Instantons and $\eta$ Meson Production near threshold in NN collisions

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Abstract

An enhancement for the $\eta$ production in proton-neutron collisions as compared with that in proton-proton scattering has been recently observed. We present a calculation for the production cross section, in proton-neutron collisions near threshold, within instanton model for the QCD vacuum and show that a specific flavor dependent non-perturbative quark-gluon interaction related to instantons is able to explain the observed enhancement.

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1 Introduction

The interest of the investigation of meson production in nucleon-nucleon and nucleon-antinucleon collisions near threshold has increased recently since different anomalies have been detected \cite{1,2,3,4}. These anomalies are characterized by unexpected enhancements in the production of the mesons which contain large admixtures of strange quarks in their wave function. The large OZI rule violations near threshold challenge most of the theoretical approaches to the dynamics of strangeness production in strong interactions (see discussion in \cite{5}).

One of this anomalous behaviors is related to the surprisingly large cross section in proton-neutron collisions near threshold for the $\eta$ production \cite{2,4}. It has been found recently \cite{4} that the ratio to the same process in p-p scattering in the same kinematical regime is about 7.

An explanation of this enhancement, based on the large contribution from $N^*(1535)$ ($S_{11}$) resonance has been suggested \cite{6}. However the final result is very sensitive to the values of the meson-$S_{11}$ coupling constants, which at present are not well known. It is therefore difficult to establish the precise contribution from the $S_{11}$ to this cross section and confirm the validity of the mechanism.

In here, we present a different explanation for the enhancement, which in our case is associated with a particular microscopic mechanism of QCD. In the vacuum of QCD there exist strong fluctuations of the gauge fields called instantons (see recent review \cite{7}). The instantons induce flavor dependent quark-quark \cite{8} and quark-gluon interactions \cite{9} whose structure is related with the flavor and helicity properties of the quark zero-modes in the presence of the instanton fields. The most famous of them is t’Hooft’s quark-quark interaction \cite{8} which has been successfully applied to describe the properties of the QCD vacuum and hadron spectroscopy \cite{10}. Besides this interaction, which contains $2N_f$ quark legs, the instantons can also lead to different type of the vertices which include arbitrary numbers of quark and gluon legs. The gluons in these vertices produce easily strange quarks. Therefore these new type of interactions should lead to large violations of the OZI rule in the strong interaction. One example of them, which leads to a non-zero value of the chromomagnetic moment of the quark, has been discussed recently \cite{9}.

In Section 2 we discuss the effective quark-multi-gluon Lagrangian induced by the instantons. In Section 3 the specific contribution of the mechanism to the cross section for the production of $\eta$ mesons in proton-neutron
collisions is estimated. Finally in Section 4 the main conclusions of the present work will be drawn.

2 Effective quark-gluon vertices induced by instantons

The effective Lagrangian induced by instantons has the following form \[10\]

\[ L_{\text{inst}} = \int \prod_q (m_q \rho - 2\pi^2 \rho^3 \bar{q}_R (1 + \frac{i}{4} \tau^a U_{aa'} \bar{\eta}_{a' \mu\nu} \sigma_{\mu\nu}) q_L) \times \exp(-\frac{2\pi^2}{g} \rho^2 U_{bb'} \bar{\eta}_{b' \gamma\delta} G_{\gamma\delta}^b \frac{d\rho}{\rho^5} d_0(\rho) d\delta), \] (1)

where \( \rho \) is the instanton size, \( \tau^a \) are the Pauli matrices associated with the generators of the \( SU(2)_c \) subgroup of the \( SU(3)_c \) color group, \( d_0(\rho) \) is the density of the instantons, \( d\delta \) stands for integration over the instanton orientation in color space, \( \int d\delta = 1 \), \( U \) is the orientation matrix of the instanton, \( \bar{\eta}_{\mu\nu} \) represent t’Hooft’s symbols. This Lagrangian describes the effective interaction between gluons and quarks arising from QCD through the color dipole moment of the instanton.

The effective quark–gluon interaction which generates the contribution to \( \eta \) production can be obtained by expanding \( \| \) in powers of the gluon strength and integrating over \( d\delta \). The effective parameter of this expansion is the ratio of the average instanton size \( \rho_c \) to the confinement radius \( R_{\text{conf}} \). This ratio, in realistic models for the QCD vacuum \[7\], is rather small, \( \rho_c^2 / R_{\text{conf}}^2 \approx 0.1 \), and therefore we can restrict the expansion to the lowest order. The corresponding approximation contains contributions to the production of mesons with definite quantum numbers.

