Study of exclusive semileptonic and nonleptonic decays of $B_c^-$ in a nonrelativistic quark model.

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We present results for different observables measured in semileptonic and nonleptonic decays of the $B_c^-$ meson. The calculations have been done within the framework of a nonrelativistic constituent quark model. In order to check the sensitivity of all our results against the inter-quark interaction we use five different quark–quark potentials. We obtain form factors, decay widths and asymmetry parameters for semileptonic $B_c^-\to c\bar{c}$ and $B_c^-\to B$ decays. In the limit of infinite heavy quark mass our model reproduces the constraints of heavy quark spin symmetry. For the actual heavy quark masses we find nonetheless large corrections to that limiting situation for some form factors. We also analyze exclusive nonleptonic two–meson decay channels within the factorization approximation.


I. INTRODUCTION

Since its discovery at Fermilab by the CDF Collaboration [1] the $B_c$ meson has drawn a lot of attention. Unlike other heavy mesons it is composed of two heavy quarks of different flavor ($b$, $c$) and, being below the $B-D$ threshold, it can only decay through weak interactions making an ideal system to study weak decays of heavy quarks.

It is known [2] that one cannot apply heavy quark symmetry (HQSS) to hadrons containing two heavy quarks: the kinetic energy term, needed in those systems to regulate infrared divergences, breaks heavy flavor symmetry. Still, there is a symmetry that survives: heavy quark spin symmetry (HQSS). This symmetry amounts to the decoupling of the two heavy quark spins since the spin–spin interaction vanishes for infinite heavy quark masses. Using HQSS Jenkins et al. [2] were able to obtain, in the infinite heavy quark mass limit, relations between different form factors for semileptonic $B_c$ decays into pseudoscalar and vector mesons. Contrary to the heavy-light meson case where standard HQSS applies, no determination of corrections in inverse powers of the heavy quark masses has been worked out in this case. So one can only test any model calculation against HQSS predictions in the infinite heavy quark mass limit.

With both quarks being heavy, a nonrelativistic treatment of the $B_c$ meson should provide reliable results. Besides a nonrelativistic model will comply with the constraints imposed by HQSS as the spin–spin interaction vanish in the infinity heavy quark mass limit. In this paper we will study, within the framework of a nonrelativistic quark model, exclusive semileptonic and nonleptonic decays of the $B_c^-$ meson driven by a $b\to c$ or $c\to d$, $\bar{s}$ transitions at the quark level. We will not consider semileptonic processes driven by the quark $b\to u$ transition. Our experience with this kind of processes, like the analogous $B\to \pi$ semileptonic decay [5], shows that the nonrelativistic model without any improvements underestimates the decay width for two reasons: first at high $q^2$ transfers one might need to include the exchange of a $B^*$ meson, and second the model underestimates the form factors at low $q^2$ or high three–momentum transfers. We will concentrate thus on semileptonic $B_c^-\to c\bar{c}$ and $B_c^-\to B$ transitions. As for–two–meson nonleptonic decay we will only consider channels with a least a $c\bar{c}$ or $B$ final meson. In the first case we will include channels with final $D$ mesons for which there is a contribution coming from an effective $b\to d$, $s$ transition. As later explained this is not the main contribution to the decay amplitude and besides the momentum transfer in those cases is neither too high nor too low so that the problems mentioned above are avoided.

The observables studied here have been analyzed before in the context of different models like the relativistic constituent quark model [4, 5, 6], the quasi-potential approach to the relativistic quark model [7, 8], the instantaneous nonrelativistic approach to the Bethe–Salpeter equation [9, 10, 11], the Bethe–Salpeter equation [12, 13], the three point sum rules of QCD and nonrelativistic QCD [14, 15, 16, 17], the QCD relativistic potential model [18], the relativistic constituent quark model formulated on the light front [19], the relativistic quark–meson model [20] or in models that use the Isgur, Scora, Grinstein and Wise wave functions [21] like the calculations in Refs. [22, 23, 24]. We will compare our results with those obtained in these latter references whenever it is possible. Besides, we will perform an exhaustive study and compile in this work all our results for exclusive semileptonic and nonleptonic $B_c^-$ decays, paying an special attention to the theoretical uncertainties affecting our predictions and providing reliable estimates for all of them.

In the present calculation we shall use physical masses taken from Ref. [25]. For the $B_c$ meson mass and lifetime we shall use the central values of the recent experimental determinations by the CDF Collaboration of $m_{B_c} =$
6285.7 ± 5.3 ± 1.2 MeV/c² [20] and \( \tau_{B_c} = (0.463^{+0.073}_{-0.065} \pm 0.036) \times 10^{-12} \) s [27]. This new mass value is very close to the one we obtain with the different quark-quark potentials that we use in this work (see below) from where we get \( m_{B_c} = 6291.6^{+13}_{-33} \) MeV.

We shall also need Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and different meson decay constants. For the former we shall use the ones quoted in Ref. [4] that we reproduce in Table I. All of them are within the ranges quoted by the Particle Data Group (PDG) [22].

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**TABLE I:** Values for Cabibbo-Kobayashi-Maskawa matrix elements used in this work.

For the meson decay constants the used values in this work are compiled in Table II. They correspond to central values of experimental measurements or lattice determinations. The results for \( f_\rho \) and \( f_{K^*} \) have been obtained by the authors in Ref. [4] using \( \tau \) lepton decay data. Our own theoretical calculation, obtained with the model described in Ref. [28], gives the values for \( f_\rho = 0.189 \sim 0.227 \) GeV, \( f_{K^*} = 0.180 \sim 0.220 \) GeV depending on the inter-quark interaction used, results which agree with the determinations in Ref. [4]. We shall nevertheless use the latter for our calculations. For \( f_{\eta_c} \) we have been unable to find an experimental result or a lattice determination. There are at least two theoretical determinations that predict \( f_{\eta_c} = 0.484 \) GeV [4] and \( f_{\eta_c} = 0.420 \pm 0.052 \) GeV [21]. Again our own calculation gives values in the range \( f_{\eta_c} = 0.485 \sim 0.500 \) GeV depending on the inter-quark interaction used. Here we will take \( f_{\eta_c} = 0.490 \) GeV.

\[
\begin{array}{ccccccc}
f_\rho & f_{\rho^0} & f_{\rho^-} & f_{K^-} & f_{K^*0} & f_{K^*+} & f_{K^*+} \\
0.1307 & 0.130 & 0.210 & 0.1598 & 0.224 & 0.224 & 0.974 & 0.0413 \\
\end{array}
\]

**TABLE II:** Meson decay constants in GeV used in this work.

The rest of the paper is organized as follows. In Sect. III we introduce our meson states and the potential models used to obtain the spatial part of their wave functions. In Sect. III we study the \( B_c \) meson semileptonic decays into various \( c\bar{c} \) channels, both for a final light charged lepton (\( e, \mu \)) and for a heavy one (\( \tau \)). In Sect. IV we study different exclusive nonleptonic two–meson decay channels of the \( B_c^- \) meson with one of the final mesons being a \( c\bar{c} \) one. In Sect. V we study semileptonic \( B_c^- \to B \) decays and in Sect. VI nonleptonic two–meson decays with one of the mesons having a \( b \) quark. We briefly summarize our results in Sect. VII. The paper also contains four appendices: in appendix A we give different sets of polarization vectors used in this paper, appendices B and C collect the expressions for all the matrix elements needed to evaluate the different observables analyzed, finally in appendix D we give the expressions for the helicity components of the hadron tensor to be defined below.

