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Anticipating the Higher Generations of Quarks from Rephasing Invariance of the Mixing Matrix

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ABSTRACT

We show that the number of invariant CP-violating parameters X_{CP} jumps from the unique universal one in three generations to nine in the four-generation case, saturating the parameter space for generation numbers higher then three. This can lead to drastically different consequences in CP-violating phenomena. We give the quark mass matrices in the three-generation case and speculate for higher generations. We also give some invariant definitions of "maximal" CP violation.

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Measurements of the b-particle lifetime and its decay properties[1] have sharply constrained the parameter space of the Kobayashi-Maskawa (KM) model.[2] The even smaller allowed region remaining after fitting the CP-violating parameter ϵ could be ruled out by a little improvement in the upper bound of $|V_{ub}|$.[3] It is simple to understand the origin of this correlation between $|V_{ub}|$ and CP-violating effects. The general argument is that if we had one of the moduli of the KM matrix equal to zero, the CP-violating phase δ could be rephased away. This point is made clear in the mixing matrix by Chau and Keung (CK),[4] in which all the imaginary parts entering the mixing matrix are in the form $|V_{ub}|e^{-i\phi}$. In general, there are (N-1)(N-2)/2 CP-violating independent phases for N generations,[5] thus an increase in the number of generations will allow enough freedom to remedy this situation.[6] Some consequences of the existence of higher generations have been discussed in the literature. Most of the effects discussed are dependent on the higher-generation quark masses.[6]

In this paper we shall discuss the consequences of the existence of higher generations, especially those effects that are independent of detailed model calculations but rely upon some intrinsic properties of the presence of more than three generations. First, we give some generic results of the quark mixing matrix from its rephasing invariant properties. We show that the number of invariant CP-violating parameters X_{CP} jumps from the unique universal one in three generations to nine in the four-generation case, and saturates the parameter space for generation number higher than three. This immediately leads to the possibility of very different implications for CP-violating effects. It is well-known that the general three-generation consequences of small D°, $\bar{D}^{\circ}$, as well as small B_d°, $\bar{B}_d^{\circ}$ mixing, but appreciable B_s°, $\bar{B}_s^{\circ}$ mixing, and small mass-matrix (superweak) CP violation, e.g., small $(\ell^+\ell^+ - \ell^-\ell^-)$ asymmetry from

D°, B° decays,[7] may change with the presence of higher generations. But these mass-matrix effects are loop effects depending on the new quark masses, as demonstrated in the explicit calculations in Ref. (6). However, there are CP-violating effects, dominated by tree graphs, thus independent of the new quark masses, and dependent only on the existence of the new generations. Especially interesting, we find that the partial decay rate differences in charm decays can be enhanced by more than an order of magnitude with respect to the three-generation case, shown in Ref. (8). All this leads to interesting consequences from high charm-particle-producing machines, as well as from high beauty-particle-producing machines.

To help in the general analysis of the CP-violating phenomena, we first introduce rephasing invariants. Given V, the mixing matrix, we know that physical observables are independent of the quark phase convention, i.e., physics is invariant under the rephasing property $V \to V' = D_1 \; V \; D_2$, where $D_{1,2}$ are arbitrary diagonal unitary matrices. Following the seminal paper of Greenberg,[9] we define the following rephasing invariants (RI)[10]

$$T_i^\alpha \equiv \mathrm{Tr}\{V^+\Delta^\alpha V\Delta_i\} = |V_{\alpha i}|^2 , \qquad (1)$$

$$T_{ij}^{\alpha\beta} \equiv \mathrm{Tr}\{V^+\Delta^\alpha V\Delta_i V^+\Delta^\beta V\Delta_j\} = V^*_{\alpha j}\; V_{\alpha i}\; V^*_{\beta i}\; V_{\beta j} ,$$

where $(\Delta^\alpha)_{jk} = \delta_{j\alpha}\delta_{k\alpha}$, the indices α, β, indicate the charge-2/3 quarks, and the subindex i indicates the charge-(-1/3) quarks. It can be shown[10] that all physical quantities can be written in terms of these RI's. $T_{ij}^{\alpha\alpha}$ are real, $T_{ij}^{\alpha\beta} = (T_{ij}^{\beta\alpha})^*$ and $T_{ij}^{\alpha\beta} = T_{ji}^{\beta\alpha}$ so that the CP-violating observables are controlled by Im $(T_{ij}^{\alpha\beta})$ $\alpha<\beta, i<j$. Using $\sum_{\alpha=1}^{N} \Delta^\alpha = \sum_{i=1}^{N} \Delta_i = I$ and unitarity of V for the N generation case, we only need to consider indices running from 1 to (N-1).

