τ EDM at Low Energies

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Low energy tau pair production, at B factories and on top of the Υ resonances, allows for a detailed investigation on the CP violation at the electromagnetic tau pair production vertex. High statistic available at low energies offers the opportunity for an independent analysis of CP-violation in the τ lepton physics. We show that stringent and independent bounds on the τ electric dipole moment, competitive with the high energy measurements, can be established in low energies experiments.

The electric dipole moment (EDM) has been extensively investigated in particle physics [1,2,3]. Nowadays the most precise bound is the one on the electron EDM, $d^e_\gamma = (0.07\pm0.07)\times10^{-26}\, e\, cm$, while the loosest is on the τ EDM [1], $Re(d^\tau_\gamma) > -3.1$ and $< 3.1 \times 10^{-16}\, e\, cm$. The PDG quoted bound on d^e_γ for the τ comes from a CP-even observable: the total cross section for $e^+e^- \to \tau^+\tau^-\gamma$. This should be superseded with the measurements of CP-odd observables, and this is what we propose in what follows. Low energy experiments provide for independent constraints on the EDM and allow to separate the effects coming from the electric and weak-electric dipole moments. A non zero measurement of an EDM is a time reversal odd signal; CPT theorem for quantum field theories states that this is equivalent to CP violation. These dipole moments are generated in the standard model only at three loops but extended models can induce an EDM not far from present experimental sensitivities[4]. The tau EDM and weak-EDM have been studied in detail at LEP in spin correlation observables [5] and also in spin linear terms [6]. Most of the statistics for the tau pair production was dominated by LEP but nowadays the situation has evolved. High luminosity B factories and their upgrades have a large τ pair sample.

We parametrized deviations from the standard model, at low energies, by an effective Lagrangian built with the standard model particle spectrum, having as zero order term just the standard model Lagrangian, and containing higher dimension gauge invariant operators suppressed by the scale of new physics, Λ [8,7]. The leading non-standard effects that contribute to the EDM and weak-EDM come from dimension six operators:

$$\mathcal{O}_B = \frac{g'}{2\Lambda^2}\overline{L_L}\varphi\sigma_{\mu\nu}\tau_R B^{\mu\nu} \quad (1)$$

$$\mathcal{O}_W = \frac{g}{2\Lambda^2}\overline{L_L}\vec{\tau}\varphi\sigma_{\mu\nu}\tau_R \vec{W}^{\mu\nu}\ . \quad (2)$$

Here $L_L = (\nu_L, \tau_L)$ is the tau leptonic doublet, φ is the Higgs doublet, $B^{\nu\nu}$ and $\vec{W}^{\mu\nu}$ are the $U(1)_Y$ and $SU(2)_L$ field strength tensors, and g' and g are the gauge couplings.

The effective Lagrangian is

$$\mathcal{L}_{eff} = i\alpha_B\mathcal{O}_B + i\alpha_W\mathcal{O}_W + \text{h.c.} \quad (3)$$

where the couplings α_B and α_W real. After spontaneous symmetry breaking, the Higgs gets a vacuum expectation value $<\varphi^0>= u/\sqrt{2}$ with $u = 1/\sqrt{\sqrt{2}G_F} = 246$ GeV, and the interactions (3) can be written in terms of the gauge boson mass eigenstates A^{μ} and Z^{μ}. Thus, the Lagrangian for the EDM, written in terms of the mass eigenstates, is[1]

$$\mathcal{L}_{eff}^{\gamma,Z} = -i\frac{e}{2m_\tau} F_\gamma^\tau \overline{\tau}\sigma_{\mu\nu}\gamma^5\tau F^{\mu\nu} - i\frac{e}{2m_\tau} F_Z^\tau \overline{\tau}\sigma_{\mu\nu}\gamma^5\tau Z^{\mu\nu} \quad (4)$$

where $F_{\mu\nu}$ and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ are the Abelian field strength tensor of the photon and the Z gauge boson, respectively. As usual, we have defined the following dimensionless couplings

$$F_\gamma^\tau = (\alpha_B - \alpha_W)\frac{u m_\tau}{\sqrt{2}\Lambda^2} , \quad (5)$$

