NEUTRON POLARIZABILITY AND THE n-e SCATTERING LENGTH

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ABSTRACT

It is shown that the neutron polarizability contribution to the n-e scattering length is within the present level of precision. The experimental result on this interaction is consistent with a value of the neutron polarizability similar to the proton one.
The best determinations of the neutron-electron spin averaged scattering length $a$ are \((-1.34 \pm 0.03) \times 10^{-3} \text{ fm} \) \(^1\) and \((-1.56 \pm 0.04) \times 10^{-3} \text{ fm} \) \(^2\) by two independent methods.

The one photon exchange contribution to the scattering length is

$$a^{\text{1ph}} = e^2 2 M \left. \frac{d F_{\text{in}}}{d (-q^2)} \right|_{q^2 = 0} = -e^2 2 M \left[ \left. \frac{d F_{\text{in}}}{d (-q^2)} \right|_{q^2 = 0} - \frac{\mu_n}{4M^2} \right]$$

in terms of the Sachs and Dirac form factors. Hence the neutron magnetic moment contributes \(-1.47 \times 10^{-3} \text{ fm} \) \([\text{the Foldy term} \] \(^3\)) and it correctly describes the main part of the scattering length. In addition to a possible contribution from $F_{\text{in}}$, the (n-e) scattering can also be influenced by an "intrinsic" (n-e) interaction.

It is the aim of the present paper to estimate that part of the intrinsic interaction which is due to the coupling between the induced neutron dipole moment and the electric field of the electron. It will be shown that at present level of precision this effect cannot be neglected.

The neutron induced dipole moment in a static and uniform electric field $E_k$ is

$$d_i = \alpha_{i,k} E_k$$

where we will only consider the diagonal part of the polarizability, that is,

$$\alpha_{i,k} = \alpha_n \delta_{i,k}.$$ In the static limit the Coulomb field of the scattered electron has $E = \frac{e}{r^2} \hat{r}$, so that its interaction with the induced dipole moment corresponds to an induced potential

$$V(r) = -\frac{l}{2} \alpha_n \frac{e^2}{r^4}$$

This potential gives a divergent contribution to the neutron electron scattering length, which in reality is finite because of the neutron form factor as well as the non-static electric field. A very crude estimate [to be improved below] of the polarizability contribution to the (n-e) scattering length is obtained by
using Born approximation and the radial cut-off $R$

$$\delta a \approx -M \alpha_n \frac{e^2}{R}$$  \hspace{1cm} (4)

A reasonable cut-off is $R \approx 1$ fm (a smaller cut-off will give larger estimates). The neutron polarizability is unfortunately very poorly known. The only experimentally quoted results are obtained from neutron scattering on high $Z$ elements at 1 to 8 MeV energies \(^4\). The value obtained in this way is

$$\alpha_n = 0.26 \pm 0.10 \text{ fm}^3$$  \hspace{1cm} (5)

with 99% confidence limit claimed. Inserted in Eq. (4), this gives

$$\delta a \approx 7.5 \times 10^{-3} \text{ fm},$$

which is five times larger than the observed value! Since the value (5) is 250 times larger than the proton polarizability

$$\alpha_p = (9 \pm 2) \times 10^{-4} \text{ fm}^3 \hspace{1cm} (5)$$

the latter may give a better indication of $\alpha_n$. The value (5) is also in conflict with the polarizability expected from photo-neutron processes

$$\frac{1}{2\pi^2} \int_0^\infty \frac{\sigma_{\text{EL}}(\omega)}{\omega^2} \, d\omega < \alpha_n < \frac{1}{2\pi^2} \int_0^\infty \frac{\sigma_{\text{tot}}(\omega)}{\omega^2} \, d\omega$$  \hspace{1cm} (6)

Using $\sigma_{\text{EL}}$ as determined from recent analyses of single pion photoproduction $\gamma + n \rightarrow \pi + N \hspace{1cm} (6)$ as well as $\sigma_{\text{tot}}$ as determined from deuteron data \(^7\), we have

$$5 \times 10^{-4} \text{ fm}^3 < \alpha_n < 14 \times 10^{-4} \text{ fm}^3$$  \hspace{1cm} (7)

which indeed is similar to the proton value. Using $\alpha_n = \alpha_p$ we find

$$\delta a = -3 \times 10^{-5} \text{ fm}$$

which is of the order of the experimentally quoted errors for the (n-e) scattering length.

