Neutrino Mass and Missing Momentum Higgs Boson Signals

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In the simplest scheme for neutrino masses invoking a triplet of Higgs scalars there are two CP-even neutral Higgs bosons $H_i$ ($i=1,2$) and one massive pseudoscalar $A$. For some choices of parameters, the lightest $H_1$ may be lighter than the Standard Model Higgs boson. If the smallness of neutrino mass is due to the small value of the triplet expectation value, as expected in a seesaw scheme, the Higgs bosons may decay dominantly to the invisible neutrino channel. We derive limits on Higgs masses and couplings that follow from LEP I precision measurements of the invisible Z width.

Neutrino mass constitutes one of the deepest open issues in the Standard Model (SM) of particle physics, which now finds some observational support. Neutrino masses in the few eV range may also be crucial in explaining the large scale structure of the universe. In many $SU(2)\otimes U(1)$ extensions of the SM neutrino masses are generated from the spontaneous violation of a global lepton number symmetry leading to the existence of a physical Goldstone boson - the majoron. In such models the majoron acts as the tracer of the neutrino mass generation mechanism and may have, depending on the details of the model, many interesting phenomenological implications. If the breaking of lepton number occurs at the weak scale the lightest Higgs boson can decay dominantly into the weakly interacting majorons. Since these escape detection, this decay is called invisible.

Here we consider a more direct way in which neutrino mass physics can show up as a missing momentum Higgs boson signature. As our illustrative model we consider the simplest triplet model for generating neutrino masses as first proposed, in the majoron-less form. The model contains a complex $SU(2)$ triplet of scalar bosons $\Delta$, in addition to the standard Higgs doublet $\phi$

$$
\phi = \begin{bmatrix} \phi^0 \\ \phi^- \end{bmatrix}, \quad \Delta = \begin{bmatrix} \Delta^0 \\ \Delta^+ / \sqrt{2} \\ \Delta^{++} / \sqrt{2} \end{bmatrix},
$$

where we have used the $2 \times 2$ matrix notation for the Higgs triplet. Apart from obvious terms in $\phi$ and $\Delta$, the scalar potential contains two mixed quartic terms and one tri-linear term given as

$$
\lambda_3 \phi^T \phi \phi^T \phi + \lambda_5 (\phi^T \Delta \phi) - \frac{\kappa}{\sqrt{2}} (\phi^T \Delta \phi + \text{h.c.})
$$

where the $\lambda$’s are dimensionless coupling constants and $\kappa$ has mass dimension and should lie at the weak scale or less. Note that the smallness $\kappa$ is natural according to ’t Hooft’s criterion, since the symmetry of the model increases as $\kappa \to 0$. We assume that the fields $\phi$ and $\Delta$ acquire nonzero real vacuum expectation values (VEVS) $v_2$ and $v_3$, respectively. It is easy to verify explicitly that, for many choices of its parameters, the potential has indeed minima for nonzero values of $v_2$ and $v_3$. According to this, we shift the fields in the following way

$$
\phi^0 = \frac{v_2}{\sqrt{2}} + \frac{R_2 + iI_2}{\sqrt{2}},
\Delta^0 = \frac{v_3}{\sqrt{2}} + \frac{R_3 + iI_3}{\sqrt{2}}
$$

(3)

Notice that the existence of a cubic term in the scalar potential breaks explicitly the lepton number symmetry, avoiding the Gelmini–Roncadelli triplet majoron - now ruled out experimentally by the measurement of the invisible Z width at LEP I.

As a result of the Yukawa coupling of the Higgs triplet to the leptons $\ell \Delta T$ a Majorana neutrino mass matrix will be generated, as follows

$$
m_\nu = h_\nu v_3
$$

(4)

In order to comply with cosmological limits the magnitude of the Yukawa couplings and/or $v_3$ is substantially restricted. We are interested in the limit of small values of the triplet VEV $v_3$. In this limit the smallness of neutrino mass in eq. (4) can be ascribed to the smallness of $v_3$ without having to invoke tiny Yukawa couplings. This is exactly what naturally happens in a seesaw scheme.