To second order in \( G_{\gamma\delta}^b \) we found out that effective Lagrangian, which is responsible for \( \eta \) production, has the following form

\[ \Delta L_\eta = - \int L_{\text{t’Hooft}} \frac{\pi^3 \rho^4 d\rho}{8\alpha_s(\rho)} G_{\mu\nu}^a \bar{G}_{\mu\nu}^a, \] (2)

where \( L_{\text{t’Hooft}} \) is t’Hooft’s interaction which for massless \( u \) and \( d \) quarks is

\[ For \text{ anti-instantons one should interchange } R \leftrightarrow L. \]
\[ \mathcal{L}_{t'\text{Hooft}} = \frac{16\pi^4 d_0(\rho)\rho}{9} (\bar{u}_R u_L \bar{d}_R d_L + \frac{3}{8} (\bar{u}_R t^a u_L \bar{d}_R t^a d_L - \frac{3}{4} \bar{u}_R \sigma_{\mu \nu} t^a \bar{u}_L \bar{d}_R \sigma_{\mu \nu} t^a d_L)), \]

This quark-gluon vertex is presented in Fig.1. Within the simplest version of the instanton liquid model for the QCD vacuum, the size of instantons is fixed

\[ d_0(\rho) \sim \delta(\rho - \rho_c). \]

\( \rho_c \sim 1.6 \div 2.0 \text{ GeV}^{-1} \) is the average size of the instantons in the QCD vacuum. This value is obtained from hadron spectroscopy \[4\]. However this size is not very well defined, for example, recent lattice calculations \[11\] give a value for it which is larger, \( \rho_c \sim 2.5 \text{ GeV}^{-1} \). We will therefore in our calculation consider a large interval for \( \rho_c \sim 1.6 \div 2.5 \text{ GeV}^{-1} \).

By using the low energy theorem, based on the chiral anomaly, one can calculate the following matrix element \[12\], \[13\]

\[ < 0 | \frac{3\alpha_s}{4\pi} G_{\mu \nu}^a G_{\mu \nu}^a | \eta > \approx \sqrt{3} f_\pi m_{\eta}^2, \]

where we have neglected \( m_u \) and \( m_d \) with respect to \( m_s \). Under the assumptions of ref. \[13\] the quark-\( \eta \) meson interaction from \(2\) becomes

\[ \mathcal{L}_\eta = \frac{2\pi^6 \rho_c^6 f_{\pi} m_{\eta}^2}{\sqrt{3} \alpha_s^2(\rho_c)} (\bar{u}_R u_L \bar{d}_R d_L + \frac{3}{8} (\bar{u}_R t^a u_L \bar{d}_R t^a d_L - \frac{3}{4} \bar{u}_R \sigma_{\mu \nu} t^a \bar{u}_L \bar{d}_R \sigma_{\mu \nu} t^a d_L)) \phi_\eta. \]
where $\phi_\eta$ is the $\eta$-field. It should be mentioned that this effective interaction can only be used if the frequency of gluon fields is small. This condition however holds for the production of the particles near threshold.

3 Contribution of the instantons to the $\eta$ production cross section near threshold

A very unique feature of the interaction (3) is its strong flavor dependence. Indeed it is non zero only if quarks have different flavors. Therefore, one can expect that the contribution of it to the $\eta$ meson production in proton-neutron collision is larger then in proton-proton and neutron-neutron ones, because in the former the flavor structure favors the probability of interaction.

The contribution of the instanton interaction is determined by the matrix element of interaction (3)

$$< NN|L_\eta|NN\eta > \propto < NN|L'_t\text{Hooft}|NN > .$$

Due to the strong spin-flavor dependence of t'Hooft's interaction (3), the final result is very sensitive to the spin-flavor structure of the nucleon wave function. We will use a model for hadron structure which has proved to be consistent with the instanton picture and which will favor the proposed mechanism.

