Critical review of $[K^- ppn]$ bound states

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We make a thorough study of the process of three body kaon absorption in nuclei, in connection with a recent FINUDA experiment which claims the existence of a deeply bound kaonic state from the observation of a peak in the $Ad$ invariant mass distribution following $K^-$ absorption on $^6Li$. We show that the peak is naturally explained in terms of $K^-$ absorption from three nucleons leaving the rest as spectators. We can also reproduce all the other observables measured in the same experiment and used to support the hypothesis of the deeply bound kaon state. Our study also reveals interesting aspects of kaon absorption in nuclei, a process that must be understood in order to make progress in the search for $K^-$ deeply bound states in nuclei.


I. INTRODUCTION

The possibility of having deeply bound $K^-$ states in nuclei is drawing much attention both theoretically and experimentally. The starting point to face this problem is obviously the understanding of the elementary $KN$ interaction, and lots of efforts have been devoted to this topic, mostly using unitary extensions of chiral perturbation theory [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The recent determination of the $K^-p$ scattering length from the study of $K^-p$ atoms in DEAR at DAΦNE [11] has stimulated a revival of the interest on this issue and several studies have already incorporated chiral Lagrangians of higher order [12, 13, 14, 15] in addition to the lowest order one used in [2, 3, 4].

Much work has also been done along these lines in order to study the interaction of kaons with nuclei, deducing $K^-$ nucleus optical potentials with a moderate attraction of about 50 MeV at normal nuclear matter density [14, 17, 18, 19, 20]. The selfconsistency of the calculation is an important requirement for the construction of the potential, due to the presence of the $\Lambda(1405)$ resonance below threshold, and is responsible for a fast transition from a repulsive potential in the $tt$ approximation at very low densities to an attraction at the densities felt by measured kaonic atom states. This “shallow” theoretical potential was shown to reproduce satisfactorily the data on shifts and widths of kaonic atoms [21]. However, reduced chi-squared values were obtained from phenomenological fits to kaonic atoms which favored strongly attractive potentials of the order of $-200$ MeV at the center of the nucleus [22]. A combination of theory and phenomenology was attempted in Ref. [23], where an excellent fit to the full set of kaonic atom data was found with a potential that deviated at most by 20% from the theoretical one of [17]. This potential also generated deeply bound $K^-$ nuclear states having a width of the order of 100 MeV, much bigger than the corresponding binding energy. The bound states would then overlap among themselves and with the continuum and, consequently, would not show up as narrow peaks in an experiment.

Other phenomenological potentials of sizable attraction (with potential depths around 100–200 MeV at $\rho_0$) that could in principle accommodate deeply bound states, have been discussed in [24, 25, 26, 27, 28, 29, 30, 31]. In these latter works a relativistic mean field approach is followed, introducing $\sigma$ and $\omega$ fields which couple to kaons and nucleons to obtain the $K$ nucleus optical potential. Less attractive potentials within this framework are also found in [32, 33, 34]. A new look at these relativistic mean field potentials from the perspective of the microscopic chiral unitary approach is presented in [35].

The opposite extreme has been represented by some highly attractive phenomenological potentials with about 600 MeV strength in the center of the nucleus [36, 37]. These potentials, leading to compressed nuclear matter of ten times nuclear matter density, met criticisms from [38] and more recently from [10]. The work of [38] met criticism from [39] concerning the ”narrow peak” predicted in [38], but actually the width of the peak was not calculated in [38]. It was calculated in [10] showing that it was not narrow and indeed agrees with the revised experiment of the KEK work as we shall discuss below. The criticisms of [39] were rebutted in [40] and more recently in [41].

Predictions of deeply bound $K^-$ states for few nucleon systems have been first done in [36, 37]. More recently, Faddeev-type calculations were made for the $\bar{K}NN$ system using phenomenological input in [42, 43] and a leading-order chiral interaction in [44]. Both studies found a $K^-NN$ quasibound state above the $\pi\Sigma N$ threshold with a relatively large width. A variational approach with phenomenological local potentials has also
been applied in [45] to study the $\bar{K}NN$ system, leading to a bound state at about 50 MeV below the $\bar{K}NN$ threshold. A more recent variational calculation [46,47] emphasizes the important role of the repulsive $NN$ interaction at short distances and obtains preliminary results having smaller bindings and larger widths than those found by the other earlier approaches.

