Scalar $\Lambda N$ and $\Lambda\Lambda$ interaction in a chiral unitary approach

K. Sasaki, E. Oset, and M. J. Vicente Vacas

Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC.

(Dated: July 25, 2013)

We study the central part of $\Lambda N$ and $\Lambda\Lambda$ potential by considering the correlated and uncorrelated two-meson exchange besides the $\omega$ exchange contribution. The correlated two-meson is evaluated in a chiral unitary approach. We find that a short range repulsion is generated by the correlated two-meson potential which also produces an attraction in the intermediate distance region. The uncorrelated two-meson exchange produces a sizeable attraction in all cases which is counterbalanced by $\omega$ exchange contribution.

PACS numbers: 13.75.Ev, 12.39.Fe.

I. INTRODUCTION

The scalar isoscalar potential plays an important role in the nucleon nucleon interaction providing an intermediate range attraction in all channels which is demanded by the data. In models of the NN interaction using one boson exchange (OBE) this part of the interaction was accounted for by allowing the exchange of a "$\sigma$" particle in as early papers as 1. The exchange of a scalar particle has been a constant in other OBE models 2. In some models a broad $\sigma$(760) was advocated 3,4 which has been used later on in more recent models 5,6. The particle data group 7 refers to the lightest scalar meson as $f_0(600)$ or $\sigma$. The nature of this particle, and even its mere existence, has been a source of controversy 8, but in recent years a strong convergence towards the idea that the $\sigma$ is not a genuine meson state made up from a constituent $q\bar{q}$ pair has been witnessed 2,10. This idea has obtained further support from unitarized chiral perturbation theory where the $\sigma$ appears as a dynamically generated resonance from the $\pi\pi$ interaction 11,12,13,14,15,16,17,18,19. Other models that start from a seed of $q\bar{q}$, but couple this state to meson meson components in a unitary approach, converge to the same idea by showing that the meson cloud is essential in building up the low lying scalar resonances 20,21,22,23.

The picture of the $\sigma$ as a dynamically generated resonance called for a new interpretation of the $\sigma$ exchange in the NN interaction and this work was performed in 23. In this work the traditional $\sigma$ exchange was substituted by the exchange of two interacting mesons within the chiral unitary framework of 13, and an intermediate attraction was found together with a repulsion at short distances, which makes the picture qualitatively different from the ordinary, always attractive, $\sigma$ exchange. The exchange of two interacting pions, although nonperturbative, was considered in 24 and shown to reproduce well the NN peripheral partial waves with $L > 2$. A recent work studying the isoscalar contact $NN$ interactions retakes the unitarization of the $\pi\pi$ amplitude in the two pion exchange using the Omnes representation 25.

The work of 23 was complemented in 26, where in addition to the interacting two pion exchange, the contribution of the uncorrelated two pion exchange and the repulsive contribution of the $\omega$ exchange were considered, leading altogether to a good reproduction of the empirical scalar isoscalar interaction of 27,28.

The purpose of this work is to extend this to the strange sector evaluating the $\Lambda N$ scalar isoscalar interaction.

Empirical evaluations of the $YN$ scalar isoscalar interaction are done in several works allowing the exchange of a scalar meson and making fits to data. As quoted above, the Nijmegen group makes use of the exchange of a heavy scalar meson and there are different fits available in the literature 1,2,26,31. A recent work of the group shows an interesting feature which is the improvement of the results by using a form factor incorporating a zero 31, which leads to qualitative features of the scalar meson exchange similar to those observed in 27. The exchange of various scalar mesons is also considered in 32 as well as correlated two pion exchange, which however is treated phenomenologically. Another approach to the problem is the chiral quark model in which the $\pi$ and a $\sigma$ are allowed to be exchanged between constituent quarks 33,34. In the same line, in the works of Refs. 38,39 a SU(3) nonet of scalar mesons is exchanged between the quarks.

The closest work to our approach is the theoretical work of 37, following the line of the Juelich model 37,37, where the uncorrelated and correlated two pion exchange are considered explicitly. The approach to the correlated two pion (and two kaon) exchange is done rather differently by evaluating theoretically the $BB \rightarrow \pi\pi, KK$ amplitudes and using then unitarity and dispersion relations to relate these amplitudes to the correlated two meson exchange contribution to the $BB \rightarrow BB$ interaction. Our approach evaluates directly the correlated two pion exchange by explicitly using the chiral unitary approach to deal with the pion pion interaction and using appropriate triangle diagrams to account for the coupling of the two pions to the baryons. The success of this approach providing the scalar isoscalar $NN$ interaction provides solid grounds to extend these ideas to the case of the $\Lambda N$ interaction, which

...
we present in this work.

II. CORRELATED TWO-MESON EXCHANGE BETWEEN BARYONS

We follow closely Ref. [23] and consider the correlated two-meson exchange between baryons. In order to evaluate these diagrams we use the lowest order chiral Lagrangeans

\[ \mathcal{L}_2 = \frac{1}{6f_\pi^2} \text{Tr}[\Phi \partial^\mu \Phi \partial_\mu \Phi - \Phi \Phi \partial^\mu \partial_\mu \Phi] + \frac{1}{12f_\pi^2} \text{Tr}[M \Phi^4] \]  

\[ \mathcal{L}_B = \frac{D + F}{\sqrt{2}f_\pi} \text{Tr}[\bar{B} \gamma_5 \gamma^\mu \partial_\mu \Phi B] + \frac{D - F}{\sqrt{2}f_\pi} \text{Tr}[\bar{B} \gamma_5 \gamma^\mu B \partial_\mu \Phi] \]  

where \( \Phi \) and \( B \) are the standard \( SU(3) \) matrices for the octets of pseudoscalar mesons and baryons respectively [40, 41, 42, 43, 44, 45, 46]. The mass matrix of the mesons octet is defined by \( M \equiv \text{diag}(m_{\pi}^2, m_{\pi}^2, 2m_{\pi}^2 - m_N^2) \). From there, one obtains the different pion-pion (or \( K\bar{K} \)) lowest order amplitudes, which can be found in [13] and the \( MBB \) vertices which for convenience we show in the appendix.