So in the three-generation case we conclude that all CP-violating observables must be proportional to $X_{CP} = \text{Im}\, T_{12}^{12}$. This is a general proof of the theorem shown in Ref. 4. To be more specific, from unitarity

$T_{32}^{13} = -T_{32}^{12} - T_{32}^{11} = + T_{22}^{12} + T_{12}^{12} - T_{32}^{11}$, we have

$$X_{CP} = \text{Im}\, T_{12}^{12} = \text{Im}\, T_{32}^{13} = c_1 c_2 c_3 s_1^{\,2} s_2 s_3 s_\delta = c_x c_y c_z^{\,2} s_x s_y s_z s_\phi \,, \tag{2}$$

where c_i, s_i are the parameters of the KM and CK mixing matrices, respectively.[11] Like all other physical observables, X_{CP} can be expressed in terms of $T_1^1, T_2^1, T_1^2, T_2^2$, up to the sign.

In the four-generation case, the same reduction procedure used to get Eq. (2) tells us that all CP-violating observables are controlled by nine independent anaalogs to X_{CP}, that is,

$$(X_{CP})_{ij}^{\alpha\beta} = \text{Im}\, T_{ij}^{\alpha\beta} \,, \tag{3}$$

where α, β, i, j run from 1 to (N-1), with $\alpha<\beta$, $i<j$. Note that there are $[(N-1)(N-2)/2]^2$ $(X_{CP})_{ij}^{\alpha\beta}$, i.e., for N=2, there is zero X_{CP}; for N=3, there is one X_{CP}; for N=4, there are nine X_{CP}'s and it can be proved to be independent[10]; for N>3 the number of X_{CP}'s saturates the parameter space of the mixing matrix. So it is a unique feature of the three generation case that there is one universal CP-violating parameter. This qualitatively distinct feature forms the basis of our model-independent observation for clues to the existence of more than three generations of quarks. In particular, partial decay rate differences are proportional to X_{CP} in the three-generation case, and to $(X_{CP})_{ij}^{\alpha\beta}$ in the four-generation case. So there is the chance that these kinds of observables will show important differences. We will return to this point later on.

To be specific, we shall use the four-generation case as an example, and choose the following criteria for constructing explicitly the mixing matrix: i) imposing a simple three-generation limit, ii) setting parameters with straightforward experimental correspondences. In the three-generation case, ii) has been accomplished by the CK parametrization,[4,12] so it seems natural to choose a generalization of this parametrization. Since this is of the Maiani type,[12] we will have

$$V = D^{+} \prod_{i<j} \omega_{ij} (\theta_{ij}, \phi_{ij}) D , \tag{4}$$

where D is an arbitrary diagonal unitary matrix that can be suitably chosen and $\omega_{ij}(\theta_{ij},\phi_{ij})$ is a complex rotation between the i and j generations with argument θ_{ij} and phases ϕ_{ij}. In order to give an explicit parametrization for V in (4), we must still decide the particular order we pick for the ω_{ij} products and how we choose the phases of D in order to eliminate as many ϕ_{ij} complex phases as possible. The effect of D^{+} and D in Eq. (4) can be completely absorbed by redefining ϕ_{ij}.[12,14] Nevertheless, there is a general result that constrains the ϕ_{ij} that can be rephased away: given three generations, $i<j<k$ $(\phi_{ij} + \phi_{jk} - \phi_{ik})$ is independent of D. This means that for three given values of i,j,k the three phases entering into the previous equations cannot be rephased away. So consistent with requirements i) and ii), our parametrization is

$$V = \left[\omega_{34}(\theta_u)\omega_{24}(\theta_v,\phi_3)\omega_{14}(\theta_w,\phi_2)\right]\left[\omega_{23}(\theta_y)\omega_{13}(\theta_z,\phi_1)\omega_{12}(\theta_x)\right] . \tag{5}$$

