Inclusive Neutrino Scattering in $^{12}\text{C}$: Implications for $\nu_\mu$ to $\nu_e$ Oscillations

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Abstract

We study inclusive $\nu_e$ and $\nu_\mu$ cross sections in $^{12}\text{C}$ in a theory that takes into account significant nuclear renormalization of strengths. Our calculation is in excellent agreement with the measured inclusive muon capture rate and the flux-averaged $\nu_e$ cross section, but overestimates the flux-averaged $\nu_\mu$ inclusive cross section. These reactions are of crucial relevance to the issue of possible $\nu_\mu$ to $\nu_e$ oscillations. We also calculate the flux-averaged cross sections in $^{13}\text{C}$ and $^{27}\text{Al}$, which are found to be consistent with the available experimental result.

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I. INTRODUCTION

Extracting an interesting physics signal of some rare events from an experiment crucially depends on our ability to understand the physics background in that setting. A good example of this is the search for neutrino oscillations, and hence physics beyond the standard model, in the Liquid Scintillator Neutrino Detector (LSND) related experiments, with profound implications for particle physics, nuclear physics and astrophysics. The LSND group have advanced evidence for the $\nu_\mu \rightarrow \nu_e$ oscillations in a recent experiment with neutrinos from pion decays in flight [1]. Our trust in this claim depend crucially upon the performance of these experiments in benchmark reactions on nucleons and nuclei initiated by neutrinos. Some of these benchmark reactions are exclusive ones on proton, $^{12}$C and $^{16}$O targets, while others are inclusive processes in complex nuclei. Given separate claims of agreements with experiment and lack of it in recent theoretical calculations [2, 3, 4, 5, 6] of inclusive neutrino reactions in $^{12}$C, this is of topical interest and deserves a careful examination.

In this paper, we are going to discuss inclusive neutrino reactions in nuclear targets $^{12}$C, $^{13}$C and $^{27}$Al, concentrating on the theory of the reactions [7]

\begin{align*}
^{12}C + \nu_e &\rightarrow X + e^- \\
^{12}C + \nu_\mu &\rightarrow X + \mu^-,
\end{align*}

where the produced $X$ is not observed. Thus, generically, we can indicate the reactions (1,2) as $^{12}C(\nu_l, l^-)X$ where $X$ is the unspecified nuclear state and $l$ is the charged lepton. At low energies, the reaction (1) has been measured by the E225 [8] and LSND [9, 10] experiments at the Los Alamos Meson Physics Facility (LAMPF) and the experiments by KARMEN [11, 12] collaboration at the ISIS facility, with $\nu_e$ beam from muon decays at rest (DAR), given by the Michel spectrum. The flux-averaged cross sections from the various experiments are given in Table I. Particularly interesting to us are the most recent values of the cross section for the reaction (1)

\[ \sigma(\nu_e) = (14.8 \pm 0.7 \pm 1.1) \times 10^{-42} \text{ cm}^2, \] (3a)

from the LSND experiment [10] and

\[ \sigma(\nu_e) = (15.2 \pm 1.0 \pm 1.3) \times 10^{-42} \text{ cm}^2, \] (3b)

from the KARMEN collaboration [12]. In Eq.(3), the inclusive cross sections are obtained by adding the cross sections leading to ground state and excited states, with their errors added in quadrature. Within their errors, these experimental results overlap. The reaction (2), on the other hand, has been measured over the last five years at LAMPF [13, 14, 15] using $\nu_\mu$ beam from pion decays in flight (DIF). The earlier E764 experiment of Koetke et al. [13] used a neutrino beam of slightly higher energy than used in later experiments and gave a cross section for the reaction (2) too large when compared to the theoretical predictions [2, 3, 4, 5, 6]. This reaction was further studied by Albert et al. [14] with more massive detectors and larger
exposure than used in \[13\], using a beam of neutrinos with energy around 180 MeV. These authors obtained considerably smaller cross section than that by Koetke et al. The recent studies by the LSND collaboration \[15\] have improved the experimental situation, providing the latest measured value for this cross section. We give, in Table I, the results of all of these experiments, but use in our theoretical discussion the latest result, reported by the LSND collaboration \[15\]

\[
\sigma(\nu_\mu) = (11.2 \pm 0.3 \pm 1.8) \times 10^{-40} \text{cm}^2. \tag{4}
\]

We also calculate the flux-averaged neutrino cross-sections in $^{13}C$ and $^{27}Al$ as benchmarks for our theory. These are found to be reasonably consistent with the available experimental results \[8\].