On the experimental side the situation is still at a very early stage. Initial hopes that a peak seen in the $(K_{\text{stop}}, p)$ reaction on $^4$He [48] could be a signal of a $K^-$ bound in the trinucleon with a binding of 195 MeV gradually lost a support. First, an alternative explanation of the peak was presented in [38], showing that a peak with the strength claimed in the experiment was coming from $K^-$ absorption on a pair of nucleons going to $p\Sigma$, leaving the other two nucleons as spectators. This hypothesis led to the prediction that such a peak should be seen in other light or medium nuclei where it should be narrower and weaker as the nuclear size increases. This was confirmed with the finding of such a peak in the $(K_{\text{stop}}, p)$ reaction on $^6$Li, which already fades away in $^{12}$C nuclei at FINUDA [49]. In [38] the $K^-$ absorption was described as taking place from (np) pairs of the Fermi sea. In [49] the same explanation was given for the peaks suggesting that the (np) pairs would be correlated in "quasi"-deuteron clusters. The final development in this discussion has come from a new experiment of the KEK reaction of [48] reported in [50] where, performing a more precise measurement, which subsaned deficiencies in the efficiency corrections, the relatively narrow peak seen in [48] disappears and only a broad bump remains around the region where the peak was initially claimed. The position and width of this broad bump are in agreement with the estimations done in [40, 41] based on the kaon absorption mechanism of [38].

The second source of initial hope came from the experiment of the FINUDA collaboration [51], where a peak seen in the invariant mass distribution of $\Lambda p$ following $K^-$ absorption in a mixture of light nuclei was interpreted as evidence for a $K^- pp$ bound state, with 115 MeV binding and 67 MeV width. However, it was shown in [52, 53] that the peak seen could be interpreted in terms of $K^-$ absorption on a pair of nucleons leading to a $\Lambda p$ pair, followed by the rescattering of the $p$ or the $\Lambda$ on the remnant nucleus.

More recently, a new experiment of the FINUDA collaboration [54] found a peak on the invariant mass of $\Lambda d$ following the absorption of a $K^-$ on $^6$Li, which was interpreted as a signature for a bound $KNNN$ state with 58 MeV binding and 37 MeV width. These results are puzzling, since the bound state of the $K$ in the three nucleon system has significantly smaller values for the binding and width than those claimed for the bound state of the $K$ in the two nucleon system [51]. These unexpected results require serious thoughts but no discussion was done in [54].

About the same time as the FINUDA experiment [54] a similar experiment was performed at KEK [55], looking also at the $\Lambda d$ invariant mass following $K^-$ absorption but on a $^4$He target. The authors of this latter work do not share the conclusions of [54] concerning the association of the peak to a $\bar{K}$ bound state, and claim instead that the peak could be a signature of three body absorption.

In the present work we perform detailed calculations of $K^-$ absorption from three nucleons in $^6$Li and show that all features observed in the experiment of [54] can be well interpreted in the picture of three body kaon absorption, as suggested in [55], with the rest of the nucleons acting as spectators.

## II. MECHANISM FOR $K^-$ THREE BODY ABSORPTION

In the $K^- A \rightarrow \Lambda d A'$ reaction [54], at least three nucleons must participate in the absorption process. Two body $K^-$ absorption processes of the type $K^- NN \rightarrow \Sigma N(\Lambda N)$ have been studied experimentally in [54] and their strength is seen to be smaller than that of the one body absorption $K^- N \rightarrow \pi \Sigma(\pi \Lambda)$ mechanisms. This result follows the argument that it is easier to find one nucleon than two nucleons together in the nucleus. This is also the case in pion absorption in nuclei, where extensive studies, both theoretical [57] and experimental [58], obtain the direct two and three body absorption rates with the former one dominating over the later, particularly for pions of low energy. We follow here the same logics and assume the process to be dominated by direct three body $K^-$ absorption, the four body playing a minor role.