The isoscalar amplitudes, which contain only \( s \)-wave, have been obtained by employing Lagrangean \( \mathcal{L}_2 \) in [13]. The lowest order, tree level, amplitudes of meson-meson scattering can be written as

\[ t^{(J=0,L)}_{\pi\pi\to\pi\pi} = -\frac{1}{f_\pi^2} s - \frac{m_{\pi}^2}{2} + \frac{1}{3f_\pi^2} \sum_i (p_i^2 - m_i^2) \]  

\[ t^{(J=0,L)}_{K\bar{K}\to\pi\bar{K}} = -\frac{3s}{4f_\pi^2} \sum_i (p_i^2 - m_i^2) \]  

\[ t^{(J=0,L)}_{\pi\pi\to\pi\bar{K}} = \frac{\sqrt{3}s}{4f_\pi^2} \sum_i (p_i^2 - m_i^2) \]  

where the \( L \) on the superscript stands for the leading order amplitude of meson-meson scattering and we have employed the convenient unitary normalization and the isospin phase convention (\( |\pi^+\rangle = -(1,1), |K^-\rangle = -(1/2,-1/2) \)):

\[ |\pi\pi, (I = 0)\rangle = \frac{1}{\sqrt{6}} |\pi^0\pi^0 + \pi^+\pi^- + \pi^-\pi^+\rangle \]  

\[ |K\bar{K}, (I = 0)\rangle = \frac{1}{\sqrt{2}} |K^0\bar{K}^0 + K^+\bar{K}^-\rangle. \]  

In eqs. (3-5) we have separated the lowest order interaction into a part which provides the on-shell contribution and another term (the one with \( (p_i^2 - m_i^2) \)) which contributes only for off-shell mesons.

As shown in [12, 23, 26], the off-shell part of the meson-meson amplitudes does not contribute to our calculation. In fact, for the meson-meson loops, this contribution is absorbed into the physical mass and the coupling. As for the coupling to the baryons, there is a cancellation of the off-shell part of the meson-meson amplitude in eq. (3) with the diagrams of the type of Fig. 11. This fact is valid not only for the \( NN \) case but also for \( YN \) and \( YY \) case. Thus, hereafter, we only consider the on-shell part of the meson-meson amplitude.

This on-shell treatment enables us to separate the on-shell meson-meson amplitude from the triangle loop integration that couples the mesons to the baryons. Thus we can define the correlated two-meson potential as

\[ V_{B_1B_2}^{\text{Cor}}(q) = \sum_{ij} N_{ij} \Delta_{B_1}^{i} \Delta_{B_2}^{j} \]
where $\Delta$ indicates the triangle loop contribution of two-meson for baryon $B_k$ and $N_{ij}$ is a factor from the isospin summation, concretely $N_{\pi\pi,\pi\pi} = 6$, $N_{\pi K KK} = N_{K K K K} = 2$. For concreteness, the $\Delta$ function in the correlated two-pion potential for $NN$ channel is given by

$$\Delta^{(\pi\pi)}_N = \left( \frac{D + F}{2f_\pi} \right)^2 V_{NN}^{(\pi\pi)}(q)$$

(9)

where $V_{NN}^{(m_1 m_2)}(q)$ is the vertex function which is already evaluated in [23] and given in a generalized form as

$$V_{NN}^{(m_1 m_2)}(q) = \int \frac{d^3p}{(2\pi)^3} \frac{M_{B'}}{E_{B'}(\vec{p})} \frac{(\vec{p} + \vec{q}) \cdot \vec{p}}{2\omega_1\omega_2(\omega_1 + \omega_2)} \frac{\omega_1 + \omega_2 + E_{B'}(\vec{p}) - M_B}{(\omega_1 + E_{B'}(\vec{p}) - M_B)(\omega_2 + E_{B'}(\vec{p}) - M_B)}$$

(10)

with

$$E_{B'}(\vec{p}) = \sqrt{\vec{p}^2 + M_{B'}^2}; \quad \omega_1 = \sqrt{\mu_1^2 + \vec{p}^2}; \quad \omega_2 = \sqrt{\mu_2^2 + (\vec{p} + \vec{q})^2}.$$  

(11)

This is calculated with the variables corresponding to Fig. 2(a) and where, as in [23], we have put the initial momentum at rest. We introduce a static form factor in order to regularize the triangle loop function. The form factor employed in this calculation is

$$F(\vec{p})F(\vec{p} + \vec{q}) = \frac{\Lambda^2}{\Lambda^2 + \vec{p}^2} \frac{\Lambda^2}{\Lambda^2 + (\vec{p} + \vec{q})^2}$$

(12)

where the cutoff is chosen as $\Lambda = 1.0$ GeV.

A. Lowest order contribution in the isoscalar exchange in the $\Lambda N \to \Lambda N$ interaction

For this case the potential generated by the correlated two-pion diagrams shown in Fig. 3 is given by

$$V_{\pi\pi \to \pi\pi} = 6 \left[ \frac{D}{\sqrt{2}f_\pi} \right]^2 V_{\Sigma \Lambda}^{(\pi\pi)}(q) \left[ \frac{D + F}{2f_\pi} \right]^2 V_{NN}^{(\pi\pi)}(q)$$

(13)

where $V_{\Sigma \Lambda}^{(\pi\pi)}(q)$ is the vertex function defined in eq. (10) with the same form factor. Furthermore, as discribed in section II C we substitute $t_{\pi\pi \to \pi\pi}^{(I = 0, L)}$ by the full unitarized amplitude. From now on we consider directly the full meson-meson amplitudes in all cases.

