ABSTRACT

Low energy effects of generic extensions of the Standard Model can be comprehensively parametrized in terms of higher dimensional effective operators. After the success of all the recent precision tests on the Standard Model, we argue that any sensible description of these extensions at the Z-scale must be stable under higher order quantum corrections. The imposition of $SU(2)_L \times U(1)_Y$ gauge invariance seems to be the simplest and most natural way to fulfill this requirement. With this assumption, all the possible deviations from the standard triple gauge boson vertices can be consistently parametrized in terms of a finite set of gauge invariant operators. We deal here with those operators that do not give any tree level effect on present experimental observables and constrain them by computing their effects at the one-loop level. We conclude that for a light Higgs boson, the direct measurement at LEP200 can improve present bounds on these "blind directions", while for a heavy Higgs it is most unlikely to provide any new information.
1 Introduction

Half a dozen of the Standard Model (SM) predictions are already tested to first order in their quantum corrections\(^1\). For these quantum corrections to be finite, subtle cancellations occur, which require the algebraic relations imposed by the theory’s gauge invariance on the various couplings to be exactly satisfied. In this situation, the natural hypothesis is to assume that theories beyond the SM must be such that they generate an effective theory, which preserves at least the gauge symmetry $SU(2) \times U(1)$ over the symmetry breaking scale $v$. Other assumptions need to prove their ”quantum consistency”. As emphasized in \([1, 2]\), this has often been overlooked in the literature, leading to overly optimistic expectations concerning the ”new physics” sensitivity of future machines.

The leading low energy effects of a generic ”meta-theory” at a scale $\Lambda (\geq v)$ can be comprehensively and systematically parametrized in terms of a linear combination of higher dimensional operators, constructed out of the light fields. If the low energy limit of this ”meta-theory” has the symmetries and light fields of the minimal SM (it takes care of its own ultraviolet divergences), the higher dimensional operators must also preserve those symmetries. Besides, even if the low energy effective theory at the $Z$-scale is not the renormalizable linear SM (this is for instance the case of a theory with a strongly interacting symmetry breaking sector), the successes of the minimal SM still imply that $\Lambda$ cannot be independent of $v$ and that the pattern of the symmetry breaking must be $SU(2)_L \times SU(2)_C \rightarrow SU(2)$, at least\(^2\) to a precision of 1%.

In \([1]\) the effects of ”new-physics” on the structure and strength of the triple gauge-boson vertices (TGVs) were systematically analysed in an effective Lagrangian parametrization. The basis of this method is discussed in sections 2 and 3. It was found that all the operators affecting the TGVs can be expressed as a linear combination of six independent ones, which were chosen to be those giving tree level effects on present observables. All the rest can be expressed in terms of these by means of the equations of motion. However, some combinations of the basis operators are such that their tree level effects on present observables exactly cancel. These are the so called ”blind directions”. Even when there is no known symmetry or dynamical reason why a meta-theory would be so contrived as to generate a low effective Lagrangian pointing exclusively in these directions, in order to leave out any theoretical prejudice, we do not exclude this possibility. In this paper, we constrain them directly from present data through their one-loop effects. In section 4, we present the results of our computation, which has also been partially carried out in \([3]\). In section 5, we compare LEP-1 and LEP-2 sensitivities. We conclude in section 6 that the better chances for LEP-2 concerning an anomalous correction to the triple gauge boson vertices in these directions, come from a relatively light Higgs (as expected also for many other reasons), while for a heavy Higgs LEP-1 constraints are already considerably better.

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\(^1\) Quantitatively only the running of QED and QCD couplings has been significatively tested up to now, although including also the proper weak corrections one obtains a better agreement with experimental data. We acknowledge discussions on this point with L. Maiani and L. Okun.

\(^2\) This pattern is not the most general \([3]\), but it doesn’t seem very sensible at present to adopt the point of view of Burguess and London \([4]\) of substituting the hypothesis of a general symmetry principle ($SU(2)_C$) by an unnatural set of fine tunnings.
2 Effective Lagrangian Parametrization

Recently, there has been some controversy in relation with the "uses and abuses" of Effective Lagrangian parametrizations in the search for non-standard effects in present and future experiments [6]. Questions like what are the symmetries one must impose on an Effective Lagrangian or whether the latter can be used in higher orders of perturbation theory without loosing their predictive power, seem to be misunderstood even though the principles of this approach were settled long ago [7, 8]. We briefly review the ideas behind the Effective Lagrangian parametrization of New Physics and how they can consistently be used in higher orders of perturbation theory.

The Operator Product Expansion [9] is an example of the factorization property of renormalizable theories, by which, at a given energy scale, the effects on physical observables of the much higher energy modes, can be factorized in the effective couplings of the light ones. In any renormalizable theory in which the typical scale of some fields (given for example by their mass) becomes large, the effect of this heavy sector on Feynman’s amplitudes between light states can be expanded in perturbation theory as a sum of local operators of the light fields (multiplied by the appropriate dimensional coefficients which depend on the heavy scale) [3].

Suppose that at very high energy we have a renormalizable theory in which the masses of some fields are much higher than the energy of our experiment (if the masses are generated by spontaneous symmetry breaking we assume that the vev for that sector is large). If the light sector is symmetric under a given gauge subgroup (then it is renormalizable), obviously the full theory must also be symmetric and necessarily the operators of the large mass expansion also preserve this gauge symmetry [4]. In neglecting the contributions vanishing as $\Lambda \rightarrow \infty$, we are left with local operators of light fields of dimension $d \leq 4$, whose coefficients may contain positive powers of $\Lambda$ times logarithms. However, as long as they preserve the symmetries of the light sector, they just produce a finite renormalization of the light couplings and all the non-vanishing dependence on $\Lambda$ is physically unobservable. The heavy fields decouple [12]. This is in fact the case in many physically interesting situations where the new physics comes from extended gauge groups or larger symmetries. We will refer to this situation as the decoupling case.

