Quark Exchange in Deep Inelastic Scattering

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Abstract

We use a model for baryons that links the constituent structure to the deep inelastic (current) properties. The approach consists in a laboratory partonic description (based on a model of hadron structure), to which a low momentum scale $Q_0$ is ascribed, which is evolved to high momenta by means of the renormalization group. A generalization of the model by means of the hadronic quark cluster decomposition, provides a description of the structure functions of nuclei and is the starting point to study the effects that the antismuonization at the quark level has on the structure function of a model deuteron. The analysis contains conventional and high momentum partonic components. We next study quark Pauli blocking scenarios by including exotic delta nuclei in our analysis. By assuming that nuclei are composed of quasi-deuteron clusters, i.e., the deuteron parameters are density dependent, we analyze these effects on the structure functions of heavier nuclei.

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1 Introduction

At the initial stages of the parton model the need to connect this light cone description to a laboratory one arose as a way to identify the partons [1]. Nowadays, the need to establish a link between models of hadron structure and the high energy data has revived interest in this problem.

Two main lines of approaches have been developed. One of them proceeds to calculate the deep inelastic properties by means of partons within a laboratory frame description. The intricacies of the data imply complex wave functions and normalizations [2]. The other assumes a parametrization (or model) for the initial distribution of partons at a low energy scale and perturbative QCD evolution to approach the high energy data [3]. This procedure was applied initially to the Bag model, and justification for its suitability in the case of twist two operators was provided [4]. More recently it has been also implemented in the non relativistic quark model [5].

These schemes have been intensively used to study the nucleon structure functions for conventional, as well as, polarized scattering [6]. Moreover the appearance of the EMC effect, provided a renewed interest in nuclear physics in the deep inelastic regime [7] with particular emphasis in how the properties of nucleons change due to the nuclear medium. The description of the behavior of nucleons in a medium has led to the possibility of new interesting phenomena like the change of the nucleon size in a nucleus (swelling) [8], which has found also other interpretations, more in line with effective chiral theories, in the form of scaling of coupling constants [9].

In the framework of non relativistic quark models the implementation of the QC D evolutive scenario has been quite succesful in the description of conventional, as well as, polarized nucleon data [5,6]. More than its relative success, what makes the formalism extremely appealing is the simplicity with which one is able to connect properties of the model wave functions with the asymmetric structure functions. We have recently extended this description to the study of deuterons properties and through a quasi-deuteron picture [10], i.e., the properties of the deuteron are taken to be density dependent, to heavier nuclei [11]. The model allows by construction a detailed analysis of quark exchange effects and this subject centers the interest of the present article.

We devote the rest of the introduction to recall those ingredients of our previous work, which should serve to clarify the sections that follow. In Section 2 we discuss the quark exchange contributions applied to conventional degrees of freedom. In section 3 we discuss certain components of the wave function, hereafter called exotic nuclei, which should produce, if experimentally isolated, clear signals of quark exchange phenomena. Finally in Section 4 we recapitulate our findings and extract some conclusions.

Our starting point is the deuteron wave function which can be written in terms of quarks (for the spin up component) in the general form [11,12]

\[ \Psi_s(123456) = A(d \uparrow (123;456)) \]

where \( A \) is the quark antisymmetrizer, 1,2,... represent the quark degrees of freedom (u,d,\( \bar{d} \), \( \bar{u} \), red, blue, white) and

\[ d \uparrow (123;456) = \mathcal{R}(123;456) \frac{1}{\sqrt{2}} (P \uparrow (123) N \uparrow (456) - N \uparrow (123) P \uparrow (456)) \]

Here \( P \) and \( N \) characterize the internal proton and neutron wave function in terms of quarks respectively.

For the spatial part of the wave function we take [12],

\[ \mathcal{R}(123;456) = \exp[-B r_1^2 - B r_2^2 - A \sum_{i=1}^{a} r_i^2] \]

where \( r_i \) the radial coordinates of the quarks and \( r_1 \) and \( r_2 \) the coordinates of the center of mass of the clusters. The parameters \( A \) and \( B \) have a well defined meaning:

\( A \) determines the size of the baryons, \( B \) the strength of the nuclear harmonic force.

