Prospects for the extraction of $|V_{cb}|$ from $B_c \rightarrow J/\psi \mu^+ \nu$ decays at LHC

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September 27, 2013

Abstract

We discuss the possibility of extracting $|V_{cb}|$ from the future measurement at LHC of the $B_c \rightarrow J/\psi \mu^+ \nu$ decay rate followed by the leptonic decay $J/\psi \rightarrow \mu^+ \mu^-$. We examine in detail the influence of the measurement of the muons’ momenta and vertex reconstruction in the final uncertainty of $|V_{cb}|$, as well as the required theoretical inputs at both the production and weak decay level, and their reliability in this regard. We stress the relevance of a rigorous description of the fragmentation contributions to inclusive production at large transverse momentum of heavy quarkonia in $pp$ collisions.

1 Introduction

The interplay between theory and experiment, inherent to all branches of science, becomes crucial in high-energy physics at present. Indeed, despite any (always welcome) unexpected discovery, the required investment of money and time demands well-defined aims and physical objectives from the theoretical side long time before the start of real data-taking. This is particularly apparent at LHC experiments, in one of which (ATLAS) one of us (M.A.S.L.) is currently involved. On the other hand, it is a task of theorists to survey all those theoretical topics compatible with the ultimate goals of an experiment, keeping connection with a “realistic” experimental point of view. This is, in fact, one of the underlying motivations of the present work.

In particular, it has been recently pointed out the feasibility of the observation of $B_c$ mesons in the ATLAS experiment\textsuperscript{4}. With respect to their production rate, setting the $b\bar{b}$ cross section equal to 500 $\mu$b and assuming the fragmentation probability of a $b$ quark into a $B_c$ meson of the order of $10^{-3}$\textsuperscript{5}, the expected number of $B_c$ mesons at “low” luminosity\textsuperscript{1} is about $10^{10}$ per year ($10^7$ s) corresponding to an integrated luminosity of 10 $fb^{-1}$.

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\textsuperscript{4}It is foreseen that during the first years, LHC will run at $\mathcal{L} \approx 1\times10^{34}$ cm$^{-2}$ s$^{-1}$. Later on, the luminosity will increase
In this paper we shall argue that in LHC experiments the decay channel

$$B_c \rightarrow J/\psi \mu^+ \nu_\mu$$

followed by the leptonic decay of the $J/\psi$ into a pair of oppositely charged muons, could provide a reliable extraction of the mixing matrix element $|V_{cb}|$. The expected semileptonic branching ratio $\simeq 2\%$ for channel (1) [3] together with the leptonic branching ratio for the $J/\psi$ into two muons $\simeq 6\%$ [4], yields an overall branching fraction of order $10^{-3}$.

We next combine the last $BF$ with the acceptance $\mathcal{A}$ for the muon trigger $\simeq 15\%$ (where the three possible triggering muons per decay have been taken into account.) Besides, the detection of the two remaining particles (i.e. the non-triggering muons satisfying some $p_\perp$ and $\eta$ cuts [6]) amounts to an acceptance for signal events of $\simeq 12\%$. Finally, we assume an identification efficiency for each muon of 0.8 [5] yielding a combined value of 0.5.

Thereby, the expected number of useful $B_c$ decays $\mathcal{N}$ could reach several $10^5$ per year at low luminosity.

Such large statistics together with the foreseen precision in the muon momentum measurement should permit the experimental access to the kinematic region near zero-recoil of charmonium (with respect to the $B_c$ meson) with a good energy/momentum resolution. Moreover, very stringent cuts can be put on events and hence, expectedly, background could be almost entirely removed from the event sample permitting a very precise determination of the $B_c$ lifetime. We explore all these issues in the following section.

### 2 Extraction of $|V_{cb}|$

In recent times, an attempt to derive the value of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ from $B \rightarrow D^* \ell \nu$ in a (more or less) model-independent way has been proposed [7] within the framework of the heavy quark effective theory (HQET) [8]. This method basically relies on the existence of a universal form factor, the so-called Isgur-Wise function $\xi(w)$, depending on $w = v_1 \cdot v_2$, variable representing the Lorentz factor of the final hadron in the initial hadron’s frame. (Hereafter subindices 1, 2 will denote initial, final hadronic quantities.)

Near zero recoil

$$\lim_{w \rightarrow 1} \frac{1}{\sqrt{w^2 - 1}} \frac{d\Gamma(B \rightarrow D^* \ell \nu)}{dw} = \frac{G_F^2}{4\pi^3} (m_1 - m_2)^2 m_2^2 \eta_A^2 \xi^2(1) |V_{cb}|^2$$

where $\eta_A$ denotes a perturbative coefficient [8]. On the other hand, the $\xi(w)$ form factor absorbs those corrections arising from inverse powers of the heavy quark masses, thus

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2Hereafter, we mainly rely on technical details contained in the Technical Proposal of the ATLAS Collaboration [5]. Those aspects directly related to the generation of the $B_c$ signal and its background, the trigger and reconstruction efficiencies are left to a separate publication [6]. Let us only mention that first-level trigger muons are required to have a transverse momentum higher than 6 GeV/c and pseudorapidity $|\eta| < 2.5$. The other two muons are required to pass the cuts $p_\perp > 3$ GeV/c and $|\eta| < 2.5$.

