DOUBLE $\Delta$ PRODUCTION IN THE

$$\gamma d \rightarrow pn\pi^+\pi^-$$

REACTION.

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Abstract

We have studied the $\gamma d \rightarrow \Delta^+\Delta^-$ reaction which requires the collaboration of the two nucleons in deuteron. By means of a model previously developed for the $\gamma p \rightarrow p\pi^+\pi^-$ reaction, the two body exchange currents leading to double delta creation are derived. A fair agreement is obtained with a recent experiment, but more precise measurements and the extension to higher photon energies look advisable in order to see the limits of the present theoretical approach.
Processes involving necessarily two nucleons in nuclei are particularly relevant and through them one expects to get insight into nuclear correlations, together with the reaction mechanisms. This should help us get a unified picture of reactions like photon or pion absorption and their relationship to meson exchange currents (MEC), which evidence themselves in a variety of reactions, like deuteron photodisintegration \[1\], $e^-$ elastic and inelastic scattering \[2\], etc. In this latter reactions MEC appear as corrections to the leading impulse approximation. The study of reactions where the MEC, or two body mechanisms, are the leading term provides a good laboratory to check our theoretical ideas and have these MEC mechanisms under control for application in a variety of reactions. In this respect the reaction $\gamma d \to \Delta \Delta$ offers one such opportunity since necessarily the two nucleons in the deuteron are involved and excited to $\Delta$ states. The recent measurement of this reaction \[3\] offers a good opportunity to test these ideas, and this is the purpose of the present work.

Our two body mechanisms for the process are constrained from the reaction $\gamma p \to p \pi^+ \pi^-$. Two body MEC are automatically generated by means of the $\gamma N \to N \pi^+ \pi^-$ reaction in one nucleon followed by the absorption of one of the pions in the second nucleon.

A thorough study of the $\gamma p \to p \pi^+ \pi^-$ reaction has been done in \[4\] and reproduces fairly well the experimental cross sections, invariant mass distributions \[5, 6\], etc. From this model we choose the dominant diagrams in which the final $\pi N$ system comes from the decay of a $\Delta$. The pion absorbed in the second nucleon excites a $\Delta$ and thus we are led to the diagrams depicted in fig. 1 for the two $\Delta$ excitation process.

There we depict the $\Delta^{++} \Delta^-$ excitation for $\gamma$ scattering on a $pn$ pair. The $T = 0$ wave function of the deuteron can be written as $|(pn)\rangle - |(np)\rangle)/\sqrt{2}$.

Fig. 1 corresponds to $\gamma$ scattering with the first isospin component $|(pn)\rangle$ of the wave function with the photon being absorbed either by the $p$ or the $n$. The diagrams corresponding to $\gamma$ scattering with the second component $|(np)\rangle$ would be identical to those in fig. 1 by exchanging the $p$ and $n$, the $\Delta^{++}$ and $\Delta^-$ and replacing the $\pi^-$ by a $\pi^+$. Since now the final state is $\Delta^- \Delta^{++}$, instead of $\Delta^{++} \Delta^-$ from the first component, these two sets of diagrams do not interfere, they contribute the same amount to the cross section and for practical purposes one evaluates the cross section with the diagrams of fig. 1 ignoring the $1/\sqrt{2}$ factor of the isospin wave function. The contribution of the $N^*$ (1520) to the $\gamma N \to N \pi^+ \pi^-$ reaction was made manifest in \[4\]. It interferes with the direct $\Delta$ production process and is responsible for the resonant like bump appearing in the $\gamma p \to p \pi^+ \pi^-$ cross section, which does not appear in the absence of the $N^*$ term.

Assuming for the moment the $\Delta$’s as stable particles we obtain the cross section

$$\sigma = \frac{M_d}{(s - M_d^2)} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3} \frac{M_\Delta}{E_\Delta(p)} \frac{M_\Delta}{E_\Delta(p')} \frac{\bar{\Sigma}\Sigma|T|^2}{2}$$
\[(2\pi)^4 \delta^4(k + p_d - p - p') \tag{1}\]

where \(M_d, M_\Delta\) are the deuteron and \(\Delta\) masses, \(E_\Delta(\vec{p})\) the energy \(\sqrt{M_\Delta^2 + \vec{p}^2}\) and \(k_1, p_d, p, p'\) the fourmomenta of the photon, deuteron, \(\Delta^{++}\) and \(\Delta^-\) respectively. \(T\) is the matrix element for the reaction from the model of fig. 1 in Mandl and Shaw normalization \([7]\).