We also include the exchange of $K\bar{K}$ in the approach. The diagrams to take into account are shown in Fig. 4. By using the couplings in the appendix, we evaluate the potential generated by the correlated $K\bar{K}$ contributions in a similar way as before obtaining

$$V_{K\bar{K} \to K\bar{K}} = 2 \Delta \frac{K\bar{K}}{K\bar{K}} \left[ \frac{D + F}{2f_\pi} \right]^2 V_{NN}^{(\pi\pi)}(q)$$

(14)
with the triangle kaon-loop contribution given by

\[
\Delta^{(K\bar{K})}_\Lambda = \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right)^2 V_{N\Lambda}^{(K\bar{K})}(q) + \left( \frac{3F - D}{2\sqrt{3}f_\pi} \right)^2 V_{\Xi\Lambda}^{(K\bar{K})}(q)
\]

(15)

\[
\Delta^{(K\bar{K})}_N = \frac{3}{2} \left( \frac{D - F}{2\sqrt{3}f_\pi} \right)^2 V_{\Sigma\Lambda}^{(K\bar{K})}(q) + \frac{1}{2} \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right)^2 V_{\Lambda\Lambda}^{(K\bar{K})}(q)
\]

(16)

The factors in the brace of \(\Delta^{(K\bar{K})}_N\) come from the \(I = 0\) projection of the kaon-couplings.

Now one must consider the mixed terms with vertices with \(\pi\) or \(K\) which involve the \(\pi\pi \rightarrow K\bar{K}\) transition amplitude. The diagrams to consider are shown in Fig. 4. The potential in this case is given by

\[
V_{\pi\pi \rightarrow K\bar{K}} = 2\sqrt{3}\Delta^{\pi\pi}_\Lambda t_{\pi\pi \rightarrow K\bar{K}}^{(I=0,L)} \Delta^{K\bar{K}}_\Lambda + 2\sqrt{3}\Delta^{K\bar{K}}_N t_{K\bar{K} \rightarrow \pi\pi}^{(I=0,L)} \Delta^{\pi\pi}_N
\]

(17)

with the triangle meson-loop contribution shown before.

B. Contribution of \(\Delta, \Sigma^*, \Xi^*\) intermediate states

Next we wish to include the contribution of the intermediate \(\Delta, \Sigma^*, \Xi^*\) states. In the block of diagrams of Fig. 3 we can introduce \(\Sigma^*\) in the left triangular vertex, or \(\Delta\) in the right triangular vertex, or both.

The coupling of the decuplet to the octet of mesons and baryons is given by

\[
\mathcal{L}^{Dcc} = \frac{\sqrt{3}}{f_\pi} C \sum_{a,b,c,d,e} \epsilon^{abc} \left( (T_{ade}\Phi^d B^e_\pi) \bar{S} \cdot (-\bar{q}) + (B^e_\pi T_{ade}) \bar{S} \cdot (-\bar{q}) \right)
\]

(18)

for an outgoing meson with momentum \(q\). The \(C\) is determined from the \(\Delta N\pi\) coupling constant. \(T\) is the decuplet baryon field shown in the appendix. This Lagrangean gives rise to couplings of the type

\[
\gamma_{MBB'} f_{\pi N\Lambda}^* \frac{\bar{S} \cdot \bar{q}}{m_\pi}
\]

(19)

for outgoing mesons of momentum \(\bar{q}\). The \(\gamma_{MBB'}\) coefficients can be found in the appendix. We use \(f_{\pi N\Lambda}^* = 2.12 f_{\pi NN}\) to obtain the correct width of the \(\Delta(1232)\) resonance.

Altogether, the contribution involving the \(\pi\pi \rightarrow \pi\pi\) amplitude is given by

\[
V^{(\pi\pi \rightarrow \pi\pi)} = 6 t_{(I=0,L)}^{(\pi\pi \rightarrow \pi\pi)} \left[ \left( \frac{D}{\sqrt{3}f_\pi} \right)^2 V_{\Sigma\Lambda}^{(\pi\pi)}(q) + \frac{2}{3} \left( \frac{f_{\pi N\Lambda}^*}{\sqrt{2}m_\pi} \right)^2 V_{\Sigma\Sigma}^{(\pi\pi)}(q) \right]
\]

\[
\times \left[ \left( \frac{D + F}{2f_\pi} \right)^2 V_{N\Lambda}^{(\pi\pi)}(q) + \frac{4}{9} \left( \frac{f_{\pi N\Delta}^*}{m_\pi} \right)^2 V_{\Delta\Delta}^{(\pi\pi)}(q) \right]
\]

(20)
where the factor two-thirds comes from the difference of spin and the extra two-thirds in front of $V^{(\pi\pi)}_{\Delta N}$ from the change of isospin. This equation shows how the triangle loop contribution is modified by the excited baryon in the intermediate state. Thus, the modified triangle loop contribution can be written as

$$\tilde{\Delta}^{(\pi\pi)}_\Lambda = \Delta^{(\pi\pi)}_\Lambda + \frac{2}{3} \left( \frac{f_{\pi N \Delta}}{\sqrt{2} m_\pi} \right)^2 V^{(\pi\pi)}_{\Sigma^* N}(q)$$

$$\tilde{\Delta}^{(\pi\pi)}_N = \Delta^{(\pi\pi)}_N + \frac{4}{9} \left( \frac{f_{\pi N \Delta}}{m_\pi} \right)^2 V^{(\pi\pi)}_{\Delta N}(q).$$

In the same way we take into account the contributions of $\Sigma^*$, $\Xi^*$ in the $K\bar{K}$ triangle contribution by means of the couplings of the appendix, and the results are given by

$$\tilde{\Delta}^{(K\bar{K})}_\Lambda = \Delta^{(K\bar{K})}_\Lambda + \frac{2}{3} \left( \frac{f_{\pi N \Delta}}{\sqrt{2} m_\pi} \right)^2 V^{(K\bar{K})}_{\Sigma^* \Lambda}(q)$$

$$\tilde{\Delta}^{(K\bar{K})}_N = \Delta^{(K\bar{K})}_N + \frac{2}{3} \left( \frac{f_{\pi N \Delta}}{\sqrt{2} m_\pi} \right)^2 V^{(K\bar{K})}_{\Sigma^* N}(q).$$