$$F_Z^\tau = -(\alpha_B s_W^2 + \alpha_W c_W^2)\frac{u m_\tau}{\sqrt{2}\Lambda^2}\frac{1}{s_W c_W} \quad (6)$$

where $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$ sine and cosine of the weak angle. Notice that, in the effective Lagrangian approach, exactly the same couplings that contribute to processes at high energies also contribute to the electric dipole moment form factors, $F^{new}(q^2)$, at $q^2 = 0$. The difference $F^{new}(q^2) - F^{new}(0)$ only comes from higher dimension operators whose effect is suppressed by powers of q^2/Λ^2, as long as $q^2 \ll \Lambda^2$ as needed for the consistence of the effective Lagrangian approach. For this reason we make no distinction between electric dipole moment and electric form factor.

The electric and weak-electric dipole moment are

$$d_\gamma^\tau = \frac{e}{2m_\tau}F_\gamma^\tau, \quad (7)$$

$$d_Z^\tau = \frac{e}{2m_\tau}F_Z^\tau \quad (8)$$

and are usually expressed in units of ecm.

The $e^+ e^- \longrightarrow \gamma, \Upsilon \longrightarrow \tau^+\tau^-$ cross section has contributions coming from the standard model and the effective Lagrangian Eq.(4). At low energies tree level contributions come from direct γ exchange (off the Υ peak) or Υ (at the Υ peak) exchange while interference $\gamma - Z$ (or $\Upsilon - Z$ at the Υ peak) and $Z - Z$ diagrams are suppressed by q^2/M_Z^2. At tree level then, the relevant diagrams are shown in Fig.1. from the standard (a), (b) and beyond the standard (c), (d) amplitudes.

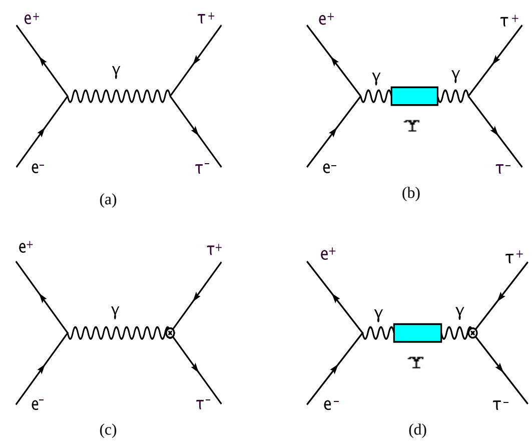

Figure 1. Diagrams (a) direct γ exchange (b) Υ production (c) EDM in γ exchange (d) EDM in Υ production

We are interested in the differential cross section for $e^+e^- \longrightarrow \tau^+(s_+)\tau^-(s_-)$ and we will retain only up to linear terms in the dipole moments. In this way, the only proportional terms to the EDM one gets from diagrams (c) and (d) for the $e^+ e^- \longrightarrow \tau^+\tau^-$ cross section come in the spin-spin correlation. For details, see for example the cross section formulas in [9].

All contributions to polarization terms are determined by the discrete symmetry properties: P, CP, T and helicity flip. In our hypothesis the EDM does not contribute to the single spin dependent terms. In fact it appears in the normal P-even, T-odd $(s_+ - s_-)_N$ (to the scattering plane) spin-linear terms. Taking into accout that the EDM effective Lagrangian is P and T-odd, we find that the EDM contributes to this single spin term but only through the interference with the axial part of a Z exchange. This term is proportional to the electron or the fermion axial coupling to the Z. As such, it is doubly suppress

[1]Similar results, but for the magnetic moments, are found in [7] where the notation is the same.

by q^2/M_Z^2 at low energies and also by the axial coupling $(1/4 - s_W^2)$.

The EDM contributes to the spin-spin correlation terms. The T-odd normal-transverse $(\boldsymbol{s}_+ \times \boldsymbol{s}_-)_{N,T}$ and normal-longitudinal $(\boldsymbol{s}_+ \times \boldsymbol{s}_-)_{N,L}$ correlation terms are proportional to the EDM. This last term also receives standard model contributions through absorptive parts generated in radiative corrections. At tree level it is the imaginary part of the Z propagator that produces a contribution to this correlation with the interference of the amplitudes of direct γ and Z exchange. This term is suppressed at low energies, and has been calculated in [10] and subtracted if necessarily.

