\Lambda(1520) and \Sigma(1385) in the nuclear medium

Murat M. Kaskulov\textsuperscript{\dagger} and E. Oset\textsuperscript{\dagger}

Departamento de Física Teórica and IFIC,
Centro Mixto Universidad de Valencia-CSIC,
Institutos de Investigación de Paterna,
Aptd. 22085, 46071 Valencia, Spain
(Dated: January 27, 2014)

Abstract

Recent studies of the \Lambda(1520) resonance within chiral unitary theory with coupled channels find the resonance as a dynamically generated state from the interaction of the decuplet of baryons and the octet of mesons, essentially a quasibound state of $\pi\Sigma^*(1385)$ in this case, although the coupling of the \Lambda(1520) to the $\bar{K}N$ and $\pi\Sigma$ makes this picture only approximate. The $\pi\Sigma^*(1385)$ decay channel of the \Lambda(1520) is forbidden in free space for the nominal mass of the $\Sigma^*(1385)$, but the coupling of the $\pi$ to $ph$ components in the nuclear medium opens new decay channels of the \Lambda(1520) in the nucleus and produces a much larger width. Together with medium modifications of the $\bar{K}N$ and $\pi\Sigma$ decay channels, the final width of the \Lambda(1520) at nuclear matter density is more than five times bigger than the free one. We perform the calculations by dressing simultaneously the \Lambda(1520) and the $\Sigma^*(1385)$ resonances, finding moderate changes in the mass but substantial ones in the width of both resonances.
INTRODUCTION

The $\Lambda(1520)$ is an intriguing resonance which has captured much attention and is easily produced in $K^-$ induced reactions [1, 2, 3, 4] or photon induced reactions [5]. Two recent different initiatives have brought this resonance to be again a focus of attention. On the one hand, this resonance appears, and is an important reference, in experiments that try to see the pentaquark $\Theta^+$ [6] (see [7] for a detailed reference of papers on this issue and [8] for a recent review). In fact, the large background appearing in the tail of the $\Lambda(1520)$ at energies higher than the nominal mass is one of the issues to be clarified when trying to make analyses on the $\Theta^+$. The issue of the large background has already been addressed in [10] and it is found to be associated to the large coupling of the resonance to the $\pi \Sigma^*(1385)$ channel.

The other initiative concerning the $\Lambda(1520)$ has been the study of the interaction of the decuplet of baryons with the octet of pseudoscalar mesons [11, 12], which has brought as an output that many of the low lying $3/2^-$ resonances are dynamically generated from the interaction of the coupled channels of these two multiplets. In particular, the $\Lambda(1520)$ appears basically as a quasibound state of the $\pi \Sigma^*(1385)$ system. The small free width of the $\Lambda(1520)$ of 15 MeV comes from the decay into $\bar{K}N$ and $\pi \Sigma$, since the decay into $\pi \Sigma^*(1385)$ is forbidden for the nominal mass of the $\Sigma^*(1385)$. Of course, the coupling of the $\Lambda(1520)$ to $\bar{K}N$ and $\pi \Sigma$ makes the picture of the $\Lambda(1520)$ more elaborate, with $\pi \Sigma^*(1385)$ being a very important component but with also sizable admixtures of $\bar{K}N$ and $\pi \Sigma$ [13, 14].

The change of resonance properties in the nuclear medium is also a field that captures permanent attention, and basic symmetries can be tested through medium modification of particle properties [15, 16, 17, 18, 19, 20, 21, 22, 23]. The decay of the $\Lambda(1520)$ in the nuclear medium bears resemblance to the one of the $\Delta(1232)$ [24]. The $\Delta$ decays into $\pi N$ and the $\pi$ gets renormalized in the medium by exciting $ph$ and $\Delta h$ components, as a consequence of which the $\Delta$ is renormalized and its pion (photon) induced excitation in nuclei incorporates now the mechanisms of pion (photon) absorption in the medium. In the present case, the $\Lambda(1520)$ decay into $\pi \Sigma^*(1385)$, only allowed through the $\Sigma^*(1385)$ width, gets drastically modified when the $\pi$ is allowed to excite $ph$ and $\Delta h$ components in the nucleus, since automatically the phase space for the decay into $ph \Sigma^*(1385)$ gets tremendously increased. This fact, together with the large coupling of the $\Lambda(1520)$ to the $\pi \Sigma^*(1385)$ channel predicted by the chiral theory, leads to a very large width of
the Λ(1520) in nuclei. Similar nuclear effects will modify the πΣ decay channel and the K N will be analogously modified when K is allowed to excite hyperon-hole excitations. All these channels lead to a considerable increase of the width of the Λ(1520) in the nucleus.

Concerning the medium corrections of the πΣ*(1385) channel, this has already a precedent in physics in the decay of an ordinary Λ(1115) in nuclei. The free Λ(1115) decay into πN through weak interactions, mesonic decay, is largely suppressed by Pauli blocking in nuclei. However, the pion can excite ph components in the medium leading to a new Λ(1115) decay mode Λ(1115) → N ph, or equivalently Λ(1115)N → NN, non mesonic decay. This new decay channel is far bigger than the mesonic decay in the nucleus and as large as the free one [25, 26, 27, 28, 29, 30].

The confirmation of the large width of the Λ(1520) in the medium, more than a factor five times larger than the free one as predicted here, would provide a strong support for the nature of the Λ(1520) as a dynamically generated resonance from πΣ*, πΣ and K N channels. Clear indications that this might be the case can be seen in the analysis of Λ(1520) production in heavy ion reactions [31, 32].

We have organized the paper as follows. In Sections II and III the model for the Λ(1520) and Σ*(1385) self energies in the nuclear medium is described. The result and discussions are presented in Section IV and V. Finally, the conclusions are given in Section VI.

RENORMALIZATION OF THE Λ(1520)

In this section we discuss the formalism used in the present work for the description of in-medium properties of the Λ(1520) hyperon. Here we follow the standard approach where the nuclear medium is described by the noninteracting Fermi sea and the baryonic resonances get modified in the nuclear medium in the dressing procedure by coupling the mesons in the loops to baryon - and hyperon - hole excitations.