Current generation of calculations of the above reactions can be grouped into two classes, depending on their predictions for the flux-averaged cross sections for the process (2): those that substantially exceed this observed cross section \[2, 3, 7\] and others that find agreement \[4, 6\]. The deficit of the flux-averaged cross sections may be: (1) a manifestation of theoretical problem, of not being able to do a correct enough nuclear calculation; (2) an experimental problem of not doing a precise and reliable enough experiment. Since both theoretical calculations and experimental analysis are involved in the determination of the excess events in the $\nu_e$ channel in the LSND neutrino oscillation experiment \[1\], it is important to have a clear understanding of the nuclear physics related uncertainties in this reaction. The purpose of this paper is to narrow down the options by examining the first point very critically from our point of view \[7\].

A reaction which can be regarded as a benchmark in the context of the processes (1) and (2) in general, and (2) in particular, is the nuclear capture of muons (NMC) from the atomic $1S$ state by the charged weak current \[16, 17\]:

\[
^{12}C + \mu^- (1S) \rightarrow X + \nu_\mu. \tag{5}
\]

This process serves as an excellent check, in the low and intermediate energy transfer region, in our ability to control the theoretical uncertainties \[17\]. The inclusive NMC rate $\Lambda_c$ for the process (5) is very accurately known. Taking the world average of the best experimental determinations of the inclusive muon capture rate $\Lambda_c$, \[18, 19\] with their errors added in quadrature, we obtain \[17\],

\[
\Lambda_c = (3.80 \pm 0.10) \times 10^4 \text{s}^{-1}. \tag{6}
\]

Thus, we have here a weak reaction rate, which is closely related to the processes (1) and (2) and is known at an accuracy of about 2.5%, posing a tremendous challenge to the theoretical approaches to weak nuclear reactions in nuclei.

Various theoretical approaches to calculate reactions (1) and (2) \[2, 3, 4, 5\], when applied to the inclusive muon capture, reproduce the NMC rate $\Lambda_c$ quoted in Eq.(6) rather well, within the limits of their theoretical accuracy \[4, 18, 21\]. In the case of inclusive neutrino reactions, however, the situation is different. The calculations of Kolbe et al. reproduce the $(\nu_e, e^-)$ rather well, but overestimate the $(\nu_\mu, \mu^-)$ by about
Auerbach et al., on the other hand, use the set of parameters for their model, which explain the inclusive muon capture rate. They predict cross sections for the neutrino reactions, which are 20% and 15% larger than the experimental values for the reactions (1) and (2) respectively, quoted in Eqs.(3) and (4). These overestimates are from the maximum experimental values allowed within the quoted errors. Therefore, they represent a real discrepancy with the experiment. It is possible to explain the observed neutrino cross sections in the calculations of Auerbach et al. with another version of their residual nuclear forces by varying the model parameters, but this version predicts a NMC capture rate of $3.09 \times 10^4 s^{-1}$, which is rather small compared to the value quoted in Eq.(6).

Finally, to complete this survey, we would like to mention that a Fermi gas model calculation with a Fermi momentum $k_F = 225 \text{ MeV}$ gives a much higher value of $24.1 \times 10^{-40} \text{ cm}^2$ for the flux-averaged $(\nu_\mu, \mu^-)$ cross section. This is reduced to $22.7 \times 10^{-40} \text{ cm}^2$, when the effects of meson exchange currents are taken into account [3]. In another calculation, the method of so-called “elementary particle model”, extended to inclusive reactions, has been used, to obtain a cross section of $13.1 \times 10^{-40} \text{ cm}^2$ [3], which is in good agreement with the experimental value. However, the extension of the elementary particle model to the inclusive reactions makes use of several assumptions, which have not been tested in the energy region of the LSND experiments. Here this method is expected to underestimate the inclusive cross sections [3, 4, 22].

This survey brings us to our calculation, which we briefly describe in section II and discuss the results and conclusions in section III.

