Prospects for probing the gluon density in protons using heavy quarkonium hadroproduction

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Abstract

We examine carefully bottomonia hadroproduction in colliders as a way of probing the gluon density in protons. To this end we develop some previous work, getting quantitative predictions and concluding that our proposal can be useful to perform consistency checks of the parameterization sets of different parton distribution functions.

Keywords: Gluon density; Quarkonia production; Bottomonium; NRQCD; Tevatron; LHC
1 Introduction

With the advent of large hadron colliders, a precise knowledge of the structure of hadrons has become compulsory if experimental data on elementary particle physics are expected to be analysed and ultimately interpreted with the desired high accuracy and reliability. In particular, one of the goals of the LHC project is to perform precise tests of the Standard Model of strong, weak and electromagnetic interactions and the fundamental constituents of matter. Actually the LHC machine can be viewed as a gluon-gluon collider to a large extent and many signatures (and their backgrounds) of physics, both within and beyond the Standard Model, involve gluons in the initial state [1]. Therefore an accurate knowledge of the gluon density in protons acquires a special relevance. Currently there are three major groups - namely CTEQ, MRST and GRV - providing regular updates of the partonic structure of protons as new data and/or theoretical improvements become available [2].

The parton distribution functions (PDFs) are phenomenologically determined by global analyses of a wide class of hard processes involving initial-state hadrons, making use of the QCD parton framework. Presently, the most precise determinations of the gluon momentum distributions in the proton basically come from data on deep-inelastic scattering (DIS), in particular through the analysis of the scaling violations of the structure function $F_2$. However, this represents an indirect method since the gluon density is obtained by means of the QCD evolution (DGLAP) equations. On the other hand, hadron-hadron scattering processes with prompt photon production or jets in the final state should be (and actually already are) extremely adequate to probe “directly” the gluon distribution in hadrons [3]. Such “direct” determinations can be viewed as complementary to indirect analyses, providing independent tests of perturbative QCD with distinct systematic errors. Let us also remark that, in colliders like the Fermilab Tevatron or the LHC [4], gluon densities are - or can be - probed at similar (and higher) $x$-values as in DIS but at significantly larger energy scales.

In this paper, we examine the possibility of using heavy quarkonia inclusive hadroproduction to probe the gluon density of protons, extending in a more quantitative way the ideas earlier presented in Refs. [5, 6]. In particular we are focusing on bottomonia production in proton-proton collisions at the LHC. Our claim is that, ultimately, the measurement of bottomonia cross sections could provide a useful consistency check of different PDF sets and their energy-scale evolution.

2 Heavy quarkonia inclusive hadroproduction

At very high transverse momentum, fragmentation mechanisms should expectedly become dominant in the cross section of bottomonium hadroproduction - as in any other single-particle production channel. Specifically, gluon fragmentation into a $b\bar{b}$ pair followed by its non-perturbative evolution yielding a $\Upsilon$ final state by emission of soft gluons, should play the leading role according to the colour-octet mechanism (COM) [7]. Such a production mechanism constitutes the relativistic generalization of the colour-singlet model (CSM) and one of the most natural explanations of the excess of heavy quarkonia production observed at the Tevatron [8, 9, 10]. Moreover, the COM can be cast into a rigorous framework based on NRQCD [11], an effective theory coming from first principles.
Furthermore, as long as we focus on quarkonium hadroproduction at large transverse momentum, the factorization assumption, underlying the NRQCD description, should be justified by the large scale $p_T$; for bottomonia in particular the heavy quark mass is likely large enough to factorize the short and long distance physics in the partonic interaction itself, as it is well known from decay processes. Whether similar arguments can be applied to charmonium resonances has to be checked, for example analysing the transverse polarization of the resonance [12].

Therefore one can expect, on solid grounds, that one of the main production channels of bottomonia at the LHC should correspond to the partonic subprocess:

$$g \ g \rightarrow g^* \ g$$

and the subsequent gluon fragmentation into a $\Upsilon(nS)$ state accompanied by light hadrons $X_s$:  

$$g^* \rightarrow \Upsilon(nS) \ X_s \ \ ; \ \ (n = 1, 2, 3)$$

produced through the already mentioned colour-octet mechanism.