The former assumption means in practice that the other three nucleons not directly involved in the absorption process will be spectators in the reaction. These three spectator nucleons have to leave the nucleus, but they were bound in $^6$Li. The nuclear dynamics takes care of this since there is a distribution of momenta and energies in the nucleus, and the ejection of either three nucleons, a $nd$ pair or tritium, implies that the absorption is done in the most bound nucleons.

The other element of relevance is the atomic orbit from which the kaon is absorbed. This information is provided by the last measured transition in the X-ray spectroscopy of $K^-$-atoms, which occurs precisely because absorption overcomes the $\gamma$ ray emission. In the case of $^6$Li this happens for the $2p$ atomic state [21, 22].

Following the line of studies done for pion absorption and other inclusive reactions [61], we describe the nucleus in terms of a local Fermi sea with Fermi momentum $k_F(r)$. The nucleons move in a mean field given by the Thomas Fermi potential

$$V(r) = -\frac{k_F^2(r)}{2m_N}, \quad k_F(r) = \left(\frac{3\pi^2}{2}\rho(r)\right)^{1/3},$$

where $m_N$ is the nucleon mass and $\rho(r)$ is the local nucleon density inside the nucleus.
Thus the energy of the Λ system, the other three nucleons acting as spectators.

K mass which, for each K

cisely the invariant mass of the corresponding

dons. And we determine ∆ = 7 MeV. In the above expression

m_{K^-} , M_t , M_{Li\text{ }}are the masses of the corresponding

and nuclei.

The probability of K^- absorption by three nucleons will be determined from the third power of the nuclear density as

\[ \Gamma \propto \int d^3r |\Psi_{K^-}(r)|^2 \rho^3(r) , \]

where |\Psi_{K^-}(r)| is the K^- atomic wave function. In order to take into account the Fermi motion we write the density as \[ \rho(r) = 4 \int \frac{d^3p}{(2\pi)^3} \Theta(k_F(r) - |p|) \] and then we obtain

\[ \Gamma \propto \int d^3r d^3p_1 d^3p_2 d^3p_3 |\Psi_{K^-}(r)|^2 \times \]

\[ \times \Theta(k_F(r) - |p_1|) \Theta(k_F(r) - |p_2|) \Theta(k_F(r) - |p_3|) . \]

From this expression we can evaluate all observables of the reaction. Let us first concentrate on the Λd invariant mass which, for each K^-NNN → Λd decay event, is precisely the invariant mass of the corresponding K^-NNN system, the other three nucleons acting as spectators. Thus the energy of the Λd pair is obtained from

\[ E_{\Lambda d} = E_{K^-NNN} \equiv E_{K^-} + E_N + E_{N_2} + E_{N_3} \]

\[ = m_{K^-} + 3m_N + \frac{p_1^2}{2m_N} + \frac{p_2^2}{2m_N} + \frac{p_3^2}{2m_N} - 3 \frac{k_F^2(r)}{2m_N} - 3\Delta \]

and the momentum from

\[ P_{\Lambda d} = P_{K^-NNN} = p_1 + p_2 + p_3 , \]

and, correspondingly,

\[ M_{\Lambda d} = E_{\Lambda d} - \frac{P_{\Lambda d}^2}{2E_{\Lambda d}} . \]

One may also easily obtain the invariant mass of the residual system, \( M^* \), from

\[ M^* = E^* - P^* \frac{2}{2E^*} , \]

with

\[ E^* = m_{K^-} + M_{6Li} - E_{K^-NNN} , \quad P^* = -P_{\Lambda d} . \]

Each event in the multiple integral of Eq. (4), done with the Monte Carlo method, selects particular values for \( r, p_1, p_2 \) and \( p_3 \) which, in turn, determine the value of the corresponding \( \Lambda d \) invariant mass from Eqs. (5)–(7).

Since the minimum obvious invariant mass of the residual three-nucleon system is \( M^* = M_t \), corresponding to the emission of tritium, the cut \( \Theta(M^* - M_t) \) is also imposed for each event. A compilation of events provides us with the \( \Lambda d \) invariant mass distribution. We also directly obtain the distribution of total \( \Lambda d \) momentum, Eq. (6), to be directly compared with the \( \Lambda d \) momentum measured in [54].