Therefore the total leading-order potential generated by the correlated two-meson contribution with the excited baryons in the intermediate states is given by substitution of the $\Delta$ to the $\tilde{\Delta}$ as

$$V^{Cor}_{\Lambda N}(q) = \sum_{i,j} N_{ij} \tilde{\Delta}^{i}_{\Lambda} t_{i\rightarrow j} \tilde{\Delta}^{j}_{N}.$$  

## C. Unitarization of the amplitudes

Here we follow Ref. [13] and iterate the meson-meson potential to infinite order by using a Lippmann-Schwinger type equation in coupled $\pi\pi$ and $K\bar{K}$ channels. As it is shown in [13, 14], the Lippmann-Schwinger equation can be reduced under the on-shell factorization to the algebraic relation

$$T = [1 - V G]^{-1} V$$

where $V \equiv t$ used in the former eqs. (2-5), and $G$ is the meson-meson loop function. The $G$ function is given by

$$G(s) = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2^2 [s - (\omega_1 + \omega_2)^2 + i\epsilon]}$$

where $\omega_i = \sqrt{q_i^2 + m_i^2}$. It is regularized with the cutoff scheme which is different from the expression used in the previous paper [22]. One advantage of the usage of a cutoff is that it does not produce undesirable poles as mentioned in [22]. For the region of interest to us, one can use both cutoff or dimensional regularization to evaluate $G$. Some caveats about the use at unreasonably low negative values of $s$ are discussed in [22], which set restrictions on the results at very short distances.

We have also included the $g$ channel but their effect is small and can be approximately reabsorbed in the $\pi\pi$, $K\bar{K}$ channels by redefining $q_{max}$ [13]. We obtain good results for the $\pi\pi$ phase shift up to 1.2 GeV by using the $\pi\pi$, $K\bar{K}$ channels and $q_{max} = 1.0$ GeV.

The final expression for the $\Delta N$ scalar potential is given by summing the expressions in section II eq. (25) and substituting $t_i$ by the unitarized amplitude $T_i$.

## III. UNCORRELATES TWO MESON EXCHANGE

We consider both the direct and crossed diagrams of the uncorrelated two-pion and two-kaon exchange contributions shown in Fig. 4. We follow here the procedure of [24] and use the variables of the diagrams as shown in Fig. 2(b), (c).

The uncorrelated two-meson potential is given after performing analytically the $p^0$ integration in terms of the integrals

$$V^{(i,M_i M_2)}_{B_1, B_2}(q) = - \int \frac{d^3p}{(2\pi)^3} \frac{M_1 M_2}{E_1 E_2} \left( p^2 - q^2 \right)^2 R_i(\cdot)$$

(28)
where $M_1$ and $M_2$ are the baryon masses and $i$ stands for the direct ($D$) or crossed ($C$) terms and

$$R_{D} = \frac{(\omega_1 + \omega_2)(E'_1 + E'_2)^2 + 2\omega_1\omega_2}{2\omega_1\omega_2(E'_1 + E'_2)(\omega_1 + E'_1)(\omega_2 + E'_2)}$$

$$R_{C} = \frac{(\omega_1 + \omega_2)(E'_1 + E'_2)^2 + \omega_1^2 + \omega_2^2 + \omega_1\omega_2 + \omega_1E'_1 + \omega_2E'_2}{2\omega_1\omega_2(E'_1 + E'_2)(\omega_1 + E'_1)(\omega_2 + E'_2)}$$

(29)

(30)

with $E'_i = E_i - p_0^0$. It is worth mentioning that, if we compare without coupling constants, the crossed contribution is much smaller than the box type contribution. Furthermore, both $\mathcal{V}^D$ and $\mathcal{V}^C$ are largely suppressed by the mass of the exchanged meson. As a result of this rough estimation, we expect that the uncorrelated two-kaon contribution is much smaller than the uncorrelated two-pion contribution.

By taking the coupling of the vertices in the different diagrams into account we get the contribution to the ΛN potential with considering the contribution of the $\Delta$, $\Sigma^*$, $\Xi^*$ in the intermediate states as

$$v_{\Lambda N}^{(D,\pi\pi)} = 3 \left( \frac{D}{\sqrt{3}f_\pi} \right)^2 \left( \frac{D + F}{2f_\pi} \right)^2 V_{\Sigma\Sigma}^{(D,\pi\pi)}(q) + 2 \left( \frac{D + F}{2f_\pi} \right)^2 \left( \frac{f_{\pi N\Delta}}{\sqrt{2m_\pi}} \right)^2 V_{\Sigma^* \Xi^*}^{(D,\pi\pi)}(q)$$

$$+ \frac{4}{3} \left( \frac{D}{\sqrt{3}f_\pi} \right)^2 \left( \frac{f_{\pi N\Delta}}{m_\pi} \right)^2 V_{\Sigma\Delta}^{(D,\pi\pi)}(q)$$

$$+ \frac{8}{9} \left( \frac{f_{\pi N\Delta}}{2m_\pi} \right)^2 V_{\Sigma^* \Delta}^{(D,\pi\pi)}(q)$$

(31)

where we write explicitly the left and right baryon in the diagrams and the couple of mesons exchanged.

Similarly for the $\mathcal{V}^{(D,\pi\pi)}$ diagrams we calculate the uncorrelated two-kaon potential as

$$v_{\Lambda N}^{(D,\pi\pi)} = 3 \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right)^2 \left( \frac{D - F}{2f_\pi} \right)^2 V_{\Lambda N}^{(D,\pi\pi)}(q) + \frac{1}{3} \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right)^2 \left( \frac{f_{\pi N\Delta}}{m_\pi} \right)^2 V_{\Lambda N}^{(D,\pi\pi)}(q)$$

$$+ 2 \left( \frac{3F - D}{2\sqrt{3}f_\pi} \right)^2 \left( \frac{f_{\pi N\Delta}}{\sqrt{6m_\pi}} \right)^2 V_{\Xi^* \Sigma^*}^{(D,\pi\pi)}(q)$$

$$+ \frac{4}{3} \left( \frac{3F - D}{2\sqrt{3}f_\pi} \right)^2 \left( \frac{f_{\pi N\Delta}}{\sqrt{2m_\pi}} \right)^2 V_{\Xi^* \Delta^*}^{(D,\pi\pi)}(q)$$