Taking into account the fact that this model contains one doubly-charged and one singly-charged scalar boson, in addition to the one charged unphysical $SU(2) \otimes U(1)$ Goldstone mode (longitudinal $W_\pm$), it follows that the neutral Higgs sector of this model is composed by four real fields. Due to CP invariance they split into two un-mixed sectors. For example, for $v_3 \ll \kappa$ and $v_3 \ll v_2$ the CP–even Higgs mass squared matrix may be written as

$$
M_R^2 = \begin{bmatrix}
\gamma m_H^2 & \frac{\gamma m_H^2 - 2m_A^2}{v_2} \\

& m_A^2 + O(v_3^2)
\end{bmatrix}
$$

(5)
where $\gamma$ is a ratio of $\lambda$’s, $m_{\gamma}^2$ is the SM Higgs mass, and $m_{\lambda}^2$ is the physical pseudoscalar boson mass given by

$$m_{\lambda}^2 = \frac{1}{2} \kappa \frac{v_3^2 + 4v_3^2}{v_3}$$  \hspace{1cm} (6)$$

This mass results from diagonalizing the $2 \times 2$ CP–odd Higgs mass squared matrix via a rotation matrix $O_T$ defined by a small angle

$$\sin \beta = \frac{2v_3}{\sqrt{(v_2^2 + 2v_3^2)}}$$  \hspace{1cm} (7)$$

and obeying $O_T M_T^T O_T^T = \text{diag}(0,m_{\lambda}^2)$, so that the first state is the unphysical Goldstone boson. On the other hand $O_R M_R^T O_R^T = \text{diag}(m_{H_1}^2, m_{H_2}^2)$ where we take, by definition, $m_{H_1} \leq m_{H_2}$, so that the first state corresponds to the lightest CP–even Higgs boson. The corresponding matrix $O_R$ is given in terms of an angle $\alpha$. The parameter $\sin \alpha$ determining the projection of $H_1$ along the triplet can be large when $m_{A} < m_{H}$, as we will see below.

Note that none of the CP-even or CP-odd Higgs boson masses obtained in our model lie at the scale $v_3$. Consequently, even though the model suffers from the usual hierarchy problem, this may be avoided by supersymmetrization, as in the SM. This would estabilize all Higgs masses at the weak scale.

The W and Z masses come from the kinetic part of the scalar Lagrangian

$$L_0 = (D_\mu \phi)^\dagger D^\mu \phi + \text{tr} [(D_\mu \Delta)^\dagger D^\mu \Delta]$$  \hspace{1cm} (8)$$

where the covariant derivative is defined by

$$D = \partial^\mu + ig T \cdot W^\mu + i g' Y V^\mu$$  \hspace{1cm} (9)$$

where $g$ and $g'$ are the $SU(2)$ and $U(1)$ gauge couplings respectively. The generators act on the scalars fields as

$$T \phi = \frac{1}{2} \tau \phi, \hspace{1cm} T \Delta = -\frac{1}{2} \tau \Delta - \frac{1}{2} \Delta \tau$$

$$Y \phi = -i \phi, \hspace{1cm} Y \Delta = 2 \Delta$$  \hspace{1cm} (10)$$

With these definitions we have $T_3 \phi^0 = \frac{1}{2} \phi^0$ and $T_3 \Delta^0 = -1 \Delta^0$. The W mass is given by

$$m_W^2 = \frac{1}{4} g^2 (v_2^2 + 2 v_3^2)$$  \hspace{1cm} (11)$$

so that $\sqrt{v_2^2 + 2 v_3^2} \approx 246$ GeV. From the measurement of the $\rho$ parameter one has

$$\rho = 1 + \frac{2 v_3^2}{v_2^2 + 2 v_3^2} = 1.001 \pm 0.002.$$  \hspace{1cm} (12)$$

which implies in practice that $v_3 \lesssim 9.5$ GeV and $v_2$ almost fixed, leading to $\sin \beta \lesssim 10^{-2}$. This restriction on $v_3$ is automatically fulfilled for the values we are dealing with.

