

Valentina Mantovani-Sarti · Alessandro Drago ·
Vicente Vento · Byung-Yoon Park

The baryon number two system in the Chiral Soliton Model

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Abstract We study the interaction between two $B = 1$ states in a Chiral Soliton Model where baryons are described as non-topological solitons. By using the hedgehog solution for the $B = 1$ states we construct three possible $B = 2$ configurations to analyze the role of the relative orientation of the hedgehog quills in the dynamics. The strong dependence of the intersoliton interaction on these relative orientations reveals that studies of dense hadronic matter using this model should take into account their implications.

Keywords non-topological soliton · nucleon-nucleon interaction

One important issue in the hadron physics is to understand the properties of hadronic matter under extreme conditions, e.g., at high temperature as in relativistic heavy-ion physics and/or at high density as in compact stars. The phase diagram of hadronic matter turns out richer than what has been predicted by perturbative Quantum Chromodynamics (QCD) [1]. Because of the well-known “sign problem” in Lattice QCD [2] one cannot explore the whole phase space by the direct calculations from the fundamental theory with quark and gluon degrees of freedom. Thus, we must rely on effective field theories which are defined in terms of hadronic fields [3].

The Skyrme model [4] is an effective low energy meson theory rooted in large N_c QCD, where a baryon is described by a topological soliton [5]. This model has been shown to be successful in describing not only single baryons but even heavy baryons and multi-baryon systems [6]. Furthermore, this model has been applied to study dense and hot baryonic matter and the consequent modifications of meson properties in such a medium have been obtained [7, 8]. Since this model does not contain explicit quark and gluon degrees of freedom, the main interest of these studies has been the phase transition associated with the chiral symmetry restoration.

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V. Mantovani-Sarti and A. Drago
Dipartimento di Fisica, Università di Ferrara and INFN sezione di Ferrara, 44100 Ferrara, Italy
E-mail: smantovani@fe.infn.it, drago@fe.infn.it

V. Vento
Departamento de Física Teórica and IFIC, Universidad de Valencia - CSIC E-46100 Burjassot (Valencia), Spain
E-mail: vicente.vento@uv.es

B.-Y. Park
Department of Physics, Chungnam National University, Daejeon 305-764, Korea
E-mail: bypark@cnu.ac.kr

Another description which has been applied to the study of hadronic properties is the non topological soliton model introduced by Friedberg and Lee [9]. In this description both quark fields and meson fields are used to obtain the properties of hadrons, although the color degrees of freedom remain hidden. The model has also been applied to the study of hadronic matter and the chiral phase transition by using a single Wigner-Seitz cell [10]. This formalism, very different from the one used in similar studies in the Skyrme model, disregards some of the features associated with the long range interaction, which play an essential role in describing the phase transition in the Skyrme model.

We present here our study of the two body dynamics in the Chiral Soliton Model, focusing on the properties arising from the soliton (non topological) structure of the theory [11]. The understanding of the two body forces, will allow us to describe hadronic matter taking into account the long range effects of the intersoliton dynamics. The new feature in the present approach, in comparison with the Skyrme model, is the description of the quark degrees of freedom.

We use for our analysis the simple chiral quark model studied in ref.[12]. The model Lagrangian density reads

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - g(\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}\gamma_5))\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi} \cdot \partial^\mu\boldsymbol{\pi}) - V(\sigma, \boldsymbol{\pi}), \quad (1)$$

where σ represents a scalar isosinglet meson, $\boldsymbol{\pi}$ are isotriplet pion fields and ψ describes the isodoublet (u and d) quark fields. $V(\sigma, \boldsymbol{\pi})$ is a potential energy that should provide symmetry breaking and we use the model of ref.[12]. This Lagrangian is a generalization of the Friedberg-Lee model which implements the appropriate realization of chiral symmetry. By choosing the vacuum at $(\sigma_0 = f_\pi, \mathbf{0})$, the model Lagrangian describes mesons and quarks with masses m_σ , m_π and $m_q = gf_\pi$, respectively. For these parameters, we take the following values $m_\sigma = 550$ MeV, $m_\pi = 138$ MeV, $g = 5$ and $f_\pi = 93$ MeV.