3Semi-electronic $B_c \rightarrow J/\psi e^+ \nu$ decays, followed by the $J/\psi$ decay into $\mu^+ \mu^-$ can be considered as well. In addition, if the muon from the semi-muonic decay (1) gives the trigger, the event can be selected with the mode $J/\psi \rightarrow e^+ e^-$. The trigger acceptance for the former is $\simeq 12\%$ whereas for the latter is $\simeq 3\%$ [4].
removing its universal character. In the infinite quark mass limit, \( \hat{\xi} \) coincides with the Isgur-Wise function.

Experimentally, one can obtain the product \( \hat{\xi}(w)\cdot|V_{cb}| \) from the measured differential rate \( (1/\tau_B)dBr/dq^2 \), where \( q = p_1 - p_2 \) is the four-momentum transfer to the leptonic system (see figure 1). As developed by Neubert \([7]\), the zero recoil point is especially suitable for the extraction of \( |V_{cb}| \) in \( B \rightarrow D^{*}\ell\nu \) decays. This is because heavy-quark flavour symmetry and Luke’s theorem determine the normalization of \( \hat{\xi}(1) \) up to second-order power corrections: \( \hat{\xi}(1) = 1 + \delta_{1/m^2} \) where the last coefficient stands for the long-distance correction \([8]\). The theoretical control of these uncertainties permits a reliable determination of \( |V_{cb}| \) at this kinematic point where, however, there are no data due to vanishing phase space. An extrapolation to this endpoint is thus required and the final precision depends both on the experimental accuracy, background removal and theoretical corrections as well.

In order to apply a similar approach to the semileptonic channel (1) of \( B_c \) mesons we will characterize the kinematic state of hadrons by means of their four-velocity instead of four-momenta as well. This will make sense in our later theoretical approach as we are dealing with weakly bound states where the constituent heavy quarks are not too far offshell, as pointed out elsewhere \([10]\) \([11]\).

For clarity of presentation let us make below some remarks to be developed one by one in the following:

a) The accessible \( w \) range in \( B_c \rightarrow J/\psi\mu^+\nu \) decays (see figure 2) is limited to the interval \((1, 1.26)\), whereas in \( B \rightarrow D^{*}\ell\nu \) decays the range is significantly wider \((1, 1.5)\). The full kinematic reconstruction of the decay is required in order to measure \( w \) for each sampled event.

b) The precision in the measurement of the muon’s momentum in \( B \)-physics at LHC experiments is of order 2% \([3]\) or less \([4]\), representing an uncertainty of about 2% or less in the reconstructed momentum of the \( J/\psi \).

c) Background from processes like \( B \rightarrow J/\psi K^\pm \) must be taken into account.

d) Contamination from \( B_c \rightarrow X\mu^+\nu \) decays, where \( X = \psi(2S),\chi_c(1P) \) stands for a higher charmonium resonance, will be addressed as well.

e) We shall consider combinatorial background coming from \( b\bar{b} \) pairs subsequently decaying into muons.

f) A rigorous fragmentation framework recently developed by Bodwin, Braaten and Lepage \([4]\) could provide a reliable evaluation of the (high-\( p_T \)) production cross section of heavy quarkonia in \( pp \) collisions at LHC energy.

g) Precise predictions for the decay rate in the vicinity of zero-recoil could be obtained in the framework of lattice QCD \([5]\) or even potential models \([6]\) of hadrons.
2.1 Signal

In this section we develop items a) and b):

a) In order to get the \( w \) differential distribution of the \( B_c \) semileptonic decays, their complete kinematic reconstruction (event by event) is required. This can be achieved despite the unknown energy of the initial \((B_c)\) particle (whose mass is assumed to have been previously measured from the \(B_c \to J/\psi \pi \) decay at the LHC \( \text{[1]} \), or even earlier at the Fermilab Tevatron) and the unknown momentum of the final \( \nu \), if the initial direction of the former can be experimentally determined. Indeed, this should be the case by means of the vertexing capability of high performance inner detectors at LHC experiments.

In fact, a precise secondary vertex resolution for B physics has been established as an experimental goal in the ATLAS experiment \( \text{[5]} \). This fact together with the accurate reconstruction of the primary vertex at low luminosity should permit the measurement of the \( B_c \)'s flight-path.