For our spherically symmetric wave function the structure function in the scaling limit is calculated according to the formula [5]

\[ F_2(x) = \frac{5}{9} \int_{k^2_{min}}^{\infty} dk^2 n(k^2) \]

where

\[ k_{min} = \frac{M x}{2 \left(1 - \frac{m^2}{M^2 z^2}\right)} \]

In Eqs. (4) and (5) \( m \) represents the constituent quark mass (\( m = m_u = m_d \)), \( M \) the deuterium mass, \( z \) the deuteron scaling variable (\( z \approx \frac{x}{-z} \) where \( x \) is the so called Bjorken scaling variable) and \( n(k^2) \), the quark momentum distribution in the deuteron system, can be obtained as [5]
where $n(k) = \frac{1}{(2\pi)^3} \int d^3p d^3q e^{i(k-p-q)\cdot r}\rho(r,r')$.

In terms of $x$ the structure function for our model deuteron becomes

$$F_2(x) = \frac{15\pi m F_D^p(x)}{N_D} \left( 1 + \frac{1}{N_D} \right) \left( 1 + \frac{1}{N_D} \right)$$

which contains two very distinct contributions. One associated with the structure of each individual nucleon, called direct, which we have written in Eq.(7) as the factor outside the parenthesis. The other, due to quark antisymmetrization between quarks of different hadrons, called exchange, which we have written in the same equation as ratios which are non factorizable [12]. The latter represent non convolution contributions to the structure function [13]. The mathematical expressions for these terms will be given in the next section.

The formulation just presented does not lead to the right support. Nevertheless one can correct for this by applying the prescription [14]

$$F_2^p(x) = \frac{1}{(1-x)^2} F_2^A (x)$$

In Eq.(8) the so called flux factor, arising from relativistic kinematics, has been incorporated [15].

To get the structure function at high momentum transfer $Q$ we do perturbative QCD evolution starting from a low momentum scale $Q_0$, to which we adscribe the model calculated structure function [3,4].

Since the deuteron is an isosinglet, an SU(2) approximation to the evolution, and the assumption of the non existence of gluons at $Q_0$, an ingredient of the present model, lead to [16].

$$M_0^D(Q^2) = \int d\bar{x} e^{-2}(x\Sigma(\bar{x} , Q^2)) = a_{11}(\alpha) M_0^N(Q^2)$$

$$M_1^D(Q^2) = \int d\bar{x} e^{-2}(x\Sigma(\bar{x} , Q^2)) = a_{11}(\alpha) M_1^N(Q^2)$$

where $\Sigma$ and $G$ label the singlet and the gluon distributions respectively and the matrix elements $a_{11}$ and $a_{21}$ are calculated perturbatively as an expansion in powers of the coupling constant $a_s$.

Two deuteron like systems (D1 and D2) will have the same evolution properties determined by their isosinglet nature and from Eq.(9) we obtain

$$M_2^{D1}(Q^2) = \frac{M_2^{D1}(Q_0^2)}{M_2^{D1}(Q_0^2)} M_0^{D1}(Q^2)$$

(11)

to all orders of perturbation theory.

To generate from the above formalism the structure function at high momentum for a quasi-deuteron system we take one of the deuteron structure functions as known, i.e., the moments $M_0^{D1}$ at $Q^2$ are known. Then we apply our model to determine the ratio $\frac{M_2^{D1}}{M_2^{D1}(Q_0^2)}$ and use Eq.(11) to determine the moments at $Q^2$ for the other quasi-deuteron D1. From them we reconstruct the D1 structure function by the inverse Mellin transform [3,5].

For example, in the realistic case, we take the experimental parametrization of ref. [17]

$$F_2^{D1} = \frac{9}{5} (Q^2 = 5 GeV^2) = \hat{F}_2^{D1}(x) + \hat{F}_2^{D1}(x)$$

(12)

as the known structure function of a loosely bound quasi-deuteron system (rms radius > 5 fm).

In the case of using experimental data the inverse Mellin transform turns out to be appropriate for values of $x < 1$. For $x > 1$, the values of the structure function are very small, and we have not found an adequate procedure for reconstruction (the inverse Mellin transform oscillates wildly). In order to envisage, in these circumstances the physics beyond $x = 1$, we have performed the analysis also with a toy experimental structure function for the deuteron which extends much more than the realistic one beyond this point (see Fig.1).

The procedure just described assumes the same evolution scale for different Quasi Deuteron systems and, consequently, the same evolution scale for all nuclei at least at the level of their Quasi Deuteron subclusters. From the physical point of view it imposes on the input data the parton momentum flow of the low en-
ergy model, avoiding effects associated with the truncation of the matrix elements
expansion in powers of the coupling constant.