We must clearly state that our later development relies on the hypothesis that the main contribution to heavy quarkonium hadroproduction at high transverse momentum comes from a fragmentation mechanism as in (2) whose final hadronization stage is governed by a single non-perturbative parameter. Strictly speaking, the colour-octet production mechanism is not necessarily required; for example, another possibility is gluon fragmentation into a heavy resonance via a colour-singlet channel. However, this mechanism actually corresponds to a higher order contribution since colour conservation and charge conjugation require the emission of two extra gluons in the subprocess (2) [13]. Therefore, the colour-octet fragmentation mechanism should play the leading role at high transverse momentum in bottomonium hadroproduction [6] if the COM turns out to be correct. In this regard, the different leading and subleading contributions and their relative importance will be very briefly reviewed in section 2.1.

Ideally, the final-state gluon ($g$) in Eq.(1) will give rise to a recoiling jet ($g \rightarrow jet$), sharing, in principle, the same transverse momentum as the heavy resonance - neglecting initial and final state gluon radiation. On the other hand, the leptonic decay of the resonance is obviously the best way to have a clean signal of its formation among a huge hadronic background. Of particular interest is the muonic channel, since such muons would very likely pass the first level trigger - consisting (in ATLAS) of a muon with transverse momentum larger than 6 GeV and (absolute) pseudo-rapidity less than 2.5 [1].

Hence, events would topologically consist of an almost isolated muon pair from the decay of the heavy resonance recoiling against a single jet, thereby providing a suitable tag for the production mechanism represented in Eqs.(1-2). Indeed this topology is the expected one since the mass difference between the intermediate coloured and final states should be quite small, allowing the emission of at most a few light hadrons via soft gluon radiation - denoted by $X_s$ - accompanying the resonance at the final hadronization stage. Moreover, in order to remove possible background, events with (more than one) mini-jets should be discarded from the selected sample.
Figure 1: Sketch of a parton-level process leading to a resonance recoiling against a gluon yielding a final-state jet. Notice that if the two interacting partons had different fractions $x$ of the total momenta of the colliding protons, the event would not show a back-to-back topology between the dimuon and the recoiling jet. Besides, initial and final state radiation can affect the event topology similarly. We consider an event compatible with a back-to-back topology if the rapidity difference in absolute value of the final-state resonance and jet is less than a predetermined value set by $2y_{cut}$.

2.1 Differential cross section

In this Section we reproduce the main points presented in Ref. [6] for the sake of clarity in our later more technical proposal and critical discussion.

According to the collinear approximation, employed in our analysis, the triple differential cross section for the inclusive production process $pp \rightarrow \Upsilon X$ can be written as

$$\frac{d^3\sigma}{dy_{\Upsilon}dy_{jet}dp_T} = 2p_T \sum_{ab} x_a x_b f_{a/p}(x_a, Q^2) f_{b/p}(x_b, Q^2) \frac{d\hat{\sigma}_{ab}(Q^2)}{dt}$$

where $p_T$ denotes the transverse momentum of both the resonance and the jet; $y_{\Upsilon}$ and $y_{jet}$ represent the corresponding rapidities in the Lab frame, respectively; $f_{a,b/p}(x_{a,b}, Q^2)$ denotes the parton-$a,b$ density in the proton at Feynman $x_{a,b}$ and typical hard interaction scale $Q^2$. (Let us note that all factorization, fragmentation and renormalization scales have been taken equal for simplicity.)