The square bracket to the right is a trivial extension to four generations of the CK matrix V_{CK}. We put this factor on the right, and the remaining ordination of ω_{ij} to simplify at maximum the first two rows. Requirement (ii) imposes the location of ϕ_2 and ϕ_3 because we will see later on that θ_w and θ_v are restricted to be relatively small and θ_u will be completely free. Then the mixing matrix is

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_u & s_u \\ 0 & 0 & -s_u & c_u \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_v & 0 & s_v e^{-i\phi_3} \\ 0 & 0 & 1 & 0 \\ 0 & -s_v e^{i\phi_3} & 0 & c_v \end{bmatrix} \begin{bmatrix} c_w & 0 & 0 & s_w e^{-i\phi_2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_w e^{i\phi_2} & 0 & 0 & c_w \end{bmatrix} \left[\begin{array}{ccc|c} & & & 0 \\ & V_{CK} & & 0 \\ & & & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] =$$

$$= \left[\begin{array}{l|l|l|l}
c_w c_x c_z & c_w s_x c_z & c_w s_z e^{-i\phi_1} & s_w e^{-i\phi_2} \\
\hline
\begin{array}{l} -s_v s_w c_x c_z e^{i(\phi_2-\phi_3)} \\ -c_v s_x c_y - c_v c_x s_y s_z e^{i\phi_1} \end{array} & \begin{array}{l} -s_v s_w s_x c_z e^{i(\phi_2-\phi_3)} \\ +c_v c_x c_y - c_v s_x s_y s_z e^{i\phi_1} \end{array} & \begin{array}{l} -s_v s_w s_z e^{i(\phi_2-\phi_3-\phi_1)} \\ + c_v s_y c_z \end{array} & s_v c_w e^{-i\phi_3} \\
\hline
\begin{array}{l} -s_u c_v s_w c_x c_z e^{i\phi_2} \\ +s_u s_v s_x c_y e^{i\phi_3} + s_u s_v c_x s_y s_z e^{i(\phi_3+\phi_1)} \\ +c_u s_x s_y - c_u c_x c_y s_z e^{i\phi_1} \end{array} & \begin{array}{l} -s_u c_v s_w s_x c_z e^{i\phi_2} \\ -s_u s_v c_x c_y e^{i\phi_3} + s_u s_v s_x s_y s_z e^{i(\phi_3+\phi_1)} \\ -c_u c_x s_y - c_u s_x c_y s_z e^{i\phi_1} \end{array} & \begin{array}{l} -s_u c_v s_w s_z e^{i(\phi_2-\phi_1)} \\ -s_u s_v s_y c_z e^{i\phi_3} + c_u c_y c_z \end{array} & s_u c_v c_w \\
\hline
\begin{array}{l} -c_u c_v s_w c_x c_z e^{i\phi_2} \\ +c_u s_v s_x c_y e^{i\phi_3} + c_u s_v c_x s_y s_z e^{i(\phi_3+\phi_1)} \\ -s_u s_x s_y + s_u c_x c_y s_z e^{i\phi_1} \end{array} & \begin{array}{l} -c_u c_v s_w s_x c_z e^{i\phi_2} \\ -c_u s_v c_x c_y e^{i\phi_3} + c_u s_v s_x s_y s_z e^{i(\phi_3+\phi_1)} \\ +s_u c_x s_y + s_u s_x c_y s_z e^{i\phi_1} \end{array} & \begin{array}{l} -c_u c_v s_w s_z e^{i(\phi_2-\phi_1)} \\ -c_u s_v s_y c_z e^{i\phi_3} - s_u c_y c_z \end{array} & c_u c_v c_w
\end{array}\right] \qquad (6)$$

To set bounds to the mixing angles we will use the experimental information[1] $|V_{ud}| = 0.9734$, $|V_{us}| = 0.224$, $|V_{ub}| \leq 0.008$, $|V_{cd}| = 0.21 - 0.23$ $|V_{cs}| \geq 0.81$, $|V_{cb}| \simeq 0.05$, and from unitarity $|V_{ub'}| \leq 0.05$ and $|V_{cb'}| \leq 0.55$. From this value we get $s_x = 0.224$, $s_y = 0.055$, $s_z \leq 0.008$, $s_w \leq 0.05$, $s_v \leq 0.55$; but s_u remains a free parameter. We must stress that the bounds on s_v depend essentially on the $|V_{cs}|$ bound. We have assigned $|V_{cs}|$ a rather conservative value, so one must keep in mind the possibility of having a slightly smaller upper bound for s_v. In this pattern, one extremely interesting possibility that immediately comes to mind is that the mixing between the three and four generations can be of the order of $s_u \simeq 1$. In this case, the submatrix of these two generations would be essentially off-diagonal, giving rise to a long lifetime for the t quark and a suppression of the decay mode of W into t, depending of course on the mass of the b' quark.