**II. MESON STATES AND INTER-QUARK INTERACTIONS**

Within a nonrelativistic constituent quark model, the state of a meson \( M \) is given by

\[
\left| M, \lambda \vec{P} \right>_{NR} = \int d^3p \sum_{\alpha_1, \alpha_2} \phi^{(M, \lambda)}_{\alpha_1, \alpha_2}(\vec{p}) \times \frac{(-1)^{(1/2)-s_2}}{(2\pi)^{3/2} \sqrt{2E_{f_1}(p_1)2E_{f_2}(p_2)}} \left| q_1 \alpha_1 \vec{p}_1 = \frac{m_{f_1}}{m_{f_1} + m_{f_2}} \vec{P} - \vec{p} \right> \left| q_2 \alpha_2 \vec{p}_2 = \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \vec{P} + \vec{p} \right>
\]

where \( \vec{P} \) stands for the meson three momentum and \( \lambda \) represents the spin projection in the meson center of mass. \( \alpha_1 \) and \( \alpha_2 \) represent the quantum numbers of spin \( s \), flavor \( f \) and color \( c \) (\( \alpha \equiv (s, f, c) \)), of the quark and the
antiquark, while \((E_f, (\vec{p}_f), \vec{p}_f), m_f\), and \((E_f, (\vec{p}_f), \vec{p}_f), m_f\), are their respective four–momenta and masses. The factor \((-1)^{(1/2)-s_2}\) is included in order that the antiquark spin states have the correct relative phase. The normalization of the quark and antiquark states is

\[
(\alpha' \bar{\alpha}' | \alpha \bar{\alpha}) = \delta_{\alpha' \alpha} (2\pi)^3 2E_f(\vec{p}) \delta(\vec{p}' - \vec{p})
\]

Furthermore, \(\hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p})\) is the momentum space wave function for the relative motion of the quark-antiquark system. Its normalization is given by

\[
\int d^3p \sum_{\alpha_1, \alpha_2} \left( \phi_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}) \right)^* \phi_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}) = \delta_{\lambda', \lambda}
\]

and, thus, the normalization of our meson states is

\[
NR \langle M, \lambda' \bar{\vec{p}}' | M, \lambda \vec{p} \rangle_{NR} = \delta_{\lambda', \lambda} (2\pi)^3 \delta(\vec{p}' - \vec{p})
\]

In this calculation we will need the ground state wave function for scalar \((0^+), \) pseudoscalar\((0^-), \) vector \((1^-), \) axial vector \((1^+), \) tensor \((2^+), \) and pseudotensor \((2^-)\) mesons. Assuming always the lowest possible value for the orbital angular momentum we will have for a meson \(M\) with scalar, pseudoscalar and vector quantum numbers:

\[
\hat{\phi}_{\alpha_1, \alpha_2}^{(M(0^+))}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \phi_{(s_1, f_1), (s_2, f_2)}^{(M(0^+))}(\vec{p})
\]

\[
\hat{\phi}_{\alpha_1, \alpha_2}^{(M(0^-))}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \left( \hat{\phi}_{f_1, f_2}^{(M(0^-))}(\vec{p}) \right) \sum_m (1/2, 1/2, 1; s_1, s_2, -m) (1, 1, 0; m, -m, 0) Y_{1m}(\vec{p})
\]

\[
\hat{\phi}_{\alpha_1, \alpha_2}^{(M(1^-), \lambda)}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \left( -1 \right) \phi_{f_1, f_2}^{(M(1^-), \lambda)}(\vec{p}) \left( 1/2, 1/2, 0; s_1, s_2, 0 \right) Y_{00}(\vec{p})
\]

where \((j_1, j_2, j_3; m_1, m_2, m_3)\) are Clebsch-Gordan coefficients, \(Y_{lm}(\vec{p})\) are spherical harmonics, and \(\hat{\phi}_{f_1, f_2}^{(M)}(\vec{p})\) is the Fourier transform of the radial coordinate space wave function.

For axial mesons we need orbital angular momentum \(L = 1\). In this case two values of the total quark–antiquark spin \(S_{qq} = 0, 1\) are possible, giving rise to the two states:

\[
\hat{\phi}_{\alpha_1, \alpha_2}^{(M(1^+, S_{qq} = 0), \lambda)}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \phi_{(s_1, f_1), (s_2, f_2)}^{(M(1^+, S_{qq} = 0), \lambda)}(\vec{p})
\]

\[
\hat{\phi}_{\alpha_1, \alpha_2}^{(M(1^+, S_{qq} = 1), \lambda)}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \left( -1 \right) \phi_{f_1, f_2}^{(M(1^+, S_{qq} = 1), \lambda)}(\vec{p}) \left( 1/2, 1/2, 1; s_1, s_2, \lambda - m \right) (1, 1, 1; m, \lambda - m, \lambda) Y_{1m}(\vec{p})
\]

\[\text{Note that under charge conjugation (C) quark and antiquark creation operators are related via } Cc_1^\dagger(\vec{p})C = (-1)^{(1/2)-s} d_1^\dagger(\vec{p}).\]

This implies that the antiquark states with the correct spin relative phase are not \(d_1^\dagger(\vec{p})|0\rangle = |\bar{q}, \alpha \bar{p}\rangle\) but are given instead by \((-1)^{(1/2)-s} d_1^\dagger(\vec{p})|0\rangle = (-1)^{(1/2)-s} |\bar{q}, \alpha \bar{p}\rangle\).
Finally for tensor and pseudotensor mesons we have the wave functions:

$$\phi_{\alpha_1, \alpha_2}^{(M(2^+), \lambda)}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \phi_{s_1, f_1, s_2, f_2}^{(M(2^+), \lambda)}(\vec{p})$$

$$= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \phi_{f_1, f_2}^{(M(2^+), \lambda)}(|\vec{p}|) \sum_m (1/2, 1/2, 1; s_1, s_2, \lambda - m) (1, 1, 2; m, \lambda - m, \lambda) Y_{1m}(\vec{p})$$

$$\phi_{\alpha_1, \alpha_2}^{(M(2^-), \lambda)}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \phi_{s_1, f_1, s_2, f_2}^{(M(2^-), \lambda)}(\vec{p})$$

$$= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} (-1) \phi_{f_1, f_2}^{(M(2^-), \lambda)}(|\vec{p}|) \sum_m (1/2, 1/2, 1; s_1, s_2, \lambda - m) (2, 1, 2; m, \lambda - m, \lambda)Y_{2m}(\vec{p})$$

All phases have been introduced for later convenience.