Please note that the model presented here is a straightforward generalization (from two nucleon to three nucleon \( K^- \) absorption) of the one used in Refs. [52, 53], however here we concentrate on the primary reaction peak, while in Refs. [52, 53] the authors were more interested in the peak generated by the final state interactions, i.e. by the collisions of the primary produced Λ and p on their way out of the nucleus. Since the two nucleon \( K^- \) absorption, discussed in [52, 53], was measured for heavier nuclei [51] the final state interaction peak was stronger than that of the primary reaction, contrary to the reaction studied in this work.

Other observables measured in [54] require an additional work. One is the angular correlation of \( \Lambda d \) pairs, and the other is the missing mass assuming a residual \( nd \) system, apart from the measured \( \Lambda d \) pair, namely

\[ T_{miss} = m_{K^-} + M_{6Li} - m_{\Lambda} - m_n - 2M_d - (T_{\Lambda} + T_d) \]

where \( m_{\Lambda}, M_d \) and \( T_{\Lambda}, T_d \) are the masses and the kinetic energies of the Λ and the d, correspondingly. These two observables require the evaluation of the individual Λ and d momenta in the laboratory frame. Their value in the center of mass (CM) frame of the \( \Lambda d \) pair is given in terms of the known invariant mass but their direction in this frame is arbitrary. We take this into account by obtaining Λ and d momenta in the CM frame

\[ p_{\Lambda}^{CM} = p_{\Lambda}^{CM} (\sin \Theta \cos \phi, \sin \Theta \sin \phi, \cos \Theta) , \]

\[ p_{d}^{CM} = -p_{\Lambda}^{CM} , \]

with

\[ p_{\Lambda}^{CM} = \frac{\lambda^{1/2}(M_{\Lambda d}^2, m_{\Lambda}^2, M_d^2)}{2M_{\Lambda d}} , \]

where the events are now generated according to the distribution provided by the integral

\[ \int d\cos \Theta \int d\phi \int d^3r d^3p_1 d^3p_2 d^3p_3 |\Psi_{K^-}(r)|^2 \]
about 36 MeV, as reported in the experiment. Note that we are exploring in the present work. We obtain a width of 3280 MeV. As we can see in the figure, the distribution is strongly peaked backward and the agreement with experiment is very good.

The angular correlations between the emitted \( \Lambda \) and \( d \) can be seen in the distribution displayed in Fig. 2, where, as in the experimental analysis, we consider only those events which fall in the region \( 3220 \text{ MeV} < M_{\Lambda d} < 3280 \text{ MeV} \). As we can see in the figure, the distribution is strongly peaked backward and the agreement with experiment is very good.

The distribution of the total \( \Lambda d \) momentum in the mass range of the bump is shown in Fig. 3. The experimental paper does not show a distribution but quotes that it peaks around 190 MeV/c, which is precisely the region where the peak of our calculated spectrum lies.

Our results for the missing mass distribution, defined by Eq. (10), are compared with the experimental data in Fig. 4. As we can see, the agreement with experiment is reasonably good within the large experimental errors. We should remark here that the peak in Fig. 4 was associated in \cite{54} to the mechanism of \( K^- \) absorption in a \( ^4\text{He} \) cluster, namely \( K^- + \alpha(d) \rightarrow \Lambda d \alpha(d) \), motivated by the assumption that the \( ^6\text{Li} \) nucleus is largely made of a \( \alpha \) particle and a deuteron. As we can see, our approach, which relies upon three body absorption, reproduces the data which, thus, cannot be taken as evidence for the mechanism claimed in \cite{54}.

\begin{equation}
\times \Theta(k_F(r) - |p_1|) \Theta(k_F(r) - |p_2|) \Theta(k_F(r) - |p_3|) \\
\times \Theta(M^* - M_1). \tag{13}
\end{equation}