The self-consistent $B = 1$ solution using the hedgehog Ansatz is given by,

$$\begin{aligned} \sigma_{B=1}(\mathbf{r}) &= \sigma(r) \quad , \quad \boldsymbol{\pi}_{B=1}(\mathbf{r}) = \pi(r)\hat{r} \\ \psi_{B=1}(\mathbf{r}) &= \frac{1}{\sqrt{4\pi}} \begin{pmatrix} u(r) \\ i\boldsymbol{\sigma} \cdot \hat{r}v(r) \end{pmatrix} \frac{1}{2}(|u \downarrow\rangle - |d \uparrow\rangle). \end{aligned} \quad (2)$$

The solution is stabilized, *not* by a topological constraint, *but* by the energy which becomes lower than three free quark masses. The solution for the the sigma field develops a bag-like spatial structure where the quark fields become localized. Making such bag-like structure costs more than 800 MeV in meson field energy, which is compensated by the binding energies of the quarks. In total, the whole system has a binding energy of about 400 MeV. The rms radius of the baryon number distribution is about 0.7 fm.

In order to study the soliton-soliton interaction, we need to construct a $B = 2$ system where two solitons are separated by some distance. Suppose that we have two solitons whose centers are at \mathbf{r}_1 and \mathbf{r}_2 . When two solitons are sufficiently far apart, the ‘‘product Ansatz’’ from the Skyrme model is useful to construct the two-soliton system [7]. In this scheme the σ and $\boldsymbol{\pi}$ configuration for the $B = 2$ system can be approximated by

$$\begin{aligned} &\left(\frac{\sigma_{B=2}(\mathbf{r}) + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}_{B=2}(\mathbf{r})}{f_\pi} \right) \\ &= \left(\frac{\sigma_{B=1}(\mathbf{r} - \mathbf{r}_1) + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}_{B=1}(\mathbf{r} - \mathbf{r}_1)}{f_\pi} \right) C \left(\frac{\sigma_{B=1}(\mathbf{r} - \mathbf{r}_2) + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}_{B=1}(\mathbf{r} - \mathbf{r}_2)}{f_\pi} \right) C^\dagger, \end{aligned} \quad (3)$$

where f_π is used to have the proper dimensions. As was done in the Skyrme case, a constant $SU(2)$ matrix C can be inserted to introduce the relative orientation of the second soliton with respect to the first one in isospin space. We choose $\mathbf{r}_{1,2} = \pm(d/2)\hat{z}$ and consider three different configurations

- Configuration A : $C=1$, i.e., two unrotated hedgehog solitons,
- Configuration B : $C = e^{i\tau_z\pi/2} = i\tau_z$, i.e. , the second soliton is rotated by an angle π about the axis that is parallel to the line joining two centers,
- Configuration C : $C = e^{i\tau_x\pi/2} = i\tau_x$, i.e. , the second soliton is rotated by an angle π about the axis that is perpendicular to the line joining two centers.

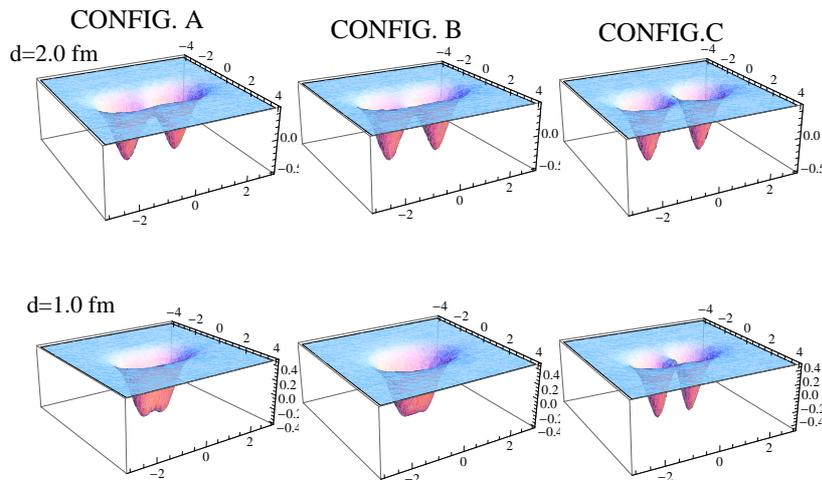


Fig. 1 The structure of the σ field for $B=2$, $\sigma_{B=2}$, for the three configurations studied at large (upper figures) and small (lower figures) separation parameter d .