The partonic differential cross section can be written as:

$$\frac{d\hat{\sigma}_{ab}(Q^2)}{dt} \equiv \frac{d\hat{\sigma}}{dt}(ab \rightarrow \Upsilon X_h) = \frac{1}{16\pi s^2} \sum_n \sum |A(ab \rightarrow [b\bar{b}]_n X_h)|^2 \langle \mathcal{O}_{\Upsilon n}^X \rangle$$

consisting of a short-distance (and calculable) part, i.e. $A(ab \rightarrow [b\bar{b}]_n X_h)$ where $X_h$ denotes the hadron bulk from the hard interaction, as depicted in Fig.1, and a long-distance piece $\langle \mathcal{O}_{\Upsilon n}^X \rangle$ which can be identified as a NRQCD matrix element [11]. The barred summation represents an average over initial spin and colour for the short-distance partonic process, and there is another summation running over channels corresponding to different quantum
numbers of the intermediate state (labelled by \( n \)) evolving into final heavy quarkonium. Using the spectroscopic notation for the \([\bar{b}b]_n\) state: \( ^{2S+1}L_J^{(1,8)} \), where the superscript \((1, 8)\) stands for either a colour-singlet or a colour-octet state, the different contributions are: \( ^{1S_0(1)} \), \( ^{3S_1(8)} \), \( ^{1S_0(8)} \), \( ^{3P_J^{(8)}} \), \ldots The relative importance of each contribution is basically given by its \( p_T \) dependence, the order of the strong coupling constant \( \alpha_s \) and the so-called velocity scaling rules of NRQCD [11] (a nice overview can be found in [13]). In this work, we shall concentrate only on direct production at high-\( p_T \), (dominated by the \( ^{3S_1(8)} \) contribution) without feedown from \( \chi_{bJ} \) states.

The basic kinematics of the partonic subprocess (see Figure 1) is determined by the transverse momentum \( p_T \) and the rapidities of the resonance \( y_\Upsilon \) and of the recoiling jet \( y_{\text{jet}} \), respectively [1]; therefore \( y_T + y_{\text{jet}} \) is twice the rapidity of the partonic centre-of-mass system \( y_0 \) (i.e. \( y_0 = (y_T + y_{\text{jet}})/2 \)) and \( \delta y = y_T - y_{\text{jet}} \) denotes the rapidity difference.

At large transverse momentum one can write:

\[
x_{a,b} \simeq \frac{2p_T \ e^{y_0}}{\sqrt{s}} \ \cosh \frac{\delta y}{2}
\]  

where the measured transverse momentum of the resonance \( p_T \) is assumed to coincide at leading order with the transverse energy of the jet. Eq.(5) can be seen as a particular application of well-known formulae for dijet hadroproduction (see [15] and references therein).

At relatively high \( p_T \) the dominant partonic subprocess for \( \Upsilon \) hadroproduction should be the gluon-gluon interaction [6]. Thereby, the following combination appears in the cross section of Eq.(3) particularizing to the case of gluon colliding partons \( a \) and \( b \): \( f_{g/p}(x, Q^2) \equiv g(x, Q^2) \),

\[
x_a x_b \ g(x_a, Q^2) \ g(x_b, Q^2)
\]

calculated at the common scale \( Q^2 \), basically set by \( p_T^2 \) as we shall see.

In particular, requiring \( y_0 = 0 \) (i.e. \( |y_T| = |y_{\text{jet}}| \) but with opposite signs) implies considering only those values \( x \equiv x_a = x_b \) (within the uncertainty interval to be discussed below) given by

\[
x = x_T \ \cosh y_T
\]

with \( x_T = 2p_T/\sqrt{s} \). Thus the theoretical and experimental analysis simplifies notably: only the transverse momentum and rapidity of the \( \Upsilon \) resonance enter into Eq.(6), both determined from the experimental measurement of the transverse momenta and pseudorapidities of the muons from its decay.

On the other hand, the Feynman \( x \) can be known with a precision limited by the rapidity uncertainty of the resonance; indeed, for a fixed \( p_T \) value, the typical uncertainty \( \Delta x \) is given by [6]

\[
\frac{|\Delta x|}{x} < y_{\text{cut}}
\]  

\footnote{According to energy/momentum leading-order balance of the parton-level interaction, we assume that the rapidity of the parent gluon equals the jet rapidity which can be obtained from its measured pseudo-rapidity \( \eta_{\text{jet}} \approx y_{\text{jet}} \) (for technical details about the measurement of the jet pseudorapidity in the hadronic calorimeter we refer the reader to [1]).}
where $y_{\text{cut}}$ represents the half-width of the allowed rapidity interval for the resonance, corresponding to a back-to-back topology as shown in Fig. 1.

