(2), 0.4(1)\} \times 10^{-2}.$$

For a more detailed discussion, including complete list of numerical results, see ref. [4].

2. $B^0 - \bar{B}^0$ mixing parameter

We now use the results of the previous section to compute $B_1^{RI}(\mu)$ which parametrises the matrix element of the operator $\hat{O}_1(\mu)$ as

$$\langle \bar{B}_q | O_1(\mu) | B_q \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_1(\mu). \quad (6)$$

Its value is extracted from the plateau of the ratio

$$\frac{\langle \sum_{\vec{x}, \vec{y}} P^\dagger(\vec{x}, -t_x) \hat{O}_1(\vec{0}, 0; \mu) P^\dagger(\vec{y}, t_y) \rangle}{\frac{8}{3} \langle \sum_{\vec{x}} A_0(x) P^\dagger(0) \rangle \langle \sum_{\vec{y}} A_0(y) P^\dagger(0) \rangle} \Big|_{t_x, t_y \gg 0} B_1(\mu),$$

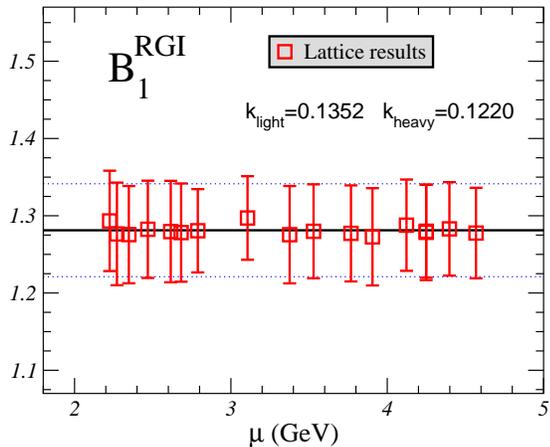


Figure 3. Verification of the scale independence of the parameter B_1^{RGI} defined in eq. (7).

where A_μ and P are the heavy-light axial current and the pseudoscalar density, respectively. Our sample contains 100 independent gauge field configuration generated on a $32^3 \times 70$ lattice in the quenched approximation. We work with 4 light and 6 heavy quark masses. The directly accessed heavy-light pseudoscalar mesons (with light quarks around the strange quark mass) are in the range $m_P \in [1.7, 3.6]$ GeV. In the above ratio we fix $t_x = 21$ and find the plateau in the interval $t_y \in [45, 54]$, for all combinations of our light and heavy quark masses. We compute $B_1(\mu)$ for 17 values of the renormalisation scale μ , which we then convert to the renormalisation group invariant form (RGI) at NLO accuracy, as

$$B_1^{RGI} = \alpha_s(\mu) \frac{\gamma_0}{2\beta_0} \left(1 + \frac{\alpha_s(\mu)}{4\pi} J_{RI} \right) B_1^{RI}(\mu), \quad (7)$$

where β_0 , γ_0 and J_{RI} are already defined after eq. (5). In fig. 3 we show the quality of our B_1^{RGI} .

In order to get the value of the B -parameter relevant for the $B^0 - \bar{B}^0$ mixing amplitude, we need to extrapolate to the physical point corresponding to $1/m_{B_{d,s}}$. For that purpose we construct the quantity $\Phi(m_P)$:

$$\begin{aligned} \Phi(m_P) &= \left(\frac{\alpha_s(m_P)}{\alpha_s(m_B)} \right) \frac{\gamma_0}{2\beta_0} B_1^{RGI} \\ &= \alpha_0 + \frac{\alpha_1}{m_P} + \frac{\alpha_2}{m_P^2} + \dots, \end{aligned} \quad (8)$$

where the argument in $\Phi(m_P)$ indicates that the B -parameter is computed for the pseu-

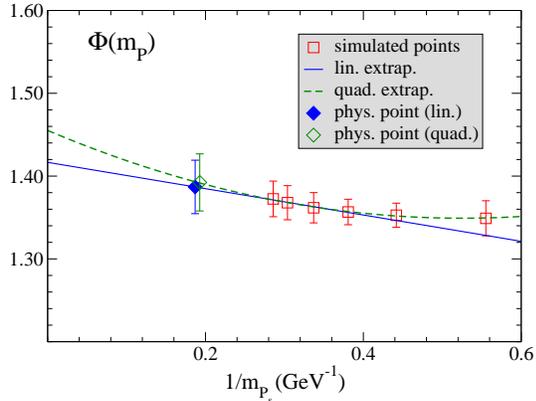


Figure 4. Heavy quark (meson) extrapolation to $1/m_{B_s}$. The linear extrapolation is made with 4 heavier mesons only, whereas for the quadratic one all 6 masses are used.

doscalar meson of mass m_P . The rôle of the evolution factor in (8) is to resum the terms $\propto \log(m_B/m_P)$, thus removing them from the expansion in $1/m_P$. The extrapolated point, $\Phi(m_B)$, is simply B_1^{RGI} , the one that is needed for the $B^0 - \bar{B}^0$ mixing. The $1/m_P$ -extrapolation is illustrated in fig. 4. Our preliminary results read,

$$B_{B_d}^{RGI} = 1.46(7)(1), \quad B_{B_s}^{RGI} = 1.38(3)(1),$$

where the first error is statistical and the second is the difference between the results of the linear and quadratic extrapolations.

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