The S-wave decay of the Λ(1520) → πΣ*(1385)

In the study of [10, 11, 13] the Λ(1520) is generated dynamically from the πΣ*(1385) and K Ξ* channels interacting in S-wave. In [13] the K N and πΣ(1189) channels in D-wave are added in order to produce the proper width of the Λ(1520). Of all these channels the most important one
is the $\pi\Sigma^*(1385)$, but the width into this channel is very small due to largely reduced phase space for the $\Lambda(1520)$ decay into $\pi\Sigma^*(1385)$, only possible through the tail of the $\Sigma^*(1385)$ when its width is considered. But the relevance of the $\pi\Sigma^*(1385)$ channel and the fact that in the nuclear medium the phase space for decay into this and associated channels becomes very large, makes the renormalization of this channel very important in studying the $\Lambda(1520)$ in the nuclear medium.

The in-medium renormalization of $\Lambda(1520)$ in the $\pi\Sigma^*(1385)$ channel can be represented by the first three diagrams in Fig. 1. In these diagrams the $\Sigma^*(1385)$ arises as an intermediate state but will be also dressed in its relevant decay channels. Hence, in the present work we address simultaneously the dressing of the $\Sigma^*(1385)$ and $\Lambda(1520)$ in the nuclear matter.

In the following we specify the in-medium propagators of the hyperons $\tilde{D}_Y$ and pions $\tilde{D}_\pi$. The kaon propagation in the nuclear medium will be addressed separately. In terms of a dispersion relation representation

$$\tilde{D}_Y(K, \rho) = \int_0^\infty dW \frac{S_Y(W, K, \rho)}{K^0 - W + i0^+}, \quad (1)$$

$$\tilde{D}_\pi(k, \rho) = \int_0^\infty d\omega (2\omega) \frac{S_\pi(\omega, k, \rho)}{(\omega^2 + i0^+)} \quad (2)$$

Here $K(k)$ are the four momenta and $S_Y(\pi)$ are the spectral functions of hyperons (pions), $\rho$ is the nuclear matter density and Eq. (1) accounts for the positive energy part of the fermionic propagator only. For $S_Y(\pi)$ we have

$$S_Y(\pi) = -\frac{1}{\pi} \text{Im}[\tilde{D}_Y(\pi)] \quad (3)$$

where

$$\text{Im}[\tilde{D}_Y(W, K, \rho)] = \frac{M_Y}{E_Y(K)} \frac{\text{Im}[\Sigma_Y(W, K, \rho)]}{[W - E_Y(K) - \text{Re}\Sigma_Y(W, K, \rho)]^2 + [\text{Im}\Sigma_Y(W, K, \rho)]^2} \quad (4)$$

$$\text{Im}[\tilde{D}_\pi(\omega, k, \rho)] = \frac{\text{Im}[\Pi_\pi(\omega, k, \rho)]}{[\omega^2 - \tilde{\omega}^2(k) - \text{Re}\Pi_\pi(\omega, k, \rho)]^2 + [\text{Im}\Pi_\pi(\omega, k, \rho)]^2} \quad (5)$$

In Eqs. (4) and (5) $E_Y(K) = \sqrt{K^2 + M_Y^2}$ and $\tilde{\omega}(k) = \sqrt{k^2 + m_\pi^2}$ are the on-mass-shell energies of hyperons and pions, respectively, and the in-medium self energy $\Sigma_Y$ is the subject of the present calculations. The $P$-wave pion polarization operator $\Pi$ is given by

$$\Pi_\pi(k, \rho) = \left(\frac{D + F}{2f_\pi}\right)^2 k^2 U(k, \rho) \left[1 - \left(\frac{D + F}{2f_\pi}\right)^2 g' U(k, \rho)\right]^{-1} \quad (6)$$
where $D$ and $F$ are the axial vector coupling constants and $f_\pi = 93$ MeV is the pion decay constant. One finds $F \simeq 0.51$, $D \simeq 0.76$ and the axial coupling constant used in present calculations is $g_A = D + F \simeq 1.27$. Also in Eq. (5) $g' = 0.7$ is the Landau-Migdal parameter and $U(k, \rho) = U^d(k, \rho) + U^c(k, \rho)$ is the Lindhard function including the direct and crossed contributions of $p - h$ and $\Delta - h$ excitations with the normalization of the appendix of Ref. [27]. These are the conventional definitions. Latter on we shall modify the formalism to account for the short-range (SR) correlations relevant for the in-medium dressing of the $\Sigma^*(1385)$.

The $S$-wave character of the $\Lambda(1520) \to \pi \Sigma^*(1385)$ decay requires the following transition amplitude

$$-i\ell_{\Lambda(1520) \to \pi \Sigma^*} = -ig_{\Lambda^* \Sigma^* \pi^*}$$

(7)

where $g_{\Lambda^* \Sigma^* \pi}$ already accounts for the three charge states in the isospin $I = 0$ channel, see Eq. (8), in the decay $\Lambda(1520) \to \pi \Sigma^*(1385)$. Alternatively, one can use the isospin decomposition (the
convention for phases is from Ref. [13])

\[
\Lambda(1520) \rightarrow |\pi\Sigma^*(1385); I = 0|
\]

\[
= \frac{1}{\sqrt{3}} \left[ |\pi^-\Sigma^{*+}) - |\pi^+\Sigma^{*-}) - |\pi^0\Sigma^{*0}) \right]
\]

and multiply the coupling constant \( g_{\Lambda^*\Sigma^*\pi} \) by Clebsch-Gordan coefficients \( \pm \sqrt{1/3} \) in the corresponding vertices. The sum of partial decay width into these channels equals to the \( I = 0 \) contribution provided by the single coupling of Eq. (7).