To control orders of magnitude, it is convenient to use the Wolfenstein-type parametrization,[15] $s_x = \lambda$; $s_y' = \lambda^2$; $s_z \le \lambda^3$; $s_w \le \lambda^2$; $s_v \le 2\lambda$, taking into account these orders of magnitude, we found the following approximate form, to the order of λ^4, for V:

$$V = \left[\begin{array}{l|l|l|l}
c_x & s_x & s_z e^{-i\phi_1} & s_w e^{-i\phi_2} \\
\hline
\begin{array}{l} -s_x c_v \\ -s_v s_w e^{+i(\phi-\phi_3)} \\ -s_y s_z e^{i\phi_1} \end{array} &
\begin{array}{l} c_v c_x \\ -s_v s_w s_x e^{+i(\phi_2-\phi_3)} \\ -s_x s_y s_z e^{i\phi_1} \end{array} &
\begin{array}{l} s_y c_v \\ -s_v s_w s_z e^{-i(\phi_1-\phi_3-\phi_2)} \end{array} &
s_v e^{-i\phi_3} \\
\hline
\begin{array}{l} -s_u s_w e^{i\phi_2} \\ +s_u s_v s_x e^{i\phi_3} \\ +c_u s_x s_y \\ -c_u s_z e^{i\phi_1} \end{array} &
\begin{array}{l} -s_u s_v c_x e^{i\phi_3} \\ -c_u s_y \\ -s_u s_w s_x e^{i\phi_2} \\ -c_u s_x s_z e^{i\phi_1} \end{array} &
\begin{array}{l} c_u + \\ -s_u s_v s_y e^{i\phi_3} \\ -s_u s_w s_z e^{i(\phi_2-\phi_1)} \end{array} &
s_u c_v \\
\hline
\begin{array}{l} -c_u s_w e^{i\phi_2} \\ +c_u s_v s_x e^{i\phi_3} \\ -s_u s_x s_y \\ s_u s_z e^{i\phi_1} \end{array} &
\begin{array}{l} -c_u s_v c_x e^{i\phi_1} \\ s_u s_y \\ -c_u s_w s_x e^{i\phi_2} \\ s_u s_x s_z e^{i\phi_1} \end{array} &
\begin{array}{l} -s_u \\ -c_u s_v s_y e^{i\phi_3} \\ -s_u s_w s_z e^{i(\phi_2-\phi_1)} \end{array} &
c_u c_v
\end{array}\right] \qquad (7)$$

This matrix represents a simplified version of (6) dictated by experimental information, and it can be safely used to perform numerical analysis in involved processes. In (7) we have included terms of order λ^6 when they are the leading ones proportional to some of the complex phases.

Having at hand a rather restricted form of the mixing matrix in the four-generation case, it is of interest to look for the different possibilities for the mass matrix in the four-generation case. This is useful as a guideline for model builders, and also in the search for any kind of regularity among masses and mixing angles. In the standard model, the quark mass matrix is

$$L_{MASS} = -\bar{\psi}^U_L M^U \psi^U_R - \bar{\psi}^D_L M^D \psi^D_R + \text{H.c.} ; \tag{8}$$

because the right-handed fields are singlet, it is possible to reduce M^U and M^D to Hermitian matrices[16] (through the polar decomposition of a matrix into an Hermitian one times a unitary one and absorbing the unitary transformation into the right-handed fields) or symmetric matrices. This is always possible even outside the standard model, provided the right-handed currents are absent or sufficiently suppressed at low energies.[17] So without loss of generality in the standard model, one can start with the most general M^U and M^D matrices being Hermitian or symmetric. But consistently one can go one step further. Let U_1 be the unitary matrix that diagonalizes the Hermitian M^U matrix. Then if we transform all the quark fields (up, down, left and right) with U_1^+, we change only the mass term of the Lagrangian. But $U_1^+ M^D U_1$ is Hermitian, being M^D Hermitian, and now M^U is diagonal. Finally, we can apply a suitable unitary diagonal transformation to all the fields in order to eliminate as many phases as possible from $U_1^+ M^D U_1$. So without loss of generality, the most general form of M^U and M^D in the standard model is M^U diagonal and M^D Hermitian with some removable phases. Picking the Hermitian case, the mass matrix becomes:

$M_H \equiv V M^D (\text{diag.}) V^+ =$

$$\simeq \left[\begin{array}{c|c|c} m_d + m_s s_x^2 & m_s s_x + m_b s_y s_z e^{-i\phi_1} & m_b s_z e^{-i\phi_1} - m_s s_x s_y \\ \hline m_s s_x + m_b s_y s_z e^{i\phi_1} & m_s c_x^2 + m_b s_y^2 & \begin{matrix}(m_b - m_s) s_y \\ -m_s s_x s_z e^{-i\phi_1}\end{matrix} \\ \hline m_b s_z e^{i\phi_1} - m_s s_x s_y & \begin{matrix}(m_b - m_s) s_y \\ -m_s s_x s_z e^{i\phi_1}\end{matrix} & m_b c_y^2 \end{array}\right] \qquad (9)$$

To get (9), we have used the smallness of s_y, s_z and $m_d/m_b \leq \lambda^4$, $m_s/m_b \leq \lambda^2$. Similarly, we can get the symmetric mass matrix VM^D (diag.) V. Equation (9) is accurate up to order $m_b\lambda^4$. Of course, (9) reproduces the pattern noted by Frampton and Jarlskog[16] (FJ) $M^U/m_t = M_H/m_b + O(\lambda^2)$, although it makes it clearer that the origin of this pattern comes essentially from the great variety of scales in the quark masses. The analysis of the four generation case in this pattern with the Petra limit $m_b/m_{b'} \leq \lambda$ gives all the possible mass matrices allowed by nature. Instead of writing down M_H in the general case, we will present some interesting particular cases. It is evident that unless we suppress some mixing angle, $m_{b'}$ will dominate almost all the matrix elements, so if we impose the condition that not all the elements in the first two rows be dominated by $m_{b'}$ we obtain the pattern $s_w \leq \lambda^3$ and $s_v \leq \lambda^2$. Now we will concentrate on two rather orthogonal illustrative examples: the case in which the mixing among the third and fourth generations is big or small. In the case $s_u \leq \lambda^2$ and writing down only the leading terms of every matrix element, we get:

$$M_H \simeq \begin{bmatrix} m_d + m_s s_x^2 & m_s s_x & m_b s_z e^{-i\phi_1} & m_{b'} s_w e^{-i\phi_2} \\ m_s s_x & m_s & m_b s_y & m_{b'} s_v e^{-i\phi_3} \\ m_b s_z e^{+i\phi_1} & m_b s_y & m_b & m_{b'} s_u \\ m_{b'} s_w e^{+i\phi_2} & m_{b'} s_v e^{+i\phi_3} & m_{b'} s_u & m_{b'} \end{bmatrix} . \qquad (10)$$

Clearly Eq. (10) represents a straightforward generalization of (9), picking the leading terms. So $s_w \leq \lambda^3$, $s_v \leq \lambda^2$ with $s_u \lesssim \lambda^2$ provides a natural pattern generalizing the three-generation case. A more exotic situation would be with $c_u \leq \lambda^2$, in this case

$$M_H \simeq \begin{bmatrix} m_d + m_s s_x^2 & m_s s_x & m_b' s_w e^{-i\phi_2} & -m_b e^{-i\phi_1} \\ m_s s_x & m_s & m_b' s_v e^{-i\phi_3} & -m_b s_y \\ m_b' s_w e^{+i\phi_2} & m_b' s_v e^{+i\phi_3} & m_b' & m_b' c_u \\ -m_b s_z e^{+i\phi_1} & -m_b s_y & m_b' c_u & m_b \end{bmatrix} . \qquad (11)$$

This second possibility is remarkable in that if we interchange the third and fourth generations, the result turns out to be exactly the same as (10) except for the presence of unobservable signs and the change ($c_u \leftrightarrow s_u$). Of course, this is a consistancy check of the simple fact that by relabeling the generation number we can convert an off-diagonal coupling into a diagonal one. Equations (10) and (11) represent two simple and extreme situations of the type of mass matrix we can expect in the four-generation case. So far, nature has followed the pattern that the order of generation is the order of masses, but in four generations this pattern may change as in Eq. (11), where, the generation order 3 and 4 is reversed with respect to the mass order 3 and 4.