An effective Lagrangian parametrization is natural here, because the renormalizability of the light theory provides a power counting which controls the non-renormalizable interactions coming from the heavy sector. If we want to consider the effects up to some given order in $1/\Lambda$, we can truncate the expansion at that order and the theory so obtained takes care of its own ultraviolet divergences [13].

Thus, in parametrizing as generally as possible the leading effects of any decoupling new physics, we will consider a combination of light operators of the smallest possible dimension greater than 4,

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3 Some care is needed when considering a theory in which masses are generated by spontaneous symmetry breaking, for in this case masses are given by some dimensionless couplings $\lambda$ times the vacuum expectation value (vev) of a scalar field. When the mass is large due to a large Yukawa coupling, while the vev is light, the large mass expansion does not exist in general [10].

4 In fact the operators must be BRS-invariant, so in principle, there could be operators involving ghost fields. However, as shown in [11] the Faddeev-Popov procedure can be done in two steps, firstly for the "heavy subgroup" when integrating the heavy sector and secondly for the "light" subgroup when defining the effective theory. This way no extra interactions appear in the "light" ghost sector.
which preserve the gauge symmetries of the light sector:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{light}} + \sum_j \alpha_j \frac{Q^d_j}{\Lambda^{d_j - 4}} \]  

(1)

This is the most general large mass expansion of a renormalizable meta-theory whose low energy limit is \( \mathcal{L}_{\text{light}} \). The quantum corrections of this lagrangian are well defined order by order in \( 1/\Lambda \), which implies, in particular, that there is no physical divergence coming out at the one loop level.

On the contrary, if the light theory is non-renormalizable, there must be some physical cutoff related to the heavy scale. The effects of the heavy fields cannot decouple, as they must compensate the tower of divergent light counterterms needed to make the full theory finite. Even though these effects can also be written as a sum of local operators of the light fields, this is in general of no use since there are now contributions with positive powers of \( \Lambda \) that can not be renormalized in \( \mathcal{L}_{\text{light}} \) and, a priori, it is not clear that there is any useful expansion to sum the leading effects in \( \Lambda \), in terms of a finite set of couplings. The quantum corrections in this case contain non-renormalizable divergences and, contrary to what is claimed in [4], this has a clear physical meaning: the cutoff cannot be sent to infinity, furthermore, there must be some natural relation between the light and heavy scales. One example of the fact that these divergences are physical is the Minimal Standard Model. Suppose that an elementary Higgs do exist, but its mass is larger than \( M_Z \) and \( M_W \). Then, at LEP energies, we can integrate it out and work in the resulting effective theory, which is a non-linear sigma model. The non-renormalizable divergences that appear in the perturbative computation of quantum corrections in the effective theory, turn out to be the corrections in \( M_H \) computed in the full renormalizable theory [14], they do not disappear! In fact, it is through these divergences that one is able to estimate the natural scale of the couplings in the effective theory (contrary to the case of a decoupling sector, for which there is no naturality argument to estimate \( \Lambda \) and it can only be determined experimentally).

3 Anomalous Triple Gauge Boson Vertices

The literature abounds in estimates of the LEP-2 sensitivity to the structure of the gauge boson vertex, based on a general Lorentz invariant parametrization of that vertex which does not preserve the gauge symmetry [15]. As we have seen, the effect of these new interactions is to break the renormalizability of the light theory (as the gauge symmetry is lost) and consequently its predictivity. This leads to very optimistic expectations concerning the ”new physics” sensitivity of future machines, but makes it difficult to understand how it can be that half a dozen of the predictions of the SM are already succesfully tested to first order in their quantum corrections [16].

In [4] a general Effective Lagrangian parametrization of novel effects in the structure of the TGVs was considered. Two main possibilities were analysed:

1) The case of Decoupling type of New Physics. In this case the hypothesis is that the Minimal Standard Model is the correct theory to describe physical phenomena at the Z scale. Novel effects would then come from extra particles, larger gauge symmetries, compositeness... characterized by a mass scale \( \Lambda \) distinct from the scale \( v \). In particular,
this means that the unspecified heavy objects are assumed not to acquire their masses from the standard machinery of symmetry breaking, though they may well be involved in the mechanisms that trigger it. The leading effects were shown to come from d=6 operators, which by the previous reasoning must be SU(2)xU(1) invariant. In \[1\], the basis operators and the blind direction \(O_W \equiv i \vec{W}_\mu^a \times \vec{W}_\nu^\lambda \cdot \vec{W}_\mu^\lambda\) were studied in detail. Here we deal with the remaining blind-directions. In the decoupling case, we have two more of them:

\[
O_{B\Phi} \equiv i B^{\mu\nu}(D_\mu \Phi)^\dagger D_\nu \Phi, \quad (2)
\]

\[
O_{W\Phi} \equiv i \vec{W}^{\mu\nu}(D_\mu \Phi)^\dagger \vec{\sigma} D_\nu \Phi. \quad (3)
\]

where \(W_{\mu\nu}^a \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_\epsilon_{abc} W^b_\mu W^c_\nu\), and \(B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu\).

2) The case of a strongly interacting symmetry breaking sector does not fit in the previous picture, because the light theory is in this case a non-linear sigma model which is not perturbatively renormalizable. We assume that the interactions responsible for the generation of intermediate vector boson masses have a global \(SU(2)_L \times SU(2)_C\) symmetry, with \(SU(2)_C\) the accidental custodial symmetry [17] of the standard potential of scalar doublets. This is the only natural situation given the experimental fact that \(\rho \simeq 1\) to a precision of a percent. Without any further assumption, the most general Lagrangian is that of a nonlinearly realized \(SU(2)_L \times SU(2)_C\) which breaks to \(SU(2)_L\). After switching on the gauge fields, this implies the \(SU(2)_L \times U(1)\) gauge symmetry [18].