The value of the coupling constant \( g_{\Lambda^*\Sigma^*\pi} = 1.57 \) was determined in Ref. [13] by means of the residue of the scattering amplitude near the pole position of the \( \Lambda(1520) \). A value of \( g_{\Lambda^*\Sigma^*\pi} = 1.21 \) was used in Ref. [10] where the \( \bar{K}N \) and \( \pi\Sigma \) channels are not considered. Recent studies which combine simultaneous description of both the photoproduction and \( \bar{K}N \) scattering data results in \( g_{\Lambda^*\Sigma^*\pi} = 0.89 \) [14]. The latter value is used in the present work. Actually in Ref. [14] there is a discussion of many reactions involving the \( \Lambda(1520) \). However, for the purpose of setting the strength of \( g_{\Lambda^*\Sigma^*\pi} \) it suffices to quote that this coupling is needed to interpret the \( K^-p \rightarrow \pi^+\pi^-\Lambda \) reaction [39] using the formalism of Ref. [13] and the empirical width of 15 MeV for the \( \Lambda(1520) \). (The \( K^-p \rightarrow \pi^+\pi^-\Lambda \) cross section is twice as big as that of \( K^-p \rightarrow \pi^0\pi^0\Lambda \) studied in [13]).

Using Eq. (7) the one-loop contribution to the \( \Lambda(1520) \) self-energy from \( \pi\Sigma^* \) intermediate state takes the form

\[
- i \left[ \Sigma(P, \rho) \right]_{\pi\Sigma^* (1385)}^{\Lambda(1520)} = g_{\Lambda^*\Sigma^*\pi}^2 \int \frac{d^4k}{(2\pi)^4} \bar{D}_{\Sigma^*}(P-k, \rho) \bar{D}_\pi(k, \rho)
\]

where \( \bar{D}_{\Sigma^*} \) and \( \bar{D}_\pi \) are in-medium propagators of the \( \Sigma^*(1385) \) and pion, respectively. The in-medium renormalization of the \( \Sigma^*(1385) \) which enters Eq. (9) will be addressed in Section .

The \textbf{D-wave decay of the} \( \Lambda^*(1520) \rightarrow \pi\Sigma + \bar{K}N \)

The conventional \textbf{D-wave decay of the} \( \Lambda(1520) \) into the \( \bar{K}N \) and \( \pi\Sigma \) channels accounts for practically all of the \( \Lambda(1520) \) free width [40]. The diagrams responsible for the renormalization of the \( \Lambda(1520) \) in these decay channels are shown in Fig. (d-i). The pertinent \textbf{D-wave transition operator} is given by

\[
- i t_{\pi\Sigma \rightarrow \Lambda^* (\bar{K}N \rightarrow \Lambda^*)} = -i g_{\Lambda^*\pi\Sigma (\Lambda^*\bar{K}N)} (S^\dagger \cdot k)(\sigma \cdot k)
\]

where \( k \) is the pion three momentum and \( S^\dagger \) is the \( 2 \times 4 \) transition operator from spin \( 1/2 \) to spin \( 3/2 \) fulfilling the relation \( S_i S^\dagger_j = 2\delta_{ij}/3 - i\epsilon_{ijk}\sigma_k/3 \). Both coupling constants \( g_{\Lambda^*\pi\Sigma} \) and \( g_{\Lambda^*\bar{K}N} \) are
adjusted to reproduce the free decay branches of the $\Lambda(1520)$. Using Eq. (9) the partial decay width of the $\Lambda(1520)$ into the $\bar{K}N$ or $\pi\Sigma$ decay channels can be calculated using the formula

$$\Gamma_{\Lambda^*\rightarrow\Sigma(N)^+\pi(K)}(s) = -2\text{Im} \left[ \frac{\Sigma(s)}{\Sigma_{\pi\Sigma(K\bar{N})}} \right]$$

(10)

where $|k_{CM}| = \lambda^{1/2}(s, M_{\pi\Sigma}^2, m_{\pi(K)})/(2\sqrt{s})$ with $s = P^2$ and $\lambda$ is the Kálen function. At the nominal pole position $P^2 = M_{\Lambda(1520)}$ we get $g_{\Lambda^*\pi\Sigma} = 10.75$ GeV$^{-2}$ and $g_{\Lambda^*\bar{K}N} = 16.01$ GeV$^{-2}$. In non-dimensional limits, comparable to $g_{\Lambda^*\pi\Sigma}$ in Eq. (7)

$$\left[ g_{\Lambda^*\pi\Sigma(K\bar{N})}^2 k_{CM}^2 \right]/3 \equiv \tilde{g}_{\Lambda^*\pi\Sigma(K\bar{N})}$$

(11)

the value of these couplings at the pole position of the $\Lambda^*(1520)$ would be $\tilde{g}_{\Lambda^*\pi\Sigma} = 0.44$ and $\tilde{g}_{\Lambda^*\bar{K}N} = 0.54$, which show that the coupling $g_{\Lambda^*\pi\Sigma}$ is still the largest one.

$\Lambda^*(1520) \rightarrow \pi\Sigma(1189)$ channel: In the nuclear medium there are peculiarities which enforce us to consider the $\pi\Sigma$ and $\bar{K}N$ channels separately. The $\pi\Sigma$-channel is a simplest one in the conventional decay of the $\Lambda^*(1520)$ hyperon. The self energy loop integral in this decay channel reads

$$\left[ \Sigma(P, \rho) \right]_{\pi\Sigma}^{\Lambda^*} = g_{\Lambda^*\pi\Sigma}^2 \frac{i}{3} \int \frac{d^3k}{(2\pi)^3} k^4 \tilde{D}_\Sigma(P - k, \rho) \tilde{D}_\pi(k, \rho)$$

(12)

The imaginary part of Eq. (12) is meaningful by themselves and can be obtained using the Cutkosky rules

$$\Sigma(P, \rho) \rightarrow 2\text{Im} \left[ \Sigma(P, \rho) \right],$$

$$\tilde{D}_Y(K, \rho) \rightarrow 2\text{Im} \left[ \tilde{D}_Y(K, \rho) \right] \cdot \theta(K^0),$$

$$\tilde{D}_\pi(k, \rho) \rightarrow 2\text{Im} \left[ \tilde{D}_\pi(k, \rho) \right] \cdot \theta(k^0).$$

(13)

From this it is given by

$$\text{Im} \left[ \Sigma(P, \rho) \right]_{\pi\Sigma}^{\Lambda^*} = g_{\Lambda^*\pi\Sigma}^2 \frac{i}{3} \int \frac{d^3k}{(2\pi)^3} \frac{k^4 M_{\Sigma}}{E_{\Sigma}(P - k)}$$

$$\times \text{Im} \left[ \tilde{D}_\pi(\omega, k, \rho) \right] \cdot \theta(\omega) \bigg|_{\omega = P_0 - E_{\Sigma}(P - k) - V_{\Sigma}(\rho)}$$

(14)

where $V_{\Sigma}(\rho)$ is the binding correction for the $\Sigma(1189)$ hyperon, see Eq. (33).