(32)

In this case the $\mathcal{V}^{(D,\pi\pi)}$ diagrams have more variety than for the $\pi\pi$ case, but the potential does not change so much because the contribution of $\mathcal{V}^{(C,\pi\pi)}$ is quite small as explained before.
Here we take into account the $\omega$ exchange potential which is known as one of main sources of short range repulsion of the baryon-baryon interaction. The $\omega$ exchange potential for $\Lambda N$ in momentum space is given by

$$V_{\omega\Lambda N}(q) = \frac{g_{\omega\Lambda}\lambda_{\omega NN}}{q^2 + m^2_{\omega}} \left( \frac{\Lambda^2_{\omega} - m^2_{\omega}}{\Lambda^2_{\omega} + q^2} \right)^2$$

where we choose $g_{\omega NN} = 13$ and $\Lambda_{\omega} = 1.4\text{GeV}$ \[25\]. The ideal mixing for $\omega$ and $\phi$ leads the relation, $g_{\omega\Lambda\Lambda} = \frac{2}{3}g_{\omega NN}$, which is deduced from the quark contents of the hadrons. For simplicity we assume the same form factor for both the $\omega NN$ and $\omega\Lambda\Lambda$ vertices.

V. THE ISOSCALAR EXCHANGE IN THE $\Lambda\Lambda$ INTERACTION

We easily extend this method to the $\Lambda\Lambda$ interaction. For this we simply replace the triangle contribution of the nucleon by that of the $\Lambda$ in eq. \[26\]

$$V_{\Lambda\Lambda}^{C\text{or}}(q) = \sum_{i,j} N_{ij} \bar{\Delta}_i^{(I=0,L)} \bar{\Delta}_j$$

where all contributions are already shown before.

The uncorrelated two-meson exchange potential is given by

$$v_{\Lambda\Lambda}^{(D,\pi\pi)} = 3 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 V^{(D,\pi\pi)}(q) + 3 \left( \frac{f^*_{\pi NN}}{\sqrt{2}m_{\pi}} \right)^2 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 V^{(\Sigma_\pi,\Sigma_\pi)}(q)$$

$$+ 3 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 \left( \frac{f^*_{\pi NN}}{\sqrt{2}m_{\pi}} \right)^2 V^{(\Sigma_{\pi},\Sigma_{\pi})}(q) + 3 \left( \frac{f^*_{\pi NN}}{\sqrt{2}m_{\pi}} \right)^2 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 V^{(D,\pi\pi)}(q)$$

$$v_{\Lambda\Lambda}^{(C,\pi\pi)} = 3 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 V^{(C,\pi\pi)}(q) + 3 \left( \frac{f^*_{\pi NN}}{\sqrt{2}m_{\pi}} \right)^2 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 V^{(\Sigma_{\pi},\Sigma_{\pi})}(q)$$

$$+ 3 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 \left( \frac{f^*_{\pi NN}}{\sqrt{2}m_{\pi}} \right)^2 V^{(\Sigma_{\pi},\Sigma_{\pi})}(q) + 3 \left( \frac{f^*_{\pi NN}}{\sqrt{2}m_{\pi}} \right)^2 \left( \frac{D}{\sqrt{3}f_{\pi}} \right)^2 V^{(C,\pi\pi)}(q)$$

\[35\]
The two-kaon exchange is given by

$$v^{(D,K)}_{\Lambda\Lambda} = 2 \left( \frac{3F - D}{2\sqrt{3}f_{\pi}} \right)^2 \left( \frac{D + 3F}{2\sqrt{3}f_{\pi}} \right)^2 V^{(D,K)}_{\Xi N}(q) + 2 \left( \frac{f^{*}_{\pi N\Lambda}}{\sqrt{2m_{\pi}}} \right)^2 \left( \frac{D + 3F}{2\sqrt{3}f_{\pi}} \right)^2 V^{(D,K)}_{\Xi^{*} N}(q)$$

$$+ 2 \left( \frac{D + 3F}{2\sqrt{3}f_{\pi}} \right)^2 \left( \frac{3F - D}{2\sqrt{3}f_{\pi}} \right)^2 V^{(D,K)}_{\Xi^{*} N}(q) + 2 \left( \frac{f^{*}_{\pi N\Lambda}}{\sqrt{2m_{\pi}}} \right)^2 \left( \frac{D + 3F}{2\sqrt{3}f_{\pi}} \right)^2 V^{(D,K)}_{\Xi^{*} N}(q)$$

$$+ 2 \left( \frac{D + 3F}{2\sqrt{3}f_{\pi}} \right)^4 V^{(C,K,K)}_{\Xi N}(q)$$

(36)

The ω exchange potential is also considered by substituting $g_{\omega NN}$ to $g_{\omega \Lambda\Lambda}$ in eq. (36).

**VI. SCALAR πK EXCHANGE IN THE κ CHANNEL**

In the interactions of the octet of mesons in the πK channel one also finds a very broad resonance [14], the kappa with $S = -1$, $I = 1/2$, $J^{P} = 0^{+}$, around 800 MeV, somewhat controversial [17, 18, 19, 50, 51]. Its exchange is also accounted for in the recent model of [52].