By changing the size and the interaction of our model nucleons, governed by
the parameters $A$ and $B$, we can study different scenarios going from almost non-
overlapping nucleons to a six quark quasi-deuteron (the six quarks in the same
potential well, $B=3A$). One can analyze what these choices imply on the input
data in the region where the two nucleon correlations become important, i.e., we
investigate the behavior of the structure functions for $x$ around 1. In ref.[11] we
studied the behavior of the dominant direct terms, in here we proceed to describe
the quark-exchange terms.

In order to study other types of scenarios we allow ourselves some additional
degrees of freedom. For example, the structure function of our deuteron before
evolution teaches us that high momentum components can be introduced into
the model by changing $k_a$. We therefore add an additional parameter to Eq.(8), which
displaces the maximum of the structure function before evolution towards higher
$x$, in order to study the effect of high momentum components of the nucleon wave
function on the quark exchange contributions. Furthermore we introduce into the
model exotic components associated with delta excitations, what we have called
some time ago delta nuclei [18]. These components tend to produce Pauli blocking
scenarios which magnify the consequences of the antisymmetrization principle at
the quark level.

2 Quark exchange contributions

In general, one can safely assert, that quark-exchange effects will not be large in
nuclear systems unless, in the kinematical region under study, the two body corre-
lations are not only qualitatively relevant but also quantitatively significant. There
are two different, though interconnected, types of correlation. On the one hand
the dynamical ones whose ultimate origin is the basic dynamical theory (QCD),
parametrized in our model in terms of harmonic oscillator wave functions. On the
other hand, quark Pauli correlations, associated with the spin-statistics theorem,
a general property of any constituent in conventional field theories. They lead to
effects that are large for ensembles of particles, which contain constituents with the
same internal quantum numbers (quark Pauli blocked systems). The prohibition
of bringing the particles close together produces changes in the spatial part of the
wave function, which in all scales of physics where they occur lead to important
observational effects. However, this is not the case for conventional nuclear systems
as the deuteron. In this case the number of quark degrees of freedom, twelve (spin-
isospin-color), is more than sufficient to accommodate all the quarks in the ground
state. This is clearly manifest if we write explicitly the structure function obtained
with our model, Eq.(7). In this case, the direct contribution is given by

$$\frac{F_D(x)}{N_D} = \frac{1}{2\pi^2} \sqrt{\frac{3}{4A + B}} \exp \left( -\frac{3k_N^2}{4A + B} \right)$$

(13)

while the exchange one can be calculated from

$$\frac{N^E}{N_D} = \left( \frac{27AB}{(6A + B)(3A + 2B)} \right)^{1/4}$$

(14)

and

$$\frac{F^E(x)}{F^D(x)} = \sqrt{3(4A + B)} \frac{N^E}{N_D} \left\{ \sqrt{\frac{6A + B}{63A^2 + 36AB + 2B^2}} \exp \left( -\frac{3(3A - B)k_N^2}{(4A + B)(63A^2 + 36AB + 2B^2)} \right) \right. \right.$$

$$\left. + \frac{1}{2} \sqrt{18A^2 + 45AB + 4B^2} \exp \left( -\frac{6(3A - B)k_N^2}{(4A + B)(18A^2 + 45AB + 4B^2)} \right) \right\}$$

$$= R_1 + \frac{1}{2} R_2$$

(15)

The last line just introduces notation for later purposes.

The situation under analysis is clearly one of non Pauli blocking. A glance at
Eqs. (13) through (15) shows that all contributions are additive a signature of small
overlap contributions [18].

Let us discuss Eq.(15), which is very illuminating. For fixed $A$ the allowed
values of $B$ are $0 \leq B \leq 3A$. $B = 0$ corresponds to two infinitely separated
clusters, while $B = 3A$ to the One Well Limit, i.e., all six quarks are in the same
harmonic oscillator well. As can be seen from the equation, in both cases, the
structure function consists only of the direct term. In the first case the exchange
contributions vanish. In the One Well Limit, the exchange contributions, which are
both large, vanish in the quotient because numerator and denominator are equal.
Thus in the case of a non Pauli blocking scenario the most extreme overlap situation

3We shall assume from now on that we have corrected for support but drop the primes for
covinence.
turns out to produce a non-exchange scenario. This is a well-known result, which is exactly realized in the harmonic oscillator model. For values of $B$ away from the extremes the exchange contributions are non-vanishing. Due to the additivity of the several exchange contributions and small spin-color factors, they are not dominant. However, they become sizeable for large $z$. In this limit the $z$-dependent exchange contribution of the numerator becomes small, while the $z$-independent of the denominator remains unaltered. Thus the full structure function becomes smaller than the direct one by amounts which can be up to ten percent for specially chosen parameters.