To evaluate the coordinate space wave function we use five different inter-quark interactions, one suggested by Bhaduri and collaborators [34], and four others suggested by Silvestre-Brac and Semay [35, 36]. All of them contain a confinement term, plus Coulomb and hyperfine terms coming from one-gluon exchange, and differ from one another in the form factors used for the hyperfine terms, the power of the confining term or the use of a form factor in the one gluon exchange Coupled potential. All free parameters in the potentials had been adjusted to reproduce the light ($\pi$, $\rho$, $K$, $K^*$, etc.) and heavy-light ($D$, $D^*$, $B$, $B^*$, etc.) meson spectra. These potentials also lead to good results for the charmed and bottom baryon ($\Lambda_{c,b}$, $\Sigma_{c,b}$, $\Sigma^*_{c,b}$, $\Xi_{c,b}$, $\Xi^*_{c,b}$, $\Omega_{c,b}$ and $\Omega^*_{c,b}$) masses [35, 36], for the semileptonic $\Lambda_b^0 \to \Lambda^+_c l^-\bar{\nu}_l$ and $\Xi_b^0 \to \Xi^+_c l^-\bar{\nu}_l$ decays [36], for the decay constants of pseudoscalar $B, D$ and vector $B^*, D^*$ mesons and the semileptonic $B \to D$ and $B \to D^*$ decays [36], for the $B \to \pi$ semileptonic decay [36], and for the strong $\Sigma_c \to \Lambda_c\pi$, $\Sigma^*_c \to \Lambda_c\pi$ and $\Xi^*_c \to \Xi_c\pi$ decays [36]. Preliminary results for the spectrum of doubly heavy baryons in Ref. 37 show excellent agreement with previous Faddeev calculations and lattice results. For more details on the inter-quark interactions see Ref. 37 or the original works 34, 57, 36.

The use of different inter-quark interactions will provide us with a spread in the results that we will consider, and quote, as a theoretical error added to the value obtained with the AL1 potential or Refs. 35, 36 that we will use to get our central results. Another source of theoretical uncertainty, that we can not account for, is the use of nonrelativistic kinematics in the evaluation of the orbital wave functions and the construction of our states in Eq. (1) above. While this is a very good approximation for the $B_c$ itself it is not that good for mesons with a light quark. That notwithstanding note that any nonrelativistic quark model has free parameters in the inter-quark interaction that are fitted to experimental data. In that sense we think that at least part of the ignored relativistic effects are included in an effective way in their fitted values.

III. SEMILEPTONIC $B_c^- \to c\bar{c}$ DECAYS

In this section we will consider the semileptonic decay of the $B_c^-$ meson into different $c\bar{c}$ states with $0^+$, $0^-$, $1^+$, $1^-$, $2^+$ and $2^-$ spin–parity quantum numbers. Those decays correspond to a $b \to c$ transition at the quark level which is governed by the current

$$J^{c\bar{c}}_\mu(0) = J^{c\bar{c}}_V(0) - J^{c\bar{c}}_A(0) = \overline{\Psi}_c(0)\gamma_\mu(1 - \gamma_5)\Psi_b(0)$$

with $\Psi_f$ a quark field of a definite flavor $f$.

A. Form factor decomposition of hadronic matrix elements

The hadronic matrix elements involved in these processes can be parametrized in terms of a few form factors as

$$\langle c\bar{c}(0^-), \vec{P}_{c\bar{c}} | J^{c\bar{c}}_\mu(0) | B_c, \vec{P}_{B_c} \rangle = \langle c\bar{c}(0^-), \vec{P}_{c\bar{c}} | J^{c\bar{c}}_V(0) | B_c, \vec{P}_{B_c} \rangle = P_\mu F_+(q^2) + q_\mu F_-(q^2)$$

$$\langle c\bar{c}(1^-), \lambda \vec{P}_{c\bar{c}} | J^{c\bar{c}}_\mu(0) | B_c, \vec{P}_{B_c} \rangle = \langle c\bar{c}(1^-), \lambda \vec{P}_{c\bar{c}} | J^{c\bar{c}}_V(0) - J^{c\bar{c}}_A(0) | B_c, \vec{P}_{B_c} \rangle$$

$$= \frac{-1}{m_{B_c} + m_{\bar{c}\bar{c}}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\alpha\beta}_{(\lambda)} (\vec{P}_{c\bar{c}}) P^\mu q^\nu V(q^2)$$

$$- i \left\{ (m_{B_c} - m_{\bar{c}\bar{c}}) \varepsilon^{\alpha\beta}_{(\lambda)} (\vec{P}_{c\bar{c}}) A_0(q^2) \right\}$$
\[ \left\{ \frac{P \cdot \varepsilon_{(\lambda)}(\vec{P}_{c\bar{c}})}{m_{B_c} + m_{c\bar{c}}} \left( P_\mu A_+(q^2) + q_\mu A_-(q^2) \right) \right\} \\
\left\{ \langle c\bar{c}(2^+), \lambda \vec{P}_{c\bar{c}} | J_{\mu}^{-}(0) | B_c, \vec{P}_{B_c} \rangle + \langle c\bar{c}(2^+) \lambda \vec{P}_{c\bar{c}} \right| J_{\mu}^{+}(0) \left| B_c, \vec{P}_{B_c} \rangle \\
= \varepsilon_{\mu \nu \alpha \beta} \varepsilon_{(\lambda)}^{(\nu) \ast} \langle \vec{P}_{c\bar{c}} \rangle P_\beta P_\delta q^\beta T_4(q^2) \\
- i \left\{ \varepsilon_{(\lambda) \mu \delta} \langle \vec{P}_{c\bar{c}} \rangle P_\delta T_1(q^2) \\
+ P_\nu P_\delta \varepsilon_{(\lambda) \mu \delta} \langle \vec{P}_{c\bar{c}} \rangle (P_\mu T_2(q^2) + q_\mu T_3(q^2)) \right\} \right\} \tag{9} \]