Let us write \( w \) as

\[
\gamma_1 \gamma_2 (1 - |\vec{v}_1| \cdot |\vec{v}_2| \cos \theta)
\]  

where \( \gamma_i = (1 - |\vec{v}_i|^2)^{-1/2} \), \( |\vec{v}_i| \) stands for the modulus of the three-velocity of particle-\( i \) and \( \theta \) denotes the angle between the \( J/\psi \) and \( B_c \) (reconstructed) trajectories. The influence of the geometric uncertainty on \( w \) through the cosine in the above quantity near zero-recoil \((\gamma_1 \simeq \gamma_2 = \gamma)\) can be inferred from the following expression

\[
\delta w \simeq |\vec{v}| \gamma \sqrt{2 \frac{w - 1}{w}} \delta \theta = \sqrt{2 \frac{w - 1}{w}} \delta \theta_0
\]  

where \( \delta \theta = \sigma/(n + 1)\gamma|\vec{v}|\tau \) and \( \delta \theta_0 = \sigma/(n + 1)c\tau \). We shall take \( \sigma \approx 100 \mu m \) as a reasonable and safe estimate for the combined primary+secondary vertex resolution. (The vector joining the production and decay vertices in space could be measured with a precision of (of the order of) \( 100 \mu m \).) The \( B_c \) average decay length has been expressed in (4) as \( n + 1 \) “lifetime-units” where \( n \) denotes the cut on the proper decay length, i.e. we consider only decays with proper times larger than \( n\tau \). We shall set \( \tau \approx 0.75 \text{ ps} \) as the expected \( B_c \)’s lifetime. This is halfway between potential model predictions (0.5 ps) \( \text{[3]} \) and QCD sum rules expectations (1 ps) \( \text{[17]} \).

It is quite remarkable that the accuracy of \( w \) becomes independent of kinematic factors since a more precise measurement of the angle due to a longer decay length because a larger Lorentz factor, is compensated by a greater influence on \( \delta w \). On the other hand, notice that the uncertainty \( \delta w \) increases with \( w \). This behaviour may be qualitatively understood since a lower recoil implies a smaller \( \theta \) angle (\( \theta = 0 \) in the limit \( w = 1 \)), thus a smaller uncertainty through \( \cos \theta \).

Tentatively we shall set \( n = 7 \) (thus \( \delta \theta_0 \approx 56 \text{ mrad} \)) amounting to a reduction factor \( 10^{-3} \) of the original sample of triggered and reconstructed events. In fact, this choice represents a compromise: the requirement of longer \( B_c \) flight-paths by imposing higher

\footnote{It is foreseen the installation of an additional precision layer at low radius during the low luminosity running}
\( n \) values should certainly diminish the experimental uncertainty on \( \theta \) but would imply a further loss of statistics, thereby increasing statistical errors.

b) With regard to the influence on \( w \) from the muon momentum resolution we obtain

\[
\delta w \simeq 2 (w - 1) \frac{\delta \gamma}{\gamma}
\]  

(5)

Setting \( \delta \gamma/\gamma \simeq 2\% \), we notice that numerically its actual influence on \( w \) is smaller than that due to the uncertainty through the angle \( \theta \).

2.2 Background

We address below those aspects related to contamination of the signal (1) by either \( B \) decays or combinatorial background:

c) Background from decays like \( B \to J/\psi K^\pm \) may fake the \( B_c \) signal if the \( K \) is misidentified as a muon. This could happen due to punchthrough, for example, if the kaon decays to a muon in flight before it reaches the calorimeter. Let us remark, however, that such muon could be rejected from the stand-alone measurement of its momentum in the air-core toroid muon detector of ATLAS, once compared to the (larger) momentum of the parent kaon independently measured in the inner detector.

In addition, there are some possible strategies to remove this kind of contamination by firstly requiring coplanarity among the direction of the parent meson, the reconstructed trajectory of the \( J/\psi \) and the \( \mu \) track. In fact, this requirement amounts to no missing transverse momentum with respect to the direction of the decaying particle. Besides, the invariant mass of the final particles would be compatible with the \( B \) decay hypothesis.

There is, however, another potential source of contamination from \( B \to J/\psi K^* \) decays occurring again at a comparable rate as the signal. The \( K^* \) subsequent decay into \( K^\pm \) and an undetected \( \pi^0 \) would make useless the last invariant mass condition. Nevertheless, since the momentum of the \( K^\pm \) in the \( K^* \) rest frame is 291 MeV [4], a lower cut of about 300 MeV on the component of the \( K^\pm \) momentum perpendicular to the plane defined by the decaying particle and the \( J/\psi \) should remove all that background with a good acceptance (\( \simeq 80\% \)) for the signal [8]. In fact, this cut is compatible with the precision on the transverse momentum due to the flight-path reconstruction. Likewise, other sources of background from channels like \( B \to X_c K^\pm \) where \( X_c \) denotes charmonium resonances subsequently decaying into \( J/\psi \) and neutrals, should be rejected as well by means of this cut.

d) Contamination coming from \( B_c \to X \mu^+\nu \) decays where \( X = \psi(2S), \chi_c(1P) \), followed by the resonance radiative decay into a \( J/\psi \), would contaminate the signal (1). Indeed, one cannot get rid of this contamination by imposing to the semileptonic decays a missing mass constraint, contrary to \( \overline{B} \to D^* \ell \nu \) in an \( e^+e^- \) collider. However, it may be possible to detect the photon in the electromagnetic calorimeter, with typical energy of few GeV, predominantly emitted within a certain cone along the direction of the \( J/\psi \). In this way those events yielding an invariant mass compatible with any of the above-mentioned
resonances would be removed from the event sample. Nevertheless, before any serious simulation analysis about this possibility had been performed, we prefer to stay at more conservative grounds and consider provisionally the emitted $\gamma$ as useless. Therefore we must evaluate the degree of contamination of the signal sample.