As shown by Weinberg [19], a loop expansion in this theory is equivalent to a momentum expansion, so that only a finite number of operators are needed to describe the physical phenomena at low energies (this number increasing with the order in the momentum expansion). There is a natural dimension-full parameter which suppresses these non-renormalizable terms (\(\Lambda = 4\pi v\) [21]) and the effective Lagrangian is just a Taylor expansion in \(U\), \(D_\mu /\Lambda^2\) and \(B_{\mu\nu} /\Lambda^2\), where \(U\) is the unitary matrix describing the longitudinal gauge-boson degrees of freedom, transforming as a \((2,2)\) of \(SU(2)_L \times SU(2)_C\). The covariant derivative acting on \(U\) is the usual one:

\[
D_\mu U \equiv \partial_\mu U + ig \frac{\vec{\sigma}}{2} \vec{W}_\mu U - ig U \frac{\sigma_3}{2} B_\mu. \quad (4)
\]

It is important to remark that in the gauge sector, the loop expansion is not necessarily a low-energy expansion.

The leading non-standard effects on TGVs come from operators of \(d_\chi = 4\). After reducing the list with the use of the equations of motion, we are left [1] with three independent ones and three blind directions. Here, we will consider the effect of these blind directions:

\[
\mathcal{L}_2 \equiv ig \beta_2 B^{\mu\nu} Tr\{T[(D_\mu U)^\dagger, (D_\nu U)^\dagger]\}, \quad (5)
\]

\[
\mathcal{L}_3 \equiv ig \beta_3 Tr\{\vec{W}^{\mu\nu}_\mu \vec{\sigma}[(D_\mu U)^\dagger, (D_\nu U)^\dagger]\}, \quad (6)
\]

\[
\mathcal{L}_9 \equiv ig \beta_9 Tr\{TW^{\mu\nu}\} Tr\{T[(D_\mu U)^\dagger, (D_\nu U)^\dagger]\}. \quad (7)
\]

\footnote{In Gasser-Leutwyler notation [20], the \(\mathcal{L}_9\) operator corresponds to a combination of these operators (\(\beta_2 = \beta_3\)).}
where \( T \equiv U_{3}U^{\dagger} \). Those operators containing \( T \) are not custodial preserving. Even when we assume that the symmetry breaking sector is symmetric under \( SU(2)_{C} \), the coupling to hypercharge breaks this symmetry and operators containing \( T \) can appear, although they always have a factor \( g' \).

It turns out that in practice, the effects of these operators are proportional to the effects of equivalent operators in the linear realization with the identification \( M_{H} \sim \Lambda \) (the linear model with a Higgs scalar is a regulator for the nonlinear model [14]):

\[
\mathcal{L}_2 \Rightarrow \frac{8g'\beta_2}{v^2} O_{B\Phi}|_{(\Phi \to v)},
\]

\[
\mathcal{L}_3 \Rightarrow \frac{8g\beta_3}{v^2} O_{W\Phi}|_{(\Phi \to v)},
\]

\[
\mathcal{L}_9 \Rightarrow \frac{-32ig\beta_3}{v^4} (\Phi^{\dagger} W_{\mu\nu} \Phi)(D_{\mu}\Phi)^{\dagger}D_{\nu} \Phi|_{(\Phi \to v)}.
\]

We see that the first two operators on the right-hand side are two of the three blind directions of the decoupling case, while the third one is an operator of \( d=8 \), which was not leading in that case. This shows the different power counting of the two parametrizations.

In fact, at any given dimension, one expects more operators in the non-decoupling case. If we consider for example the Standard Model with a higgs in the intermediate region (not so heavy as to render the theory non-renormalizable, but heavier than the energies of LEP-1 and LEP-2 experiments), we can work in both the linear and non-linear realizations. When we integrate the higgs out, all the contributions that appear in the linear case as finite corrections in \( M_{H} \) should appear after its integration as new effective operators. The linear treatment would be more adequate then, because there are fewer degrees of freedom.

### 4 Present Data Constraints

Following [1], we will use \( \alpha, G_{F} \) and \( M_{z} \) as the input parameters for the minimal Standard Model, because they are the most accurately measured quantities. Then, for constraining the previous operators from present data we will use as observables the \( W \) mass, the leptonic and hadronic widths of the \( Z \), the forward-backward asymmetry in leptonic \( Z \) decays, the \( \tau \) polarization \( P_{\tau} \) and the ratio of inclusive neutral to charged-current neutrino cross sections on aproximately isoscalar targets \( R_{\nu} \):

\[
M_{W} = 80.13 \pm 0.31 GeV [22]
\]

\[
\Gamma_{l} = 83.52 \pm 0.33 MeV [23]
\]

\[
\Gamma_{h} = 1742 \pm 8 MeV [23]
\]

\[
A'_{FB} = 0.0157 \pm 0.003 [23]
\]

\[
P_{\tau} = -1.140 \pm 0.024 [23]
\]

\[
R_{\nu} = 0.308 \pm 0.002 [24]
\]

To the experimental error in \( R_{\nu} \) we have added a theoretical error of 1% to reflect the theoretical uncertainty associated with the charm threshold in the charged currents,
discussed and estimated in [23]. We adopt the safe recipe of adding linearly the theoretical uncertainties to the experimental errors. We shall choose to perform our analysis at the 2 $\sigma$ level, the inputs to our limits on novel effects are then twice the quoted errors.