Then we can write as a first approximation,

$$
\frac{d^3\sigma}{dy_Tdy_{\text{jet}}dp_T} = 2p_T \ x^2 g(x,Q^2) \ \frac{d\hat{\sigma}_{gg}(Q^2)}{d\hat{t}}
$$

(8)

On the other hand we can choose the typical hard scale as the momentum transfer $Q^2 = -\hat{t}$, where $\hat{t}$ stands for the Mandelstam variable of the partonic subprocess. This quantity can be estimated by means of the expression

$$
Q^2 = 2p_T^2 \ \cosh^2 \frac{\delta y}{2} \left(1 - \tanh \frac{\delta y}{2}\right) = 2p_T^2 \ \cosh^2 y_T \ (1 - \tanh y_T)
$$

(9)

the last step coming since $\delta y = 2y_T$ as we have assumed $y_0 = 0$.

Let us remark that $x$ and $Q^2$ are not completely independent since the centre-of-mass energy of the partonic interaction $\hat{s} = x^2 s$ can be expressed as $\hat{s} = 4p_T^2 \ \cosh^2 y_T$, neglecting masses. Therefore the following relation is satisfied

$$
Q^2 = \frac{s}{2} x^2 \ (1 - \tanh y_T)
$$

(10)

Moreover, keeping $x$ fixed and averaging over the full rapidity range, we get

$$
\langle Q^2 \rangle = \frac{s}{2} x^2
$$

(11)

In fact, in our later development we will integrate cross sections over the full available rapidity range and this average value for $Q^2$ will be extensively used.

3 Probing the gluon density in the proton

In Ref. [6], we proposed to test different parameterizations of the gluon distribution $g(x,Q^2)$ in the proton by studying the ratios

$$
\frac{x_j^2 g(x_j,Q_j^2)}{x_i^2 g(x_i,Q_i^2)} = \left(\frac{d\hat{\sigma}_{gg}/d\hat{t}_i}{d\hat{\sigma}_{gg}/d\hat{t}_j}\right) \times \left(\frac{p_{TI}}{p_{Tj}}\right) \times \left(\frac{d^3\sigma/dy_Tdy_{\text{jet}}dp_T}{d^3\sigma/dy_Tdy_{\text{jet}}dp_T}\right)_{y_{\text{jet}}=-y_T}
$$

(12)

corresponding to several $x_i,x_j$ pairs of the parameterization set under consideration. Subindices $i,j$ denote discrete values or points of the $x$ variable associated to bins whose relative widths are set by Eq.(7); $Q_i^2$ stands for the (average) transverse scale according to Eq.(11) for

\[2\] We have assumed that the uncertainty in the determination of the Feynman $x$ value provided by Eq.(7) determines the binning of this variable. Thus, the $x$-bin size is ultimately set by the value of $y_{\text{cut}}$, used to impose the back-to-back condition on the events (i.e. the measured rapidities of the resonance and jet should differ in absolute value by that twice $y_{\text{cut}}$). We have explicitly checked that the difference, event by event, between the “observed” $x_{a,b}$ (through Eq.(5)) after applying ATLAST jet and $\Upsilon$ reconstruction \[3\] and the corresponding values selected by PYTHIA in the hard partonic interaction, is of the order of 10%, compatible with our choice $y_{\text{cut}} = 0.2$. (we are indebted to F. Camarena for his valuable help in this calculation).

\[3\] We have neglected masses only in the estimate of $Q^2$, not in the generation of events with PYTHIA. Actually for $p_T^2 \gg M_T^2$, this leads to no significant difference in the former case.
Actually the ratio given by Eq.(12) involves the so-called effective gluon distributions \[14\], since quark distributions can contribute to some extent via the partonic scattering channel \(gq \to g^*q\), and to a much lower extent via the channel \(q\bar{q} \to g^*g\) \[6\].