II. FORMALISM

The matrix element for the neutrino nucleon reaction for a neutrino of flavor $l, (l = e, \mu)$, i.e.

$$\nu_l(k) + n(p) \rightarrow l^-(k') + p(p'),\quad (7)$$

is given by

$$T = \frac{G\cos \theta}{\sqrt{2}} \bar{u}(k') \gamma^\mu (1 - \gamma_5) u(k) J_\mu,\quad (8)$$

where

$$J_\mu = \bar{u}(p') [F_1^V(q^2) \gamma_\mu + F_2^V(q^2) i\sigma_{\mu\nu} \frac{q^\nu}{2M} + F_A^V(q^2) \gamma_\mu \gamma_5 + F_P^V(q^2) q_\mu \gamma_5] u(p).\quad (9)$$

In Eq. (9), $q_\mu = k_\mu - k'_\mu$, is the four momentum transfer, $F_1^V, F_2^V, F_A^V$ and $F_P^V$ are the known weak nucleon form factors. The double differential cross section $\sigma_0(q^2, k')$, is then given by

$$\sigma_0(q^2, k') = \frac{k'}{4\pi EE'} \frac{M^2}{E_n E_p} \sum |T|^2 \delta(E - E' + E_n - E_p),\quad (10)$$
where $\Sigma \Sigma |T|^2$ represents the sum and the average respectively over the final and the initial spins of the leptons and the nucleons and is evaluated exactly, using the matrix element $T$ defined in Eq.(8). Its analytic expression is given in ref. [1]. In a nucleus, the neutrino scatters from a neutron moving in the finite nucleus of neutron density $\rho_n(\vec{r})$, with a local neutron occupation number $n_n(\vec{p}, \vec{r})$. Then the cross section in the local density approximation and in the free nucleon picture is given by

$$\sigma(q^2, k') = 2 \int d\vec{r} \int d\vec{p} \frac{n_n(\vec{p}, \vec{r})}{(2\pi)^3} \sigma_0(q^2, k'),$$

where the neutron energy $E_n$ and the proton energy $E_p$ in the expression of $\sigma_0(q^2, k')$, given in Eq.(10), are replaced by $E_n(p)$ and $E_p(p')$ respectively, $p$ and $p'$ being the momenta of the neutron and proton in the nucleus. However, neutrons and protons are not free and their momenta are constrained to satisfy the Pauli principle i.e. $p < p_{Fn}$ and $p' > p_{Fp}$, where $p_{Fn}$ and $p_{Fp}$ are the local Fermi momenta, given by

$$p_{Fn} = [3\pi^2 \rho_n(r)]^{1/3} \text{ and } p_{Fp} = [3\pi^2 \rho_n(r)]^{1/3}.$$  \hspace{1cm} (12)

Moreover, in the finite nucleus, there is a threshold energy for the reaction to proceed, also called the Q-Value, and this should be taken into account. Finally, the charged lepton produced in reactions (1) and (2) moves in the nucleus and its energy is modified by the Coulomb interaction, which should be accounted for. In our approach, these effects are incorporated by modifying the argument of the $\delta$ function in Eq. (10), from $E - E' + E_n - E_p$ to $E - E' - V_c(\vec{r}) + E_n(p) - E_p(p')$, and replacing the factor $\int \frac{d\vec{p}}{(2\pi)^3} n_n(\vec{p}, \vec{r}) \frac{M^2}{E_n E_p} \delta(E - E' + E_n - E_p)$ occurring in Eq. (11) by $-\frac{1}{2} \text{Im} U(q_0, \vec{q})$, where

$$q_0 = E - E' - V_c - Q + Q'.$$  \hspace{1cm} (13)

In Eq.(13), $V_c(\vec{r})$ is the Coulomb energy of the lepton and $Q' = E_{Fp} - E_{Fn}$, is introduced to take into account the unequal Fermi sea in the case of $N \neq Z$ nuclei. $U(q_0, \vec{q})$ is the Lindhard function given by

$$U(q_0, \vec{q}) = \int \frac{d^3p}{(2\pi)^3} \frac{n_n(\vec{p})[1 - n_p(\vec{p} + \vec{q})]}{q_0 + E_n(\vec{p}) - E_p(\vec{q} + \vec{p}) + i\epsilon} \frac{M^2}{E_n E_p}.$$  \hspace{1cm} (14)