$\Lambda^*(1520) \rightarrow \bar{K}N$ channel: The proper treatment of the $\bar{K}N$ channel is a more subtle problem. First, we consider the modification of the antikaon propagator in the nuclear medium. In its
particle-antiparticle decomposition, the dispersion relation representation of the $\bar{K}$ propagator is given by

$$\tilde{D}_{\bar{K}}(k^0, \mathbf{k}; \rho) = \int_0^\infty d\omega \frac{S_{\bar{K}}(\omega, \mathbf{k}; \rho)}{k^0 - \omega + i0^+} - \int_0^\infty d\omega \frac{S_{\bar{K}}(\omega, \mathbf{k}; \rho)}{k^0 + \omega - i0^+}$$

(15)

where $S_{\bar{K}}(K) = -\text{Im} \tilde{D}_{\bar{K}(K)}/\pi$ is the spectral function of the $\bar{K}(K)$ meson which depends on $\bar{K}(K)$ in-medium self energy $\Pi_{\bar{K}(K)}$ as provided by Eqs. (3) and (5). As is well known, the interactions of $\bar{K}$ and $K$ with the nucleons of the nuclear medium are rather different and, in principle, it is necessary to treat them separately.

In the calculation of the in-medium selfenergy we need the in-medium nucleon propagator. It is given by

$$\tilde{D}_N(p, \rho) = \frac{M_N}{E_N(p) - E_N(p) - V_N(\rho) + i0^+}$$

$$+ \frac{M_N}{E_N(p) - E_N(p) - V_N(\rho) - i0^+}$$

(16)

where $n(|p|) = \theta(k_F - |p|)$ and $k_F$ is the Fermi momentum. The first term in Eq. (16) describes the Pauli blocked propagation of nucleons and the second term is the hole propagator. The effect of the nucleon binding is accounted for by the mean field potential $V_N(\rho) \simeq -60\rho/\rho_0$ MeV. The selfenergy integral, after the integration over the $k^0$ component takes the following form

$$\left[\Sigma(P, \rho)\right]_{KN}^\Lambda = g_{KN}^2 \frac{i}{3} \int \frac{d^4 k}{(2\pi)^4} k^4 \tilde{D}_N(P - k, \rho) \tilde{D}_{\bar{K}}(k, \rho)$$

$$= -g_{KN}^2 \frac{1}{3} \int_0^\infty d\omega \int \frac{d^3 k}{(2\pi)^3} \frac{M_N}{E_N(\mathbf{P} - \mathbf{k})} \frac{k^4[1 - n(|\mathbf{P} - \mathbf{k}|)]S_{\bar{K}}(\omega, \mathbf{k}; \rho)}{P^0 - E_N(\mathbf{P} - \mathbf{k}) - \omega - V_N(\rho) + i0^+}$$

$$- g_{KN}^2 \frac{1}{3} \int_0^\infty d\omega \int \frac{d^3 k}{(2\pi)^3} \frac{M_N}{E_N(\mathbf{P} - \mathbf{k})} \frac{k^4n(|\mathbf{P} - \mathbf{k}|)S_K(\omega, \mathbf{k}; \rho)}{P^0 - E_N(\mathbf{P} - \mathbf{k}) + \omega - V_N(\rho) - i0^+}$$

(17)
In Eqs. (17) and (12) a static regulating form factor will be introduced in the results section.

Although the evaluation of Eq. (17) is straightforward, in the present work we neglect the contribution of the second term which gives no contribution to the imaginary part and only a small one to the real part. So we consider the spectral function of antikaons only. The corresponding imaginary part of the $\Lambda^*(1520)$ selfenergy is given by

$$\text{Im} \left[ \Sigma(P,\rho) \right]_{\bar{K}N}^{\Lambda^*} = -g_{\Lambda^*\bar{K}N}^2 \frac{\pi}{3} \int \frac{d^3k}{(2\pi)^3} \frac{k^4 M_N}{E_N(P-k)} \times [1 - n(|P-k|)] S_K(\omega, k, \rho) \cdot \theta(\omega) \bigg|_{\omega = P_0 - E_N(P-k) - V_N(\rho)}$$

(18)

In the following we shall briefly discuss how the antikaon spectral function is obtained. Here we follow closely Ref. [21].

The $P$-wave contribution to the $\bar{K}$ selfenergy comes from the coupling of the $\bar{K}$ meson to hyperon particle-nucleon hole ($YN^{-1}$) excitations. The corresponding many-body mechanisms are shown in Fig. 2. Because of strangeness conservation, only direct terms, Fig. 2(a), are permitted for the $\bar{K}$ excitations. Conversely, the $K$ selfenergy arises from the crossed terms, Fig. 2(b). The $K^-$ meson can couple to $p\Lambda, p\Sigma^0$ or $n\Sigma^-$ and the $\bar{K}^0$ to $n\Lambda, n\Sigma^0$ or $p\Sigma^-$. The vertices $\bar{K}NY$ are derived from the $D$ and $F$ terms of the chiral Lagrangian given in Appendix I (see Eq. (38)), expanding the unitary $SU(3)$ matrix $U$ up to one meson field. Using a non-relativistic reduction of the $\gamma^\mu \gamma^5$ matrix, one finds

$$-it_{\bar{K}NY} = C_{\bar{K}NY} (\sigma \cdot k)$$

$$= \left[ \alpha_{\bar{K}NY} \frac{D + F}{2f} + \beta_{\bar{K}NY} \frac{D - F}{2f} \right] (\sigma \cdot k)$$

(19)

where $k$ is the incoming $\bar{K}$ three momentum, $f = 1.15 f_\pi$ and $\alpha_{\bar{K}NY}, \beta_{\bar{K}NY}$ are the $SU(3)$ coefficients given in [21].