In order to have an idea of the expectations of the model for the various Higgs boson masses we diagonalize the exact mass matrices and impose the potential minimisation conditions, checking the positivity of the physical CP-even and CP-odd eigenvalues. In Fig. (1) we show the CP–odd Higgs boson mass in our model versus $\kappa$ for different $v_3$ values. The allowed region lies above the curve corresponding to $v_3 \approx 9.5$ GeV.

![FIG. 1. Lightest CP–odd Higgs boson mass versus $\kappa$ for different $v_3$ values.](image1)

Similarly in Fig. (2) we show $m_{H_1}$, the mass of the lightest CP–even Higgs boson in our model, as a function of the SM Higgs mass $m_{H}$ for different $m_{A}$ values. For example, if we fix $m_{A}$ at 200 GeV we find that $H_1$ lies along the solid diagonal line and is almost the same as the SM doublet Higgs boson up to $m_{H} \sim 200$ GeV. Past this value the $H_1$ mass lies on the horizontal line at $m_{H_1} = 200$ GeV and $H_1$ is therefore mostly triplet. As we will see later, it will decay mostly to neutrinos instead of to b-quarks. Another feature worth noticing is that there can be a substantial mixing in the CP–even sector and, as a result, the $H_1$ mass may also be lower than the SM Higgs mass $m_{H}$, especially for small $m_{A}$ values. Finally, we can also see that $m_{H_1} < m_{A}$, so that the decay $H_1 \rightarrow AA$
and a CP–odd Higgs boson, and the associated production of a CP–even Higgs and a CP–odd Higgs $A$. The relevant couplings are given by

$$L_{H_a A} = \frac{g}{2c_w} Z^\mu \left[ R_2 \partial^\mu I_2 - 2 R_3 \partial^\mu I_3 \right]$$

$$= \frac{g}{2c_w} Z^\mu [\sin \beta O_{a2}^R - 2 \cos \beta O_{a3}^R] H_a \partial^\mu A.$$  \hspace{1cm} (13)

where $c_W \equiv \cos \theta_W$ and $H_a$ is any of the two CP–even neutral Higgs bosons. The parameter defined by

$$\epsilon_A = \sin \beta \cos \alpha + 2 \cos \beta \sin \alpha \approx 2 \sin \alpha$$  \hspace{1cm} (14)

will determine the strength of the $H_1 A Z$ coupling. From eq. (5) we find that the $H_a AZ$ couplings are

$$L_{H_a Z} = \frac{g m_Z}{4 v_W} Z^\mu Z^\nu \left[ \cos \beta O_{a2}^R + 2 \sin \beta O_{a3}^R \right] H_a,$$  \hspace{1cm} (15)

and correspondingly we define

$$\epsilon_B = \cos \beta \cos \alpha - 2 \sin \beta \sin \alpha \approx \cos \alpha$$  \hspace{1cm} (16)

as a measure of the strength of the $H_1 ZZ$ coupling.

Notice the factor of 2 in the expressions for the parameters $\epsilon_A$ and $\epsilon_B$ which determine the Bjorken and the associated production cross sections, respectively. It comes from the hypercharge of the triplet.