It is important to note that the number of independent \(x\) values is basically limited by \(\Delta x\) (i.e. by \(y_{cut}\)), and hence the number of available \(x_i, x_j\) pairs in the ratio (12); therefore this condition cannot be released too much. At the same time, the foreseen statistics does not allow one to decrease \(y_{cut}\) too much either, and a compromise should be reached (our particular choice has been \(y_{cut} = 0.2\)).

In sum, the keypoint of our proposal is to consider the l.h.s. of Eq.(12) as an input corresponding to different sets of the gluon distribution for the proton, whose \(x\) dependence is hence assumed to be “known”, and in fact would be tested. On the other hand the r.h.s. corresponds to an input from experimental data and some theoretical factors likely under control.

For the sake of clarity let us rewrite expression (12) as:

\[
\frac{x_j^2 g(x_j,Q_i^2)^2}{x_i^2 g(x_i,Q_j^2)^2} = R_{\text{theo}} \times R_{\text{exp}}
\]

where

\[
R_{\text{theo}}(y_{\Upsilon i}, y_{\Upsilon j}, p_{T i}, p_{T j}; Q_i^2, Q_j^2) = \frac{d\hat{\sigma}_{gg}/d\hat{t}_i}{d\hat{\sigma}_{gg}/d\hat{t}_j}
\]

In the high \(p_T\) limit and zero rapidity \((y_{\Upsilon i} = y_{\Upsilon j} = 0)\),

\[
R_{\text{theo}} \to \frac{\alpha_s(Q_i^2)}{\alpha_s(Q_j^2)} \frac{p_{T j}^4}{p_{T i}^4}
\]

explicitly showing that \(\alpha_s(Q^2)\) is entangled in the gluon density determination. Indeed, the determination of \(\alpha_s\) from the analysis of inclusive jet cross sections at hadron colliders can be viewed as deeply related - in a complementary way - to the study of the sensitivity of the hadronic production cross sections to different PDF sets \[15\].

Since at high \(p_T\) only the \(3S_1^{(8)}\) channel will be considered (see next Section), the dependence on the NRQCD matrix elements does cancel in \(R_{\text{theo}}\), but there is a dependence on the scales \(Q_i^2\) and \(Q_j^2\) which should match the corresponding dependence in the left hand side. Let us stress that to achieve the cancellation of NRQCD matrix elements in the quotient, only a single channel should contribute dominantly to the cross section. This fact excludes including subleading contributions, preventing us, in principle, from considering any possible check of next-to-leading PDFs according to our method, and restricting the test to leading-order gluon distributions.

On the other hand the experimental input reads as the ratio

\[
R_{\text{exp}}(p_{T i}, p_{T j}; y_{\Upsilon}) = \left(\frac{p_{T i}}{p_{T j}}\right) \times \left(\frac{d^2\sigma/dy_{\Upsilon}dy_{jet}dp_{T j}}{d^2\sigma/dy_{\Upsilon}dy_{jet}dp_{T i}}\right)_{y_{jet}=-y_{\Upsilon}}
\]

which can be obtained directly from experimental data \[4\].

\[4\] Let us note that in the condition \(y_{jet} = -y_{\Upsilon}\) imposed to the events - within the uncertainty interval - could be removed, that is not requiring a back-to-back topology. This would amount to extend the study of the ratio (12) over the region of the plane \(x_a, x_b\) beyond the diagonal region \(x_a = x_b\) there by allowing additional cross-checks of the PDF under scrutiny; however, we have not included this study in the present paper.
Figure 2: Predicted $\Upsilon(1S) + \Upsilon(2S) + \Upsilon(3S)$ weighted contributions to bottomonia inclusive production at the LHC in the rapidity interval $|y_\Upsilon| < 0.2$ and $p_T > 10$ GeV. Only direct production is considered (no feed-down from $\chi_b$ states). Left panel: differential cross section; right panel: integrated cross section (see Ref. [6]). Let us note that once the colour-octet MEs have been determined from the Tevatron data, using either CTEQ4L or CTEQ5L as the input PDF, the respective predictions for the LHC are very similar [17].