Following Ref. [34] we consider the $\bar{K}NY$ interaction in combination with the $\bar{K}N\Sigma^*(1385)$ transition where the expression for vertex function is given by

$$-it_{\bar{K}N\Sigma^*} = C_{\bar{K}N\Sigma^*} (S^\dagger \cdot k) = A_{\bar{K}N\Sigma^*} \frac{2\sqrt{6}}{5} \frac{D + F}{2f} (S^\dagger \cdot k).$$

(20)

The $SU(3)$ coefficients $A_{\bar{K}N\Sigma^*}$ are given in Ref. [34]. These couplings were evaluated by first using the $SU(6)$ quark model to relate the $\pi NN$ coupling to the $\pi N\Delta$ one and then using $SU(3)$ symmetry to relate the $\pi N\Delta$ coupling to the $\bar{K}N\Sigma^*$ one, since the $\Sigma^*(1385)$ belongs to the $SU(3)$ decuplet of the $\Delta$-isobar.
FIG. 3: In-medium renormalization of the \( \Sigma^*(1385) \) in the \( \pi \Sigma + \pi \Lambda \) channels. The last four diagrams account for the short-range correlations.

The \( P \)-wave \( \bar{K} \) self energy in symmetric nuclear matter can then be summarized as

\[
\Pi_K^{(p)}(\omega, k, \rho) = \frac{1}{2} \left[ C_{K^*-p\Lambda}^2 R_{\Lambda}^2 \right] k^2 U_\Lambda(\omega, k, \rho) + \frac{3}{2} \left[ C_{K^*-p\Sigma}^2 R_{\Sigma}^2 \right] k^2 U_\Sigma(\omega, k, \rho) + \frac{1}{2} \left[ C_{K^*-p\Sigma^*}^2 R_{\Sigma^*}^2 \right] k^2 U_{\Sigma^*}(\omega, k, \rho)
\]

where the Lindhard function \( U_Y(q) \) (\( Y = \Lambda, \Sigma, \) or \( \Sigma^* \)) accounts for the direct term only, see Fig. 2 (a). Its explicit expression can be found in Ref. [34]. In Eq. (21) \( R_{\Lambda(\Sigma)} = (1 - \omega/2M_{\Lambda(\Sigma)}) \), \( R_{\Sigma^*} = (1 - \omega/M_{\Sigma^*}) \) are the relativistic recoil vertex corrections [21]. In addition, we use the static form-factors at the antikaon-baryon vertices of monopole type, \( \Lambda^2/(\Lambda^2 + k^2) \), with \( \Lambda = 1 \) GeV.

Finally, we take into account the short-range correlations in the hyperon-hole \( YN^{-1} - YN^{-1} \) channels using a standard prescription

\[
\Pi_K(\omega, k, \rho) = \frac{k^2 \Pi_K^{(p)}(\omega, k, \rho)}{1 - g' \Pi_K^{(p)}(\omega, k, \rho)} \tag{22}
\]

where we assume the same value of the Landau-Migdal parameter \( g' = 0.7 \) as in Eq. (6).

**RENORMALIZATION OF THE \( \Sigma^*(1385) \)**

As we have seen from Eq. (9), the problem of the in-medium modification of the \( \Lambda(1520) \) in the \( \pi \Sigma^*(1385) \) channel can be reduced to the proper description of the properties of pions and \( \Sigma^*(1385) \) in the nuclear medium. The renormalization scheme which we employ for the \( \Sigma^*(1385) \) is essentially the same as in the previous case except for the \( P \)-wave nature of the hadronic \( \Sigma^*(1385) \) decay. This implies some peculiarities, for instance, the proper treatment of short range correlations.
\[ \Sigma^*(1385) \rightarrow \pi \Lambda + \pi \Sigma \text{ channel:} \] We consider the following \( P \)-wave decay branches of the \( \Sigma^*(1385) \) hyperon: \( \Sigma^*(1385) \rightarrow \Lambda(1115) + \pi \) and \( \Sigma^*(1385) \rightarrow \Sigma(1189) + \pi \). In what follows we shall further dress the octet states \( \Lambda(1115) \) and \( \Sigma(1189) \) using the phenomenological optical potentials. The vertex functions describing the transitions are given by

\[ -it_{\pi Y \rightarrow \Sigma^*} = -C_{\pi Y \Sigma^*}(S^\dagger \cdot k) \]  

(23)

where \( Y = \Sigma(1189) \) or \( \Lambda(1115) \). For coupling constants \( C_{\pi Y \Sigma^*} \) we follow Ref. 34 and use the quark model values

\[ C_{\pi \Lambda \Sigma^*} = \frac{6 \, D + F}{5 \, 2f_\pi}, \quad C_{\pi \Sigma \Sigma^*} = -\frac{2\sqrt{3} \, D + F}{5 \, 2f_\pi} \]  

(24)

From this, the explicit expressions for \( \Sigma_{\pi Y} \) are given by

\[ \left[ \Sigma(P, \rho) \right]_{\pi \Lambda(1115)}^{\Sigma^*(1385)} = i \left( C_{\pi \Lambda \Sigma^*} \right)^2 \frac{1}{3} \int \frac{d^4k}{(2\pi)^4} k^2 \tilde{D}_\Lambda(P - k, \rho) \tilde{D}_\pi(k, \rho) \]  

(25)

\[ \left[ \Sigma(P, \rho) \right]_{\pi \Sigma(1189)}^{\Sigma^*(1385)} = i \left( C_{\pi \Sigma \Sigma^*} \right)^2 \frac{2}{3} \int \frac{d^4k}{(2\pi)^4} k^2 \tilde{D}_\Sigma(P - k, \rho) \tilde{D}_\pi(k, \rho) \]  

(26)

The additional factor 2 in Eq. (26) comes from contribution of two possible charge states.