We tentatively adopt experimental data on $B$ decays by making the reasonable assumption that the $B_c$ should decay semileptonically into higher resonances and $J/\psi$ with a relative ratio of order $2.7/7$ \(^4\). Further, using crudely an “average” $BF \simeq 0.16$ for the decay of the charmonium resonances into $J/\psi$ and neutrals \(^4\), we conclude that the expected contamination of the signal from all the above channels represents about $6\%$ hence below the statistical fluctuations of the sampled signal events as we shall see.

e) Combinatorial background is generated by the simultaneous coexistence of three muons in events. For instance this could happen if a $J/\psi$ is produced from a $B$ whereas the other muon comes from a different $b$ quark (thus from two spatially well separated vertices). This kind of contamination should be drastically suppressed by means of the seven-lifetimes cut imposed to the reconstructed secondary vertices. Furthermore, prompt $J/\psi$'s produced at the primary $pp$ interaction should be practically entirely removed as a background source by requiring a combined cut on the $\chi^2$ of the fitted secondary vertex and the decay length \(^3\) \(^3\).

### 2.3 Theory inputs and determination of $|V_{cb}|$

Next we examine in detail the possibility of extracting $|V_{cb}|$ from experimental data.

f) Let us start by recalling that

$$\frac{d\Gamma(B_c\rightarrow J/\psi \, \mu^+\nu)}{dw} = \frac{1}{\tau} \frac{dBr(B_c\rightarrow J/\psi \, \mu^+\nu)}{dw} \tag{6}$$

The differential branching ratio $dBr/dw$ can be written as

$$\frac{dBr(B_c\rightarrow J/\psi \, \mu^+\nu)}{dw} = \frac{1}{N(B_c)} \frac{dN(B_c\rightarrow J/\psi \, \mu^+\nu)}{dw} \tag{7}$$

where $dN/dw$ represents the number of semileptonic decays per unit of $w$ for a certain integrated luminosity, once corrected by detection efficiency; $N(B_c)$ is the corresponding total yield of $B_c$'s produced for such integrated luminosity. (Notice that $\tau$ could be determined accurately from the same collected sample of semileptonic $B_c$ decays.)

Let us observe that the exclusive decay mode $B_c\rightarrow J/\psi\pi$ to be detected at LHC \(^1\) cannot be employed for normalization purposes as far as its $BF$ would not be (obviously yet) experimentally known. On the other hand, $B_c$ decay modes involving the $c$-quark (which in principle would permit the measurement of ratios of CKM matrix elements in combination with $b$ decays) will be hardly observable among the huge background of a hadron collider.

Therefore, in order to get $N(B_c)$ it will be convenient to compare the $B_c$ production rate to the prompt $\psi'$ yield in $pp$ collisions, for the same integrated luminosity. Therefore
\[
N(B_c) = \frac{\sigma(pp\rightarrow B_c + X)}{\sigma(pp\rightarrow \psi' + X)} \times \frac{N(\psi'\rightarrow \mu^+\mu^-)}{\epsilon_{\psi'}BF(\psi'\rightarrow \mu^+\mu^-)}
\]  
(8)

where \(N(\psi'\rightarrow \mu^+\mu^-)\) stands for the number of prompt \(\psi'\) experimentally observed through the muonic decay mode and \(\epsilon_{\psi'}\) denotes its detection efficiency. Contamination from weak decays of \(B\) mesons into \(\psi'\) states should be efficiently removed by means of the vertex capability of the inner detector \([1] [3]\).

Therefore we suggest normalizing the \(B_c\) sample with the aid of the \(\psi'\) yield through expression (8). The \(\psi'\) state is preferable to the \(J/\psi\) mainly because the former expectedly should not be fed down by higher charmonium states \([5]\). Instead, data released by Tevatron shows that the \(J/\psi\) indirect production through \(\chi_c\) intermediate states may easily overwhelm direct production. Let us also remark that the production of the \(\psi'\) resonance in \(p\bar{p}\) collisions is more than one order of magnitude larger than initially expected \([19] [20]\).

At sufficiently large \(p_{\perp}\) one reasonably expects that the main contribution to heavy quarkonia production comes from the splitting of a heavy quark or a gluon. Indeed, even though the fragmentation process is of higher order in \(\alpha_s\) than the "conventional" leading order diagrams \([21]\), the former is enhanced by powers of \(p_{\perp}/m_Q\) relative to the latter \([4] [22]\). Although the situation is still controversial in the literature, there are strong indications from very detailed calculations (see \([23] [24]\) and references therein) that fragmentation indeed dominates for large enough \(p_{\perp}\). In the following, we first examine some color-singlet mechanisms, expected to contribute largely to the high-\(p_{\perp}\) inclusive production of prompt heavy quarkonia.