Let us now analyze the experimental signatures of the presence of higher generations, especially effects independent of the new quark masses. The first one to check is the unitarity condition in the three-generation case.[18] The other phenomena that are independent of the new-quark masses are the partial decay rate differences (PDRD) in nonleptonic decays dominated by tree graphs. These PDRD are proportional to the RI Im $T_{ij}^{\alpha\beta}$. To simplify the discussion, let us point out that in three generations $X_{CP} \lesssim \lambda^6 \sin(\phi_1)$; in four generations some of them are:

$$(X_{CP})^{12}_{12} \lesssim 2\lambda^4 \sin(\phi_2-\phi_3), \quad (X_{CP})^{13}_{12} \lesssim s_u^{\,2}\lambda^4 \sin(\phi_2-\phi_3),$$
$$(X_{CP})^{23}_{12} \lesssim s_u^{\,2}\lambda^4 \sin(\phi_2-\phi_3), \quad (X_{CP})^{23}_{23} \lesssim (s_u c_u)\lambda^3 \sin(\phi_3)\ , \tag{12}$$

This expression can be read out from Eqs. (1), (3), and (7). The implications of these $(X_{CP})^{\alpha\beta}_{ij}$ in Eq. (12) being potentially some order of magnitude larger than X_{CP} in the three generation case is that it permits a much wider range for PDRD given the constraint by ε and ε'/ε. For example, for $F \to K^0\pi$ the PDRD Δ_F is defined by

$$\Delta_F = \frac{\Gamma(F^+\to K^0\pi^+)-\Gamma(F^-\to \bar{K}^0\pi^-)}{\Gamma(F^+\to K^0\pi^+)+\Gamma(F^-\to \bar{K}^0\pi^-)}\quad , \tag{13}$$

since this decay is dominated by two tree graphs with a relative strong phase of $\pi/2$, we have

$$\Delta_F(\text{4-generations}) = \Delta_F(\text{3-generations})\ \frac{(X_{CP})^{12}_{12}}{X_{CP}}\quad . \tag{14}$$

So this PDRD in four generations can be as big as $-(1.8 \times 10^{-2}$ to $10^{-1})$ using the calculation of Ref. (8); in the three-generation case the value is $-(3.6 \times 10^{-4}$ to $2 \times 10^{-3})$. The other large $(X_{CP})^{\alpha\beta}_{ij}$ in Eq. (12) can only contribute to PDRD dominated by tree graphs in top decay; in addition, we also have $(X_{CP})^{13}_{13} \lesssim \lambda^5 \sin\phi_1$, an enhancement even in the case $\phi_3 = \phi_2 = 0$. All this implies that also the PDRD in top decays will have a much wider allowed range in the case of four generations.

It must be stressed that the possible enhancement found through Eq. (14) comes from an enhancement in the numerator in Eq. (13). In b decays, this kind of enhancement is not possible because, for example $(X_{CP})^{12}_{23} = \text{Im}\, T^{12}_{23}$ is of the order of magnitude of the experimental value $|T^{12}_{23}| = |V_{12}||V_{13}||V_{22}||V_{23}|$. Of course there can be significant deviations in PDRD dominated by loop graphs, but these results are dependent on the mass of the fourth-generation quarks.

At this point, one may wonder if the set of parameters (12) can be too big both for ε and ε'/ε. Nevertheless, we must stress that the results (12) must be contemplated as upper bounds and that by picking a suitable scenario, i.e., fixing m_t, $m_t{}'$, s_u, B_k and $B_k{}'$,[7,19] one can try to adjust ε and ε'/ε without spoiling (12) too much. Note that for ε and ε'/ε, we got a contribution from $(X_{CP})^{12}_{12}$, $(X_{CP})^{13}_{12}$, $(X_{CP})^{23}_{12}$. It must be stressed that in general s_u does not have to be too small and that this λ^4 bound comes from using $s_w s_v \backsim 2\lambda^3$.

