THE ROLE OF FINAL STATE INTERACTIONS IN $\varepsilon'/\varepsilon$

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The Standard Model prediction for $\varepsilon'/\varepsilon$ is updated, taking into account the chiral loop corrections induced by final state interactions. The resulting value, $\varepsilon'/\varepsilon = (17 \pm 6) \times 10^{-4}$, is in good agreement with present measurements.

1. Introduction

The CP–violating ratio $\varepsilon'/\varepsilon$ constitutes a fundamental test for our understanding of flavour–changing phenomena. The present experimental world average $\text{Re}\,(\varepsilon'/\varepsilon) = (19.3 \pm 2.4) \cdot 10^{-4}$, provides clear evidence for a non-zero value and, therefore, the existence of direct CP violation.

The theoretical prediction has been rather controversial since different groups, using different models or approximations, have obtained different results. In terms of the $K \to \pi\pi$ isospin amplitudes, $A_I = A_I \epsilon^{I, I} (I = 0, 2)$,

$$\frac{\varepsilon'}{\varepsilon} \approx e^{\Phi} \frac{\omega}{\sqrt{2}\varepsilon} \left[ \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right], \quad \Phi \approx \delta_2 - \delta_0 + \frac{\pi}{4} \approx 0,$$

(1)

where $\omega = \text{Re} A_2/\text{Re} A_0 \approx 1/22$. The CP–conserving amplitudes $\text{Re} A_I$, their ratio $\omega$ and $\varepsilon$ are usually set to their experimentally determined values. A theoretical calculation is then only needed for the quantities $\text{Im} A_I$.

Since $M_W \gg M_K$, there are large short–distance logarithmic contributions which can be summed up using the Operator Product Expansion and the renormalization group. To predict the physical amplitudes one also needs to compute long–distance hadronic matrix elements of light four–quark operators $Q_i$. They are usually parameterized in terms of the so-called bag parameters $B_i$, which measure them in units of their vacuum insertion approximation values.

To a very good approximation, the Standard Model prediction for $\varepsilon'/\varepsilon$ can be written (up to global factors) as

$$\frac{\varepsilon'}{\varepsilon} \sim \left[ B_6^{(1/2)} (1 - \Omega_{I\beta}) - 0.4 B_8^{(3/2)} \right], \quad \Omega_{I\beta} = \frac{1}{\omega} \frac{\text{Im} A_2}_{I\beta} \text{Im} A_0.$$

(2)

Thus, only two operators are numerically relevant: the QCD penguin operator $Q_6$ governs $\text{Im} A_0 (\Delta I = 1/2)$, while $\text{Im} A_2 (\Delta I = 3/2)$ is dominated by the electroweak...
penguin operator $Q_8$. The parameter $\Omega_{IB}$ takes into account isospin breaking corrections; the value $\Omega_{IB} = 0.25$ was usually adopted in all calculations. Together with $B_i \sim 1$, this produces a numerical cancellation leading to values of $\varepsilon'/\varepsilon \sim 7 \times 10^{-4}$. This number has been slightly increased by a recent Chiral Perturbation Theory (\chiPT) calculation at $O(p^4)$ which finds $\Omega_{IB} = 0.16 \pm 0.03$.

2. Chiral Loop Corrections

Chiral symmetry determines the low–energy hadronic realization of the operators $Q_i$, through a perturbative expansion in powers of momenta and quark masses. The corresponding chiral couplings can be calculated in the large–$N_C$ limit of QCD. The usual input values $B_8^{(3/2)} \approx B_6^{(1/2)} = 1$ correspond to the lowest–order approximation in both the $1/N_C$ and \chiPT expansions.

The lowest–order calculation does not provide any strong phases $\delta_I$. Those phases originate in the final rescattering of the two pions and, therefore, are generated by higher–order chiral loops. Analyticity and unitarity require the presence of a corresponding dispersive effect in the moduli of the isospin amplitudes. Since the S–wave strong phases are quite large, specially in the isospin–zero case, one should expect large unitarity corrections.

The one–loop analyses of $K \rightarrow 2\pi$ show in fact that pion loop diagrams provide an important enhancement of the $A_0$ amplitude. This chiral loop correction destroys the accidental numerical cancellation in eq. (2), generating a sizeable enhancement of the $\varepsilon'/\varepsilon$ prediction. The large one–loop correction to $A_0$ has its origin in the strong final state interaction (FSI) of the two pions in S–wave, which generates large infrared logarithms involving the light pion mass. Using analyticity and unitarity constraints, these logarithms can be exponentiated to all orders in the chiral expansion. For the CP–conserving amplitudes, the result can be written as

$$A_I = (M_K^2 - M_\pi^2) a_I(M_K^2) = (M_K^2 - M_\pi^2) \Omega_I(M_K^2, s_0) a_I(s_0),$$

where $a_I(s)$ denote reduced off-shell amplitudes with $s \equiv (p_{\pi_1} + p_{\pi_2})^2$ and

$$\Omega_I(s, s_0) \equiv e^{i\delta_I(s)} \Re_I(s, s_0) = \exp \left\{ \frac{(s - s_0)}{\pi} \int \frac{dz}{(z - s_0)(z - s - i\epsilon)} \delta_I(z) \right\}$$

provides an evolution of $a_I(s)$ from an arbitrary low–energy point $s_0$ to $s = M_K^2$. The physical amplitude $a_I(M_K^2)$ is of course independent of $s_0$.

Taking the chiral prediction for $\delta_I(z)$ and expanding the exponential to first order, one just reproduces the one–loop \chiPT result. Eq. (3) allows us to get a much more accurate prediction, by taking $s_0$ low enough that the \chiPT corrections to $a_I(s_0)$ are small and exponentiating the large logarithms with the Omnès factor $\Omega_I(M_K^2, s_0)$. Moreover, using the experimental phase-shifts in the dispersive integral one achieves an all–order resummation of FSI effects. The numerical accuracy of this exponentiation has been successfully tested through an analysis of the scalar pion form factor which has identical FSI than $A_0$.  

3. Numerical Predictions

At \( s_0 = 0 \), the chiral corrections are rather small. To a very good approximation, we can just multiply the tree–level \( \chi \)PT result for \( a_I(0) \) with the experimentally determined Omnès exponentials:

\[
\Re_0 \equiv \Re_0(M_K^2, 0) = 1.55 \pm 0.10, \quad \Re_2 \equiv \Re_2(M_K^2, 0) = 0.92 \pm 0.03. \tag{5}
\]

Thus, \( B_6^{(1/2)} \approx \Re_0 \times B_6^{(1/2)} \bigg|_{N_C \to \infty} = 1.55, \quad B_8^{(3/2)} \approx \Re_2 \times B_8^{(3/2)} \bigg|_{N_C \to \infty} \approx 0.92 \)

and \( \Omega_{IB} \approx 0.16 \times \Re_2/\Re_0 = 0.09 \). This agrees with the result \( \Omega_{IB} = 0.08 \pm 0.05 \), obtained recently with an explicit chiral loop calculation.

The large FSI correction to the \( I = 0 \) amplitude gets reinforced by the mild suppression of the \( I = 2 \) contributions. The net effect is a large enhancement of \( \varepsilon'/\varepsilon \) by a factor 2.4, pushing the predicted central value from \( 3.7 \times 10^{-4} \) to \( 17 \times 10^{-4} \). A more careful analysis, taking into account all hadronic and quark–mixing inputs gives the Standard Model prediction:

\[
\varepsilon'/\varepsilon = (17 \pm 6) \times 10^{-4}, \tag{6}
\]

which compares well with the present experimental world average.

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References