In the above expressions \( P = P_{B_c} + P_{c\bar{c}}, q = P_{B_c} - P_{c\bar{c}}, \) being \( P_{B_c} \) and \( P_{c\bar{c}} \) the meson four–momenta, \( m_{B_c} \) and \( m_{c\bar{c}} \) are the meson masses, \( \varepsilon_{\mu \nu \alpha \beta} \) is the fully antisymmetric tensor for which we take the convention \( \varepsilon^{0123} = +1, \) and \\
\( \varepsilon_{(\lambda) \mu}(\vec{P}) \) and \( \varepsilon_{(\lambda) \mu \nu}(\vec{P}) \) are the polarization vector and tensor of vector and tensor mesons respectively.² The latter can be evaluated in terms of the former as

\[ \varepsilon_{\mu(\lambda)}(\vec{P}) = \sum_m (1, 1, 2 ; m, \lambda - m, \lambda) \varepsilon_{\nu(\lambda)}(\vec{P}) \varepsilon_{\nu(\lambda)-m}(\vec{P}) \tag{10} \]

Different sets of \( \varepsilon_{(\lambda)}(\vec{P}) \) used in this work appear in appendix \( \text{A} \)

Besides the meson states in the Lorenz decompositions of Eq. 9 are normalized such that

\[ \left\langle M, \lambda' | \vec{P}' \rangle \left| M, \lambda \vec{P} \right. \right\rangle = \delta_{\lambda', \lambda} (2\pi)^3 2 E_M(\vec{P}) \delta(\vec{P}' - \vec{P}) \tag{11} \]

being \( E_M(\vec{P}) \) the energy of the \( M \) meson with three–momentum \( \vec{P}. \) Note the factor \( 2E_M \) of difference with Eq. 4

For the \( 0^+, 1^+ \) and \( 2^- \) cases the form factor decomposition is the same as for the \( 0^-, 1^- \) and \( 2^+ \) cases respectively, but with \( -J_{\mu}^{+}(0) \) contributing where \( J_{\mu}^{-}(0) \) contributed before and vice versa.

The different form factors in Eq. (9) are all relatively real thanks to time–reversal invariance. \( F_+, F_-, V, A_0, A_+, A_- \) and \( T_1 \) are dimensionless, whereas \( T_2, T_3 \) and \( T_4 \) have dimension of \( E^{-2}. \) They can be easily evaluated working in the center of mass of the \( B_c \) meson and taking \( \vec{q} \) in the \( z \) direction, so that \( \vec{P}_{c\bar{c}} = -\vec{q} = |\vec{q}|\vec{k}, \) with \( \vec{k} \) representing the unit vector in the \( z \) direction.

1. \( B_c^- \rightarrow \eta_c \bar{l}^\prime \bar{l}_t, \chi_{c0} \bar{l}^\prime \bar{l}_t \) decays

Let us start with the \( B_c^- \) decays into pseudoscalar \( \eta_c \) and scalar \( \chi_{c0} \) \( c\bar{c} \) mesons. For \( B_c^- \rightarrow \eta_c \) transitions the form factors are given by:

\[ F_+(q^2) = \frac{1}{2m_{B_c}} \left( V^0(|\vec{q}|) + \frac{V^3(|\vec{q}|)}{|\vec{q}|} \left( E_{\eta_c}(-\vec{q}) - m_{B_c} \right) \right) \]

\[ F_-(q^2) = \frac{1}{2m_{B_c}} \left( V^0(|\vec{q}|) + \frac{V^3(|\vec{q}|)}{|\vec{q}|} \left( E_{\eta_c}(-\vec{q}) + m_{B_c} \right) \right) \tag{12} \]

whereas for \( B_c^- \rightarrow \chi_{c0} \) transitions we have:

\[ F_+(q^2) = -\frac{1}{2m_{B_c}} \left( A^0(|\vec{q}|) + \frac{A^3(|\vec{q}|)}{|\vec{q}|} \left( E_{\chi_{c0}}(-\vec{q}) - m_{B_c} \right) \right) \]

\[ F_-(q^2) = -\frac{1}{2m_{B_c}} \left( A^0(|\vec{q}|) + \frac{A^3(|\vec{q}|)}{|\vec{q}|} \left( E_{\chi_{c0}}(-\vec{q}) + m_{B_c} \right) \right) \tag{13} \]

with \( V^\mu(|\vec{q}|) \) and \( A^\mu(|\vec{q}|) \) \( (\mu = 0, 3) \) calculated in our model as

\[ V^\mu(|\vec{q}|) = \left\langle \eta_c, -|\vec{q}| \vec{k} \right| J_{\mu}^{c\bar{c}}(0) \left| B_c^-, 0 \right. \right\rangle = \frac{2m_{B_c} 2E_{\eta_c}(-\vec{q})}{\sqrt{2m_{B_c} 2E_{\eta_c}(-\vec{q})}} \left\langle \eta_c, -|\vec{q}| \vec{k} \right| J_{\mu}^{c\bar{c}}(0) \left| B_c^-, 0 \right. \right\rangle \]

\[ A^\mu(|\vec{q}|) = \left\langle \chi_{c0}, -|\vec{q}| \vec{k} \right| J_{\mu}^{A}(0) \left| B_c^-, 0 \right. \right\rangle = \frac{2m_{B_c} 2E_{\chi_{c0}}(-\vec{q})}{\sqrt{2m_{B_c} 2E_{\chi_{c0}}(-\vec{q})}} \left\langle \chi_{c0}, -|\vec{q}| \vec{k} \right| J_{\mu}^{A}(0) \left| B_c^-, 0 \right. \right\rangle \tag{14} \]

² Note we have taken \( \lambda \) to be the third component of the meson spin measured in the meson center of mass.
which expressions are given in appendix [35].

In Fig. 1 we show our results for the $F_+$ and $F_-$ form factors for the semileptonic $B_c^- \rightarrow \eta_c$, $\chi_{c0}$ transitions. The minimum $q^2$ value depends on the actual final lepton and it is given, neglecting neutrino masses, by the lepton mass as $q_{\text{min}}^2 = m_l^2$. The form factors have been evaluated using the AL1 potential of Refs. [35, 36]. For decays into $\eta_c$, and for the sake of comparison, we also show the results obtained with the potential developed by Bhaduri and collaborators in Ref. [34] (BHAD). As seen in the figures the differences between the form factors evaluated with the two inter-quark interactions are smaller than 10%.