At this point we would like to mention that in a realistic calculation one would have to add strong repulsive forces at short distances which would generate short-range correlation. The correlations of this type of the interaction would effectively modulate the in-medium \( \pi \) exchange interaction \[37, 38\], introducing the correlation parameter \( g' \). The denominator in Eq. (25) takes into account this effect between \( P \)-wave bubbles in the diagrams of Fig. 3 but not between the external hyperon and the contiguous bubble. To account for this we make the separation between the longitudinal \( \mathcal{V}_l \) and transverse \( \mathcal{V}_t \) parts of the pion effective interaction \[35, 36\] in the \( P \)-wave loop integrals

\[ \left( \frac{D + F}{2f_\pi} \right)^2 \frac{k_i k_j}{(k^0)^2 - k^2 - m_\pi^2 + i0^+} \rightarrow \mathcal{V}_l(k) \hat{k}_i \hat{k}_j + \mathcal{V}_t(k)(\delta_{ij} - \hat{k}_i \hat{k}_j) \]  

(27)

Here \( k_i \) is the Cartesian component of the unit vector \( \hat{k} = k/|k| \) and

\[ \mathcal{V}_l(k) = \left( \frac{D + F}{2f_\pi} \right)^2 \left[ \frac{k^2}{(k^0)^2 - k^2 - m_\pi^2 + i0^+} + g' \right] F(k)^2 \]  

\[ \mathcal{V}_t(q) = \left( \frac{D + F}{2f_\pi} \right)^2 g' F(k)^2 \]  

(28)

This procedure can be represented by the last four diagrams in Fig. 3. In Eqs. (28) and (29) \( F(k) \) is a static form factor \( \Lambda^2/(\Lambda^2 + q^2) \) with the cut off scale \( \Lambda = 1 \) GeV. Hence, we make the following substitution in the selfenergy of the \( \Sigma^*(1385) \), see Eqs. (28) and (29)

\[ \frac{(D + F)}{2f_\pi} k^2 \tilde{D}_\pi(k, \rho) \rightarrow W(k, \rho) = \frac{\mathcal{V}_l(k)}{1 - \mathcal{U}(k, \rho)\mathcal{V}_l(k)} + \frac{2 \mathcal{V}_l(k)}{1 - \mathcal{U}(k, \rho)\mathcal{V}_l(k)}. \]  

(29)
Using the Cutkosky rules Eqs. (13) supplemented by

\[ W(k, \rho) \rightarrow 2i \text{Im} \left[ W(k, \rho) \right] \cdot \theta(k^0) \]  

one may calculate the imaginary part of the loop integrals which are given by

\[
\text{Im} \left[ \Sigma(P, \rho) \right]_{\pi \Lambda} = \left( \tilde{C}_{\pi \Lambda} \right)^2 \frac{1}{3} \int \frac{d^3k}{(2\pi)^3} \frac{M_{\Lambda}}{E_{\Lambda}(P - k)} \text{Im} \left[ W(k, \rho) \right] \cdot \theta(k^0) \bigg|_{k^0 = P^0 - E_{\Lambda}(P - k)}
\]

\[
\text{Im} \left[ \Sigma(P, \rho) \right]_{\pi \Sigma} = \left( \tilde{C}_{\pi \Sigma} \right)^2 \frac{2}{3} \int \frac{d^3k}{(2\pi)^3} \frac{M_{\Sigma}}{E_{\Sigma}(P - k)} \text{Im} \left[ W(k, \rho) \right] \cdot \theta(k^0) \bigg|_{k^0 = P^0 - E_{\Sigma}(P - k)}
\]

where \( \theta \) is the step function and \( \tilde{C}_{\pi \Sigma} \) are the reduced coupling constants obtained from Eq. (24) by omitting the factor \((D + F)/2f_\pi\).

The vacuum subtracted expression for \( W(k, \rho) \) reads

\[ \delta W(k, \rho) = W(k, \rho) - W(k, 0) = \frac{U(k, \rho)\nu_f^2(k)}{1 - U(k, \rho)\nu_i(k)} + \frac{2U(k, \rho)\nu_f^2(k)}{1 - U(k, \rho)\nu_i(k)} \]  

From this the subtracted versions of the selfenergy integrals Eqs. (25) and (26) take the form

\[
[\delta \Sigma(P, \rho)]_{\pi \Lambda} = i \left( \tilde{C}_{\pi \Lambda} \right)^2 \frac{1}{3} \int \frac{d^4k}{(2\pi)^4} \tilde{D}_{\Lambda}(P - k, \rho) \delta W(k, \rho)
\]

\[
[\delta \Sigma(P, \rho)]_{\pi \Sigma} = i \left( \tilde{C}_{\pi \Sigma} \right)^2 \frac{2}{3} \int \frac{d^4k}{(2\pi)^4} \tilde{D}_{\Sigma}(P - k, \rho) \delta W(k, \rho)
\]

Using Eqs. (32) and (33) we can evaluate the in-medium modification of both the real and imaginary parts. We also take into account the phenomenological binding corrections to \( \Sigma \) and \( \Lambda \) in nuclei to which the \( \Sigma^*(1385) \) decays. These corrections are accounted for by

\[ V_\Lambda(\rho) = \text{Re} \Sigma_\Lambda = V_\Sigma(\rho) = \text{Re} \Sigma_\Sigma = -30 \rho / \rho_0 \]  

\( \Sigma^*(1385) \rightarrow K^0 N \) channel: The decay modes considered in the previous sections are allowed in the free space. But there are channels like \( \Sigma^*(1385) \rightarrow K^0 N \) which may open up in the medium because of the additional phase space. Using the vertex function given by Eq. (20) the expression for the selfenergy integral reads

\[
[\Sigma(P, \rho)]_{K^0 N}^{\Sigma^*(1385)} = (C_{K^0 N \Sigma^*})^2 \frac{i}{3} \int_0^\infty d\omega \int \frac{d^4k}{(2\pi)^4} k^2 \tilde{D}_N(P - k, \rho) \left[ \frac{S_K(\omega, k; \rho)}{k^0 - \omega + i0^+} - \frac{S_K(\omega, k; \rho)}{k^0 + \omega - i0^+} \right]
\]
FIG. 4: The imaginary parts (a) and (b) of the Σ∗(1385) selfenergy in the (πΣ + πΛ) and ¯KN channels, respectively, at several densities ρ = ρ0 (dashed curve), ρ0/2 (dot-dashed curve) and ρ0/4 (dot-dot-dashed curve) as a function of PμΣ∗(1385) = (P0, 0). The vacuum subtracted real part (c) of the Σ∗(1385) selfenergy. The dotted vertical line indicates the Σ∗(1385) pole position.