It is well known that a significant violation of $SU(6)$ symmetry in the nucleon wave function is required to explain some properties of nucleons: $\Delta$-nucleon mass splitting, enhancement of $u$ quark distribution with respect to $d$ quark distribution at large Bjorken $x$ region in deep-inelastic scattering, behavior of nucleon form factors, etc... (see [14]). It was shown [15] that the instanton induced interaction can explain the nucleon-$\Delta$ mass difference, due to the formation of a quasi-bound scalar $ud$ diquark inside nucleon. At present, the instanton induced diquark configuration is widely under discussion, not only in connection with hadron spectroscopy, but also as a possible scenario for the formation of a color superconducting state in quark-gluon matter [16]. Therefore we feel encouraged to use this simplifying scenario.

In the quark-diquark model, proton and neutron consist of a scalar $ud$ diquark and a $u$-quark or $d$-quark, respectively. It is evident that due to specific flavor properties of the instanton vertex only the interaction between quarks with different flavors, which are not included into diquark configuration, can lead to the $\eta$ meson production in nucleon-nucleon scattering (see
Therefore the instanton induced $\eta$ meson production in proton-neutron interaction within pure quark-$ud$-diquark model should be very small in comparison with proton-neutron scattering. The experimental observation of a sizeable enhancement of the $\eta$ production in proton-neutron scattering, gives arguments in favor of a large diquark component in the nucleon wave function, however the proposed dynamical mechanism favors the enhancement in other scenarios, e.g., bag model or naive quark model, as well.

Let us to estimate the cross section for $\eta$-meson production in proton-neutron collisions. We follow the notation:

$$p(p_1) + n(p_2) \rightarrow p(p'_1) + n(p'_2) + \eta(p_\eta).$$

(8)

The cross section is given by

$$d\sigma = \frac{dPS^3}{4\sqrt{(p_1.p_2)^2 - m_p^4}} \sum_{spin} |M|^2,$$

(9)

Figure 2: The instanton contribution to the $\eta$ meson production in proton-neutron interaction

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2 The diquark-quark interaction mediated by one instanton vanishes, as determined by t’Hooft’s mechanism, since the two quarks that participate in the interaction have the same flavor. A diquark-quark interaction, through this mechanism, arises only if mediated by at least two different instantons, and is suppressed by the smallness of the density of instantons in the QCD vacuum.

3 If we neglect the production of the $\eta$ meson in the diquark-quark interaction and contributions from other mechanisms ($S_{11}, \ldots$) then the ratio of cross sections becomes $\sigma_{pp}/\sigma_{pn} \approx 0$.

4 In the case of the pure $SU(6)$ nucleon wave function one can expect the ratio $\sigma_{pp}/\sigma_{pn} \approx 1/4$, based on the result of [17] where the single instanton contribution to the ratio of amplitudes of elastic pp and pn scattering has been obtained, $\Phi(pp)/\Phi(pn) = -8/17$. 

where
\[ dPS^3 = \frac{d^3p_1'd^3p_2'd^3p_\eta}{(2\pi)^92E_1'2E_2'2E_\eta}(2\pi)^4\delta(p_1 + p_2 - p_1' - p_2' - p_\eta), \]
(10)
is the 3-body phase space volume. \( M \) is the matrix element of the interaction \( (7) \) and \( \sum_{\text{spin}} \) represents the spin summation over the final and spin averaging over the initial nucleon states.

Only the colorless part of the instanton induced interaction can contribute to the cross section of the reaction \( (8) \). Within the quark-di-quark model one can factorize the matrix element and obtains using a nonrelativistic approximation for the quark wave functions of the nucleon,
\[
M \sim <PN|\bar{u}_R u_L \bar{d}_R d_L|PN> \approx <P|\bar{u}_R u_L|P><N|\bar{d}_R d_L|N>,
\]
(11)
The final result for the matrix element is
\[
\sum_{\text{spin}} |M|^2 = \frac{\pi^{12} \rho_c^{12} f_\pi^2 m_p^4}{54 \alpha_s^4(\rho_c)^2} (8m_p^4 - 2m_p^2(t_1 + t_2) + t_1 t_2) F_N(t_1)^2 F_N(t_2)^2, \]
(12)
where
\[
t_1 = (p_1 - p_1')^2, \\
t_2 = (p_2 - p_2')^2,
\]
and \( F_N(t) \) is the strong form factor of the nucleon for which we will use the electromagnetic form factor \[ F_N(t) = \frac{4m_p^2 - 2.79t}{(4m_p^2 - t)(1 - t/0.71)^2}. \]
(13)