None of these observables are affected at tree level by the insertion of blind operators, their effects start at the one-loop level. Our calculation has been done in a generic $\xi$-gauge, and we have been able to check the cancellation of the $\xi$-dependence in physical observables. We have used two regularization procedures, dimensional regularization and a simple momentum cutoff. Both give the same results with the identification of quadratic divergences with poles at $d = 2$ and logarithmic ones with poles at $d = 4$. As discussed before, in the decoupling case all divergences but logarithms must get renormalized in the couplings of the SM, so they must also cancel in physical observables. The coefficients of the logarithmic divergences are the coefficients of the renormalization group logarithms that appear in the running of the effective Lagrangian coefficients from the scale $\Lambda$ to the $Z$-scale. Our computation is valid up to these logarithms, not including constant terms. In the nonlinear case, however, the leading contribution comes from quadratic divergences (poles in $d = 2$) that do not cancel. These clearly correspond to the terms in $M^2_H$ of the linear case. There are also some leading contributions coming from finite terms in $M^2_H$.

We express the effect of the insertion of blind operators in terms of the following dimensionless parameters:

$$\delta_{B\Phi} \equiv \frac{g_s}{c} M_W^2 \frac{M_W^2}{(4\pi)^2 \Lambda^2} \alpha_{B\Phi},$$  \hspace{1cm} (12) \\
$$\delta_{W\Phi} \equiv \frac{gs^2}{c^2} M_W^2 \frac{M_W^2}{(4\pi)^2 \Lambda^2} \alpha_{W\Phi},$$  \hspace{1cm} (13) \\
$$\delta_9 \equiv -8 s^2 g^4 \beta_9 \frac{1}{c^2 (4\pi)^2}.$$  \hspace{1cm} (14)

where, from now on, $s = \sin \theta_W$ and $c = \cos \theta_W$.

The quantum effects of these operators appear either as boson self-energies $\Pi^{\gamma\gamma}$, $\Pi^{\gamma Z}$, $\Pi^{Z Z}$ and $\Pi^{W W}$, or as corrections to the $Z f f$ vertex ($\delta c_f^L$) and the $W l \nu$ vertex ($\delta g_{W l \nu}$). We collect all this quantum corrections in the appendix A. In terms of these objects, the shifts induced in the renormalization parameters are:

$$\frac{\Delta \alpha}{\alpha} = \frac{\Pi^{\gamma\gamma}}{q^2} |_{q^2=0},$$  \hspace{1cm} (15) \\
$$\frac{\Delta M_Z^2}{M_Z^2} = \frac{\Pi^{Z Z}(q^2)}{M_Z^2} |_{q^2=M_Z^2},$$  \hspace{1cm} (16) \\
$$\frac{\Delta G_F}{G_F} = \left[ 2 \delta g_{W l \nu} \frac{g_{W l \nu}}{M_W^2} - \frac{\Pi^{W W}(q^2)}{M_W^2} \right] |_{q^2=0}. $$  \hspace{1cm} (17)

All the physical observables can now be expressed in terms of the preceding objects.

\footnote{A common misconception is the statement that there are no quadratic divergences in dimensional regularization. There are, and they correspond to poles at $d=2$.}
We parametrize the shifts in the widths in terms of $\delta \gamma_f$, $\delta \kappa_f$, which also contain the main contribution from the standard radiative corrections:\footnote{The radiatively-corrected standard predictions on $\Gamma_f$ can be cast in the form: 

$\Gamma_f \simeq \frac{G_F M_W^2}{3\pi \sqrt{2}} N_c (1 + \delta \gamma_f) (|c_f^R|^2 + |c_f^L|^2)$, with $c_f^\ell = -(1 + \delta \kappa_f) Q_f \sin^2 \theta_Z$ and $c_f^\ell = T_f^\ell + c_f^\ell$.}

\begin{align}
\delta \gamma_f &= - \frac{\Delta G_F}{G_F} - \frac{\Delta M_Z^2}{M_Z^2} + \text{Re} \left( \frac{\Pi_{ZZ}(q^2) - \Pi_{ZZ}(M_Z^2)}{q^2 - M_Z^2} \right) + 2 \frac{\delta c_L}{(c_L f - c_R f)}, \\
\delta \kappa_f &= \frac{c^2}{(c^2 - s^2)} \left( \frac{\Delta \alpha}{\alpha} - \frac{\Delta M_Z^2}{M_Z^2} \right) - \frac{\text{Re} \left( \Pi_{\gamma Z}(q^2) \right)}{s} + \frac{\delta c_R f}{(c_L f - c_R f)}. 
\end{align}

The observables we will use to constrain the blind operators are:

\begin{align}
\frac{\delta M_{W^2}}{M_{W^2}} &= \frac{\Pi_{WW}(M_{W^2})}{M_{W^2}} - \frac{\Delta M_Z^2}{M_Z^2} + \frac{s^2}{(c^2 - s^2)} \left( \frac{\Delta \alpha}{\alpha} - \frac{\Delta M_Z^2}{M_Z^2} \right), \\
\frac{\delta \Gamma_{\ell}}{\Gamma_{\ell}} &= \delta \gamma_{\ell}(M_Z^2) - 0.250 \delta \kappa_{\ell}(M_Z^2), \\
\frac{\delta \Gamma_h}{\Gamma_h} &= \delta \gamma_{q}(M_Z^2) - 0.318 \delta \kappa_{q}(M_Z^2), \\
\frac{\delta A_{FB}^l}{A_{FB}^l} &= 4 \frac{s^2 \delta \kappa_{\ell}}{g_\ell} \frac{g_a^2 - g_v^2}{g_v^2 + g_a^2}, \\
\frac{\delta P_\tau}{P_\tau} &= -2 \frac{s^2 \delta \kappa_{\ell}}{g_\ell} \frac{(g_a - g_v)^2}{g_v^2 + g_a^2}, \\
\frac{\delta R_\nu}{R_\nu} &= \delta \gamma_{\nu}(0) + \delta \gamma_{\ell}(0) + 2 \frac{\delta \kappa_u(0) (c_L^u c_R^u + \frac{1}{3} c_R^u) + \delta \kappa_d(0) (c_L^d c_R^d + \frac{1}{3} c_R^d)}{c_L^\ell + c_L^d + \frac{1}{3} (c_R^\ell + c_R^d)}. 
\end{align}
find are weaker for a light Higgs and become more restrictive as $M_H$ grows, due to the quadratic dependence on $M_H$ of the loop effects. For $O_{B\Phi}$, the constraints become more restrictive as $M_H$ increases in the whole region from $M_H = 50$ GeV to 1 TeV, while for $O_{W\Phi}$ this behaviour starts only after $M_H \approx 260$ GeV. In the region between 50 and 260 GeV the constraints are weaker for a heavier Higgs due to cancellations between terms in $M_H^2$ and the other quantum corrections.

- **Case of a light Higgs boson**

In the following table we show the 2σ contraints on $\delta_{B\Phi}$ and $\delta_{W\Phi}$, as a function of $M_H$ for any value of $m_t$:

<table>
<thead>
<tr>
<th>$M_H$ (GeV)</th>
<th>$-1.5 \cdot 10^{-4} \leq \delta_{B\Phi} \leq 3.7 \cdot 10^{-4}$</th>
<th>$-1.8 \cdot 10^{-4} \leq \delta_{W\Phi} \leq 3.8 \cdot 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>$-1.6 \cdot 10^{-4} \leq \delta_{B\Phi} \leq 3.0 \cdot 10^{-4}$</td>
<td>$-2.0 \cdot 10^{-4} \leq \delta_{W\Phi} \leq 4.6 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>100</td>
<td>$-8.0 \cdot 10^{-5} \leq \delta_{B\Phi} \leq 1.8 \cdot 10^{-4}$</td>
<td>$-2.8 \cdot 10^{-4} \leq \delta_{W\Phi} \leq 1.38 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>260</td>
<td>$-4.0 \cdot 10^{-5} \leq \delta_{B\Phi} \leq 1.1 \cdot 10^{-4}$</td>
<td>$-1.4 \cdot 10^{-4} \leq \delta_{W\Phi} \leq 4.9 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

These blind operators also generate new couplings involving scalars not present at tree level in the standard model, like the $Z H_0 \gamma$ vertex. We will translate the present experimental limits on the decay $Z \to H_0 \gamma$ into new constraints on the $\delta's$ for a light Higgs ($M_H \leq M_Z$). Let $A = A_0 + A_{B\Phi} + A_{W\Phi}$ be the amplitude for $Z \to H_0 \gamma$ decay. The corresponding width is

$$\Gamma(Z \to H_0 \gamma) = |A|^2 \frac{E^3}{12\pi}. \quad (26)$$

The standard $A_0$ is dominated by the triangle graph with intermediate W’s and is equal to [26]:

$$A_0 \simeq -\frac{e\alpha}{4\pi\sin^2\theta M_W}[4.56 + 0.25(M_H/M_W)^2], \quad (27)$$

practically independent of $M_H$ for $M_H \leq M_Z$. With the same normalization:

$$A_{B\Phi} = -\frac{\alpha_{B\Phi} M_W}{\Lambda^2}, \quad (28)$$

$$A_{W\Phi} = \frac{\alpha_{W\Phi} M_W}{\Lambda^2}. \quad (29)$$

The ratio $\Gamma(Z \to H_0 \gamma)/\Gamma_0(Z \to H_0 \gamma)$ varies from $\sim 0$ to $\sim 4$, in the interval $|\delta_{B(W)\Phi}| \leq 3.8 \cdot 10^{-5}$, indicating a strong sensitivity to the new physics. The trouble is that current bounds on the branching ratio B for $Z \to H_0 \gamma$ are not overly restrictive. From L3 result [27] $B \leq 10^{-3}$ (for $48 \leq M_H \leq 86$ GeV) we get the following constraints for $M_H = 50$ GeV:

$$|\delta_{B\Phi}| \leq 2.4 \cdot 10^{-4}, \quad (30)$$

$$|\delta_{W\Phi}| \leq 2.4 \cdot 10^{-4}. \quad (31)$$
It is remarkable that these constraints, which are completely independent of the preceding ones, are of the same order of magnitude and do not depend on $m_t$. Also the dependence on $M_H$ is weaker (of course, only inside the range $M_H \leq M_Z$).

- Case of a heavy Higgs ($M_H \sim 3 TeV$)

In the case of a heavy Higgs, we take $\Lambda \simeq M_H \simeq 4\pi v (\sim 3 TeV)$, which corresponds to the natural situation in the non-linear case. For any value of $m_t$:

$$-9.4 \cdot 10^{-3} \leq \beta_2 \leq 2.2 \cdot 10^{-2},$$
$$-1.5 \cdot 10^{-2} \leq \beta_3 \leq 3.9 \cdot 10^{-2},$$
$$-1.1 \cdot 10^{-2} \leq \beta_9 \leq 4.7 \cdot 10^{-3}.$$  \hspace{1cm} (32)

These latter constraints are much more restrictive due to the existence of quadratic divergences. Obviously, these leading contributions can be renormalized in the couplings of some other non-blind operators of the same dimension. However, for this chiral expansion to be natural, we expect that the divergent part coming from the loop contribution is (but for additional powers of the couplings $g$ or $g'$) of the same order of the renormalized coupling \[14\]. Thus, we constrain these chiral operators indirectly by constraining the counterterms they necessarily generate, which are non-blind to present observables. On naturality grounds, we expect these results to be a correct estimation of the order of magnitude. It is straightforward to check that the constraints on $\beta's$ we have just derived are typically a factor $g^2$ worse than the constraints obtained in \[1\] for non-blind operators.