Here we follow the same approach as in the former section and exchange πK in the $I = 1/2, l = 0$ channel. For this we consider the diagrams of Fig. 8. By following the same rules as before we find for the sum of all diagrams the compact expression

$$3 t_{\pi\pi}^{(l=1/2, L)} \left\{ - \left( \frac{D + 3F}{2\sqrt{3}f_{\pi}} \right) \left( \frac{D + F}{2f_{\pi}} \right) V^{(\pi K)}_{\Lambda \Lambda}(q) + \left( \frac{D - F}{2f_{\pi}} \right) \left( \frac{D - F}{2f_{\pi}} \right) V^{(\pi K)}_{\Xi}(q) \right\}^2$$

(37)

which including the contribution of Σ* states gives (there are no Δ or Ξ* intermediate states now)

$$3 t_{\pi\pi}^{(l=1/2, L)} \left\{ - \left( \frac{D + 3F}{2\sqrt{3}f_{\pi}} \right) \left( \frac{D + F}{2f_{\pi}} \right) V^{(\pi K)}_{\Lambda \Lambda}(q) + \left( \frac{D - F}{2f_{\pi}} \right) \left( \frac{D - F}{2f_{\pi}} \right) V^{(\pi K)}_{\Xi}(q) \right\}^2$$

$$- \frac{2}{3} \left( \frac{f^{*}_{\pi N\Lambda}}{\sqrt{6m_{\pi}}} \right) \left( \frac{f^{*}_{\pi N\Lambda}}{\sqrt{2m_{\pi}}} \right) V^{(\pi K)}_{\Xi^{*}}(q)$$

(38)

The πK → πK amplitude in the $I = 1/2$ channel can be obtained from the appendix of [14] and we have

$$t_{\pi K → πK}^{(l=1/2, L)} = \frac{1}{4f_{\pi}^{2}} (3u - s - 2m_{\pi}^{2} - 2m_{K}^{2})$$

(39)
which after projection over \( l = 0 \) gives \([52]\)

\[
I_{\pi K \to \pi K}^{(l=1/2, L)}(l = 0) = -\frac{1}{4f_\pi^2} \left( -\frac{5}{2} s + m_\pi^2 + m_K^2 + \frac{3(m_K^2 - m_\pi^2)^2}{2s} \right).
\]

(40)

In order to avoid the singular behavior around \( s = 0 \), we take the \( SU(3)_f \) limit in the \( \pi K \) amplitude. Then we can obtain the modified \( \pi K \to \pi K \) amplitude as

\[
I_{\pi K \to \pi K}^{(l=1/2, L)}(l = 0) = \frac{1}{4f_\pi^2} \left( -\frac{5}{2} s + 2m' \right).
\]

(41)

where \( m' \) is the average mass of pion and kaon. This amplitude is also unitarized in the same way as before with only one channel.

Next we calculate the uncorrelated \( \pi K \) diagrams shown in Fig. 8 considering the decuplet excitation of the intermediate baryon. These diagrams give

\[
v_{AN(\kappa)}^{(D,\pi K)} = -3 \left( \frac{D + F}{2f_\pi} \right) \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right) \left( \frac{D - F}{2f_\pi} \right) \left( \frac{D - F^*}{2f_\pi} \right) V_{N\Sigma}^{(D,\pi K)}(q)
\]

\[
+ \frac{1}{\sqrt{3}} \left( \frac{D + F}{2f_\pi} \right) \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right) \left( f_\pi^2 \frac{N\Delta}{m_\pi} \right)^2 V_{N\Sigma}^{(D,\pi K)}(q)
\]

\[
v_{AN(\kappa)}^{(D,K\pi)} = -3 \left( \frac{D + F}{2f_\pi} \right) \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right) \left( \frac{D - F}{2f_\pi} \right) \left( \frac{D - F^*}{2f_\pi} \right) V_{\Sigma\Sigma}^{(D,K\pi)}(q)
\]

\[
+ \frac{1}{\sqrt{3}} \left( \frac{D + F}{2f_\pi} \right) \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right) \left( f_\pi^2 \frac{N\Delta}{m_\pi} \right)^2 V_{\Sigma\Sigma}^{(D,K\pi)}(q)
\]

\[
v_{AN(\kappa)}^{(C,\pi K)} = 3 \left( \frac{D + F}{2f_\pi} \right)^2 \left( \frac{D + 3F}{2\sqrt{3}f_\pi} \right)^2 V_{NN}^{(C,\pi K)}(q)
\]

\[
v_{AN(\kappa)}^{(C,K\pi)} = 3 \left( \frac{D}{\sqrt{3}f_\pi} \right)^2 \left( \frac{D - F}{2f_\pi} \right)^2 V_{\Sigma\Sigma}^{(C,K\pi)}(q) - \frac{1}{\sqrt{3}} \left( \frac{D}{\sqrt{3}f_\pi} \right) \left( \frac{D - F}{2f_\pi} \right) \left( f_\pi^2 \frac{N\Delta}{m_\pi} \right)^2 V_{\Sigma\Sigma}^{(C,K\pi)}(q)
\]

\[
- \frac{1}{\sqrt{3}} \left( \frac{D}{\sqrt{3}f_\pi} \right) \left( \frac{D - F}{2f_\pi} \right) \left( f_\pi^2 \frac{N\Delta}{m_\pi} \right)^2 V_{\Sigma\Sigma}^{(C,K\pi)}(q) + \frac{1}{9} \left( f_\pi^2 \frac{N\Delta}{m_\pi} \right)^4 V_{\Sigma\Sigma}^{(C,K\pi)}(q)
\]

(42)

where we follow this prescription for the meson pair of the superindex: the first meson corresponds to the upper one in the direct exchange and to the upper one on the left baryon for the crossed terms.

VII. RESULTS

A. \( \Lambda N \) Potential in momentum space

Fig. 9 shows the \( \Lambda N \) potential in momentum space. The correlated two-meson contribution has a peak around \( q = 400 \) MeV. A similar peak in position and magnitude was found for the \( NN \) case in Refs. \([23, 26, 52]\). It is worth discussing this shape because it is impossible to parameterize it by a single meson exchange with usual form factors, like monopole or Gaussian. This contribution could be decomposed in, at least, two parts, one a strong repulsive part and the other a weak attraction. In any case, this contribution is much smaller than the other ones, so that the main contribution comes from the uncorrelated two-meson exchange and from \( \omega \) exchange. These two potentials have opposite sign, and the uncorrelated two-meson potential is slightly stronger than the \( \omega \) exchange potential in the whole range of \( q \). Thus, the sum of these two potentials is always negative.

The total potential has positive strength around \( q = 600 \) MeV which is pushed up by the correlated two-meson potential. The correlated two-meson potential plays a more important role in this region.