In the following we show how to measure the EDM with the appropriate observables. We will follow the notation of references [7,11] and further details and results will be published elsewhere [12].

Let us consider the τ-pair production in e^+e^- collisions though direct γ exchange (diagrams (a) and (b) in Fig. 1.).

We assume from now on that the tau production plane and direction of flight can be fully reconstructed. This can be done if both τ decay semileptonicaly; this has been studied in [13] and applied in different cases by the L3-Collaboration [14] following the ideas of [6,11] for semileptonic decays of the tau.

Polarization along the directions x, y, z correspond to what is called transverse (T), normal (N) and longitudinal (L) polarizations.

The differential cross section for τ pair production can be written:

$$\frac{d\sigma}{d\Omega_{\tau^-}} = \frac{d\sigma^0}{d\Omega_{\tau^-}} + \frac{d\sigma^S}{d\Omega_{\tau^-}} + \frac{d\sigma^{SS}}{d\Omega_{\tau^-}} + \dots \qquad (9)$$

The dots symbolize higher orders in the effective Lagrangian that are beyond experimental sensitivity and not considered in this paper. The first term of Eq. (9) represents the spin independent differential cross section

$$\frac{d\sigma^0}{d\Omega_{\tau^-}} = \frac{\alpha^2}{16\ s}\beta(2 - \beta^2 sin^2\theta) \qquad (10)$$

where α is the fine structure constant, $s = q^2$ is the square of the 4-momenta carried by the photon, θ is the angle defined by the electron and τ directions, and $\gamma = \frac{\sqrt{s}}{2m_\tau}$, $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$, are the dilation factor and τ velocity, respectively. The second term $\frac{d\sigma^S}{d\Omega_{\tau^-}}$, involves spin linear contributions and is suppressed in our hypothesis. The last term in (9) is proportional to the product of the spins of both τ's and can be written as:

$$\begin{aligned}\frac{d\sigma^{SS}}{d\Omega_{\tau^-}} \quad = \quad & \frac{\alpha^2}{16s}\beta \left(s_+^x s_-^x C_{xx} + s_+^y s_-^y C_{yy} + \right. \\ & s_+^z s_-^z C_{zz} + (s_+^x s_-^y + s_+^y s_-^x) C_{xy}^+ + \\ & (s_+^x s_-^z + s_+^z s_-^x) C_{xz}^+ + \\ & (s_+^y s_-^z + s_+^z s_-^y) C_{yz} + \\ & (\boldsymbol{s}_+ \times \boldsymbol{s}_-)_x C_{yz}^- + (\boldsymbol{s}_+ \times \boldsymbol{s}_-)_y C_{xz}^- + \\ & \left. (\boldsymbol{s}_+ \times \boldsymbol{s}_-)_z C_{xy}^- \right) \qquad (11)\end{aligned}$$

where

$$\begin{aligned} C_{xx} &= (2 - \beta^2)\sin^2\theta \\ C_{xz}^+ &= \frac{1}{\gamma} sin2\theta \\ C_{yy} &= -\sin^2\theta \\ C_{xy}^- &= 2\beta \sin^2\theta\, d_\tau^\gamma \\ C_{zz} &= (\beta^2 + (2 - \beta^2)\cos^2\theta) \\ C_{yz}^- &= \gamma\beta sin^2\theta\, d_\tau^\gamma \qquad (12)\end{aligned}$$

The spin properties of the produced taus translates in the angular distribution of both tau decay products; in order to have access to the EDM one has to measure this angular distribution. It is by means of asymmetries that this may be done. In this way we select each one of the terms we are interested in. In what follows we will show how to measure the terms that contain the EDM. We will sum in all kinematic variables as possible in order to enlarge the signal. The EDM is the leading contribution to the normal-transverse $(y - x)$ and normal-longitudinal $(y - z)$ correlations. We now show how to define an observable proportional to the C_{xy}^- term. In each of the correlation terms in the cross section we have several kinematic variables to take into account: the CM angle θ of production of the tau with respect to the electron, the azimuthal ϕ_{h^+}, $\phi_{h'^-}$ and polar θ_{h^+}, $\theta_{h'^-}$ angles of the produced hadrons h^+ and h'^- in the $\tau^\pm$ rest frame. Both hadron momentum are fixed by energy conservation and the neutrino in each channel will be integrated out.