In \[5\], only the situation $\delta_{B\Phi} = \delta_{W\Phi} = \delta$ was studied. This corresponds to the effect of the operator $L_9$ in Gasser-Leutwyler’s notation (GL). In order to compare our numerical results with theirs, we have also obtained the bounds from present data in this situation. We consider the case of a light Higgs ($M_H = 60 GeV$ and $\Lambda = 300 GeV$):

$$-2.4 \cdot 10^{-4} \leq \delta \leq 8.0 \cdot 10^{-4},$$

while their constraints translate into

$$-3.0 \cdot 10^{-4} \leq \delta \leq 1.5 \cdot 10^{-3}.$$  \hspace{1cm} (35)

We find similar results for other values of $M_H$ and $\Lambda$. To our understanding the differences between our results and those in \[3\] come from the fact that they use Altarelli-Barbieri $\epsilon's$ as present data constraints instead of directly measured observables, with the unavoidable propagation of errors. Besides, they partially lose the advantage of the different dependence on $m_t$ of the single observables.

In a recent reference \[28\], it is claimed that there are quartic contributions to $\delta\rho$ for all the operators except those that break $SU(2)_C$ via a minimal coupling to hypercharge \[29\]. These quartic divergences appear at next order in the coupling (they are $\sim \delta^2$), as expected from simple power counting \[14\], and there is no physical reason to impose their cancellation on naturality grounds. Rather, if one considers at this order all the counterterms of lower chiral dimension, these quartic divergences disappear from physical observables. Besides, the minimal coupling to hypercharge is not even realized in the standard model with a heavy Higgs boson. Operators with non minimal coupling do appear \[14\] in this case, although they always contain a factor $g'$. 

9
5 LEP200 Sensitivity

Now, we turn to study the sensitivity of the future LEP-200 experiment to these operators. There, \( \sqrt{s} \approx 200\text{GeV} \) and the channel \( e^+e^- \rightarrow W^+W^- \) is opened, so any anomaly in the self-coupling of vector bosons will contribute at the tree level. We use the standard notation for the trilinear couplings:

\[
\mathcal{L}_0^{(3)}(V) = -ie_g[V[(W^\mu W^\nu - W^\nu W^\mu)]V^\nu + \kappa_V W^\mu V^\nu] - ie_g \frac{\lambda_V}{M_W^2}[V^{\mu\nu}W^\dagger_{\nu\rho}W^\rho],
\]

where \( W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu \) and \( V = \gamma, Z \). Blind operators produce tree level shifts on the couplings \( \kappa_V \) and \( g_V \).

A very sensitive direct test concerns the differential cross section \( d\sigma/d\cos\theta^+ \), with \( \theta^+ \) the \( e^+W^+ \) scattering angle. The possible non-standard effects associated with our blind directions will come through shifts in the quantities \( \kappa_Z, \kappa_\gamma, g_Z, g_\gamma, M_W, c_L \) and \( c_R \). For LEP-200 we will consider only the tree-level shifts and neglect the one-loop effects in \( M_W, c_L \) and \( c_R \) as well as the standard radiative corrections.

\[
\begin{align*}
\delta\kappa_\gamma &= \lambda_{B \Phi} + \lambda_{W \Phi} - \frac{1}{2} \lambda_g, \\
\delta\kappa_Z &= -\frac{s^2}{c^2}(\lambda_{B \Phi} + \lambda_{W \Phi}) - \frac{1}{2} \lambda_g, \\
\delta g_\gamma &= 0, \\
\delta g_Z &= \frac{1}{c^2} \lambda_{W \Phi} g_Z.
\end{align*}
\]

where we have defined:

\[
\begin{align*}
\lambda_{B \Phi} &\equiv \frac{g_c v^2 \alpha_{B \Phi}}{4s}, & \lambda_{W \Phi} &\equiv \frac{g v^2 \alpha_{W \Phi}}{4s}, & \lambda_9 &\equiv -8g^2 \beta_9.
\end{align*}
\]

We have Monte-Carlo generated \( 10^4 \) W-pairs at \( \sqrt{s} = 200\text{GeV} \), which is a generous estimation of LEP2 statistics, and performed \( \chi^2 \) tests of significance of deviations from the standard differential cross-section at various values of \( \lambda's \). The errors considered are only statistical.

In Figs.1, we show the biggest allowed domains on the planes \((m_t, \delta)\) (at \( M_H = 50\text{GeV} \) for \( O_{B \Phi} \) and at \( M_H = 260\text{GeV} \) for \( O_{W \Phi} \)) together with the expected LEP200 constraints, for \( \Lambda = 1\text{TeV} \). For the operator \( O_{B \Phi} \), LEP200 will improve the present upper bound in a factor 2-3, at most. However, the sensitivity of LEP200 to \( O_{W \Phi} \) is \( \sim 5 \) times better, and consequently there is an improvement of an order of magnitude with respect to present bounds in this case.