We now turn to the couplings relevant for the invisible decay of the lightest Higgs bosons. The pseudoscalar $A$ may decay into stable neutrinos, via the triplet Yukawa coupling $h_\nu$, of eq. (4), or into $\bar{b}b$, via its projection onto the doublet. In order to evaluate the relative importance of the two branches we need information on the neutrino mass. In the presence of the cubic lepton–number–breaking term $\kappa$ there is no efficient means of reducing the relic neutrino number density, as a result of which neutrinos in this model must obey the limit from cosmology on stable neutrino masses

$$m_\nu \lesssim 92 \Omega h^2 \text{eV}$$  \hspace{1cm} (17)

where $\Omega h^2 \leq 1$ is a basic cosmological parameter \[3\]. Present determinations give $h$ is $0.65 \pm 0.1$, while the total $\Omega$ may still be as large as one, as suggested by inflationary models.

Without adding singlet scalar bosons there is, on the other hand, no way to introduce the majoron in a phenomenologically acceptable way, which does not conflict with LEP I data on the observed width of the $Z$ into invisible channels. From eq. (6) we see that

$$\frac{\Gamma(A \rightarrow \bar{b}b)}{\Gamma(A \rightarrow \nu \bar{\nu})} = \left( \frac{3h_\nu \sin \beta}{h_\nu \cos \beta} \right)^2 \left( \frac{6m_\nu}{m_A} \right)^2 \frac{v_3^4}{v_2^2} \approx \left( \frac{6m_\nu}{m_A} \right)^2 \frac{v_3^4}{v_2^2}$$  \hspace{1cm} (18)

where $h_\nu$ here denotes the Yukawa coupling of the most massive of the neutrinos with the triplet. One sees that for $m_\nu \approx 10 \text{eV}$ and $v_3 \lesssim \text{few MeV}$, the decay of $A$ to neutrinos will be dominant.

An analogous calculation for the CP–even sector gives

$$\frac{\Gamma(H_1 \rightarrow \bar{b}b)}{\Gamma(H_1 \rightarrow \nu \bar{\nu})} \approx \left( \frac{6m_\nu}{m_A} \right)^2 \frac{v_3^4}{v_2^2} \gamma^2$$  \hspace{1cm} (19)

Clearly for $\cos \alpha \rightarrow 0$ $H_1$ is mostly triplet and therefore decays mainly to neutrinos. This corresponds to the horizontal lines in Fig. 2. In the opposite situation $\cos \alpha \rightarrow 1$ $H_1$ is mostly doublet and decays mainly to $\bar{b}b$. The price we must pay in order to have $\cos \alpha \approx 0$ is to have again a small $v_3$ as seen from eq. (19). In the last step in eq. (19) we assumed $m_A \ll m_H$ in order to obtain a conservative upper bound on $v_3$ as a function of $\gamma$. Again, one sees that for $m_\nu \approx 10 \text{eV}$ and $\gamma = 1$ we find that for $v_3$ values in the MeV range the lightest CP–even Higgs boson $H_1$ will also decay invisibly. For smaller $\gamma$ values the upper bound on $v_3$ is relaxed correspondingly.

Note that in the simplest scheme presented above the smallness of $v_3$ is put in by hand. However, the simplest model may be regarded as an effective parametrization of a more complete left-right symmetric see-saw scheme \[6\] in which lepton number is a local symmetry violated spontaneously at a large scale $v_R$. The smallness of $v_3$ would account for the smallness of neutrino mass and would arise naturally from a minimization condition of the scalar boson potential leading to $v_3 \sim m_\nu^3/v_R$.

We now perform a model independent study of the limits that can be set based on Higgs boson production in $e^+ e^-$ colliders at the $Z$ peak and its subsequent invisible decay. Consider the massive pseudoscalar $A$ and the lightest CP–even scalar $H_1$. As we have seen $H_1$ and $A$ may decay invisibly when $v_3$ is small. As seen above, for small $\gamma$ the bound on $v_3$ for $H_1$ to decay invisibly may be somewhat relaxed. The basic parameters needed to describe the implications of the production of Higgs bosons at the $Z$ peak in this model are the masses $m_A$ and $m_{H_1}$, the coupling parameters $\epsilon_A$ and $\epsilon_B$ which determine the Bjorken and associated production cross sections and the product of the visible and invisible decay branching ratios.