The C^-_{xy} term in the cross section $d\sigma(e^+e^- \to \gamma \to \tau^+\tau^- \to h^+\bar{\nu}h'^-\nu)$ can be written as:

$$\left.\frac{d\sigma^8}{d\Omega_\tau d^3q^*_- d^3q^*_+}\right|_{C^-_{xy}} = \frac{\alpha^2\beta^2}{128\pi^3 s^2} Br_+ \, Br_- \times d^\gamma_\tau \sin^2\theta \times (n^*_{+x}n^*_{-y} - n^*_{+y}n^*_{-x}) \times \delta(q^*_- - P_-)\delta(q^*_+ - P_+) \quad (13)$$

where $Br_+ = Br(\tau^+ \to h^+\nu)$, $Br_- = Br(\tau^- \to h'^-\overline{\nu})$, $\boldsymbol{n}^*_\pm = \pm\alpha_\pm\hat{\boldsymbol{q}}^*_\pm$; $\alpha_\pm$ are the polarization parameters of the τ decay, $\boldsymbol{q}_\pm$ are the momentum of the hadrons, $P_\pm = \frac{m^2_\tau - m_\pm}{2m\tau}$; all $*$ means that the quantities are referred to the τ rest frame of reference. Integrating in some of the angles we end up with

$$\frac{d^2\sigma}{d\phi^*_- d\phi^*_+} = \frac{\alpha^2\beta^2}{192s^2} Br_- Br_+ \alpha_-\alpha_+ \times \sin(\phi^*_- - \phi^*_+)\, d^\gamma_\tau \quad (14)$$

We now integrate in these angles and define the following asymmetry

$$A_{NT} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} \quad (15)$$

where

$$d\sigma_+ = \int_{\sin(\phi^*_- - \phi^*_+)>0} \frac{d^2\sigma}{d\phi^*_- d\phi^*_+}\, d\phi^*_-\, d\phi^*_+ \quad (16)$$

$$d\sigma_- = \int_{\sin(\phi^*_- - \phi^*_+)<0} \frac{d^2\sigma}{d\phi^*_- d\phi^*_+}\, d\phi^*_-\, d\phi^*_+ \quad (17)$$

A straightforward computation gives

$$A_{NT} = \frac{4\beta}{\pi}\frac{\alpha_-\alpha_+}{3-\beta^2} d^\gamma_\tau \quad (18)$$

We have verified that all other terms in the cross section, i.e. the spin independent ones, the ones coming with the linear polarization and all the other spin-spin correlation terms, are eliminated when we integrate in the way we propose. This means that the only contribution to this asymmetry is exactly the term C^-_{xy} we are interested in. In this way we have defined a normal-transverse correlation observable directly proportional to the EDM.

In a similar way, we can define an observable related to the normal-longitudinal correlation term. In this case the angular dependence on the decay product of both τ is different and there are various observables we can define. For example, we integrate in the regions where $\sin\theta^*_- \sin\phi^*_- \cos\theta^*_+$ is positive and negative to obtain

$$A_{NL} = -\frac{4}{\pi^2}\frac{\beta\gamma}{3-\beta^2}\alpha_-\alpha_+\, d^\gamma_\tau \quad (19)$$

The above expressions are linear in the spin α factors; we can sum all over the channels of both τ's in order to enlarge the asymmetry.