As we explained before, the worst constraints from present data on this operator are obtained for \( M_H \sim 260\text{GeV} \), where the allowed domain in the \((m_t, \delta_{W \Phi})\) plane extends to very large values of \( m_t \). In fact, the upper limit on \( \delta_{W \Phi} \) grows more than a factor 4 from the one obtained at \( M_H = 50\text{GeV} \), but it can only be saturated if \( m_t \) turns out to be \( \sim 350\text{GeV} \). In this situation, the bound for \( \delta_{W \Phi} \) from present data can be read from Fig. 1:
to be compared with the expected sensitivity of LEP200, also shown in Fig. 1:

\[-3.7 \cdot 10^{-5} \leq \delta_{W\Phi} \leq 3.1 \cdot 10^{-5}.\]  

(44)

If $m_t$ is lighter than 200 GeV, the present upper bound would go down to $\delta_{W\Phi} \leq 4.0 \cdot 10^{-4}$ and, for greater values, present data constrain $\delta_{W\Phi}$ to be in a band of width $\sim 4 \cdot 10^{-4}$ whose central value grows linearly with $m_t^2$. The allowed domain from LEP200 slightly intersects this region and, in this sense, future constraints will complement present ones and not supersede them (giving a new bound on the top mass in case LEP200 fails to detect a non-standard TGV).

The results for the case of a heavy Higgs are gathered in Figs. 2. They show the comparison between the $\chi^2$-test limits from LEP200 and present constraints for the three operators of (8)-(10). These results are very similar to that of non-blind operators [1]. In particular, the bounds on $\beta_2(\sim L_9^R)$ and $\beta_3(\sim L_9^L)$ at LEP1, are much better than those that can be obtained from their tree level effects in LEP200 and CDF, although not competitive with those from SSC and LHC [30].

As we have explained in the previous section, we constrain these operators indirectly by constraining the counterterms they generate, which are non-blind to present observables. In other words, present constraints imply that the sensitivity of the experiments LEP200 and CDF to the TGVs will not be enough to measure a value of $L_9$ coupling of the order which is natural for a strongly interacting symmetry breaking sector [31].

It is interesting to study whether a rise in energy to $E_{cm} = 500$ GeV at NLC, will give much better constraints compared to LEP200’s. We have generated 25000 events at $E_b = 250$ GeV (which corresponds to an integrated luminosity of $10 fb^{-1}$) and the bounds we obtain for the $\beta'$s are the following:

\[-1.8 \cdot 10^{-2} \leq \beta_2 \leq 7.2 \cdot 10^{-3}\]  

(45)

\[-7.2 \cdot 10^{-3} \leq \beta_3 \leq 5.5 \cdot 10^{-2}.\]  

(46)

NLC sensitivity to the blind directions is a factor $\sim 5$ better than that of LEP200, and competitive with present constraints in the case of a heavy Higgs. Obviously, the information obtained from direct measurement has less uncertainty than that obtained from the loop effects, so if nature has chosen any of these blind directions to "deform" the standard TGV’s, NLC will certainly improve our present knowledge.

6 Conclusions

Our computation of the one-loop effects of the "blind operators" completes the analysis started in [1] of the bounds on non-standard triple gauge boson vertices from present experimental data. We have argued, yet once again, the necessity of imposing the gauge

---

\[8\text{In terms of } L_9^R \text{ and } L_9^L, \text{ our constraints translate into } -13.9 \leq L_9^R \leq 6.0 \text{ and } -24.6 \leq L_9^L \leq 9.5.\]
symmetry on the effective Lagragian, for this approach to be stable under higher order perturbations. Present experiments are already sensitive to radiative corrections, consequently, any meaningful search for possible new physics by means of an effective Lagrangian parametrization must be such that these corrections are well defined.

We have considered two main possibilities for departures from the original version of the standard model. The first is that in which the effective theory at the Z-scale is correctly described by the minimal standard model with a relatively light Higgs and the new physics appears as larger symmetries or extended gauge groups. Secondly, we also considered the case of a spontaneous symmetry breaking sector involving some sort of strongly coupled dynamics, where the elementary scalars may play no role at all. If the Higgs is not found within the range up to $O(1\text{ TeV})$, this possibility seems more likely.

The one-loop corrections due to these operators depend quadratically on $M_H$ (or equivalently, on the cutoff in the case of a strongly interacting symmetry breaking sector). We have studied separately the limiting cases of a heavy and a light Higgs. We conclude that LEP200 is more sensitive to any new physics pointing in these blind directions if the Higgs is light: for $O_{B\Phi}$, present bounds are only a factor 2-3 worse than our conservative estimation of future LEP200 sensitivity, while for $O_{W\Phi}$ this factor grows to an order of magnitude.

On the contrary, in the case of a heavy Higgs and considering the natural situation $M_H \sim 4\pi v$, present constraints are already considerably better than those that can be obtained in LEP200. On naturality grounds, we have argued that the quadratic dependence on the cutoff is physically relevant in the non-linear realization, implying that the size of the counterterm must be that of the quadratic terms in $\Lambda$, within an order of magnitude.

Even though it is not natural to expect that an extension of the standard model will point exclusively in these blind directions, present data already constrain considerably this possibility and, still in those situations where present bounds are weaker, the allowed regions on the planes $(m_t, \delta's)$ from present and future experiments are independent to a large extent.

Although our estimation of LEP200 sensitivity is quite optimistic, this analysis can certainly be refined. The measurement of helicity amplitudes, for instance, is expected to increase the sensitivity by a factor 2. Also, the measurement of $M_W$ with much better precision, can improve the indirect constraints.

Finally we have also estimated the sensitivity of NLC to these blind directions and found an increase of a factor $\sim 5$ with respect to LEP200.