With these modifications, the total cross section $\sigma(E_\nu)$ is given as [7]:

$$\sigma(E_\nu) = -\frac{4}{\pi} \int_0^\infty r^2 dr \int_{p_{\text{min}}^0}^{p_{\text{max}}^0} k'^2 dk' \int_{-1}^1 d(\cos \theta) \frac{1}{E_\nu E_l} \times \Sigma \Sigma |T|^2 \text{Im} U[E_\nu - E_l - Q + Q' - V_c(r), \vec{q}] \Theta[E_l + V_c(r) - m_l].$$  \hspace{1cm} (15)

The kinematic limits $p_{\text{min}}^{0, \text{max}}$ for the lepton momentum $k'$ are easily computed in our special case[7]. For the numerical integrations, we use Gaussian quadrature with high enough accuracy for our purpose. The radial integration in Eq. (15) is performed up to a radius $R = c_1 + 5 \text{fm}$, where $c_1$, is the radius parameter in the two-parameter harmonic oscillator and Fermi density distributions used for the nuclei, considered in section III.
The renormalization of weak currents in the nuclear medium is taken into account by calculating the effect of propagation of the particle-hole (ph) excitations in the nuclear medium on various terms occurring in $\Sigma\Sigma |T|^2$. The ph response is then replaced by a Random Phase Approximation (RPA) response accounting for the ph and the $\Delta h$ components, which interact through an effective spin-isospin nuclear interaction described by the Landau-Migdal potential. The details of this renormalization procedure as well as those of Eq. (15) are given in [7]. However, we have made here the following improvements, which considerably reduce the theoretical uncertainties in our calculations from the previous versions of our model: (1) Our new Lindhard function makes use of a strategy [23] that avoids the pathologies of the ordinary [7] Lindhard functions in the limit of $q_0, \vec{q}$ both going to zero, $q_\mu$ being the momentum transferred to the nucleus in the processes of interest. (2) The nuclear response function is renormalized by the ph and the $\Delta h$ correlations in nuclei [7], effects of which are quite large for low and intermediate energy neutrino scattering. The physics of this renormalization depends, among other things, on the Landau-Migdal spin-isospin parameter $g'$ [24]. This itself has an uncertainty of $\pm0.1$ around its preferred value of 0.7 [25]. We take into account the theoretical uncertainties of our estimates of $\sigma(\nu_e)$ and $\sigma(\nu_\mu)$ due to this variation of $g'$. (3) Finally, the target nucleus $^{12}$C has intrinsic parametric uncertainties in the radial density function. The effect due to this uncertainty in our cross-section estimate is taken into account by repeating our calculation in several radial parametric settings [26]. Overall, we achieve a theoretical accuracy around $\pm10\%$ for $\sigma(\nu_e)$ and $\sigma(\nu_\mu)$. This significant improvement in theoretical accuracy is even better in the case of the NMC rate for the inclusive process (5) [18]. We estimate here an uncertainty of $\pm2\%$ due to nuclear radial effects and $\pm5\%$ due to the variation of the spin-isospin parameter $g'$. Treating these two uncertainties independently and adding them in quadrature, we get a theoretical error of about $\pm6\%$ and obtain [18]

$$\Lambda_c = (3.60 \pm 0.22) \times 10^4 \text{s}^{-1},$$

(16)

it in excellent agreement with the precise experimental data (Eq.6). The inclusive nuclear muon capture provides us with a critical benchmark, an independent accurate check of our ability to describe nuclear inclusive weak processes clearly related to the neutrino scattering.

In summary, our method used in this paper is essentially an RPA approach built up from single particle states of an uncorrelated local Fermi sea. This method is, in practice, found to be a very accurate tool, when the excitation energy is sufficiently large such that relatively many states contribute to the process, in particular, if a large fraction of it comes from excitation to the continuum, as it is in the present case. The adaptation of this method to finite nuclei via the local density approximation has proved to be rather advantageous to deal with inclusive reactions and has been successfully applied to the photonuclear reactions [27], electron scattering [28], deep inelastic scattering [29] and muon capture [18, 30].