In order to have the final \( \Lambda \) and \( d \) momenta in the laboratory frame, where the \( \Lambda d \) pair has momentum \( P_{\Lambda d} \), we apply the transformations

\begin{equation}
\begin{align*}
P_\Lambda &= P^{CM}_\Lambda + m_\Lambda v \\
P_d &= -P^{CM}_\Lambda + M_d v,
\end{align*} \tag{14}
\end{equation}

where \( v = P_{\Lambda d}/(m_\Lambda + M_d) \). These last equations allow us to find the cosine of the angle between the directions of \( \Lambda \) and \( d \). Therefore, generating the distribution of events according to their relative angle is straightforward. We will see, as it is also the case of the experiment, that \( P_{\Lambda d} \sim 200 \text{ MeV/c} \), while \( P^{CM}_\Lambda \sim 650 \text{ MeV/c} \), which already guarantees that the \( \Lambda d \) events will be largely correlated back-to-back.

We note that our calculations incorporate the same momentum cuts as in the experiment, namely 140 MeV/c < \( p_\Lambda < 700 \text{ MeV/c} \) and 300 MeV/c < \( p_d < 800 \text{ MeV/c} \).

### III. RESULTS

In Fig. 1 we show the results for the invariant mass of the \( \Lambda d \) system. Our distribution, displayed with a dot-dashed line, peaks around \( M_{\Lambda d} = 3252 \text{ MeV} \) as in the experiment. The shape of the distribution also compares remarkably well with the experimental histogram in the region of the peak, which is the energy range that we are exploring in the present work. We obtain a width of about 36 MeV, as reported in the experiment. Note that apart from the peak that we are discussing, the experiment also finds events at lower \( \Lambda d \) invariant masses which did not play a role in their discussion \cite{54}. These events would be generated in cases where there is final state interaction of the \( \Lambda \) or the \( d \) with the rest of the nucleons, as was discussed in \cite{52, 53}, or through other absorption mechanisms, but this is not the object of discussion here, as well as in \cite{54}.

\[ K^-^6\text{Li} \rightarrow \Lambda d A' \]

FIG. 2: (Color online) The \( \Lambda d \) angular distribution for the \( K^-\text{stop} A \rightarrow \Lambda d A' \) reaction. Histogram and error bars are from the experiment \cite{54}, while the dot-dashed curve is the result of our calculation. As in the experimental analysis, we take into account the following cuts: 3220 MeV < \( M_{\Lambda d} < 3280 \text{ MeV} \).

The angular correlations between the emitted \( \Lambda \) and \( d \) can be seen in the distribution displayed in Fig. 2 where, as in the experimental analysis, we consider only those events which fall in the region \( 3220 \text{ MeV} < M_{\Lambda d} < 3280 \text{ MeV} \). As we can see in the figure, the distribution is strongly peaked backward and the agreement with experiment is very good.

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FIG. 3: (Color online) The $\Lambda d$ momentum distribution for the $K^-\Lambda d A' \rightarrow \Lambda d A'$ reaction. The calculation implements the cut $3220 \text{ MeV} < M_{\Lambda d} < 3280 \text{ MeV}$.

FIG. 4: (Color online) The missing mass distribution for the $K^-\Lambda d A' \rightarrow \Lambda d A'$ reaction. Histogram and error bars are from the experimental paper [54], the full curve is the result of our calculation.

An alternative way to present this information in a way more closely related to the mechanism we have, is by looking at the invariant mass distribution of the residual three particle state (the spectator nucleons in our case). This is shown in Fig. 5, where the invariant mass is measured with respect to $M_t$, a natural threshold which is imposed in our formalism. We observe a peak in the mass distribution at energies around 20 MeV, clearly higher than the tritium binding energy of 8.48 MeV. This means that there is not only room for $t$ production, but also for $dn$ production, as assumed to be the case in [54], and for uncorrelated three nucleon emission.

In Fig. 6 we present the momentum distribution of the $\Lambda$, which we can also compare with the experimental observations. For the results shown in the figure we removed the momentum cuts, $140 \text{ MeV/c} < p_\Lambda < 700 \text{ MeV/c}$ and $300 \text{ MeV/c} < p_d < 800 \text{ MeV/c}$. We observe that the $\Lambda$ momentum peaks around 635 MeV/c and most of the events are contained in the region between 450 MeV/c and 700 MeV/c, as also found in the experiment. All our events are contained within the experimental window for $p_d$ momentum.