All the above ideas can be applied for e^+e^- collisions at the Υ peak where the τ pair production is $e^+e^- \to \Upsilon \to \tau^-\tau^-$. In this way we may have an important tau pair production rate. We are interested in τ pairs produced by the decays of the Υ resonances, therefore we can use $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$, but not $\Upsilon(4S)$ because it decays dominantly into $B\overline{B}$. We assume that the resonant diagrams (b) and (d) of Fig. 1. dominate the process on the Υ peak. The Υ propagates with a Breit-Wigner and the $F_\Upsilon(q^2)$ vector form factor defined as

$$\langle\Upsilon(w,\boldsymbol{q})|\bar{\psi}_b\gamma_\mu\psi_b(0)|0\rangle = F_\Upsilon(q^2)\epsilon^*_\mu(w,\boldsymbol{q}) \quad (20)$$

is related to the partial width of $\Upsilon \to e^+e^-$,

$$\Gamma_{ee} = \frac{1}{6\pi}Q^2_b\frac{(4\pi\alpha)^2}{M^4_\Upsilon}|F_\Upsilon|^2\frac{M_\Upsilon}{2} \quad (21)$$

where $Q_b = -\frac{1}{3}$ in the electric charge of the b quark. All the hadronic physics in our process is included in this form factor.

The tau pair production at the Υ peak introduces the same polarization matrix terms with respect to the production with γ exchange (diagrams (a) and (b)). The only difference is an overall factor $\left(\frac{e^2Q^2_b|F_\Upsilon|^2}{s\Gamma_\Upsilon M_\Upsilon}\right)^2 = \left(\frac{3}{\alpha}Br(\Upsilon \to e^+e^-)\right)^2$ that is introduced in the cross section ($s = M^2_\Upsilon$). The only contributions with EDM in the polarization terms come with the interference of diagrams (b) and (d) while diagram (b) squared gives the tree level terms. All the comment we did with respect to Eqs.(9), (10) and (11) are useful here, and we find that there are no changes in the asymmetries and their expression at the Υ peak remain the same as before.

For $10^7 - 10^8$ τ's the asymmetries provide bounds of the order of $10^{-17} - 10^{-18}\, e\, cm$ for the EDM. This is one or two orders of magnitude lower than the last PDG bound. To finish we would like to stress that:

1) with low energies data we may ahve an independent analysis of the EDM from that obtained with LEP data,

2) low energies data makes possible a clear separation of the effects coming from the EDM and the weak-EDM,

3) high statistics can compensate the suppression factor q^2/Λ^2 in the low energy regime for the effective operators,

4) as contrary as it is now the case in the published bounds our observables are *CP-odd* ones and the limits thus obtained are rigorous.

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REFERENCES

1. K. Hagiwara *et al.*, *Phys. Rev.* **D66** (2002) 010001.
2. I. B. Khriplovich and S. K. Lamoreaux, *Berlin, Germany: Springer (1997) 230 p.*
3. W. Bernreuther and M. Suzuki, Rev. Mod. Phys. **63** (1991) 313 [Erratum-ibid. **64** (1992) 633].
4. D.Gómez-Dumm and G.A.González-Sprinberg, *Eur. Phys. J.* **C11** (99) 293.
5. W.Bernreuther and O.Nachtmann, *Phys. Rev. Lett.* **63** (1989) 2787 and erratum *Phys. Rev. Lett.* **64** (1990) 1072.
6. J.Bernabéu, G.A.González-Sprinberg and J.Vidal, *Phys. Lett.* **B326** (1994) 168.
7. G.A.González-Sprinberg, A.Santamaria, J.Vidal, *Nucl. Phys.* **B582** (2000) 3.
8. W. Buchmuller and D. Wyler, Nucl. Phys. B **268** (1986) 621; C. N. Leung, S. T. Love and S. Rao, Z. Phys. C **31** (1986) 433; M. S. Bilenky and A. Santamaria, Nucl. Phys. B **420** (1994) 47.
9. W. Bernreuther, O. Nachtmann and P. Overmann, Phys. Rev. D **48** (1993) 78.
10. J. Bernabéu, N. Rius and A. Pich, *Phys. Lett.* **B257** (1991) 219.
11. J.Bernabéu, G.A.González-Sprinberg, M.Tung and J.Vidal, *Nucl. Phys.* **B436** (95) 474.
12. J.Bernabéu, G.A.González-Sprinberg and J.Vidal, in preparation.
13. J. H. Kuhn, Phys. Lett. B **313** (1993) 458.
14. M. Acciarri *et al.*, *Phys. Lett.* **B434** (1998) 169.