According to the set of papers in \([2]\), perturbative QCD can provide a reliable calculation for the fragmentation function of a high-\(p_{\perp}\) parton into heavy quarkonia with only few input parameters: the charm and bottom masses, and the square of the radial wave function of heavy quarkonium at the origin \(|R(0)|^2\). Note that the largest uncertainty for normalization purposes comes from the (third power of the) charm mass in the fragmentation function (see \([2]\) for explicit expressions). However, we are only interested in the ratio

\[
r = \frac{\sigma(pp\rightarrow B_c + X)}{\sigma(pp\rightarrow \psi' + X)}
\]  
(9)

where each cross section can be written as a convolution of the parton distribution functions of the colliding protons, the cross sections for the hard subprocesses leading to the fragmenting gluon or heavy quark and the respective fragmentation functions \([2] [23]\). Thus notice that the fragmentation functions \(D_{6\rightarrow B_c}, D_{g\rightarrow B_c}, D_{c\rightarrow \psi}, D_{g\rightarrow \psi}\) \([26]\), which themselves are independent of the parton-level subprocesses, appear combined as a ratio. Therefore, those uncertainties coming from the common overall factor \(m_Q^3\) automatically cancel each other. Moreover, those uncertainties introduced by the parton distribution functions also should significantly diminish in \(r\) as well, especially when a cut on the

\[\text{See Ref.} [15] \text{ for an alternative explanation of the observed } \psi' \text{ surplus found in Tevatron. However this mechanism would require an uncomfortable large radiative } BF \text{ for higher resonances}\]

\[\text{We find an average } p_{\perp} \approx 20 \text{ GeV/c for } B_c \text{ mesons simply passing the kinematics cuts on } p_{\perp} \text{ and } |\eta| \text{ of the decay muons } [3] \]
transverse momentum of heavy quarkonia is required. Still one must evaluate

$$\kappa_0 = \frac{|R_{Bc}(0)|^2}{|R_\psi(0)|^2}$$  \(10\)

In fact there is the possibility of expressing Eq. (10) according to the general factorization analysis of [14] in a more rigorous way as

$$\kappa_0 = \frac{<0|O_{Bc}^1(1S_0)|0>}{<0|O_1^{\psi'}(3S_1)|0>}$$  \(11\)

where \(O_X^n\) are local four quark operators and the matrix elements can be evaluated from NRQCD. A similar expression holds for the \(B^*_c\). (Moreover, the denominator can be determined from the measured leptonic width of the \(\psi'\).)

Recently, an additional color-octet fragmentation mechanism [27] has been suggested in order to reconcile the experimental results on inclusive \(\psi'\) production with theoretical predictions. This mechanism assumes the creation from gluon fragmentation of a \(c\bar{c}\) pair in a color-octet state, in analogy to \(\chi_c\) production [19]. Then a new nonperturbative parameter \(H_{8(\psi')}\) is required, appearing in \(r\) combined as the dimensionless factor

$$\kappa_8(\psi') = \frac{2\pi m^2 H_{8(\psi')}'}{|R_{\psi'}(0)|^2} \approx \frac{<0|O_{8}^{\psi'}(3S_1)|0>}{<0|O_{1}^{\psi'}(3S_1)|0>}$$  \(12\)

where we remark that it can be expressed in terms of NRQCD matrix elements [14].

Furthermore, if we do not neglect the contribution to the total \(B_c\) production of those orbital excitations like \(P\)-wave states, new fragmentation functions \(D_{b \rightarrow c(P)}\) come into play [30]. Accordingly, some new \(\kappa_i\) factors will appear which can be conveniently expressed, for instance in analogy to Eqs. (11) and (12), as:

$$\kappa_{1(B_c)} = \frac{2\pi \hat{m}^2 H_{1(\psi')}'}{|R_{B_c}(0)|^2} \approx \frac{1}{\hat{m}^2} \frac{<0|O_{1}^{B_c}(P_j)|0>}{<0|O_{1}^{B_c}(1S_0)|0>}$$  \(13\)

and

$$\kappa_{8(B_c)} = \frac{2\pi \hat{m}^2 H_{8(\psi')}'}{|R_{B_c}(0)|^2} \approx \frac{1}{\hat{m}^2} \frac{<0|O_{8}^{B_c}(3S_1)|0>}{<0|O_{1}^{B_c}(1S_0)|0>}$$  \(14\)

where \(\hat{m}\) is the reduced mass of the \(b\) and \(c\) quarks. The long-distance \(H_1\) parameter [29] is related to the square of the derivative of the color-singlet wave function at the origin.

Higher \(B_c\) resonances like \(D\)-wave states might be further taken into account, introducing new nonperturbative parameters [11] involving higher derivatives of the wave function at the origin, or the NRQCD matrix element analogues.

In sum, the evaluation of the \(\kappa_i\) parameters would allow one to complete the computation of the ratio \(r\) and thereby to obtain from the experimental measurement of the \(\psi'\)
yield, the total number of $B_c$ events for an integrated luminosity with the aid of expression (8).