Finally, we would like to comment on the concept of maximal CP violation. Till now, most definitions appearing in the literature[11] are parametrization dependent and contain the ironic situation of no CP violation, e.g., $\delta = \pi/2$, but $V_{ub} = 0$. From our discussions here, if maximal CP violation can be defined at all it must be defined through the invariants X_{CP}'s.

It seems to us natural to use one of the following two criteria:
i) maximize $(X_{CP})^{\alpha\beta}_{ij}$, ii) maximize $\Delta^{\alpha\beta}_{ij} = 2(X_{CP})^{\alpha\beta}_{ij} / (T^{\alpha\beta}_{ii} + T^{\alpha\beta}_{jj})$.

In the three-generation case, these definitions have the following features:

case i) the hole mixing matrix is fixed and is equivalent to maximal mixing $|V_{ij}| = 1/\sqrt{3}$:

$$V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ -1 & e^{i\pi/3} & e^{-i\pi/3} \\ -1 & e^{-i\pi/3} & e^{i\pi/3} \end{bmatrix} . \tag{15}$$

Of course this is far from having any connection with the experimental values. V^* is also a solution for maximal CP violation.[20] In the case ii) if one assumes that non weak interactions are such that they allow maximal CP violation to manifest itself completely, $\Delta_{ij}^{\alpha\beta}$ are nothing other than the previously introduced PDRD.[21] If, for example we impose $\Delta_{ds}^{tc} = +1$, it can be proved that $\Delta_{db}^{tc} = -\Delta_{sb}^{tc} = -2/3$. So in definition ii) there is the ambiguity which PDRD gets maximized. Incidentally, in definition i) all PDRD gets the same value $\Delta_{ij}^{\alpha\beta} = \pm\sqrt{3}/2$.

Having nine independent $(X_{CP})_{ij}^{\alpha\beta}$ in the four-generation case, the definition i) also suffers from ambiguities, because it is imposible to maximize simultaniously all the $(X_{CP})_{ij}^{\alpha\beta}$. In particular, if we first maximize $(X_{CP})_{12}^{12}$ and $(X_{CP})_{12}^{13}$ we get $(X_{CP})_{12}^{14} = 0$. It must also be stressed that a solution for maximal mixing is

$$V = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{i\alpha} & -e^{i\alpha} & -1 \\ 1 & -e^{i\alpha} & e^{i\alpha} & -1 \\ 1 & -1 & -1 & +1 \end{bmatrix} \tag{16}$$

This solution contains some $(X_{CP})_{ij}^{\alpha\beta} = 0$ and, what is worse, for $\alpha = 0, \pi$ it describes a CP-conserving world. So we must conclude that the nice properties found in (15) are an accident of the three-generation case and that it is hard to imagine any rephasing invariant definition (the only one acceptable) of maximal CP violation for any number of generations. Of course, one can use i) or

ii) to decide that CP is not maximally violated in any process, but this is not so useful for obtaining further insight into the weak mixing.

In conclusion, we have shown that, in contrast to the single universal CP violating parameter X_{CP} in the three-generation case, there are nine independent such invariants, X_{CP}'S of Eq. (3) in the four-generation cases, and the X_{CP}'s saturate the parameter space of the mixing matrix for anything above three generations. They can give rise to large partial decay rate differences in charm and top quark decays in addition to those in b decays, as in the three-generation case. These new signatures are independent of the masses of the higher-generation quarks. In addition, there are other new effects that, however, are loop effects, depending on the new quark masses, e.g., large D^o-$\bar{D}^o$ and B^o_d-$\bar{B}^o_d$ mixing; and large charge asymmetries due to mass-matrix CP violation. It is most important and exciting to experimentally study these new phenomena since they deviate from the predictions of three generations of quarks.

What is most lacking right now in the unification theories is information on the mass-generation mechanism. We find it interesting that we now actually have the mass matrix for the charge-(-1/3) quark in the three-generation case, Eq. (9), which is useful for theoretical model building. We have also speculated on the possible quark mass matrix in the four-generation case. Because of current interest, we have also given two possible ways of defining "maximal" CP violation. The most important point to make is that they ought to be rephasing invariant.

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