As can be seen in Fig.1 the evolution respects these features. Unfortunately we are not able to approach the large $z$ values, where the effect should be very large in ratio, because the structure functions become very small and the Mellin inversion technique becomes inadequate. In order to confirm our expectations we plot in Fig.2 the percentage of quark exchange contribution to the moments as a function of $n$, their degree. One clearly sees that the ratio grows with $n$, a signature which confirms, that the exchange contributions grow with $z$.

For the deuteron, in the region we control, the exchange effects are small. There are two reasons for this smallness. A physical one, namely that the deuteron consists of two well separated nucleon clusters. A technical one, that our harmonic oscillator model, with its very sharp gaussian wave functions, tends to diminish the effect (see Eq.15). With realistic forces one might expect ratios of up to ten percent.

In order to increase the overlaps and thereby the exchange effects, two physical mechanisms are available in the model. The first one results from increasing $B$ while keeping $A$ constant. Physically it implies that nucleons in the quasi-deuterons of heavier nuclei are closer together than those in the free deuteron, i.e., the nuclear force becomes more attractive in the medium. The second mechanism results from decreasing $A$ while keeping $B$ fixed. In this case the implication is that the size of the nucleons in the quasi-deuterons is bigger than in the free deuteron (swelling). The evolution equations tend to respect these changes in the initial boundary conditions (see Fig.1) and therefore there is an increase of the ratio already in the region where the Mellin inversion is very stable, which signals a much greater increase in percentage in the region where the structure functions are very small.

The formalism developed, thus far, lacks high momentum components in the wave function. One expects that the high momentum components, although important to describe the actual structure function, should not be affected by quark exchange contributions. We have constructed a displaced structure function centered at a higher value of $z$ at $Q_0$ and within the same formalism (see Fig.3) the exchange effects diminish. The way we construct the displaced structure function is very naive, namely we increase the quark mass artificially. As can be seen from Eqs. (7), (8) and (15), the maximum of the structure function occurs around the point where $k_n$ vanishes. The mass of the constituent quarks controls this point. For a conventional quark mass of $313$ MeV it occurs about $z \approx \frac{1}{3}$, tending towards $z \approx 1$ as the quark mass increases towards the deuteron mass. In using this mechanism we have the advantage that the exchange contributions are treated in the same way as in the initial harmonic oscillator model.

3 Exotic nuclei

A quite different situation arises when the system under consideration contains a non-negligible probability of quark Pauli blocked components. Indeed it was shown in ref.18 that exotic nuclear components like delta nuclei, with Pauli blocking terms in the totally antisymmetrized wave function, leads to increased exchange effects when studying nuclear form factors. The formalism used for calculating the structure function at low momentum $Q_0$ is very similar to the one developed for the form factors, since it finally reduces to the calculation of one body matrix elements in momentum space, and therefore we expect that also the structure function for these exotic systems will contain large contributions from the exchange terms.

Let us consider a delta-deuteron where one of the protons with spin up has been substituted by a $\Delta^+$ with spin up, then terms with up to four quarks $u \uparrow$ (and only three colors) are present. In this case a similar calculation to the one sketched in the introduction leads to

$$F_2^A(z) = \frac{10 \pi m F_2^0(z)}{9} \frac{1 - \frac{16}{27} \frac{r_2}{r_1}}{1 - \frac{2}{3} \frac{r_2}{r_1}}$$

(16)

where

$$\frac{F_2^A(z)}{F_2^0(z)} = R_1 + \frac{2}{3} R_3$$

(17)
The notation follows that previously introduced in Eq. (15).

In this case both the sign and the spin-color factors favor the contribution from the exchange. For large $x$, as can be easily seen from the above equation, the quark exchange contribution can become even dominating. The evolution does not alter this behavior as shown in Fig. (4).

Moreover one can see that two body effects become very important if these exotic components are contemplated in nuclei. The overlap of hadron clusters leads to effects which might increase enormously the quark exchange contributions (up to 20% in the figure). Certainly a small relative probability for these components could hide the effect when comparing to the data. So it would be very interesting if one could isolate experimentally, by appropriate tagging of entrance channels and quantum number determination of exit channels, these exotic components. As remarked in ref. [18] the effort may be worth while, because we are not discussing just about a subtle effect on the data, but on the existence, at low energy, of physics outside the scope of conventional nuclear theory.