In Table III we show $F_+$ and $F_-$ evaluated at $q_{\text{min}}^2$ and $q_{\text{max}}^2$ for a final light lepton ($l = e, \mu$) and compare them to the ones obtained by Ivanov et al. in Ref. [5], and, when available, by Ebert et al. in Ref. [7]. For the $B_c^- \rightarrow \eta_c$ transition we also show the corresponding values for the $F_0$ form factor defined as

$$F_0(q^2) = F_+(q^2) + \frac{q^2}{m_{B_c}^2 - m_{\eta_c}^2} F_-(q^2) \quad (15)$$

Our results for the $\eta_c$ case are in excellent agreement with the ones obtained by Ebert et al. Compared to the results by Ivanov et al. we find large discrepancies for $F_-$. 

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<th>$B_c^- \rightarrow \chi_{c0} l^- \bar{\nu}_l$</th>
<th>$q_{\text{min}}^2$</th>
<th>$q_{\text{max}}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_+$ This work</td>
<td>0.49$^{+0.01}_{-0.01}$</td>
<td>1.00$^{+0.01}_{-0.01}$</td>
<td></td>
<td>0.30$^{+0.01}_{-0.01}$</td>
<td>0.64$^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>[5]</td>
<td>0.61</td>
<td>1.14</td>
<td></td>
<td>0.40</td>
<td>0.65</td>
</tr>
<tr>
<td>$F_-$ This work</td>
<td>0.47</td>
<td>1.07</td>
<td></td>
<td>0.32</td>
<td>0.61</td>
</tr>
<tr>
<td>[5]</td>
<td>-0.11$^{+0.01}_{-0.01}$</td>
<td>-0.25$^{+0.03}_{-0.01}$</td>
<td></td>
<td>-0.62$^{+0.01}_{-0.03}$</td>
<td>-1.35$^{+0.05}_{-0.01}$</td>
</tr>
<tr>
<td>$F_0$ This work</td>
<td>0.49$^{+0.01}_{-0.01}$</td>
<td>0.91$^{+0.01}_{-0.01}$</td>
<td></td>
<td>0.47</td>
<td>0.92</td>
</tr>
<tr>
<td>[7]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE III: $F_+$ and $F_-$ evaluated at $q_{\text{min}}^2$ and $q_{\text{max}}^2$ compared to the ones obtained by Ivanov et al. [5] and Ebert et al. Ref. [7]. Our central values have been obtained with the AL1 potential. For the $\eta_c$ channel we also show $F_0$ (see text for definition). Here $l$ stands for $l = e, \mu$. 

FIG. 1: $F_+$ and $F_-$ form factors for $B_c^- \rightarrow \eta_c$ and $B_c^- \rightarrow \chi_{c0}$ semileptonic decay evaluated with the AL1 potential of Refs. [35, 36]. In the first case, and for comparison, we also show with dotted lines the results obtained with the Bhaduri (BHAD) potential of Ref. [34].
Let us now see the form factors for the semileptonic $B_c^-$ decays into vector $J/\Psi$ and axial vector $h_c$ ($S_{q\bar{q}} = 0$) and $\chi_{c1}$ ($S_{q\bar{q}} = 1$) $c\bar{c}$ mesons. For the decay into $J/\Psi$ the form factors can be evaluated in terms of matrix elements as:

$$V(q^2) = \frac{i}{\sqrt{2}} \frac{m_{B_c} + m_{J/\Psi}}{m_{B_c} |q|} V_{\lambda=-1}(|\vec{q}|)$$

$$A_+(q^2) = \frac{i}{2m_{B_c}} \frac{m_{J/\Psi}}{|\vec{q}| m_{B_c}} \left\{ -A_{\lambda=0}^3(|\vec{q}|) + \frac{m_{B_c} - E_{J/\Psi}(-\vec{q})}{|\vec{q}|} A_{\lambda=0}^3(|\vec{q}|) \right\}$$

$$A_-(q^2) = \frac{i}{2m_{B_c}} \frac{m_{J/\Psi}}{|\vec{q}| m_{B_c}} \left\{ A_{\lambda=0}^3(|\vec{q}|) + \frac{m_{B_c} + E_{J/\Psi}(-\vec{q})}{|\vec{q}|} A_{\lambda=0}^3(|\vec{q}|) \right\}$$

$$A_0(q^2) = -i\sqrt{2} \frac{1}{m_{B_c} - m_{J/\Psi}} A_{\lambda=-1}^3(|\vec{q}|)$$

(16)

with $V_\lambda^\mu(|\vec{q}|)$ and $A_\lambda^\mu(|\vec{q}|)$ calculated in our model as

$$V_\lambda^\mu(|\vec{q}|) = \langle J/\Psi, \lambda - |\vec{q}| \kappa \mid J_{\nu}^{c b \mu}(0) \mid B_c^-, \bar{0} \rangle$$

$$A_\lambda^\mu(|\vec{q}|) = \langle J/\Psi, \lambda - |\vec{q}| \kappa \mid J_{\nu}^{c b \mu}(0) \mid B_c^-, \bar{0} \rangle$$

(17)

which expressions are given in appendix H.

The form factors corresponding to transitions to the $\chi_{c1}$ and $h_c$ axial vector mesons are obtained from the expressions in Eq. (16) by just changing

$$V_\lambda^\mu(|\vec{q}|) \leftrightarrow -A_\lambda^\mu(|\vec{q}|)$$

(18)

and using the appropriate mass for the final meson. Obviously in Eq. (17) $J/\Psi$ has to be replaced by $\chi_{c1}$ or $h_c$.

In Table XIV we show the result for the different form factors evaluated at $q^2_{min}$ and $q^2_{max}$ for the case where the final lepton is light ($l = e, \mu$). For the decay into $J/\Psi$ we also show the combination of form factors:

$$\tilde{A}_0(q^2) = \frac{m_{B_c} - m_{J/\Psi}}{2m_{J/\Psi}} (A_0(q^2) - A_+(q^2)) - \frac{q^2}{2m_{J/\Psi} (m_{B_c} + m_{J/\Psi})} A_-(q^2)$$

(19)

Our results for the $B_c \rightarrow J/\Psi$ decay channel are in agreement with the ones obtained by Ebert et al. They also agree reasonably well, with the exception of $A_-$, with the ones obtained by Ivanov et al. For the other two cases the discrepancies are in general large.

All the form factors are depicted in Figs. 2 and 3.