where ˜DN is the in-medium nucleon propagator, Eq. (16), and ˜DK is the ¯K propagator which is introduced in accord with Eq. (15). The direct application of that result is not entirely correct because we deal with the P-wave kaons which are also affected by the short range correlations in this channel. For instance, in the expression for the imaginary part of the in-medium selfenergy we obtain

$$\text{Im}\left[\Sigma(P, \rho)\right]_{\Sigma N}^{\Sigma N^*} = (\tilde{C}_{\Sigma N^*})^2 \frac{1}{3} \int \frac{d^3k}{(2\pi)^3} \frac{M_N}{E_N(P-k)} \left[1 - n(|P-k|)\right] \text{Im}\left[W_K(\omega, k, \rho)\right] (\omega, k, \rho) \cdot \theta(\omega) \bigg|_{\omega = P_0 - E_N(P-k) - V_N(\rho)}$$

where \(\tilde{C}_{\Sigma N^*} = C_{\Sigma N^*}(2f_\pi)/(D + F)\) and

$$W_K = \frac{\nu_{lK}(k)}{1 - U_K(k, \rho)\nu_{lK}(k)} + \frac{2\nu_{lK}(k)}{1 - U_K(k, \rho)\nu_{lK}(k)}$$

with \(\nu_l\) and \(\nu_{lK}\) given by Eq. (28) replacing \(m_{\pi}\) by \(m_{\bar{K}}\) and using the same \(g'\). In addition, in Eq. (37) \(U_K = \tilde{\Pi}^{(p)}_K/(D + F)^2\).

RESULTS FOR THE Σ∗(1385)

Our results for the imaginary part of the Σ∗(1385), or equivalently the width \(\Gamma_{\Sigma^*} = -2\text{Im}\Sigma_{\Sigma^*}\) in the πΣ + πΛ decay channels and in the reference frame where \(P = (P_0, 0)\) are shown in Fig. 4(a)
for several densities. The vacuum value at the nominal pole position is $\Gamma_{\Sigma^*} \simeq 30$ MeV and in good agreement with its empirical value $\Gamma_{\Sigma^*} = 35 \pm 4$ [40]. Also the vacuum branching ratio $\Gamma_\Lambda/\Gamma_{\Sigma^*} \simeq 7.7$ compares well with experiment 7.5 $\pm$ 0.5. Increasing the density we observe the broadening of the $\Sigma^*(1385)$ hyperon, and as a result, at normal nuclear matter densities $\rho_0 = 0.16$ fm$^{-3}$ the width becomes $\simeq 76$ MeV. Note, that the SR correlations play here an important role. We find that, without SR correlations the effect of the medium on $\Sigma^*(1385)$ is unrealistically big and produces the increase of the width relative to the vacuum value by a factor five. This situation is similar to the behavior of the $\Delta(1232)$-isobar at finite density where the SR correlations play an important role and strongly moderate the change of mass and width of the $\Delta$ isobar [36].

The results for another related channel, namely $\Sigma^*(1385) \rightarrow \bar{K}N$ is shown in Fig. 4 (b). Because of the Pauli blocking and relatively weak interaction of $\bar{K}$ with the nuclear medium as compared with pions the impact of this channel is small and adds an additional portion $\simeq 7$ MeV to the imaginary part. This is much smaller than obtained in Ref. [41] where only the $\bar{K}N$ channel is considered and an on-shell approximation of the $P$-wave $\bar{K}N$ amplitude is done. This approximation is not good for nuclei since it ties the momentum to the energy of the $\bar{K}$, which becomes imaginary below the $\bar{K}N$ threshold. However the $k^0$ and $k$ variables in matter are independent variables and $k$ is always physical.

For the real part of the $\Sigma^*(1385)$ self energy - the in-medium mass shift - we find an attractive potential at normal nuclear matter density with a strength of about $\simeq -45$ MeV, see Fig. 4 (c).

**RESULTS FOR THE $\Lambda^*(1520)$**

Using the dressed $\Sigma^*(1385)$ discussed in the previous section we calculate the selfenergy of the $\Lambda^*(1520)$ given by Eq. (9) in the $\pi\Sigma^*(1385)$ decay channel. Our results for the in-medium mass and width of the $\Lambda(1520)$ in this novel $\pi\Sigma^*$ channel are shown in Fig. 5 (a,d). As we have already noted for the nominal masses of hyperons involved there is an energy gap $\simeq 5$ MeV which makes the decay $\Lambda(1520) \rightarrow \pi\Sigma^*(1385)$ not possible. In the nuclear medium the pions decay to $p - h$ and $\Delta - h$ and this effect allows to open the $\Lambda(1520) \rightarrow (ph)\Sigma^*(1385)$ decay channel which has a large available phase space. In Fig. 5 the dashed, dot-dashed and dot-dot-dashed curves correspond to $\rho_0$, $\rho_0/2$ and $\rho_0/4$, respectively. In this channel we get for the width of the $\Lambda(1520)$ $\Gamma_{\Lambda(1520)} \simeq 18$ MeV at the nominal pole position and normal nuclear matter density. This value
FIG. 5: In-medium renormalization of the Λ(1520) in the πΣ∗(1385) a,d) πΣ b,e) and K N c,f) channels.

The top and bottom panels are the imaginary and the vacuum subtracted real parts of the selfenergies, respectively. The vertical line indicates the Λ(1520) pole position.

is even bigger than the free width of the Λ(1520) which is ΓΛ(1520) ≃ 15 MeV. The corresponding results for the vacuum subtracted real part of the selfenergy are shown if Fig. 5 (d). Here we find relatively weak attractive potential of about ≃ −8 MeV at the resonance pole position and normal nuclear matter density.