4 Conclusion

We have defined a model for nuclei which takes into account the quark structure in a constituent picture. The model has to be seen as a quantum mechanical parametrization of sizes within a cluster approach. Its parameters reflect the two underlying scales of the theory: i) the perturbative ($A$) characterizing the hadron confinement radius; ii) the non perturbative ($B$) characterizing the long range dynamics. In this scenario we have analyzed the deep inelastic structure function by incorporating radiative gluons and sea via the Renormalization Group Evolution.

The aim of the presentation has been to study the relevance of Quark Exchange Effects in the region where two nucleon correlations and therefore nucleon overlaps become significant.

The structure function becomes very small, once we approach the two nucleon correlation region. This has proven to be a big handicap, due to the instability of the inverse Mellin transform, which has limited the values shown to the region $x < 1$. However, a toy model calculation, which has allowed extrapolation into the two hadron correlation region, and the exchange ratio of the moments, lead us to conclude very clearly, that we are facing, through the results in the $x < 1$ region, the threshold for Quark Exchange phenomena, which should become much greater, in relative magnitude, in the two nucleon correlation region.

We have commented along the exposition the main features of our results. To summarize, we can state that the exchange effects are not negligible and probably observable. In particular exotic components like deltas contribute considerably to these effects. One has to recall that hadrons in this regime have to be treated through their underlying parton constituency, and this has been the leitmotiv of our presentation. Finally high momentum components, which are very important to fix the low $x$ region, do not seem very relevant for understanding the Quark Exchange phenomena.

Our model is naive, but complete and consistent. We treat the overlaps, within the deuteron, in a fully general way (6! terms in the wave function) within a constituent picture. The fact that we use harmonic oscillator and the laboratory partonic description allows us to solve the center of mass and the support problems. Finally through the evolution equations we include some of the missing ingredients of laboratory partonic descriptions, namely gluons and sea. We hope that many of the ideas developed above can be further pursued in more realistic calculations.

From the experimental point of view it has become clear, we hope, that the two nucleon correlation region, although difficult due to the smallness of the effects, is crucial to understand the way in which the constituents of each hadron relate with the others when forming nuclei. Moreover the non nucleonic components may play a crucial role even in this domain. If the hadronic language, represented here by our cluster approach, is still valid and how QCD affects the medium properties of the theory, i.e., if perturbatively (evolution scales) or non perturbatively (long range dynamics) or both, and in the latter case how they mix and which $x$ region influences each of them, is the debate for the medium future.

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References

Figure Captions

Fig. (1): Ratio of Quark Exchange over Total contributions to the structure function as a function of the Bjorken scaling variable $x$. The solid line, describes a system with the appropriate nucleon radius ($rms = 0.86/fm$) and the appropriate deuteron radius ($rms = 2.08/fm$). The dashed line corresponds to a small quasi deuteron ($rms = 1.1/fm$); the dotted line a normal size quasi deuteron formed from swelled nucleon bubbles, i.e., big nucleons, ($rms = 1.3/fm$). The dot-dashed line corresponds to a toy model deuteron with the same parameters but a structure function which is bigger than the experimental one in the $x > 1$ region.

Fig. (2): Ratio of Quark exchange over Total contributions to the momenta of the structure function as a function of $n$, the degree of the momenta. The deuteron solid line. The small quasi-deuteron dashed line. The quasi-deuteron with swelled nucleons dotted line. The parameters as in Fig. (1).

Fig. (3): Ratio of Quark Exchange over Total contributions to the structure function for deuterons with different laboratory partonic momentum distributions as a function of the Bjorken scaling variable. The solid line corresponds to the deuteron whose momentum distribution is centered about $x \approx 0.29$. The (large)-dashed line to a momentum distribution with center at $x \approx 0.5$. The (small)-dashed line to one centered about $x \approx 0.67$. The dotted line to one centered about $x \approx 0.8$ and finally the dash dotted line to one centered about $x \approx 0.91$.

Fig. (4): Ratio of Quark Exchange over Total contributions to the structure function for delta deuterons as a function of the Bjorken scaling variable. The solid line corresponds to the deuteron. The dashed line to the delta-deuteron with deuteron parameters. The dotted line to Delta-quasi-deuterons with swelled hadrons. The dot-dashed line to small Delta-quasi-deuterons. Parameters as in Fig. (1).