We next address the hadronic transition between the initial $B_c$ and final $J/\psi$ particles.

g) From the theoretical side the hadronic transition $B_c \rightarrow J/\psi$ involves two heavy-heavy systems. Although originally HQET was only applied to heavy-light bound states, there is a growing belief that it can be cautiously applied to the former [10] [11]. Indeed, spin symmetry should be still valid as a first order approach in the dynamics of heavy-heavy hadrons. This allows the introduction of the analogue of the Isgur-Wise function for transitions involving doubly heavy hadrons, denoted by $\eta_{12}(v_1,v_2)$. At the non-recoil point $v_1 = v_2 = v$, we shall write [10]

$$< J/\psi | A^\mu | B_c > = 2 \eta_{12}(1) \sqrt{m_1 m_2} \varepsilon_2^\mu$$

where $A^\mu = \bar{c}\gamma^\mu \gamma_5 b$ stands for the axial-vector current and $\varepsilon_2^\mu$ represents the four-vector polarization of the $J/\psi$.

Following similar steps as in the $B$ decay into $D^*$, we write the analogue of equation (2) as

$$\lim_{w \rightarrow 1} \frac{1}{\sqrt{w^2 - 1}} \frac{d\Gamma(B_c \rightarrow J/\psi \mu^+\nu)}{dw} = \frac{G_F^2}{4\pi^3} (m_1 - m_2)^2 m_2^3 \eta_{12}^2(1) |V_{cb}|^2$$

It is of key importance from the theoretical side in our proposal, the existence of a single form factor in the above expression to be determined theoretically in a rigorous manner. Observe that this is valid only in a neighbourhood of $w = 1$, however. Deviations from the strict requirement of the spin symmetry lead to the appearance of new form factors in the hadronic matrix element [8]. Even though they can be absorbed in a redefinition of $\eta_{12}(w)$ this does not mean that we could ignore them if $|V_{cb}|$ has to be extracted away from zero recoil.

Note that $\eta_{12}(1)$ may be interpreted in first approximation as the overlap of the initial and final hadron wave functions [10] [32]. Due to the intrinsic nonrelativistic nature of its heavy constituents, nonrelativistic QCD on the lattice or even refined potential based models of hadrons, could be applied in order to get $\eta_{12}$ and its slope at zero recoil in a reliable way.

This fact in conjunction with the presumed accuracy of the $J/\psi$ momentum measurement suggests determining $|V_{cb}|$ in a similar way as in the method proposed by Neubert [7] through the $\bar{B} \rightarrow D^*\ell\nu$ decay.

Observe that from an experimental point of view the $D^*$ kinematic reconstruction should suffer from larger uncertainties through the cascade decays: $D^* \rightarrow D \pi$ followed by the $D$ decay into final states with only charged particles. Conversely, the $J/\psi$ decay into $\mu^+\mu^-$ is much cleaner by all means. Of course, the environment is much dirtier in a hadron collider than in a $e^+e^-$ factory [4]. Nevertheless, since the signal consists of three muons the $B_c$ decay can be disentangled from the huge hadronic background by means of adequate kinematics cuts and constraints [4], at least at low luminosity.

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8 One may conjecture about the possibility of producing $B_c\bar{B}_c$ pairs in a future $\mu^+\mu^-$ collider [3]
In order to make a crude estimate of the slope of the analogous Isgur-Wise function $\eta_{12}$ at zero recoil, we shall make use of a nonrelativistic approach. Specifically we employ the ISGW \[34\] model which should give sensible results in the case under examination. The overlap wave function then reads

$$\eta_{12}(t) = \left[ \frac{2\beta_1\beta_2}{\beta_1^2 + \beta_2^2} \right]^{3/2} \exp \left[ -\frac{m_c^2}{2m_1m_2} \frac{t_m - t}{k^2(\beta_1^2 + \beta_2^2)} \right]$$

where $t = q^2$, $t_m = (m_1 - m_2)^2$ and $\beta_i$ denotes a variational parameter corresponding to the $i$-meson obtained by adjusting the hadronic spectra; $k$ is a parameter introduced somewhat by hand, usually set equal to 0.7.

Now let us introduce the slope parameter $\rho_{12}^2$ of the $\eta_{12}(w)$ function near zero recoil as

$$\eta_{12}(w) = \eta_{12}(1) \left\{ 1 - \rho_{12}^2 (w - 1) + \mathcal{O}[(w - 1)^2] \right\}$$

Since $t_m - t = 2m_1m_2(w - 1)$, we find that $\rho_{12}^2$ can be expressed according to the ISGW model as

$$\rho_{12}^2 = \frac{m_c^2}{2\beta_{12}^2}$$

where $\beta_{12}^2 = k^2(\beta_1^2 + \beta_2^2)/2$. For transitions between heavy-light mesons, $\beta_1 = \beta_2 = \beta$ and putting $k = 1$ one quickly recovers the result of \[35\]: $\rho^2 = m_q^2/2\beta^2$. (The relativistic correction due to the boosted four momentum of the light quark $q$ should not affect $\rho_{12}^2$ so much.)

