Gluon mass and freezing of the QCD coupling

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Abstract.

Infrared finite solutions for the gluon propagator of pure QCD are obtained from the gauge-invariant non-linear Schwinger-Dyson equation formulated in the Feynman gauge of the background field method. These solutions may be fitted using a massive propagator, with the special characteristic that the effective “mass” employed drops asymptotically as the inverse square of the momentum transfer, in agreement with general operator-product expansion arguments. Due to the presence of the dynamical gluon mass the strong effective charge extracted from these solutions freezes at a finite value, giving rise to an infrared fixed point for QCD.

1. Introduction

The systematic study of Schwinger-Dyson equations (SDE) in the framework of the pinch technique (PT) has led to the conclusion that the non-perturbative QCD dynamics generate an effective, momentum-dependent mass for the gluon, while preserving the local $SU(3)_c$ invariance of the theory [1, 2, 3]. This picture is further corroborated by lattice simulation and a variety of theoretical and phenomenological works [4]. One of the most important consequences of this picture is that this dynamical mass tames the Landau singularity associated with the perturbative $\beta$ function, giving rise to a strong effective charge “freezing” at a finite value in the infrared. In this talk we report recent progress in the study of a non-linear SDE for the gluon propagator [3].

2. The non-linear SDE

The relevant SDE for $\Delta_{\mu\nu}(q)$ is shown in Fig. (1). Due to the special properties of the truncation scheme based on the PT [1, 5] (and its connection with the Feynman gauge of the background field method (BFM) [6]), this equation is gauge-invariant despite the omission of ghost loops or higher order graphs [2]. Dropping for simplicity the longitudinal momenta, i.e. setting $\Delta_{\mu\nu}(q) = -i g_{\mu\nu} \Delta(q^2)$, one looks for solutions where $\Delta(q^2)$ reaches a finite (non-vanishing) value in the deep infrared; such solutions may be fitted by “massive” propagators of the form $\Delta^{-1}(q^2) = q^2 + m^2(q^2)$, where $m^2(q^2)$ is not “hard”, but depends non-trivially on the momentum transfer $q^2$. The tree-level expressions for the three- and four-gluon vertices appearing in the two graphs of Fig. (1) are given in the first item of [6]. For the full three-gluon vertex, $\tilde{\Gamma}$, denoted by the white blob in graph (a1), we employ a gauge technique Ansatz, expressing it as a functional of $\Delta$, in such a way as to satisfy (by construction) the all-order Ward identity

$$q^\mu \tilde{\Gamma}_{\mu\alpha\beta}(q, p_1, p_2) = i[\Delta_{\alpha\beta}^{-1}(p_1) - \Delta_{\alpha\beta}^{-1}(p_2)], \quad (1)$$
characteristic of the PT-BFM. Specifically, we use the following closed form for the vertex [3]:

\[ \Pi^{\mu\alpha\beta} = \tilde{\Gamma}^{\mu\alpha\beta} + ig^{\alpha\beta} \frac{q^\mu}{q^2} [\Pi(p_2) - \Pi(p_1)] - ic_1 \frac{q^\beta}{q^2} \left( q^\alpha g^{\mu\alpha} - q^\alpha g^{\mu\beta} \right) [\Pi(p_1) + \Pi(p_2)] -ic_2 \left( q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta} \right) \left[ \frac{\Pi(p_1)}{p_1^2} + \frac{\Pi(p_2)}{p_2^2} \right], \]  

(2)

with \( \tilde{\Gamma}^{\mu\alpha\beta}(p_1, p_2) = (p_1 - p_2)\mu g_{\alpha\beta} + 2 q_{\beta} g_{\mu\alpha} - 2 q_{\alpha} g_{\mu\beta} \), and \( i\Pi(q^2) = \Delta^{-1}(q^2) - q^2 \).