The numerical evaluation of the neutrino-nucleus reaction cross section is done using Eq.(15) and the results are presented in section III below.
III. RESULTS AND CONCLUSIONS

In order to compare with the experimental results of KARMEN\[12\] and LSND \[8\] collaborations, we compute the flux-averaged cross section,

$$\bar{\sigma} = \frac{\int_{E_0}^{E_{\nu}^\text{max}} \sigma(E_\nu) \omega(E_\nu) dE_\nu}{\int_{E_0}^{E_{\nu}^\text{max}} \omega(E_\nu) dE_\nu},$$  

(17)

where the neutrino profile function \(\omega(E_\nu)\) is well-known (i.e., the Michel spectrum in \(E_{\nu_e}\) and the spectrum of \(E_{\nu_\mu}\) provided by the LSND experiment). The lower limit \(E_0\) in Eq.(17) is taken to be zero for the \((\nu_e, e^-)\) reaction and 123.1 MeV for the \((\nu_\mu, \mu^-)\) reaction \[13\].

For \(^{12}\)C, we present our results in Tables II and III, and compare with experiments and other recent theoretical works in Table IV. Here are the main points of our analysis. In both Tables II and III, the rows 1 through 4 indicate four different choices of the radial parameters for the nuclear density \[26\]. The columns represent different choices of the Landau-Migdal spin-isospin parameter \(g'\). In Table III, we have used the \(\nu_\mu\) spectrum reported in the LSND papers \[1, 13\] for a direct comparison with the experiments. The radial uncertainties are typically about 2%, while the \(g'\) variation represents a \(\pm 7\)% spread, for the \((\nu_e, e^-)\) case and \(\pm 8\)% for the \((\nu_\mu, \mu^-)\) case both around the central values corresponding to \(g' = 0.7\). Thus, the overall spread from the theoretical error, taking both of these effects in quadrature, is \(\pm 7.3\)% for the \((\nu_e, e^-)\) case and \(\pm 8.2\)% for the \((\nu_\mu, \mu^-)\) case. In Table IV, we compare the presently available theoretical results with the most recent experimental results for these reactions.

From Table IV, we can provide our best estimate of \(\bar{\sigma}(\nu_e)\) and \(\bar{\sigma}(\nu_\mu)\) as follows:

$$\bar{\sigma}(\nu_e) = (15.48 \pm 1.13) \times 10^{-42} \text{cm}^2,$$

(18)

$$\bar{\sigma}(\nu_\mu) = (16.65 \pm 1.37) \times 10^{-40} \text{cm}^2.$$

We are in excellent agreement with the experimental value of \(\sigma(\nu_e)\), but our lower limit \(\sigma(\nu_\mu)\) is 15% higher than the higher limit of \(\sigma(\nu_\mu)\) measured by the LSND collaboration.

For \(^{13}\)C, we obtain a flux averaged cross section of \(7.25 \times 10^{-41} \text{cm}^2\) for Michel spectrum, using the density distribution parameters given in \[20\]. This should be compared with the calculations of Arafune et al. \[31\], who obtain a cross section of \(9.58 \times 10^{-41} \text{cm}^2\) for the transition to the ground state and first excited state of the final nucleus, which together give 85% of the total cross section. This implies an inclusive cross section of \(11.3 \times 10^{-41} \text{cm}^2\). There could be a reduction of (10 – 15)% if the momentum dependence of the form factors are taken into account \[32\]. This value seems to be in agreement with an unpublished result of Donnelly quoted by Krakauer it et al. \[8\]. The calculations of Arafune it et al. \[31\] do not take into account the possible quenching of the weak interaction operators in nuclei, which is studied by Fukugita it et al. \[33\]. In this paper, the quenching of the matrix elements of the
Gamow-Teller (G-T) operators is obtained in an effective operator approach, which takes into account the effect of core polarization, isobar and the meson exchange current processes. This leads to a 20% reduction in the flux-averaged cross section for the ground state transition, while the cross section to the first excited state is reduced by a factor 3. Assuming, as before, that these two states together contribute 85% of the total cross section, a flux-averaged cross section of $5.4 \times 10^{-41} \text{cm}^2$ is inferred from the calculations of Fukugita et al. [3].