We can see the relevance of the two-kaon contribution to this potential by comparing the panels (A) and (B) in Fig. 9. The two-kaon contribution was studied since one kaon exchange is important in the \( \Lambda N \) interaction. However, the effect of the two-kaon contribution, suppressed due to its heavy mass, is very small and does not change much the pionic potential.
The ΛN potential in the configuration space is shown in Fig. 10. The potential in configuration space is given by

\[
V(r) = \frac{1}{2\pi^2 r} \int_0^\infty q \sin(qr) V(q) dq.
\]

In the left panel of this figure, we can see a similar correlated two-pion potential as for the \(NN\) case, see Fig. 10 in [23]. This should be expected since the definition of the potential is quite similar to the \(NN\) case except for the masses of the baryons and coupling constants. In fact both the \(NN\) and \(ΛN\) potential generated by the correlated two-pion exchange contribution pass through zero at \(r \approx 0.9\) fm and have a minimum at \(r \approx 1.3\) fm. This potential is repulsive in the short range region and, on the other hand, it is attractive beyond 1 fm. As we have discussed in the previous subsection, the strength of the correlated two-meson potential is much smaller than the other contributions.

Fig. 10 also shows that the uncorrelated two-meson generates a strong attraction and the \(ω\) produces a repulsion in the short range region. The sum of these two potentials produces a relatively strong attraction around 1 fm and leads to large cancellations in the short range region. We do not give the results below 0.5 fm since there the overlap of the baryons and quark exchange mechanisms can lead sizeable corrections.
Although the attraction in the total potential is mainly generated by the uncorrelated two-meson potential, part of the repulsion is generated by the correlated two-meson potential. This is interesting because the correlated two-meson potential is considered as a $\sigma$ meson exchange in other papers and, there, the interaction would be always attractive (see eq. (3.19) of [23]).

Here, again, we can check the effect of two-kaon exchange potential by comparing the two panels in Fig. 10. The two-kaon contribution slightly enhances the magnitude of both the correlated and uncorrelated two-meson potential, and it makes the total potential a little deeper than the pionic potential.

C. The $\kappa$ exchange $\Lambda N$ potential

Fig. 11 shows the $\kappa$ exchange contribution in the $\Lambda N$ potential. The potential in momentum space is shown in the left panel of Fig. 11. The correlated two-meson contribution has a similar shape as for the scalar-isoscalar channel, but its size is one order of magnitude smaller.

The uncorrelated two-meson exchange contribution generates an attraction which is relatively large compared to the correlated two-meson potential.

The right panel of Fig. 11 shows the potential in the configuration space. The correlated two-meson contribution produces a moderate repulsion at the short range region and ends up at around 1.0 fm. The uncorrelated two-meson potential produces a weak attraction up to 2.0 fm. Both potentials are of a quite short range compared to the isoscalar channel. This reflects the heavy mass of the $\kappa$ meson.

The total $\kappa$ channel interaction is weakly attractive and of quite short range. The strength of the potential is weaker than the isoscalar potential, almost 1/3 at 1 fm.

D. The $\Lambda\Lambda$ potential

Fig. 12 shows the central part of the $\Lambda\Lambda$ potential. Both the uncorrelated two-meson and the $\omega$ exchange contribution are weaker than for the $\Lambda N$ case, but they still have much larger magnitude than the correlated two-meson potential. These potentials drive the medium range attraction. However, the $\omega$ exchange potential alone is not enough to produce a repulsion in the shorter distances region.

We find again that the correlated two-meson potential plays a dual role in making a repulsive potential in the short range region and a small attraction in the long range region.

These properties are seen both in the left and right panel cases. The striking difference from the $\Lambda N$ interaction is the large effect produced by the two-kaon exchange contribution. Both the correlated and uncorrelated two-meson...
potential are largely enhanced by the two-kaon exchange diagrams especially in the shorter range region. This effect leads to a large reduction of the short range repulsion. Consequently, the short range repulsion in the \( \Lambda \Lambda \) potential is largely suppressed compared to the contribution of the two-pion exchange case and therefore to the \( NN \) or \( \Lambda N \) interaction where the \( K \bar{K} \) exchange effect is much weaker.

**FIG. 12**: The scalar-isoscalar \( \Lambda \Lambda \) potential in configuration space.

**VIII. CONCLUSIONS**

We have evaluated the scalar channel potential between the \( \Lambda \) and nucleon. We have considered the correlated and uncorrelated two-meson exchange contributions in this channel besides \( \omega \) exchange. The correlated two-meson exchange contribution was calculated by using a chiral unitary approach which reproduces very well the experimental meson-meson phase shift up to 1.2 GeV.

The uncorrelated two-meson exchange contribution produces a strong attraction which is similar to the \( NN \) case. The \( \omega \) exchange contribution makes a short range repulsion, also similar to the \( NN \) case, but its strength is two-thirds of the \( NN \) case due to the simple counting of non-strange quarks in the baryons. These two contributions drive the attractive potential in the medium range region and almost cancel each other at shorter distances.

The correlated two-meson exchange contribution is relatively smaller than the other two contributions. This potential produces some attraction at medium range distances and some strong repulsion in the short range region. This behavior is quite similar to the \( NN \) case which is already calculated in [23]. The striking effect is the repulsion in the short range region where the strong attraction generated by the uncorrelated two-meson potential is cancelled by the repulsion produced by the \( \omega \) meson. Thus the correlated two-meson potential plays an important role for both the medium range attraction and the short range repulsion in the \( \Lambda N \) and \( \Lambda \Lambda \) interaction.

We have also checked the contribution of two-kaon exchange diagrams. We have found that the two-kaon contribution is rather weak and it slightly enhances the magnitude of the potential without changing its main behavior for the \( \Lambda N \) potential. Therefore it does not play an important role in the scalar \( \Lambda N \) potential. On the other hand, it has a large contribution to the \( \Lambda \Lambda \) potential, especially in the short distance region. It largely enhances both the repulsion in the correlated two-meson exchange potential and the attraction in the uncorrelated one. As a result, the total potential in the \( \Lambda \Lambda \) interaction becomes more attractive than for the \( \Lambda N \) case and the short range repulsion is also reduced.