Setting the parameters in Eq. (19) equal to some typical numerical values for $B_c$ and $J/\psi$ states, $\beta_1 = 0.82$ GeV, $\beta_2 = 0.66$ GeV, $m_c = 1.5$ GeV and letting $k$ vary between 0.7 and 1, we estimate a range

$$\rho_{12}^2 \approx 2 - 4$$

representing a higher value than $\rho^2$ (of order unity) for transitions between $B$ and $D^*$ mesons \[9\], as otherwise expected \[36\]. (Of course, this range of $\rho_{12}^2$ values must be considered as merely indicative.) Note also that one expects the curvature of $\eta_{12}(w)$ at $w = 1$ for transitions between heavy-heavy hadrons to be larger than that of $\xi(w)$ between heavy-light ones. This implies the necessity of a closer access to the zero recoil point for the accuracy of the Taylor expansion of Eq. (18) as is indeed the case.

For the purpose of illustration, we have depicted in figure 3 some expected typical points and error bars for the decay $B_c \rightarrow J/\psi(\rightarrow \mu^+\mu^-) \mu^+\nu$ for a definite prediction of the ratio of production cross sections $r$ in Eq. (9).

Under this condition, let us remark that the uncertainty on $w$ (i.e the horizontal error bars) is of systematic nature, mainly coming from the $B_c$ flight-path reconstruction. With regard to the vertical coordinate $\eta_{12}(w) \cdot |V_{cb}|$, we have assumed that its uncertainty is essentially statistical, besides the contamination by decays into higher charmonium resonances which are included as well in the vertical error bars.
This work concerns $B_c$ mesons to be copiously produced at LHC next century. We have argued that it should be possible to reconstruct those semileptonic decays into a $J/\psi$ near zero recoil, followed by its consequent decay into a muon pair. The fact that the $J/\psi$ is a vector particle should lead to a mild falling off of the number of events in the kinematic region near zero recoil as in $B \to D^{\ast} \ell \nu$. (This is because of the $s$-wave contribution to these decays, absent for a final pseudoscalar hadron as in the $B \to D \ell \nu$ channel.)

The signature of the decay $B_c \to J/\psi(\to \mu^+ \mu^-) \mu^+ \nu$, consisting of three muons coming from a common secondary vertex could be disentangled from the huge hadronic background. The excellent accuracy in the measurement of the muons momenta would lead to a precise determination of the $J/\psi$ momentum and energy. In addition, a good 3D vertex reconstruction capability is determinant for the full kinematic reconstruction of events. We have shown that, in fact, it is $\delta \theta_0 = \sigma/(n + 1) c\tau$ one of the parameters that actually matters in the expected accuracy of the differential $w$ distribution of sampled events.

We also addressed some background sources, either from real $B$ decays but faking $B_c$ ones, or from true $B_c$ semileptonic decays into higher charmonium resonances. Several strategies to remove all these contaminations were proposed.

From the theoretical point of view, the heavy quark spin symmetry permits the introduction of the analogue of the Isgur-Wise function $\eta_{12}$ for transitions between doubly heavy mesons. Deviations from the spin symmetry imply the appearance of new form factors in the hadronic matrix element which, however, do not contribute at zero recoil. Hence, at this kinematic point, only a single form factor (though including itself spin breaking effects) has to be determined for the $B_c \to J/\psi$ transition.

Experimental data would permit to obtain the product $\eta_{12}(w) |V_{cb}|$ near zero recoil in a more or less accurate way according to the accuracy of the differential decay rate $d\Gamma/dw$ itself. The measurement of the latter requires the normalization of the $B_c$ semileptonic events to the total number of the $B_c$ sample. (In other words the knowledge of the absolute branching fraction for the semileptonic decay is required.) To this end, one can rely on the fragmentation approach providing in a rigorous way the ratio of the yield of $B_c$ mesons relative to the prompt $\psi(2S)$’s produced in $pp$ collisions (see Eq. (8)). As far as we are interested in their relative production rate $r$, the uncertainty should be much less dependent on theoretical or phenomenological inputs (like heavy quark masses) than in each production rate separately.

We have shown that a set of long-distance dimensionless factors (denoted by $\kappa_{0,1,8}$) involving ratios of nonperturbative parameters like $|R(0)|^2$, $H_1$ and $H'_8$ must be determined to make a reliable prediction of $r$. Even if their evaluation from NRQCD [14] is still in its infancy, lattice techniques should permit in a not too far future to go beyond model estimates, including relativistic corrections, in a systematic and reliable way.

In addition, forthcoming $e^+e^-$ factories and Tevatron running at higher luminosity should improve considerably the accuracy of the measured $BF$ of the $B$ and $\Upsilon$ decay modes involving $P$-states of charmonium, improving the experimental information on such parameters. Let us also remark that the fragmentation hypothesis can be checked with
experimental data from $pp$ or $p\bar{p}$ collisions, for instance through the transverse momentum dependence of heavy quarkonia, its polarization and association to jets of light hadrons.

Once the differential rate $d\Gamma/dw$ is achieved to be experimentally known, an extrapolation to the $w = 1$ endpoint, either linear or quadratic (i.e. including the curvature) would yield $\eta_{12}(1) |V_{cb}|$. Consequently, the knowledge of $\eta_{12}(1)$ should provide $|V_{cb}|$. Let us stress that this calculation and the evaluation of the fragmentation probability into heavy quarkonium, both involving heavy quarks, rely essentially on similar theoretical grounds and could be derived from first principles [14].