---

3 This combination is called $A_0$ by the authors of Ref. 7.
\[
\begin{array}{|c|cc|cc|}
\hline
B_c^- \rightarrow J/\Psi \ l^- \ \bar{\nu}_l & q_{\text{min}}^2 & q_{\text{max}}^2 & B_c^- \rightarrow h_\ell^- \ \bar{\nu}_l & q_{\text{min}}^2 & q_{\text{max}}^2 \\
\hline
V & -0.61_{-0.03}^{+0.01} & -1.26_{-1.34}^{+0.01} & V & -0.040_{-0.003}^{+0.003} & -0.078_{-0.003}^{+0.003} \\
[5] & -0.83^* & -1.34 & \text{This work} & -0.25^* & -0.365^* \\
[7] & -0.49 & -1.34 & [5] & -0.25^* & -0.365^* \\
\hline
A_+ & 0.50_{-0.03}^{+0.03} & 1.31_{-1.30}^{+0.03} & A_+ & -0.85_{-0.05}^{+0.01} & -1.90_{-1.80}^{+0.06} \\
\text{This work} & 0.54 & 0.97 & [5] & 0.54 & 0.97 \\
[5] & 0.73 & 1.33 & [5] & 0.73 & 1.33 \\
\hline
A_- & 0.60_{-0.03}^{+0.03} & 1.24_{-0.16}^{+0.01} & A_- & 0.12_{-0.02}^{+0.01} & 0.36_{-0.06}^{+0.06} \\
\text{This work} & -0.95 & -1.76 & [5] & 0.12_{-0.02}^{+0.01} & 0.36_{-0.06}^{+0.06} \\
[5] & -0.95 & -1.76 & [5] & 0.12_{-0.02}^{+0.01} & 0.36_{-0.06}^{+0.06} \\
\hline
A_0 & 1.44_{-0.08}^{+0.08} & 2.58_{-0.02}^{+0.01} & A_0 & 0.28_{-0.02}^{+0.02} & 0.52_{-0.02}^{+0.02} \\
\text{This work} & 1.64 & 2.50 & [5] & 0.28_{-0.02}^{+0.02} & 0.52_{-0.02}^{+0.02} \\
[5] & 1.64 & 2.50 & [5] & 0.28_{-0.02}^{+0.02} & 0.52_{-0.02}^{+0.02} \\
\hline
\tilde{A}_0 & 0.45_{-0.01}^{+0.03} & 0.96_{-0.01}^{+0.03} & \text{This work} & 0.44 & 0.54 \\
[7] & 0.44 & 0.54 & [7] & 0.44 & 0.54 \\
\hline
\end{array}
\]

TABLE IV: \( V, A_+, A_- \) and \( A_0 \) form factors evaluated at \( q_{\text{min}}^2 \) and \( q_{\text{max}}^2 \) compared to the ones obtained by Ivanov et al. \[5\] and Ebert et al. Ref. \[7\]. Our central values have been evaluated with the AL1 potential. For the \( J/\Psi \) channel we also show \( \tilde{A}_0 \) (see text for definition). Here \( l \) stands for \( l = e, \mu \). The asterisk to the right of a number means we have changed its sign to account for the different choice of \( \epsilon \).
FIG. 3: \( V \) (solid line), \( A_+ \) (dashed line), \( A_- \) (dotted line) and \( A_0 \) (dashed-dotted line) form factors for \( B_c^- \to h_c \) and \( B_c^- \to \chi_c1 \) semileptonic decay evaluated with the AL1 potential.

3. \( B_c^- \to \Psi(3836) l \bar{\nu}_l, \chi_{c2} l \bar{\nu}_l, \) decays

Finally let us see the form factors for the \( B_c^- \to \Psi(3836) l \bar{\nu}_l, \chi_{c2} l \bar{\nu}_l, \) decays into tensor \( \chi_{c2} \) and pseudotensor \( \Psi(3836) \) mesons. For the decay into \( \chi_{c2} \) the form factors can be evaluated in terms of matrix elements as:

\[
\begin{align*}
T_1(q^2) &= -i \frac{2m_{\chi_{c2}}}{m_{B_c}} |q| V^1_{T\lambda=+1}(|q|) \\
T_2(q^2) &= i \frac{1}{2m^{3}_{B_c}} \left\{ - \sqrt{3} \frac{m_{\chi_{c2}}^2}{|q|^2} T^0_{T\lambda=0}(|q|) - \sqrt{3} \frac{m_{\chi_{c2}}^2}{|q|^2} (E_{\chi_{c2}} - m_{B_c}) A^3_{T\lambda=0}(|q|) \\
&\quad + 2m_{\chi_{c2}} \frac{1}{|q|} \left( 1 - \frac{E_{\chi_{c2}} - m_{B_c}}{|q|^2} \right) T^1_{T\lambda=+1}(|q|) \right\} \\
T_3(q^2) &= i \frac{1}{2m^{3}_{B_c}} \left\{ - \sqrt{3} \frac{m_{\chi_{c2}}^2}{|q|^2} T^0_{T\lambda=0}(|q|) - \sqrt{3} \frac{m_{\chi_{c2}}^2}{|q|^2} (E_{\chi_{c2}} - m_{B_c}) A^3_{T\lambda=0}(|q|) \\
&\quad + 2m_{\chi_{c2}} \frac{1}{|q|} \left( 1 - \frac{E_{\chi_{c2}} - m_{B_c}}{|q|^2} \right) T^1_{T\lambda=+1}(|q|) \right\} \\
T_4(q^2) &= i \frac{m_{\chi_{c2}}}{m_{B_c}^2 |q|^2} V^1_{T\lambda=+1}(|q|)
\end{align*}
\]

with \( V^\mu_{T\lambda}(|q|) \) and \( A^\mu_{T\lambda}(|q|) \) calculated in our model as:

\[
\begin{align*}
V^\mu_{T\lambda}(|q|) &= \left\langle \chi_{c2}, \lambda - |q| \bar{k} \left| J_V^{c\mu} (0) \right| B_c^-, \bar{0} \right\rangle \\
&= \sqrt{2m_{B_c} E_{\chi_{c2}} (-q)} \left\langle \chi_{c2}, \lambda - |q| \bar{k} \left| J_V^{c\mu} (0) \right| B_c^-, \bar{0} \right\rangle_{NR} \\
A^\mu_{T\lambda}(|q|) &= \left\langle \chi_{c2}, \lambda - |q| \bar{k} \left| J_A^{c\mu} (0) \right| B_c^-, \bar{0} \right\rangle \\
&= \sqrt{2m_{B_c} E_{\chi_{c2}} (-q)} \left\langle \chi_{c2}, \lambda - |q| \bar{k} \left| J_A^{c\mu} (0) \right| B_c^-, \bar{0} \right\rangle_{NR}
\end{align*}
\]

Note that while the \( \Psi(3836) \) was still quoted in the particle listings of the former Review of Particle Physics [25], it has been excluded from the more recent one [33]. We shall nevertheless keep it in our study to illustrate the results to be expected for a ground state pseudotensor particle.
which expressions are given in appendix B.