The Bjorken process contribution to the invisible $Z$ width $Z \rightarrow Z^* H_1$ is, for most of the parameter space, very small compared to that of the associated production. Thus, in order to get a conservative bound, we only consider the associated process, bearing in mind that the inclusion of the Bjorken contribution would only improve our results, i.e., would exclude a slightly wider region of parameter space.

Therefore we consider in what follows the limits that can be set on associated Higgs boson production at the $Z$ peak, $e^+ e^- \rightarrow Z \rightarrow H_1 A$ when both CP-even ($H_1$) as well as CP-odd Higgs bosons ($A$) decay invisibly. One
can write the contribution to the invisible $Z$ width as:

$$\Delta \Gamma_{\text{inv}} = \frac{\epsilon_A^2}{2} \lambda^2 \left(1 - \frac{m_H^2}{m_A^2} - \frac{m_H^2}{m_Z^2}\right) \Gamma(Z \to \nu \bar{\nu}) B_{\text{inv}} A_{\text{inv}}$$

where $B_{\text{inv}}$ and $A_{\text{inv}}$ denote the invisible branching ratios of $BR(H_1 \to \nu \bar{\nu})$ and $BR(A \to \nu \bar{\nu})$, respectively, and $\lambda$ is the usual Kälken function $\lambda(a,b,c) = (a - b - c)^2 - 4bc$.

Taking into account the experimental error in the determination of the invisible $Z$ width, given by $\sigma = 2$ MeV [8], we have determined 95% CL bounds on $\epsilon_A^2$ in the $m_H$-$m_A$ plane for fixed values of the product $B_{\text{inv}} A_{\text{inv}}$. As it is well known, the integration of a Gaussian probability distribution from $-\infty$ to $+1.64\sigma$ gives 0.95, so that the 95% CL exclusion region is defined by imposing $\Delta \Gamma_{\text{inv}} > 1.64\sigma$. In Fig. 3 we show these results for a fixed value of the product $B_{\text{inv}} A_{\text{inv}} = 1$. This corresponds to both scalar and pseudoscalar decaying totally to neutrinos. This choice is meant for definiteness. The constraints corresponding to any other value of $B_{\text{inv}} A_{\text{inv}}$ may be simply obtained by rescaling the results for our reference value given in Fig. 3. In the plot we have five curves labeled by a value of $\epsilon_A^2$.

No points below each of these curves are allowed with $\epsilon_A^2$ larger than that value. We see from this plot that simply by using the neutrino counting at the $Z$ peak one can already impose important constraints on the parameters of the model. For example, for $H_1$ and $A$ masses around 20 GeV the upper bound on $\epsilon_A^2$ is a few times $10^{-2}$. Going beyond this requires a dedicated analysis of the various event topologies that are possible in this model, for example, di-jet plus missing momentum. Fortunately the same topologies are also the ones present in other models where the invisible Higgs boson decay involves majorons, considered both for LEP I as well as LEP II data [12]. The results of that analysis can easily be adapted to the present model. In any case an updated analysis of the present LEP II data by the LEP collaborations themselves would be welcome.

In short, we have illustrated, with a very simple model, how LEP experiments may shed information on the Higgs boson sectors of models of neutrino mass. In contrast, all previously considered models with invisibly–decaying Higgs bosons invoked the existence of majorons and employed only physics at the weak scale. We have shown how the invisibly decaying Higgs boson signal can arise from the existence of a small scale ($v_3$ in our model) associated to neutrino masses in a model with explicit violation of lepton number. In a more complete see-saw left-right scheme, the smallness of $v_3$ would arise naturally as $v_3 \sim m_W^2/v_R$.

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![FIG. 3. 95% CL bounds on $\epsilon_A^2$ in $m_H$-$m_A$ plane when both $H_1$ and $A$ decay to neutrinos.](image)

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