We have also investigated the \( \pi K \) exchange in the \( \kappa \) channel for the case of the \( \Lambda N \) interaction and it shows similar features to the scalar isoscalar potential for a shorter range and sizeably weaker strength.

Finally, we have found that the medium range attraction and short range repulsion is largely depend on the flavor of the baryons. This flavor dependence of the central potential between baryons could be a clue to understand certain properties of nuclear structure and reactions.
TABLE I: Particle assignment

<table>
<thead>
<tr>
<th>Decuplet baryons</th>
<th>(T_{111} = \Delta^+ )</th>
<th>(T_{112} = \Delta^0 )</th>
<th>(T_{122} = \Delta^- )</th>
<th>(T_{222} = \Delta^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_{113} = \frac{\Sigma^{++}}{\sqrt{3}} )</td>
<td>(T_{123} = \frac{\Sigma^{*0}}{\sqrt{6}} )</td>
<td>(T_{223} = \frac{\Sigma^{*0}}{\sqrt{3}} )</td>
<td>(T_{333} = \Omega^- )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Octet baryons</th>
<th>(B_1^\pm = \frac{1}{\sqrt{6}} \Lambda \mp \frac{1}{\sqrt{2}} \Sigma^0 )</th>
<th>(B_2^\pm = \frac{1}{\sqrt{6}} \Lambda \pm \frac{1}{\sqrt{2}} \Sigma^0 )</th>
<th>(B_3^\pm = n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_1^- = \Sigma^- )</td>
<td>(B_2^- = \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 )</td>
<td>(B_3^- = -\frac{\sqrt{2}}{3} \Lambda )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Octet mesons</th>
<th>(\Phi_1^0 = \frac{1}{\sqrt{6}} \eta + \frac{1}{\sqrt{2}} \pi^0 )</th>
<th>(\Phi_2^0 = \pi^+ )</th>
<th>(\Phi_3^0 = K^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Phi_1^- = \pi^- )</td>
<td>(\Phi_2^- = \frac{1}{\sqrt{6}} \eta - \frac{1}{\sqrt{2}} \pi^0 )</td>
<td>(\Phi_3^- = K^0 )</td>
<td></td>
</tr>
<tr>
<td>(\Phi_3^- = \overline{K}^- )</td>
<td>(\Phi_3^- = \overline{K}^0 )</td>
<td>(\Phi_3^- = -\frac{\sqrt{2}}{3} \eta )</td>
<td></td>
</tr>
</tbody>
</table>

**Appendix**

After a non-relativistic reduction, the meson-baryon-baryon interaction with an emitted meson of momentum \(\vec{q} \) is given by

\[
- i t^{Oc} = \left( \alpha_{MBB'} \frac{D + F}{2f_\pi} + \beta_{MBB'} \frac{D - F}{2f_\pi} \right) \vec{\sigma} \cdot \vec{q}
\]

\[
- i t^{Dc} = \gamma_{MBB} \frac{f_{\pi NN} S}{m_\pi} \vec{S} \cdot \vec{q}
\]

where the \(\sigma\) and \(S\) are spin transition operators for the octet-octet and the octet-decuplet cases. Here we define the \(B' \to BM\) process.

**Acknowledgments**

One of us, K.S., wishes to acknowledge support from the Ministerio de Educación y Ciencia and program of estancias de jóvenes doctores y tecnólogos extranjeros en España. This work is partly supported by DGICYT contract number BFM2003-00856, and the E.U. EURIDICE network contract no. HPRN-CT-2002-00311. This research is part of the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RII3-CT-2004-506078.

<table>
<thead>
<tr>
<th>p →</th>
<th>$p\pi^0$</th>
<th>$n\pi^+$</th>
<th>$\Lambda K^+$</th>
<th>$\Sigma^0 K^+$</th>
<th>$\Sigma^+ K^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{MBP}$</td>
<td>$1$</td>
<td>$\sqrt{2}$</td>
<td>$-\frac{2}{\sqrt{3}}$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\beta_{MBP}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>$1$</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>$p \to \Delta^+ \pi^-$</td>
<td>$\Delta^0 \pi^0$</td>
<td>$\Delta^0 \pi^+$</td>
<td>$\Sigma^0 K^0$</td>
<td>$\Sigma^+ K^+$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{MBP}$</td>
<td>$1$</td>
<td>$-\frac{1}{\sqrt{3}}$</td>
<td>$-\frac{1}{\sqrt{3}}$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>$-\frac{1}{\sqrt{3}}$</td>
</tr>
<tr>
<td>n →</td>
<td>$p\pi^-$</td>
<td>$n\pi^0$</td>
<td>$\Lambda K^0$</td>
<td>$\Sigma^0 K^0$</td>
<td>$\Sigma^- K^+$</td>
</tr>
<tr>
<td>$\alpha_{MBn}$</td>
<td>$\sqrt{2}$</td>
<td>$-1$</td>
<td>$\frac{2}{\sqrt{3}}$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\beta_{MBn}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>$-1$</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>n →</td>
<td>$\Delta^0 \pi^0$</td>
<td>$\Delta^+ \pi^-$</td>
<td>$\Sigma^0 K^0$</td>
<td>$\Sigma^+ K^+$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{MBn}$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>$-\frac{2}{\sqrt{3}}$</td>
<td>$-1$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>$-\frac{1}{\sqrt{3}}$</td>
</tr>
</tbody>
</table>

| $\Lambda \to$ | $\Sigma^+ \pi^-$ | $\Sigma^0 \pi^0$ | $\Sigma^- \pi^+$ | $pK^-$ | $nK^0$ | $\Xi^0 K^0$ | $\Xi^+ K^+$ |
| $\alpha_{MBA}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |
| $\beta_{MBA}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |

| $\Lambda \to$ | $\Sigma^+ \pi^-$ | $\Sigma^0 \pi^0$ | $\Sigma^- \pi^+$ | $pK^-$ | $nK^0$ | $\Xi^0 K^0$ | $\Xi^+ K^+$ |
| $\gamma_{MBA}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |

[8] See article of N. A. Tornqvist, in Ref. [2], pag. 506.