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8 Appendix A

- Self-energies:

\[ \Pi_{\gamma\gamma} = -2c^2(\delta_{B\Phi} + \delta_{W\Phi} - \frac{1}{2}\delta_9)[\Lambda^2 - (\frac{q^2}{6} + 3M_W^2)\log(\frac{\Lambda^2}{M_W^2})] \frac{q^2}{M_W^2} \]  
\hspace{1cm} (47)

\[ \Pi_{\gamma Z} = \frac{c}{s^4}(s^2\delta_{B\Phi} - c^2\delta_{W\Phi} + \frac{c^2 - s^2}{4}\delta_9)(2\frac{\Lambda^2}{M_W^2} - (\frac{q^2}{3M_W^2} + 6)\log(\frac{\Lambda^2}{M_W^2})) 
+ \frac{9}{4}(\delta_{B\Phi} + \delta_{W\Phi} - \frac{1}{2}\delta_9) - \frac{s^2}{4c^2}(\delta_{B\Phi} - \delta_{W\Phi} - \frac{1}{2}\delta_9) - 3\frac{s^2}{2}\delta_{W\Phi}\log(\frac{\Lambda^2}{M_W^2})q^2 
+ \frac{c}{s}(\delta_{W\Phi} - \delta_{B\Phi} + \frac{\delta_9}{2})\frac{M_H^2}{4M_W^2}(\log(\frac{M_H^2}{M_W^2}) - \frac{1}{2})q^2 \]  
\hspace{1cm} (48)

\[ \Pi_{ZZ} = \frac{c^4}{s^2\delta_{W\Phi} + s^2\delta_{B\Phi} + \frac{c^2}{2}\delta_9}(-2\Lambda^2 + (q^2/3)\log(\frac{\Lambda^2}{M_W^2})) \frac{q^2}{M_W^2} 
+ \frac{c^2}{s^2}\delta_{W\Phi} + \frac{c^2}{2s^2}\delta_9)[(\frac{q^2}{6c^2} + 1 - \frac{M_H^2}{2M_W^2}) 
+ \frac{3}{2c^2}(M_H^2 + M_Z^2 - \frac{q^2}{3})\log(\frac{\Lambda^2}{M_W^2}) + \frac{1}{2}(q^2 - 3M_Z^2)\frac{M_H^2}{M_W^2}(\log(\frac{M_H^2}{M_W^2}) - \frac{1}{2}) + \frac{3}{c^2}\Lambda^2] 
+ \frac{c^2}{s^2}\delta_{W\Phi}[q^2(3 + 12c^2 - \frac{2}{3c^2} - 3\xi) + (3(\xi + 1)M_Z^2 - \frac{q^2}{c^2})]\log(\frac{\Lambda^2}{M_W^2}) - \frac{1}{s^2}\delta_9\Lambda^2 \]  
\hspace{1cm} (49)

\[ \Pi_{WW} = (\delta_{B\Phi} + \frac{c^2}{s^2}\delta_{W\Phi})[-\frac{q^2}{3} + 3(\Lambda^2 + (\frac{M_Z^2}{2} - \frac{3}{2}M_W^2 - \frac{q^2}{6})\log(\frac{\Lambda^2}{M_W^2}))] 
+ \frac{c^2}{s^2}\delta_{W\Phi}([(M_H^2 + \frac{28}{3} - 3\xi) + \frac{q^2}{3} - \frac{M_H^2}{2}]\log(\frac{\Lambda^2}{M_W^2}) - 2\Lambda^2)\frac{q^2}{M_W^2} 
+ 3((\xi + 6)M_W^2 - \frac{3}{2}M_Z^2 + \frac{M_H^2}{2})\log(\frac{\Lambda^2}{M_W^2})] + \frac{1}{2}(q^2 - 3M_Z^2)(\frac{M_H^2}{M_W^2}(\log(\frac{M_H^2}{M_W^2}) - \frac{1}{2})) 
+ \frac{c^2}{s^2}\delta_9[-\frac{\Lambda^2}{2} + (\frac{3q^2}{4}\xi - \frac{13q^2}{12} + \frac{q^2}{4} + \frac{M_Z^2}{4} + \frac{5}{4}M_W^2 - \frac{3}{4}(\xi M_Z^2)\log(\frac{\Lambda^2}{M_W^2})))] \]  
\hspace{1cm} (50)

- Vertex corrections:

\[ \delta c_{L^f} = \frac{3}{2s^2}(\xi + 1)(c_{L^f} - c_{R^f})\delta_{W\Phi}\log(\frac{\Lambda^2}{M_W^2}) \]  
\hspace{1cm} (51)

\[ \frac{\delta g_{W\nu}}{g_{W\nu}} = \frac{3c^2}{4s^2}(\xi + 1)[(\delta_{W\Phi} - \frac{\delta_9}{2})(c_{L^\nu} - c_{L^{'f}} + c_{R^f}) + \frac{1}{c^2}\delta_{W\Phi}(c_{L^\nu} - c_{L^{'f}})]\log(\frac{\Lambda^2}{M_W^2}) \]  
\hspace{1cm} (52)
References


Figure Captions

Figs. 1 Allowed 2σ contours in the ($δ_i, m_i$) planes from present data and $M_H = 50 GeV / M_H = 260 GeV$ for $O_{BΦ} / O_{WΦ}$. The dashed domains subtends the values of $δ_i's$ that can not be distinguished from zero at LEP200 at the 2σ level.

Figs. 2 $χ^2$ test fo significance of the effect of $λ_i's \neq 0$ on $dσ / dcosθ$. The horizontal line shows the 2σ sensitivity for $10^4$ W-pairs at $\sqrt{s} = 200 GeV$, the projections along the vertical arrows delimit the interval of $λ_i's$ inside which a LEP-2 measurement would test the hypothesis $λ_i's \neq 0$ with less than 2σ significance. Vertical bands encompasses the values fo $λ_i's$ currently allowed by the lower-energy tests, for $M_H \sim 4πv$. 