FIG. 5: (Color online) The invariant mass distribution of the residual system, Eq. (8), for the $K^-\Lambda d A' \rightarrow \Lambda d A'$ reaction.

FIG. 6: (Color online) The $p_\Lambda$ distribution for the $K^-\Lambda d A' \rightarrow \Lambda d A'$ reaction.
IV. EMPIRICAL QUALITATIVE DISCUSSION
OF THE STRENGTH OF THE REACTION

In addition to the observables discussed in the previous section the yield of the observed peak is also given in [54] as $Y_{\Lambda d} = (4.4 \pm 1.4) \times 10^{-3}/K_{stop}$. In the former discussion we did not use a specific dynamical model for the absorption of the $K^-$ by three nucleons. This is, we did not use specific Lagrangians and a set of Feynman diagrams which would have given us the strength of the absorption process. In experimental studies the yield is simply the number of events for a particular channel per stopped $K^-$. In contrast a theoretical determination of the yield of a process, or in other words the fraction of the total rate that goes into a particular channel, requires the calculation of all possible reaction processes. Clearly this is a very hard and time demanding task. Indeed, the experimental work of $K^-$ absorption on $^4He$ [54] quotes in Table III a list of 21 reactions following $K^-$ absorption by one nucleon (mesonic) and multinucleon (nonmesonic) mechanisms, and this represents only a fraction of the total. In many cases, one has nuclei in the final state which complicates further an eventual theoretical calculation. The present theoretical situation is such that the microscopic mechanisms for two nucleon kaon absorption are only available from [17, 18]. There is no work done on three nucleon $K^-$ absorption, and the scarce theoretical work on three nucleon absorption of pions [54, 58] is a reflection of the intrinsic theoretical difficulties that any microscopic evaluation involves. This, together with the enormous amount of physical channels that one would have to evaluate to produce the relative yield of just one of them, describes clearly the horizon of such a goal.

In view of this horizon the work presented here becomes even more valuable, because it has demonstrated that the observables presented in [54] to support the idea of a kaon bound state could be reproduced with just the kinematics of the three body absorption mechanism, and the detailed dynamical mechanisms that would have allowed us an evaluation of the absolute strength of the reaction were never needed. We also note that the yield of the peak was not offered as a proof for the advocated $K^-$ bound state in [54], since the strength itself provides no information on the mass and width which are the characteristics of a physical state.

However, we shall make some instructive discussion about this yield from the empirical point of view with the only purpose to gain some knowledge on $K^-$ absorption. This will illustrate that experiments like the one about this yield from the empirical point of view with the only purpose to gain some knowledge on $K^-$ absorption. This will illustrate that experiments like the one we are discussing provide indeed valuable information on $K^-$ absorption worth giving some thoughts to.

We start from the yield of $3.5 \pm 0.2\%$ quoted in [64] for the channel $K^- \, ^4He \rightarrow \Lambda nd$, but from only two nucleon $K^-$ absorption, since it was guaranteed that the produced deuteron was a spectator. The estimated yield does not give yet any information on three body absorption. Indeed, two situations can be envisaged for the $K^- \, ^4He \rightarrow \Lambda nd$ reaction: two body absorption $K^-pn \rightarrow \Lambda n$, which produces a slow (spectator) deuteron, [64], or three body absorption $K^-ppn \rightarrow \Lambda d$, which leaves a neutron as a spectator. In this latter case the deuteron would be produced basically back to back with the $\Lambda$ and would have a relatively large momentum. In order to get the strength of the three body absorption process we need extra information, which can be found by looking to the $\Lambda$ momentum spectrum for the $K^- \, ^4He \rightarrow \Lambda nd$ reaction shown in Fig. 2a of [56], and which we reproduce in Fig. 7. In the figure we have inserted two arrows indicating the average momentum of the $\Lambda$ for the two situations described before which are about 550 MeV/c for the two body absorption and 650 MeV/c for the three body absorption. Obviously the three body absorption case is penalized dynamically for two reasons: the three body absorption amplitude should be smaller than the two body one, and forming a $d$ from an excited $\Lambda np$ system after three body absorption should be more difficult than forming a $d$ from a spectator $np$ system in the case of two body absorption. The experimental distribution, with admitted poor statistics, is still significant in as much as it shows strength below the peak for two body absorption. In order to have such events we must invoke some extra collision of the $\Lambda$ with the remnant two body spectator following the dominant two body absorption, which would remove energy from the $\Lambda$ leading to smaller $\Lambda$ momenta. The small experimental bump around 650 MeV/c for the $\Lambda$ momentum should be then attributed to the three body absorption process.