3.1 Efficiencies, statistics and expected accuracy

From an experimental point of view, one of the advantages of heavy quarkonium physics (bottomonium in particular) at the LHC is its clean (and self-triggering) signal through the muonic decay channel: $\Upsilon(nS) \rightarrow \mu^+\mu^-$ allowing a full kinematic reconstruction. However, it may happen that the discrimination among the different $\Upsilon(nS)$ states via mass reconstruction could become a difficult task at very high $p_T$ because of the uncertainty on the measurement of quite large momenta of muons. Indeed, above $p_T \simeq 50$ GeV for each individual muon momenta, the uncertainty associated to the reconstructed mass from two muons makes impossible the discrimination below the 1 GeV level [1]. Nevertheless, since we are proposing to study ratios of cross sections, we can consider the overall $\Upsilon(nS)$ direct production, without separating the different bottomonia sources - the weighted matrix element cancelling in the quotient (14) if the mass difference between the different $\Upsilon$ states is neglected [6].

In order to assess the foreseen statistics and efficiency factors, we have employed the PYTHIA [16] event generator [10] with the colour-octet production mechanism implemented as a new code in the Monte Carlo programme (a detailed account of the implementation, results and discussion can be found in Ref. [8]). For the purpose of illustration we plot in Fig.2 the combined production rate at $p_T > 10$ GeV and $|y_T| < 0.2$ for the upper and

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5The quarkonium (charmonium) sector regarding $2 \rightarrow 2$ production channels has remained unchanged in PYTHIA from the older version 5.7 - used in our earlier studies - to the current one 6.2 - leading to the same results in all cases for high $p_T$. 

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8
lower values of the colour-octet matrix element obtained in [6] from previous fits to CDF experimental data of Run IB at the Tevatron.

Note that despite the respective muonic branching ratios are already included in the plot, actual measurements require taking into account efficiencies for triggering, reconstruction and identification of particles and jets from detectors. In our study we have not performed a full simulation of the detector effects, although we have benefitted from different studies focusing on leptons and jets separately [1]. Since we are addressing the feasibility of this proposal, rather than searching for precise predictions, this approximation should suffice.

Firstly, the reconstruction of the $\Upsilon(nS)$ mass from the $\mu^+\mu^-$ pair implies a reduction factor of about 80% by applying the same cuts as for the $J/\psi$ [1], i.e. a $[-3\sigma, +3\sigma]$ symmetric window around the nominal mass. These figures do not include however the muon trigger and identification efficiencies which altogether roughly amount to 85% for the triggering muon and 95% for its partner, respectively. Therefore, the overall reconstruction factor can be estimated as

$$\epsilon_{\mu^+\mu^-} = 0.8 \times 0.85 \times 0.95 \simeq 0.65$$

Moreover, jet reconstruction efficiency, $\epsilon_{\text{jet}}$, amounts on the average to about 75% in the $p_T$ range under consideration, using an algorithm with a jet cone radius $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} = 0.7$ [18]. On the other hand, there is an additional reduction factor, denoted as $\epsilon_y$, due to the requirement of a back-to-back topology compatible with the constraint given by Eq.(7), which sets the expected accuracy in the determination of $x$; choosing $y_{\text{cut}} = 0.2$ as already mentioned, we found (see Ref. [6] for more details) that $\epsilon_y \simeq 35\%$.

In sum, the combined efficiency and reduction factor to be applied is

$$\epsilon_{\text{tot}} = \epsilon_{\mu^+\mu^-} \times \epsilon_{\text{jet}} \times \epsilon_y = 0.65 \times 0.75 \times 0.35 \simeq 0.17$$

Assuming an integrated luminosity of 10 fb$^{-1}$, corresponding to one year running ($10^7$s) of LHC at “low” luminosity ($10^{33}$ cm$^{-2}$s$^{-1}$) and sweeping all the rapidity range [-2.5,2.5] by steps of 0.4 rapidity units - according to our choice $y_{\text{cut}} = 0.2$ - we find from our simulation an expected number of about 10,000 events, corresponding to the highest Feynman $x$ bin around the value of 0.014 considered in our analysis. Multiplying this number by the total efficiency factor $\epsilon_{\text{tot}}$, we get the estimate:

$$\simeq 1,700 \text{ events per year run}$$

Hence after three years of LHC running at low luminosity [1], the accumulated statistics at the largest $x$ bin would amount to about 5,000 events, basically fixing the typical accuracy of the order of $\simeq 2.5\%$, which essentially sets the “discrimination level” to be applied to different sets of PDFs, as we shall see in the next section.