Since the \(\Delta\)'s are unstable particles we introduce the identity

\[
\int dp^0 \delta(p^0 - E_\Delta(\vec{p})) = 1 \tag{2}\]

but now we make the replacement

\[
\frac{M_\Delta}{E_\Delta(\vec{p})} \delta(p^0 - E_\Delta(\vec{p})) \rightarrow -\frac{1}{\pi} \text{Im} \frac{M_\Delta}{E_\Delta(\vec{p})} \frac{1}{p^0 - E_\Delta(\vec{p}) - i\text{Im} \Sigma_{\Delta} \frac{M_\Delta}{E_\Delta(\vec{p})}} 
\approx -\frac{1}{\pi} \text{Im} \frac{1}{\sqrt{s_1 - M_\Delta} + \frac{i\Gamma(s_1)}{2}} \tag{3}\]

and the same expressions of eqs. (2) and (3) in the \(p'\) variable, with \(s_1 = p'^0 - \vec{p}'^2\), and \(\Gamma(s_1)\) the width of the \(\Delta\) at rest with invariant mass \(\sqrt{s_1}\), \([4]\).

With the modifications introduced by eqs. (2),(3) we write the final expression for the cross section

\[
\sigma = \frac{4M_d}{(s - M_d^2)} \int \frac{dp^0}{(2\pi)^4} I_m \frac{1}{\sqrt{s_1 - M_\Delta} + \frac{i\Gamma(s_1)}{2}} I_m \frac{1}{\sqrt{s_2 - M_\Delta} + \frac{i\Gamma(s_2)}{2}} \bar{\Sigma} \Sigma |T|^2 \tag{4}\]

with \(p' = k + p_d - p\) from momentum conservation and \(s_2 = p'^0 - \vec{p}'^2\).

The nuclear matrix element \(T\) is given, with the definition for the intermediate momentum \(q\) given in fig. 1, by

\[
T = \int \frac{d^3q}{(2\pi)^3} \tilde{\phi}(\vec{q} + \vec{p}' - \vec{p} + \vec{k}) \tilde{T} \tag{5}\]

where \(\tilde{\phi}(k)\) is the deuteron relative wave function in momentum space

\[
\tilde{\phi}(k) = \int d^3x e^{i\vec{k}\vec{x}} \phi(\vec{x}) ; \int d^3x |\phi(\vec{x})|^2 = 1 \tag{6}\]

and \(\phi(\vec{x})\) is the deuteron relative wave function in coordinate space which we take from ref. \([8]\) keeping only the \(s\)-wave part. The deuteron wave function has been separated into CM and relative coordinates and the CM wave function has led to the conservation of momentum after integration over the CM variables. The particular choice of the internal variables in fig. 1 is done such
that the wave function appears with the same argument in all terms as shown in eq. (5).

The two body matrix $\tilde{T}$ corresponding to diagrams a) b) c) of fig. 1 is given by (see appendix of ref. [4] for the effective Lagrangians and Feynman rules)

$$
-i\tilde{T} = e\left(\frac{f^*}{\mu}\right)^2 F(q)^2 \{\vec{S}_1^\dagger \cdot \vec{\epsilon} \vec{S}_2^\dagger \cdot (\vec{k} + \vec{q})\} \frac{i}{(k + q)^2 - \mu^2 + i\epsilon}
$$

$$
+ \vec{S}_1^\dagger \cdot \vec{q} \vec{S}_2^\dagger \cdot \vec{\epsilon} \frac{i}{q^2 - \mu^2 + i\epsilon}
$$

$$
+ i2\vec{q} \cdot \vec{\epsilon} \vec{S}_1^\dagger \cdot \vec{q} \vec{S}_2^\dagger \cdot (\vec{k} + \vec{q}) \frac{i}{q^2 - \mu^2 + i\epsilon} \frac{i}{(q + k)^2 - \mu^2 + i\epsilon}
$$

(7)

where the variable $q^0$ is given by $q^0 = E_d/2 - p^0$, $e$ is the electron charge, $\vec{\epsilon}$ the photon polarization vector (in Coulomb gauge, $\epsilon^0 = 0, \vec{\epsilon} \cdot \vec{k} = 0$), $\vec{S}$ the transition spin matrix from 1/2 to 3/2 and $F(q)$ a monopole form factor with $\Lambda = 1.3 \text{GeV}$. Although quark models [4], or model calculations of the $NN$ interaction using correlated two pion exchange [10], suggest smaller values of $\Lambda$, our input is related to the one boson exchange model and for consistency we must use the form factor determined with these models [11]. However, we have checked the sensitivity of our results to the form factor. By using $\Lambda = 1000 \text{GeV}$ we find that the cross section is decreased by less than 5% in the whole energy range that we study.

The integral of eq. (5) with the amplitude of eq. (7) contains the pion propagator and both the principal part and the pion pole parts are evaluated.