The form factors corresponding to transitions to $\Psi(3836)$ are obtained from the expressions in Eq. (20) by just changing

$$V_{T \lambda}^\mu(\vec{q^*}) \leftarrow -A_{T \lambda}^\mu(\vec{q^*})$$

and using the appropriate mass for the final meson. Besides in Eq. (21) $\chi_{c2}$ has to be replaced by $\Psi(3836)$.

The results for the different form factors appear in Fig. 4. In Table V we show $T_1$, $T_2$, $T_3$ and $T_4$ evaluated at $q_{\min}^2$ and $q_{\max}^2$ for the case of a final light lepton, and compare them to the values obtained by Ivanov et al. [5]. For the $B_c^- \to \chi_{c2}$ transition we find a reasonable agreement between the two calculations. For $B_c^- \to \Psi(3836)$ there is also a reasonable agreement for the absolute values of the form factors but we disagree for some of the signs.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$B_c^- \to \chi_{c2} l^- \bar{\nu}_l$ & $q_{\min}^2$ & $q_{\max}^2$ \\
\hline
$T_1$ & 0.97$^{+0.08}_{-0.01}$ & 1.95$^{+0.01}_{-0.06}$ \\
This work & 1.22 & 1.69 \\
[5] & & \\
$T_2$ [GeV$^{-2}$] & -0.012$^{+0.001}_{-0.001}$ & -0.025$^{+0.001}_{-0.001}$ \\
This work & -0.011 & -0.018 \\
[5] & & \\
$T_3$ [GeV$^{-2}$] & 0.013$^{+0.001}_{-0.001}$ & 0.030$^{+0.001}_{-0.001}$ \\
This work & 0.025 & 0.040 \\
[5] & & \\
$T_4$ [GeV$^{-2}$] & 0.013$^{+0.001}_{-0.001}$ & 0.030$^{+0.001}_{-0.001}$ \\
This work & 0.021$^*$ & 0.033$^*$ \\
[5] & & \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$B_c^- \to \Psi(3836) l^- \bar{\nu}_l$ & $q_{\min}^2$ & $q_{\max}^2$ \\
\hline
$T_1$ & 0.29$^{+0.02}_{-0.01}$ & 0.22$^{+0.01}_{-0.02}$ \\
This work & 0.052 & 0.35 \\
[5] & & \\
$T_2$ [GeV$^{-2}$] & -0.010$^{+0.006}_{-0.018}$ & -0.018$^{+0.006}_{-0.018}$ \\
This work & -0.0071 & 0.0090 \\
[5] & & \\
$T_3$ [GeV$^{-2}$] & 0.025$^{+0.001}_{-0.001}$ & 0.052$^{+0.001}_{-0.001}$ \\
This work & -0.036 & -0.052 \\
[5] & & \\
$T_4$ [GeV$^{-2}$] & 0.022$^{+0.001}_{-0.001}$ & 0.045$^{+0.003}_{-0.003}$ \\
This work & -0.026$^*$ & -0.038$^*$ \\
[5] & & \\
\hline
\end{tabular}
\end{table}

TABLE V: Values for $T_1$, $T_2$, $T_3$ and $T_4$ evaluated at $q_{\min}^2$ and $q_{\max}^2$ compared to the ones obtained by Ivanov et al. [5]. Here $l$ stands for $l = e, \mu$. Asterisk as in Table IV.
B. Decay width

For a \( B_c \) at rest the double differential decay width with respect to \( q^2 \) and \( x_l \), being \( x_l \) the cosine of the angle between the final meson momentum and the momentum of the final charged lepton measured in the lepton–neutrino center of mass frame (CMF), is given by\(^5\)

\[
\frac{d^2\Gamma}{dq^2dx_l} = \frac{G_F^2}{64m_{B_c}^3} \frac{|V_{bc}|^2}{8\pi^3} \lambda^{1/2}(q^2, m_{B_c}^2, m_{\ell c}^2) \frac{q^2 - m_l^2}{q^2} H_{\alpha\beta}(P_{B_c}, P_{\ell c}) \mathcal{L}_{\alpha\beta}(p_l, p_{\nu})
\]  

(23)

where \( G_F = 1.16637(1) \times 10^{-5}\)GeV\(^{-2}\) is the Fermi constant, \( \lambda(a, b, c) = (a + b - c)^2 - 4ab \), \( m_l \) is the mass of the charged lepton, \( H_{\alpha\beta} \) and \( \mathcal{L}_{\alpha\beta} \) are the hadron and lepton tensors, and \( P_{B_c}, P_{\ell c}, p_l, p_{\nu} \) are the meson and lepton four–momenta. The lepton tensor is\(^6\)

\[
\mathcal{L}_{\alpha\beta}(p_l, p_{\nu}) = 8 \left( p_l^\alpha p_{\nu}^\beta + p_l^\beta p_{\nu}^\alpha - g_{\alpha\beta} p_l \cdot p_{\nu} + i \varepsilon_{\alpha\beta\sigma} p_l^\sigma p_{\nu}\right)
\]  

(24)

As for the hadron tensor it is given by

\[
H_{\alpha\beta}(P_{B_c}, P_{\ell c}) = \sum_\lambda h_{(\lambda)}(P_{B_c}, P_{\ell c}) h^*_{(\lambda)}(P_{B_c}, P_{\ell c})
\]  

(25)

with

\[
h_{(\lambda)}(P_{B_c}, P_{\ell c}) = \left\langle c\bar{c}, \lambda \bar{F}_{\ell c}, J_{\alpha}^{b}(0)|B_c, \bar{P}_{B_c}\right\rangle
\]  

(26)

The quantity

\[
H_{\alpha\beta}(P_{B_c}, P_{\ell c}) \mathcal{L}_{\alpha\beta}(p_l, p_{\nu})
\]  

(27)

is a scalar and to evaluate it we can choose \( \bar{F}_{\ell c} \) along the negative \( z \)-axis. This implies also that the CMF of the final leptons moves in the positive \( z \)-direction. Furthermore we shall follow Ref. [5] and introduce helicity components for the hadron and lepton tensors. For that purpose we rewrite

\[
H_{\alpha\beta}(P_{B_c}, P_{\ell c}) \mathcal{L}_{\alpha\beta}(p_l, p_{\nu}) = \mathcal{H}^{\alpha\beta}(P_{B_c}, P_{\ell c}) g_{\alpha\beta} \mathcal{L}_{\alpha\beta}(p_l, p_{\nu})
\]  

(28)

and use\(^4\)

\[
g_{\mu\nu} = \sum_{r=t,\pm 1,0} g_{rr} \varepsilon^{(r)}_\mu(q) \varepsilon^{(r)}_\nu(q) \quad ; \quad g_{tt} = 1, g_{\pm 1, 0} = -1
\]  