### 4 Testing the $x$-shape of different PDFs

As already pointed out, the l.h.s. of Eq.(12) can be considered as an input from a given parameterization set of the gluon distribution of the proton; in Fig. 3.b we show the $x$-shape for different choices (CTEQ, MRST, GRV) [19]. All distributions were normalized

\(^{6}\text{We are considering the possibility of extending our proposal to the high luminosity regime at the LHC coping with pile-up effects, as well as using the electronic decay channel of the }\Upsilon\text{ resonances, too. This would mean to increase considerably the foreseen statistics allowing to raise the }x\text{ range available in our study.}\)
Figure 3: a) $x_i g_i(x_i, Q_i^2)$ values for different leading log (LL) PDFs [19]. b) Ratios $x_i^2 g_i^2(x_i, Q_i^2)/x_5^2 g_5^2(x_5, Q_5^2)$, i.e. values of $x_i^2 g_i^2(x_i, Q_i^2)$ normalized to the rightmost point ($i = 5$); $Q_i^2$ represents here an average value over the whole rapidity according to Eq.(11) but dependent on $x_i$.

to the rightmost point of Fig. 3.a, i.e. corresponding to $x_5 = 0.014$. (Although arbitrary in principle, this particular choice seems quite appropriate since all PDFs become close each other near this point.) The points shown in the plot correspond to different values of the $x$ variable whose bin widths follow the requirement of Eq. (7) with $y_{\text{cut}} = 0.2$; $Q_i^2$ was selected as the average value of Eq. (11) for each $x$ value. For the sake of clarity, let us stress that the plots in Fig. 3 have a “general validity”, i.e. can be obtained from any PDF parameterization set under the above-mentioned conditions, without any reference to a specific experiment; the applicability to a particular case depends crucially on the expected number of events.

In this regard we show in Fig. 4 the maximum difference (in %) obtained from the curve of Fig.3.b, amounting to about 35% at most over the whole $x$ interval. Therefore the foreseen precision level of 2.5% (obtained in the previous Section) likely should permit discriminating easily among those LL PDFs at the LHC. Finally, let us note that, according to our remark in Section 2, we have not included next-to-leading order PDFs in our analysis since this would require, for consistency, further subleading contributions even at high $p_T$, and therefore new NRQCD matrix elements not completely cancelling in the ratio (14).

On the other hand, it might be possible to perform an overall consistency check on bottomonium production by considering the integrated cross section above a lower $p_{T\text{min}}$ cutoff, say 20 GeV, for different parton distributions. This is in accordance with the suggestion made in [4, 20] of looking at the uncertainties on physical quantities rather than obtaining errors associated to the PDFs. In particular, expected differences of the theoretical calculations on the integrated cross sections above $p_{T\text{min}} = 20$ GeV for bottomonia hadroproduction at the LHC, using different parameterization sets, is of order 40% respectively [4, 20].

\[ \text{For a lower } p_{T\text{min}} \text{ the difference becomes even larger; for instance, it could reach a factor 2 for } p_T > 1 \text{ GeV!} \]
Figure 4: Differences (in %) between current “normalized” LL PDFs corresponding to the upper (CTEQ) and lower (MRST) values of Fig. 3.b. A foreseen precision of $\approx 2.5\%$ should suffice to discriminate among them over the whole $x$ range shown in the plot.

5 Conclusions and last remarks

With the advent of high energy, high luminosity proton colliders, it is universally recognized the importance of knowing the uncertainties of the parton distributions to perform accurate measurements of the Standard Model parameters and carry out a tantalizing discovery programme on new physics. Keeping this interest in mind, “direct” determinations of the gluon density by means of distinct processes should permit the computation of systematic and theoretical errors, as well as putting stringent constraints to global fits where DIS data, however, are still playing a central role.