We find that our flux-averaged cross section for $^{13}C$, reported in this paper, is 35% smaller than that obtained by Arafune et al. while it is 35% larger than the results of Fukugita et al. It will be interesting to test these predictions by measuring this cross section in the low energy neutrino experiments with liquid scintillators, where $^{13}C$ forms part of natural carbon. In the experiments of Krakauer et al. [8], it is reported that

$$\sigma_{av} = 0.723\bar{\sigma}(\nu_e^{13}C) + \bar{\sigma}(\nu_e^{27}Al) < 18.3 \times 10^{-41} \text{cm}^2.$$  (20)

In our approach, we calculate the flux averaged cross section in $^{27}Al$ to be $11.48 \times 10^{-41} \text{cm}^2$ with $g' = 0.7$ and using one set of parameters from ref. [20]. This, along with the value obtained for the neutrino cross section in $^{13}C$, gives a value of $16.5 \times 10^{-41} \text{cm}^2$ for $\sigma_{av}$ in Eq.(20). We associate a theoretical uncertainty of 6% due to $g'$ and density variation of $^{27}Al$ on this average cross section. This value is consistent with the available experimental information on these reactions.

We would like to emphasize that the renormalization of nuclear strengths in our model produces a reduction of about 40% in the $(\nu_e, e^-)$ cross section to bring it in agreement with the experiment [1]. Similarly large reductions also occur in the $(\nu_\mu, \mu^-)$ case.

In summary, our calculations show no discrepancy with the measured flux-averaged $\nu_e$ cross sections in $^{12}C$, like other authors. We also nicely reproduce the measured inclusive muon capture rate, now known very accurately [19, 20]. But we see a discrepancy, at least by 15% in the flux-averaged $\nu_\mu$ cross section compared with the LSND experiment [13], the theoretical prediction being higher than the experiment. The discrepancies between the experimental and various theoretical results for $(\nu_\mu, \mu^-)$ inclusive cross sections in $^{12}C$ should be taken seriously, in view of its implications in present studies of neutrino oscillations. Our results for the case of $^{13}C$ and $^{27}Al$ are consistent with the only experimental limit available at present. A high-quality experimental measurement of the inclusive cross section, specially in $^{13}C$, will be very useful in understanding the quenching of the G-T strengths in this nucleus in the light of the wide range of theoretical predictions for this reaction.

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References


Table I: A summary of flux-averaged $\nu_e$ and $\nu_\mu$ cross sections as obtained in various experiments done with $^{12}$C target. The unit for the $\nu_e$ cross section is $10^{-42} \text{cm}^2$ and for $\nu_\mu$ cross section is $10^{-40} \text{cm}^2$.

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<th>KARMEN collab.</th>
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<td>[11]</td>
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<td>$\sigma(\nu_\mu)$</td>
<td>8.3±0.7±1.6</td>
<td>11.2±3.1±0.8</td>
<td>159±26±37</td>
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Table II: Flux-averaged $\nu_e$. Four radial parameter sets are chosen from the literature [26], with parameters $c_1$ and $c_2$ in fm. The Landau-Migdal parameter $g'$ is taken as $g' = 0.7 ± 0.1$. The unit for cross section is $10^{-42} \text{cm}^2$.

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<td>1.692</td>
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<td>16.99</td>
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<td>14.52</td>
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Table III. Flux-averaged $\nu_\mu$ cross section. Radial parameter sets and $g'$ values are as in Table II. The $\nu_\mu$ flux is taken from $S_1$ [15]. The unit for cross section is $10^{-40} \text{cm}^2$.

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Table IV. Summary of flux-averaged cross sections. Experimental results are inferred by adding ground state and excited state contributions for $(\nu_e, e^-)$. Theoretical results are from Kolbe et al. [2], Auerbach et al. [4], Umino et al. [3] and this work. The units are $10^{-42} \text{cm}^2$ for $(\nu_e, e^-)$ and $10^{-40} \text{cm}^2$ for $(\nu_\mu, \mu^-)$ cross sections.

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<td>-</td>
<td>15.48 ± 1.13</td>
<td>14.8 ± 1.0 ± 1.5 [9, 10]</td>
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<td>$\sigma(\nu_\mu)$</td>
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<td>13.5 - 15.2</td>
<td>22.7 - 24.1</td>
<td>16.65 ± 1.37</td>
<td>15.2 ± 1.4 ± 1.8 [13]</td>
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