Defining the renormalization-group invariant quantity [4] \( d(q^2) = g^2 \Delta(q^2) \), we arrive at

\[ d^{-1}(x) = K' x + \tilde{b} \sum_{i=1}^{8} \tilde{A}_i(x) + d^{-1}(0), \]  

(3)

with

\[
\begin{align*}
\hat{A}_1(x) &= -\left( 1 + \frac{6c_2}{5} \right) x \int_x^\infty dy \, y \, \mathcal{L}(y) \, d^2(y), \\
\hat{A}_2(x) &= \frac{6c_2}{5} x \int_x^\infty dy \, \mathcal{L}(y) \, d(y), \\
\hat{A}_3(x) &= -\left( 1 + \frac{6c_2}{5} \right) x \mathcal{L}(x) \int_0^x dy \, y \, \mathcal{L}(y) \, d(y), \\
\hat{A}_4(x) &= \left( - \frac{1}{10} - \frac{3c_2}{5} + \frac{5c_1}{5} \right) \int_0^x dy \, y^2 \, \mathcal{L}(y) \, d^2(y), \\
\hat{A}_5(x) &= \frac{6}{5} \left( 1 + c_1 \right) \mathcal{L}(x) \int_0^x dy \, y^2 \, \mathcal{L}(y) \, d(y), \\
\hat{A}_6(x) &= \frac{6c_2}{5} \int_0^x dy \, y^2 \, \mathcal{L}(y) \, d(y), \\
\hat{A}_7(x) &= \frac{2}{5} \mathcal{L}(x) \frac{d(x)}{x} \int_0^x dy \, y^3 \, \mathcal{L}(y) \, d(y), \\
\hat{A}_8(x) &= \frac{1}{3x} \int_0^x dy \, y^3 \, \mathcal{L}(y) \, d^2(y),
\end{align*}
\]

(4)

where \( x = q^2 \). The renormalization constant \( K' \) is fixed by the condition \( d^{-1}(\mu^2) = \mu^2/g^2 \), (with \( \mu^2 \gg \Lambda^2 \)), and \( \mathcal{L}(q^2) \equiv \tilde{b} \ln \left( q^2/\Lambda^2 \right) \), where \( \Lambda \) is QCD mass scale. Due to the poles contained in the Ansatz for \( \Pi^{\mu\alpha\beta} \), \( d^{-1}(0) \) does not vanish, and is given by the (divergent) expression

\[ d^{-1}(0) = \frac{3\tilde{b}}{\alpha^2} \left[ 2(1 + c_1) \int d^4k \, \mathcal{L}(k^2) \, d(k^2) - (1 + 2c_1) \int d^4k \, k^2 \, \mathcal{L}^2(k^2) \, d^2(k^3) \right], \]  

(5)

which can be made finite using dimensional regularization, and assuming that \( m^2(q^2) \) drops sufficiently fast in the UV [2].

Figure 1. The gluonic “one-loop dressed” contributions to the SDE.
3. Results
The way to extract from \(d(q^2)\) the corresponding \(m^2(q^2)\) and \(g^2(q^2)\) is by casting the numerical solutions into the form [1]

\[
d(q^2) = \frac{g^2(q^2)}{q^2 + m^2(q^2)}, \quad g^2(q^2) = \left(\frac{\bar{b} \ln \left(\frac{q^2 + f(q^2, m^2(q^2))}{\Lambda^2}\right)}{q^2 + m^2(q^2)}\right)^{-1}.
\]

with

\[
f(q^2, m^2(q^2)) = \rho_1 m^2(q^2) + \rho_2 \frac{m^4(q^2)}{q^2 + m^2(q^2)} + \rho_3 \frac{m^6(q^2)}{(q^2 + m^2(q^2))^2},
\]

The functional form used for the running mass is

\[
m^2(q^2) = m^2_0 \left[\ln\left(\frac{q^2 + \rho m^2_0}{\Lambda^2}\right) / \ln\left(\frac{\rho m^2_0}{\Lambda^2}\right)\right]^{\gamma_2-1},
\]

where \(\gamma_2 = \frac{4}{3} + \frac{6\epsilon_1}{5}; \rho, \rho_1, \rho_2, \) and \(\rho_3\) are adjustable constants. Evidently, \(m^2(q^2)\) is dropping in the deep ultraviolet as an inverse power of the momentum, as expected from general operator-product expansion calculations [7]. Note that \(f(q^2, m^2(q^2))\) is such that \(f(0, m^2(0)) > 0;\) as a result, \(g^2(q^2)\) reaches a finite positive value at \(q^2 = 0\), leading to an infrared fixed point [1, 8, 9].

![Figure 2. Left: dynamical mass with power-law running, for \(m^2_0 = 0.5\) GeV\(^2\) and \(\rho = 1.046\) in Eq.(5). Right: the running charge, \(\alpha(q^2) = g^2(q^2)/4\pi\).](image)

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References
[4] For an extensive list of citations, see [2].