To this aim and in view of the foreseen rates of bottomonia production at relatively large transverse momentum in the LHC [6], we propose testing the $x$-shape of the gluon density in protons. The proposal could be similarly applied to forthcoming experimental data from the high-luminosity run II at the Tevatron.

If gluon fragmentation is confirmed as the dominant production bottomonia mechanism at high transverse momentum, the topology of events would consist of an almost isolated lepton pair coming from the resonance decay, recoiling against a jet. This represents basically two advantages:

- There is a well reconstructed direction defined by the lepton pair which can be assigned to the fragmenting gluon in the final state of the hard interaction, thereby helping in the search for the partner jet with a back-to-back topology, tagging altogether the event.

However at small/moderate $p_T$ the colour-singlet mechanism would be largely dominant with quite more parameters entering the computation - such as the wave functions (and derivatives) of bottomonia states, cascade branching ratios - which would make more complex the physical interpretation of any discrepancy with the experimental result.
• According to the colour-octet production mechanism, the lepton pair should provide, by means of its measured rapidity and transverse momentum, a clean kinematic information about the partonic subprocess itself (i.e. the \( x \)-values of the colliding partons).

From inspection of the Figs. (3-4) and the expected discrimination level (\( \approx 2.5\% \)) corresponding to the foreseen statistics collected after three years of data-taking at the LHC (at “low” luminosity), we concluded that a clear discrimination between different sets of LL PDFs should be feasible. In this regard, let us stress that Monte Carlo simulations in high energy physics have definitely become an ordinary - as well as necessary - tool nowadays for the experimental analysis of data, in particular to disentangle interesting signals from background. General-purpose event generators implementing LL PDFs as default options (like PYTHIA) are widely used by the scientific community involved in the front-line analysis of experimental data. For instance, cross section measurements often require Monte Carlo estimates of the fraction of unobserved events due to detector acceptance and efficiency limits; an equivalent argument applies to background estimates. A knowledge, as precise as possible, of the dependence of the theoretical predictions on the choice of a particular PDF is mandatory.

Then, we have concluded that our proposal could be useful as a consistency check of different gluon density parameterizations for the proton, of interest for other physical processes involving gluons in the initial state, too. In particular, exciting searches for extra dimensions have been recently carried out at the Tevatron [21] and are foreseen at the LHC [22] where effective Planck scales [23] of the order of the TeV can be tested. Relevant processes in the latter case are \( gg \rightarrow G_{KK}g \), \( qg \rightarrow G_{KK}q \), ... where \( G_{KK} \) denotes a Kaluza-Klein graviton, leading to a final-state single jet and missing energy. The channel given by Eqs.(1-2) should also be helpful in the study of the detector response to a monojet and missing transverse energy, taking into account in the latter case the information provided by the recoiling muonic pair.

On the other hand, parton distributions unintegrated over the parton transverse momentum are increasingly becoming employed in the analysis of physical processes initiated by hadrons. Indeed, one of their advantages is that they correspond to the quantities entering Feynman diagrams allowing for true kinematics even at leading order. In our particular case, this would demand to modify the cross section given in Eq.(3). However, the theoretical situation regarding heavy quarkonium production is still controversial since there are important discrepancies between the collinear approximation and the so-called \( k_T \) factorization [24, 13].

A final remark is in order. Although the universality of the colour-octet matrix elements is not definitely well established [13], one could expect that the matrix elements obtained in essentially the same kind of hadronic processes (i.e. hadroproduction at the Tevatron) should become reliable enough once used at LHC energies, under the same theoretical inputs (e.g. the bottom mass, choice for the factorization and fragmentation scales, etc). Therefore, if a precise normalization of the bottomonia cross section is provided (using forthcoming data from high-luminosity Run II at the Tevatron) the possibility exists of probing different PDF sets by means of an unfolding procedure from heavy quarkonia hadroproduction at the LHC.
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References