So far we have avoided to include the $N^*(1520)$ contribution because this can be done in an easy way. Once more we refer the reader to ref. [4] for the effective Lagrangians and couplings. The terms d) e) of fig. 1 have exactly the same spin structure as the terms a) and c) in that figure and their incorporation into the amplitude is done, as shown in section 3 of ref. [4], by means of the substitution in terms a) and c) of fig. 1 (first and second terms in eq. (7))

$$
e\frac{f^*}{\mu} \vec{g} \cdot \vec{\epsilon} \rightarrow \{e\frac{f^*}{\mu} - \frac{f_{N^*\Delta\pi}}{\sqrt{s} - M_{N^*} + i\Gamma^*(s)}(\tilde{g}_\gamma - \tilde{g}_\sigma)\} \vec{g} \cdot \vec{\epsilon}
$$

(8)

with $\sqrt{s}$ the invariant $N^*$ mass in the diagrams, $\Gamma^*(s)$ the $N^*$ decay width [4]. We take the values for the coupling constants

$$
\tilde{f}_{N^*\Delta\pi} = 0.677
$$

$$
\tilde{g}_\gamma = 0.108; \tilde{g}_\sigma = -0.049 \text{ for } N^* \rightarrow \gamma p
$$

$$
\tilde{g}_\gamma = -0.129; \tilde{g}_\sigma = 0.00731 \text{ for } N^* \rightarrow \gamma n
$$

(9)
which are required to reproduce the helicity amplitudes in the $N^*(1520) \rightarrow N\gamma$
decay and the decay of $N^*(1520)$ into the $\Delta\pi$ system\[4\].

We show the results in fig. 2 and compare them to the available experimental results\[3\]. As we can see, the results agree rather well with the data in the low energy range, but in the highest measured point, the calculated cross section is smaller than the experimental one. We should point out here that the dominant terms in figs. 1 are those involving the Kroll Ruderman term, 1a, 1c. Diagram 1b gives a smaller contribution and when added to the dominant terms 1a + 1c it changes the cross section only at the level of 10% (it decreases $\sigma$ below $E_\gamma = 740 MeV$ and increases it above this energy).

We also show in the figure the effects of considering the $N^*$ propagator. A similar thing happens in the case of the $\gamma p \rightarrow p\pi^+\pi^-$ reaction and the interference is responsible for the bump in the cross section shown by the experiment. In spite of the large experimental errors\[3\] we still can see that the cross section in the experiment of fig. 2 is better reproduced by the inclusion of the $N^*$ term. In ref. [4] the coupling $N^*(1520) \rightarrow \Delta\pi$ is taken as a constant. At high energies some small momentum dependent components of this coupling could become more important. However, we have used a quark model picture to evaluate this vertex and have found that the differences between the results with the constant coupling and the ones of the quark model, providing extra momentum dependent terms, are not significant in the $\gamma p \rightarrow p\pi^+\pi^-$ reaction\[12\].

In fig. 2 we also show our results at higher energies. As we can observe, the cross section stops increasing around $E_\gamma = 1060 MeV$ and starts going down smoothly from there on. However, one should also be aware that other MEC terms generated from the model of ref. [1] and leading to $2\Delta$ excitation could become relevant in the region of $E_\gamma > 800 MeV$. We should also note that our model for $\gamma p \rightarrow \pi^+\pi^- p$ starts having discrepancies with the data at energies $E_\gamma > 800 MeV$. Furthermore, as the photon energy increases one is picking up larger momentum components of the deuteron (see the argument of $\tilde{\phi}$ in eq. (5) and the $d$-wave component could also play a role. For all these reasons our results for $E_\gamma > 800 MeV$ have larger uncertainties as the photon energy increases.

In summary we have studied the reaction $\gamma d \rightarrow \Delta^{++}\Delta^-$ which is a genuine two body process. The mechanisms for the reaction were obtained by studying previously the $\gamma N \rightarrow N\pi^+\pi^-$ reaction and choosing the diagrams where one nucleon and a pion emerge in a $\Delta$ resonant state. The second pion was absorbed by the second nucleon exciting also a $\Delta$. The two body mechanisms generated in this way reproduce fairly well the experimental cross section at low energies but the results seem to be lower than experiment at higher energies, although the experimental data show strong oscillations there. The common features of the approach used here with current microscopic approaches for photon absorption in nuclei gives extra support to these approaches and also
strengthens our confidence in the use of two body meson exchange currents in other reactions.

In order to know the limits of the present method, more accurate data and extension of the measurements to higher energies would be most welcome.

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figure captions.

fig. 1.- Terms considered in our model for the $\gamma d \rightarrow \Delta^{++}\Delta^-$ reaction.

fig. 2.- Results of the model compared to the data of ref. [3]. Dashed line, omitting the $N^*$ terms (fig. 3d, 3e). Solid line, results including all terms of the model of fig. 1.
References


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