The renormalization of the Λ∗(1520) in the πΣ(1189) channel is shown in Fig. 5 (b,e). These curves correspond to the regularization of the selfenergy integral Eq. (12) when using the form factor [F(k^2)]^2 = [Λ^2/(Λ^2 + k^2)]^2 with the cut off scale Λ = 450 ÷ 500 MeV which is needed for D-wave loops in Ref. [14]. We normalize the form factor to unity at the Λ∗(1520) pole position [F(k^2)/F(k_{on-shell}^2)]^2 where k_{on-shell} is the on-mass-shell three momenta of the meson in the loop. We use both limiting values and consider them as a sort of theoretical uncertainties. The results are presented as a band where the upper limit correspond to the Λ = 500 MeV and the lower
FIG. 6: Values with theoretical uncertainties for the width of the Λ(1520) at rest in the medium, including the free width, as function of the nuclear matter density $\rho/\rho_0$ (a). The imaginary (b) and real (c) parts of the Λ(1520) selfenergy as a function of a three momentum $|P_\Lambda|$. 

limit to the $\Lambda = 450$ MeV. For instance, for $\Lambda = 500$ MeV we get $\simeq 32$ MeV width of the $\Lambda^*(1520)$ at normal nuclear matter density $\rho_0$ and for $\Lambda = 450$ MeV we get $\simeq 26$ MeV. One should compare this result with the free decay in this channel (solid curve) where the corresponding value is $\simeq 7$ MeV only. The vacuum subtracted real part of the selfenergy is shown in Fig. 5 (e) and was calculated with cut off $\Lambda = 500$ MeV. The changes are moderate and for $\rho = \rho_0$ we get the attraction $\simeq -6$ MeV at energies near the pole position. In Fig. 5 (c,f) we show our results for the renormalization of the Λ(1520) in the $\bar{K}N$ channel. Here the results are quantitatively similar to the $\pi\Sigma$ channel. At normal density we get the width $\simeq 20$ MeV and additional attraction $\simeq -7$ MeV. With present uncertainties we give a band of values for the width of the Λ(1520) in the nuclear medium including now the free width and the in-medium renormalization from the $\pi\Sigma^*(1385)$, $\bar{K}N$ and $\pi\Sigma$ related channels. We show these results in Fig. 6 (a) as a function of $\rho/\rho_0$. As one can see, at $\rho = \rho_0$ we get a $\Lambda(1520)$ width of about $\simeq 70 \div 80$ MeV, which is about five times the free width. These results are of the same order of magnitude as those obtained in Ref. [41] with the $\bar{K}N$ channel alone. The comments made above about the approximations done in [41] hold also in the present case.

Finally, we extend the discussion to some particular kinematic relevant for possible application of the presented formalism to reactions like $\gamma p \rightarrow K^+ \Lambda(1520)$ in nuclei where the Λ(1520) hyperon is produced with large momentum. In Fig. 6 the imaginary (b) and real (c) parts of the selfenergy
are shown for the $\Lambda(1520)$ moving in nuclear matter as a function of a three momentum $|P_\Lambda|$. We can see that the imaginary part of the $\Lambda(1520)$ selfenergy is not changed much from its value at zero momentum. However, the real part changes sign from $|P_\Lambda| = 0$ to $|P_\Lambda| \approx 1000$ MeV. But in both cases these changes are relatively small. Even if the experiment quoted above would be most suited to determine the $\Lambda(1520)$ width in the nucleus, there is already experimental information which allows us to get some hint on its size. The study performed in heavy ion collisions \[31, 32\] indicates that a better agreement of theory with experimental data is obtained assuming that about half of the $\Lambda(1520)$ produced are absorbed in the nucleus. Such a reduction can only be obtained with an in-medium width of tens of MeV as one can guess from comparison to studies done in $\phi$ production in the $pA$ reaction \[42\].

**CONCLUSIONS**

We have addressed the problem of the self energy of the $\Lambda(1520)$ and $\Sigma^*(1385)$ resonances in a nuclear medium and we have found relevant changes in the medium in both the real and imaginary parts, particularly in the latter. Considering the coupled channel character of the $\Lambda(1520)$ resonance, where the $\pi \Sigma^*$, $\bar{K}N$ and $\pi \Sigma$ channels play a very important role, particularly the $\pi \Sigma^*$, one finds pronounced changes in the width in the nuclear medium when the $\pi$ component is allowed to become a $p - h$ excitation and the $\bar{K}$ component a $\Lambda - h$, $\Sigma - h$ or $\Sigma^* - h$ excitations. All the three channels when these $p - h$ or $Y - h$ excitations are allowed, increase the width by about $15 \div 20$ MeV, a larger amount than the free width, as a consequence of which one obtains at the end a $\Lambda(1520)$ width at $\rho = \rho_0$ of about four to five times the free width.

Such a spectacular change should be in principle easily observable experimentally. It will be interesting to look at suitable reactions to measure this. Recently there have been interesting developments in this direction and experiments to measure changes in the $\phi$ width in the medium have been conducted \[43\] or are being proposed \[44\] by looking at the $A$ dependence of the $\phi$ production in nuclei. Theoretical calculations of this $A$ dependence show indeed that the method is suited and offers advantage over other methods \[42, 45\]. The investigation of these medium effects would be a novel and interesting enterprise which would shed much information of the nature of the $\Lambda(1520)$ resonance.
Acknowledgments

This work is partly supported by DGICYT contract number BFM2003-00856, and the E.U. EU-RIDICE network contract no. HPRN-CT-2002-00311. This research is part of the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RII3-CT-2004-506078.

EFFECTIVE SU(3) LAGRANGIAN

The lowest order chiral Lagrangian, coupling the octet of pseudoscalar mesons to the octet of $1/2^+$ baryons, is

\[ L_1^{(B)} = \langle \bar{B}i\gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B}B \rangle \]
\[ + \frac{1}{2}D\langle \bar{B}\gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{1}{2}F\langle \bar{B}\gamma^\mu \gamma_5 [u_\mu, B] \rangle \]

where \( \langle \rangle \) denotes the trace of SU(3) matrices and

\[ \nabla_\mu B = \partial_\mu B + [\Gamma_\mu, B] \]
\[ \Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u\partial_\mu u^\dagger) \]
\[ U = u^2 = \exp(i\sqrt{2}\Phi/f) \]
\[ u_\mu = iu^\dagger \partial_\mu U u^\dagger. \]

The SU(3) matrices for the mesons and the baryons are the following

\[
\Phi = \begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\
K^- & K^0 & -\frac{2}{\sqrt{6}}\eta
\end{pmatrix},
\]

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\
\Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\
\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda
\end{pmatrix}.
\]

In Eq. (38) \( D \) and \( F \) are the axial vector coupling constants and \( f \) is the pseudoscalar meson decay constant.

* Electronic address: kaskulov@ific.uv.es
† Electronic address: oset@ific.uv.es