In the numerical calculation we use the NLO approximation for the strong coupling constant
\[
\alpha_s(\rho) = -\frac{2\pi}{\beta_1 t} (1 + \frac{2\beta_2 t}{\beta_1 t}),
\]
(14)
where
\[
\beta_1 = -\frac{33 - 2N_f}{6}, \quad \beta_2 = -\frac{153 - 19N_f}{12}
\]
(15)
We took into account in \[ (12) \] the additional factor two related to the contribution of anti-instantons.
and

\[ t = \log\left( \frac{1}{\rho_0^2 \Lambda^2} + \delta \right). \]  

In Equation (16), the parameter, \( \delta \approx \frac{1}{\rho_0^2 \Lambda^2} \), provides a smooth interpolation of the value of \( \alpha_s(\rho) \) from the perturbative (\( \rho \to 0 \)) to the nonperturbative regime (\( \rho \to \infty \)) \[19\], and for \( N_f = 3 \) we have used \( \Lambda = 230 \text{ MeV} \), and \( \rho_0 = 1.6 \text{ GeV}^{-1} \).

The exact phase space integration of (9) gives the total cross section of the reaction (8) as a function of the excess energy in c.m. system \( Q = \sqrt{S} - m_n - m_p - m_\eta \), \( S = (p_1 + p_2)^2 \), and is presented in Fig.3 for the different values of the instanton size \( \rho_c \). This dependence is in qualitative agreement with experimental data of WASA/PROMICE Collaboration at CELSIUS \[4\]. It should be mentioned that the instanton induced cross section of the \( \eta \) production in \( pn \) scattering is very sensitive to the average size of instanton in the QCD vacuum and for \( \rho_c = 2.1 \text{ GeV}^{-1} \) it describes the experimental data.

![Figure 3: Q-dependence of \( \eta \) production cross section for the different values of the average instanton size in comparison with experimental data of WASA/PROMICE Collaboration.](image)

Before ending this section it is worth stressing the main difference between our approach and the hadronic approach. In the latter quantum numbers and
energy considerations determine the intermediate meson, the $S_{11}$, whose coupling constant is at present unknown and whose properties, mass and width, have to be feed in from experiments. In our approach the calculation is basically parameter free, despite the fact that we have allowed some freedom into the size parameter reflecting the crudeness of the approximations involved. An independent determination of the $S_{11}$ coupling constant would clarify the relation between the nonperturbative microscopic mechanism presented here and the hadronic approach. The conclusions drawn thus far are only slightly affected by final state interactions [20] due to their structure and magnitude compared with that of the of the enhancement.

4 Conclusion

We have shown that a novel mechanism for $\eta$ production in nucleon-nucleon scattering might be behind the anomalous enhancement found for proton-neutron collisions. The proposed mechanism is related with the presence of strong nonperturbative fluctuations of gluons fields in the QCD vacuum, called instantons. It has been shown that the instanton induced quark-gluon interaction, due specially to its singular flavor dependence, is able to explain the large $\eta$-meson production cross section near threshold in proton-neutron scattering. The calculation has been realized within a simplified scenario of baryon structure, the diquark description, which maximizes the flavor dependence of the interaction. However it is immediate that other models will not avoid the basic mechanism, whose origin is solely in the structure of the instanton interaction.

The final result for the cross section of the $\eta$ production is very sensitive to the value of the instanton size. This observation opens the door to investigate the parameters of the QCD vacuum in the threshold production of $\eta$ (and other mesons) with high accuracy.

Naturally our proposed mechanism appears in nature entangled with other ones. Their experimental separation is very important. A possible way is the determination of the energy and angular dependence of the production processes. Due to the point-like origin of the instanton-induced interaction, its dependences are totally different from those of other mechanisms. For example in $\eta$-meson production the competing mechanism is the production of an intermediate $S_{11}$-resonance and certainly its energy and angular dependence are very different. A very clean way to disentangle the various
mechanisms is the measurement of the spin dependence of the cross section in the production processes by using polarized nucleon beams and targets. In this case the instantons give a large contribution to the double spin asymmetries due to very strong helicity dependence of t’Hooft’s interaction.

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