From the iteration of the Coulomb potential we know that the behaviour of the induced potential is $-r^2$ if $r \rightarrow 0$ and $-r^{-4}$ if $r \rightarrow \infty$. If the neutron transition density $g(r)$ is sharply localized at the "radius" $R$ we can write
\[ q(r) \propto \frac{r}{R^3} \theta(R-r) + \frac{i}{r^2} \theta(r-R) \]  

(8)

and then our contribution is given by

\[ \delta a = -e^2 M \frac{\alpha_\pi}{\pi} \frac{G}{5} \frac{K(R, \omega)}{R} \]  

(9)

where \( K(R, \omega) \) takes into account that the electric field is not static. This factor is defined as

\[ K = \frac{\int dk \frac{\omega}{\omega+k} k^2 \left| \int_0^\infty dr r^2 \frac{d}{dr} \theta(kr) \frac{g(r)}{r^2} \right|^2}{\int_0^\infty dk \frac{1}{k^2} \left| \int_0^\infty dr r^2 \frac{d}{dr} \theta(kr) g(r) \right|^2} \]  

(10)

with \( \omega \) the energy for which dipole excitations of the neutron are localized. Their most important contributions come from the pion nucleon continuum, then a safe recipe is to take values of \( \omega \) between 1.5 and 3 \( \text{fm}^{-1} \). We obtain

\[ \delta a = -(4.17 \times 10^{-2} \text{ fm}^{-1}) f(R) \alpha_\pi \]  

(11)

where Fig. 1 gives the values of the function \( f(R) \equiv \frac{K(R, \omega)}{R} \). We observe that for a large range of plausible \( R \)-values [0.4 - 1.2 fm] \( f(R) \) only changes within a factor two. Our result is thus fairly independent of the details of the neutron form factor. The \( R^{-1} \) dependence, as found in the static approximation, is reduced because the typical electron excitation energies increase as \( R \) decreases. In fact, the values of \( K \) go from 0 to 1 with \( R \) going from 0 to \( \infty \) in such a way that the divergence of \( f(R) \) with \( R \to 0 \) is only logarithmic. A harmonic oscillator quark model of the neutron suggests \( R \approx 0.7 \text{ fm} \) and then \( f(R) \approx 0.8 \text{ fm}^{-1} \). If \( \alpha_\pi = \alpha_p \), the polarizability contribution is \( \delta a = -3 \times 10^{-5} \text{ fm} \), that is, a correction of few per cent to the observed scattering length, giving support to the idea that the Foldy term accounts practically for all the interaction and that \( F_{\text{lin}}(-q^2) \approx 0 \).
One can ask how important other multi-polarities, different from $\ell = 1$, are. We show in a forthcoming paper that the sum of the $\ell > 1$ contributions is probably not higher than 30% of the dipole term.

Our investigation indicates very clearly the necessity of an independent determination of $\alpha_n$. Barring a tremendous accidental cancellation between $\frac{dF_{\ell n}}{dq^2}q^2 = 0$ and the polarizability contribution, the (n-e) scattering indicates a value for $\alpha_n^p$ at most a few times that of $\alpha_n^p$, i.e., $\alpha_n \leq (2 - 3) \times 10^{-3}$ fm$^3$. The experimental values for $\alpha_n^p$ by scattering on heavy nuclei must then be interpreted as a nuclear physics effect. This question could be settled by scattering of neutrons of rather low energy [100 eV to 1 keV] from high Z elements. While the s-wave scattering depends strongly on nuclear structure, the p-wave scattering is dominated by the polarizability as pointed out by Thaler: because of the $r^{-4}$ potential the p-wave phase shift varies like $k^2$ and not by $k^3$ as is the case for the nuclear interaction. In the angular distribution $\sigma(\theta) \propto 1 + \beta \cos \theta$, $|\beta|$ is of the order $|\beta| \sim 10^{-2} \alpha_n^p \sqrt{E}$ for heavy elements, with $\alpha_n$ in fm$^3$ and the neutron energy E in eV. For keV neutrons a polarization asymmetry of the order of a few per cent would occur if $\alpha_n^p$ is as large as indicated by Ref. 4, while p-wave nuclear scattering is still negligible. If $\alpha_n^p$, as we believe, this method is unpractical, and one would have to rely on semi-theoretical derivations of $\alpha_n$ by dispersion relations or envisage the difficult study of neutron Rayleigh scattering in a photon-neutron crossing beam experiment.

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REFERENCES


8) J. Bernabeu and T.E.O. Ericson, to be published.


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FIGURE CAPTION

Values of \( f(R) \equiv K(R, \omega)/R \) [Eq. 11] for two extreme values of the average neutron dipole excitation \( \omega \).