Of course, one may consider as well the possibility of choosing a kinematic point different from $w = 1$ to obtain $|V_{cb}|$. This requires, however, the introduction of new independent form factors (i.e. entering in the hadronic matrix element with different Lorentz structures) whereas at zero recoil one must evaluate all these corrections to a single form factor. Furthermore, from an experimental point of view the non-recoil point is also more favourable. For example, a possible choice might be the maximum recoil point of the $J/\psi$ at $w = 1.26 \ (q^2 \simeq 0)$, where several models predict the wave function overlap [37]. Nevertheless, notice from figure 3 that, despite smaller vertical (statistical) error bars, horizontal (systematic) ones are larger, hence limiting a close access to this point and thereby making more advantageous the extrapolation to the $w = 1$ end-point.

Lastly we make some final comments:

- From the experimental viewpoint, observe that if the secondary vertex resolution could be improved and/or the lifetime of the $B_c$ was closer to 1 ps than to 0.5 ps, the experimental accuracy near zero recoil improves considerably. Thus, we show in figure 4 possible plots for different angular resolutions.

- It is important to stress that the quantity to be experimentally measured with a large accuracy from $B_c \to J/\psi \, \mu^+\nu$ decays can be written as the linear combination:

$$r^{1/2} \times \eta_{12}(1) \times |V_{cb}|$$

In summary, we suggest to keep an open mind on the possibility of an alternative determination of $|V_{cb}|$ from semileptonic decays of $B_c$ mesons at LHC. Even if at the time LHC will start to run, B factories would have already provided a (still) more precise value of $|V_{cb}|$ than at present via the semileptonic $B$ decay, an independent determination of it should bring a valuable cross-check. On the other hand, a lot of activity is being devoted to the analysis of inclusive production of prompt charmonium resonances at Fermilab. Therefore we conclude that if the fragmentation approach to describe the production of heavy quarkonia at the large transverse momentum domain is accurately tested (and tuned) from experimental data, $B_c$ semileptonic decays could be competitive with the $B$ ones.

Alternatively, one can turn the question round and consider $|V_{cb}|$ as a well-known parameter thus verifying QCD calculations, either at the production or at the weak decay level. Indeed, fragmentation into heavy quarkonia offers an interesting check of perturbative QCD and a deep insight into the nonperturbative dynamics in the hadronic formation. Besides, a precise knowledge of the $B_c$ production rate is a necessary condition for the experimental measurement of the absolute branching fraction of any of its decay modes.
Acknowledgments

We are especially indebted to P. Eerola and N. Ellis, and the B physics group of the ATLAS Collaboration for comments, suggestions and an encouraging attitude. Discussions on partial aspects of this work with V. Giménez, M. Neubert, A. Pineda, O. Pène and J. Soto are acknowledged as well.
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Figure 1: $\hat{\xi}(w) - |V_{cb}|$ distribution derived from the $q^2$-spectrum of the decay $B \rightarrow D^* \ell \nu$ measured by ARGUS \cite{12} and $\eta_A \hat{\xi}(w) - |V_{cb}|$ measured by CLEO \cite{13}. 

$\hat{\xi}(w)$
Figure 2: Dalitz plot corresponding to the decay $B_c \to J/\psi \mu^+ \nu$. The leptonic momentum squared is given by $q^2 = (p_{\mu} + p_{\nu})^2 = (p_{B_c} - p_{J/\psi})^2$, and $s_{23} = (p_{J/\psi} + p_{\mu})^2$. The horizontal lines correspond to constant $w = v_{B_c} \cdot v_{J/\psi}$. The non-recoil point of the $J/\psi$ in the decaying rest frame has coordinates $s_{23} \simeq m_{B_c} m_{J/\psi} \simeq 19.5$ GeV$^2$ and $q^2 = (m_{B_c} - m_{J/\psi})^2 \simeq 10.1$ GeV$^2$. Notice the linearity between the velocity transfer $w$ and $q^2$ as a kinematic variable characterizing the form factors variation for the hadronic transition.
Figure 3: Hypothetical $\eta_{12}(w)\cdot |V_{cb}|$ distribution of the decay $B_c \rightarrow J/\psi \mu^+ \nu$ for a 500 events sample tentatively assigned to an integrated luminosity of 20 $fb^{-1}$ (i.e. two years of running at low luminosity). The available $w$ range extends over the interval $(1, 1.26)$. It was assumed an spatial resolution for flight-path reconstruction of $\sigma \approx 100$ $\mu$m and a cut of $n = 7$ lifetime units ($\delta \theta_0 \approx 56$ mrad). Horizontal bars show systematic errors basically limiting the number of experimental points. Vertical bars take into account both the statistical fluctuations of the sampled events in the bin (distributed according to the expected differential rate for a pseudoscalar to vector semileptonic decay [8]), and the uncertainty coming from contamination of higher resonances in the $B_c$ decay. Points line up with $\eta_{12}$ set equal to $-2$. 
Figure 4: The same as in figure 3 but with different values for $\delta\theta_0$ corresponding to variations of $\sigma$ between 50 and 100 $\mu$m and $\tau$ between 0.5 and 1 ps. The improvement on the number of points near zero recoil can be considerable.