In the Skyrme model, taking the product of two $B = 1$ soliton solutions is one of the most convenient ways to obtain the $B = 2$ intersoliton dynamics [7]. However, in the “linear” Chiral Soliton Model, with explicit quark degrees of freedom, since we are not restricted by a topological winding number, the *product* scheme may not be so essential, though it provides some advantages. First of all, it makes $\sigma_{B=2}$ and $\pi_{B=2}$ naturally satisfy the boundary conditions at infinity; that is,

$$\sigma(r \rightarrow \infty) \rightarrow f_\pi, \quad \pi(r \rightarrow \infty) \rightarrow 0 \quad (4)$$

without any further artificial construction. Secondly, when the separation distance between two solitons is sufficiently large, the two solitons will have their own identity. We show in Fig.1 the $\sigma_{B=2}$ field obtained by this Ansatz (3). Note that, at large and intermediate separations, the relative distance is a well defined quantity, while at short separations the baryons deform heavily, making complicated overlapping shapes, and the relative distance cannot be well defined. The meson cloud appears as a halo around the baryons and deforms as the baryons approach each other. This deformation is the energy density representation of the pion exchange potential. The strong deformation of the cores is a consequence of the repulsion mediated in reality by multiple meson exchanges.

The zeroth order solution of the quark wavefunctions, since the background field has some kind of reflection symmetry except for an isospin rotation, can be expressed as the linear combination

$$\psi_{B=2}^\pm = \frac{1}{\sqrt{2}} (\psi_L(\mathbf{r}) \pm \psi_R(\mathbf{r})), \text{ where } \psi_L(\mathbf{r}) = \psi_{B=1}(\mathbf{r} - \mathbf{r}_1), \text{ and } \psi_R(\mathbf{r}) = C\psi_{B=1}(\mathbf{r} - \mathbf{r}_2), \quad (5)$$

and $\psi_R(\mathbf{r})$ is rotated by C .

The energy of the $B = 2$ system is then calculated by substituting these approximated solutions for the mesons into the corresponding formulas and evaluating the expectation values of the Hamiltonian with respect to the $B = 2$ state where three quarks occupy each energy levels described by the wavefunctions ψ_\pm . By subtracting twice the soliton mass from the total energy of the $B = 2$ system, we obtain the soliton-soliton interaction energy as a function of the separation distance and the relative orientation.

In Fig.2 we show the interaction as a function of d for the three different configurations. There is no dramatic difference in shape between the three cases. The difference is a matter of detail. Configuration A and B are repulsive for all d and B is more repulsive at the large distance. At large distances the most stable state is in the C configuration. Thus, at large separation the behavior of the soliton interaction

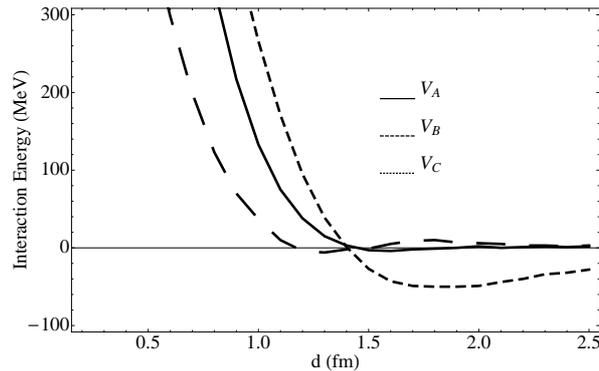


Fig. 2 Soliton-soliton interaction energy as a function of the separation distance for the three relative orientations mentioned in the text.

seems to be similar to that obtained in the Skyrme model. This result is rather predictable since when two solitons are far apart, the interaction between them is mainly through the meson exchange.

As the baryons get closer to each other there is a transition to the B configuration, just at the point where V_C rises. Configuration B seems to be the lowest energy state only where the interaction is repulsive. It seems that the quark fields play a role at short separation. However, our first order calculations, without any modifications on the quark wavefunction, are too primitive to draw any conclusion for the short distance behavior of the potential.

Let us summarize our main findings [11]. We have established the ground for the description of baryonic matter in the Chiral Soliton Model. The main difference between this description and the Skyrme model one is the existence of quark fields in the Lagrangian. Color has been eliminated in favor of a scalar isoscalar field responsible for confinement. In this way the color phase transition and the chiral phase transition are governed by the same mechanism, namely the non topological structure of the σ field soliton.

As a first step we have recovered in our Chiral Soliton Model the B=1 hedgehog solution. The solitonic structure is apparent in the solution leading to a confinement scenario, where the pion field is partially expelled from the quarkish core. Thus the center of the baryons is dominated by relativistic quarks bouncing around, in the intermediate region quarks and pions coexist and in the outer region a pion cloud extends out for quite some distance [12].

We have studied the B=2 interaction, with a product Ansatz approach, and discovered the dominating features of the baryon-baryon interaction. By looking at the energy density as the baryons approach each other, we show that the long range tail of the baryon-baryon interaction is dominated by the pion cloud. We have analyzed the importance of the quills orientation into the dynamics and have found two sensitive orientations, which should describe the two states of matter as determined by the realization of chiral symmetry, a scenario which resembles closely our Skyrme description [7].

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