We can make a rough estimate of 8/51 events for three body absorption to the total $K^- \, ^4He \rightarrow \Lambda nd$ yield, or 8/43 for the three body to two body absorption ratio. This, together with the result from [64], gives us a rate of 0.65% for the three body absorption, with large uncertainties from the poor statistics of the $\Lambda$ momentum spectrum of [56] (of the order of 40% from the counts reported). This number is also consistent with the rate
of about 1% provided for the $K^-d$ reaction with high momentum deuterons in \cite{54}, interpreted there as indicative of three body absorption.

We are aware that $^6Li$ is different from $^4He$ and the rates could vary from one nucleus to the other. Among other possibilities, the deuteron breakup in the final state in $^6Li$ could reduce this rate somehow. Therefore, the qualitative estimate that we have made for the three body absorption based upon the experimental data of \cite{56, 64} in $^4He$ agrees qualitatively with the yield of 0.44% ± 0.14% provided for the peak of the \cite{54} experiment, which we have attributed to three body absorption in our kinematical study of the former sections.

V. CONCLUSIONS

We have performed a detailed study of the kinematics following three-nucleon $K^-$ absorption in $^6Li$ to reanalyze a recent FINUDA experiment, which claims the existence of a deeply bound kaonic state from the observation of a peak in the $Ad$ invariant mass distribution.

Since we are looking at an inclusive process, where a detailed knowledge on the nuclear structure is not needed, we have used a local Fermi sea model for the nucleus, which includes the following basic features: 1) the nucleus is represented by a local Fermi sea, 2) a kaon from the atomic 2p orbit of $^6Li$ is allowed to be absorbed by three nucleons with momenta chosen randomly within the local Fermi sea, 3) these nucleons are bound by a Thomas Fermi potential with an additional gap energy, 4) the total momentum and energy of the initial chosen to respect the threshold of the reaction, 4) the Thomas Fermi potential with an additional gap energy provided for the peak of the $Ad$ experiment, which we have attributed to three body absorption in our kinematical study of the former sections.

We have been able to reproduce all the basic features observed in the experiment of Ref. \cite{54}, namely the invariant mass distribution of $Ad$ pairs, the highly correlated back-to-back angular distribution between the $\Lambda$ and the $d$, the distribution of missing mass with respect to a final $nd$ system, apart from the measured $Ad$ pair, and the momentum distributions of the individual $\Lambda$ and $d$, as well as that of the combined $Ad$ pair. In particular, the study served to show that the peak in the $Ad$ invariant mass distribution observed in the experiment could be naturally reproduced within a three-nucleon $K^-$ absorption mechanism, thus concluding that this observation cannot be used as an evidence for the existence of a $K$ bound on a tribaryon, as was done in \cite{54}.

On the positive side, the exercise served to go one step forward in the understanding of the process of kaon absorption in nuclei, in this case looking at the three body mechanism. This interesting phenomenon deserves a special attention by itself. Looking at the amount of work that was invested in the understanding of pion absorption in nuclei, both theoretical and experimental \cite{57, 59}, we can only be satisfied to see that, if some experiments have been done for reasons which could not be supported a posteriori, they are paving the road for gradually achieving a more complete understanding of the phenomenon of kaon absorption in nuclei, which is necessary for progress in the same search for possible deeply bound kaon states.

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