(29)

with \( \varepsilon^{(r)}_\mu(q) = q^\mu/q^2 \) and where the \( \varepsilon^{(r)}_\mu(q) \), \( r = \pm 1, 0 \) are the polarization vectors for an on–shell vector particle with four–momentum \( q \) and helicity \( r \). Defining helicity components for the hadron and lepton tensors as

\[
\mathcal{H}_{\epsilon\epsilon}(P_{B_c}, P_{\ell c}) = \varepsilon^{(r)}_\epsilon(q) \mathcal{H}^{\alpha\beta}(P_{B_c}, P_{\ell c}) \varepsilon^{(s)}_\beta(q)
\]

\[
\mathcal{L}_{\epsilon\epsilon}(p_l, p_{\nu}) = \varepsilon^{(r)}_\epsilon(q) \mathcal{L}_{\alpha\beta}(p_l, p_{\nu}) \varepsilon^{(s)}_\beta(q)
\]  

(30)

we have that

\[
H_{\alpha\beta}(P_{B_c}, P_{\ell c}) \mathcal{L}_{\alpha\beta}(p_l, p_{\nu}) = \sum_{r=t,\pm 1,0} \sum_{s=t,\pm 1,0} g_{rr} g_{ss} \mathcal{H}_{\epsilon\epsilon}(P_{B_c}, P_{\ell c}) \mathcal{L}_{\epsilon\epsilon}(p_l, p_{\nu})
\]  

(31)

Let us start with the lepton tensor. We can take advantage of the fact that the Wigner rotation relating the original frame and the CMF of the final leptons is the identity to evaluate the lepton tensor helicity components in this latter reference system

\[
\mathcal{L}_{\epsilon\epsilon}(p_l, p_{\nu}) = \varepsilon^{(r)}_\epsilon(q) \mathcal{L}_{\alpha\beta}(p_l, p_{\nu}) \varepsilon^{(s)}_\beta(q) = \varepsilon^{(r)}_\epsilon(q) \mathcal{L}_{\alpha\beta}(p_l, p_{\nu}) \varepsilon^{(s)}_\beta(q)
\]  

(32)

---

\(^5\) We shall neglect neutrino masses in the calculation.

\(^6\) The \( \mp \) signs correspond respectively to decays into \( l^-\bar{\nu}_l \) (for \( B_c^- \) decays) and \( l^+\nu_l \) (for \( B_c^+ \) decays).
were the tilde stands for momenta measured in the final lepton’s CMF. For the purpose of evaluation we can use
\[
\vec{p}_l^\alpha = (E_l(|\vec{p}_l|), -|\vec{p}_l| \sqrt{1 - x_l^2}, 0, -|\vec{p}_l| x_l)
\]
\[
\vec{p}_\nu^\alpha = (|\vec{p}_\nu|, |\vec{p}_\nu| \sqrt{1 - x_\nu^2}, 0, |\vec{p}_\nu| x_\nu)
\]
with $|\vec{p}_l|$ the modulus of the lepton three-momentum measured in the leptons CMF.

The only helicity components that we shall need are the following.
\[
L_{1+}(p_l, p_\nu) = L_{1-}(p_l, p_\nu) = -4x_l \frac{m_1^2(q^2 - m_1^2)}{q^2}
\]
\[
L_{1-1}(p_l, p_\nu) = (q^2 - m_1^2) \left( 4(1 \pm x_l) - 2(1 - x_l^2) \frac{q^2 - m_1^2}{q^2} \right)
\]
\[
L_{00}(p_l, p_\nu) = 4(q^2 - m_1^2) \left( 1 - x_l^2 \frac{q^2 - m_1^2}{q^2} \right)
\]
(33)

As for the hadron tensor it is convenient to introduce helicity amplitudes defined as
\[
h_{(\lambda)} r(P_{Bc}, P_{c\bar{c}}) = \varepsilon^*_{(r)}(q) h_{(\lambda)}^r(P_{Bc}, P_{c\bar{c}}) \quad , \quad r = t, \pm 1, 0
\]
in terms of which
\[
\mathcal{H}_{rs}(P_{Bc}, P_{c\bar{c}}) = \sum_{\lambda} h_{(\lambda)} r(P_{Bc}, P_{c\bar{c}}) h_{(\lambda)* s}(P_{Bc}, P_{c\bar{c}})
\]
(36)

We now give the expressions for the helicity amplitudes evaluated in the original frame.

- Case $0^- \rightarrow 0^-, 0^+$. 
\[
h_t(P_{Bc}, P_{c\bar{c}}) = \frac{m_{Bc}^2 - m_{c\bar{c}}^2}{\sqrt{q^2}} F_+(q^2) + \sqrt{q^2} F_-(q^2)
\]
\[
h_0(P_{Bc}, P_{c\bar{c}}) = \frac{\lambda^{1/2}(q^2, m_{Bc}^2, m_{c\bar{c}}^2)}{\sqrt{q^2}} F_+(q^2)
\]
\[
h_{+1}(P_{Bc}, P_{c\bar{c}}) = h_{-1}(P_{Bc}, P_{c\bar{c}}) = 0
\]
(37)

- Case $0^- \rightarrow 1^-, 1^+$. 
\[
h_{(\lambda)t}(P_{Bc}, P_{c\bar{c}}) = i \delta_{\lambda 0} \frac{\lambda^{1/2}(q^2, m_{Bc}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}} \sqrt{q^2}} \left( (m_{Bc} - m_{c\bar{c}}) (A_0(q^2) - A_+(q^2)) - \frac{q^2}{m_{Bc} + m_{c\bar{c}}} A_-(q^2) \right)
\]
\[
h_{(\lambda)+1}(P_{Bc}, P_{c\bar{c}}) = -i \delta_{\lambda -1} \left( \frac{\lambda^{1/2}(q^2, m_{Bc}^2, m_{c\bar{c}}^2)}{m_{Bc} + m_{c\bar{c}}} V(q^2) + (m_{Bc} - m_{c\bar{c}}) A_0(q^2) \right)
\]
\[
h_{(\lambda)-1}(P_{Bc}, P_{c\bar{c}}) = -i \delta_{\lambda +1} \left( \frac{\lambda^{1/2}(q^2, m_{Bc}^2, m_{c\bar{c}}^2)}{m_{Bc} + m_{c\bar{c}}} V(q^2) + (m_{Bc} - m_{c\bar{c}}) A_0(q^2) \right)
\]
\[
h_{(\lambda)0}(P_{Bc}, P_{c\bar{c}}) = i \delta_{\lambda 0} \left( (m_{Bc} - m_{c\bar{c}}) \frac{m_{Bc}^2 - q^2 - m_{c\bar{c}}^2}{2m_{c\bar{c}} \sqrt{q^2}} A_0(q^2) - \frac{\lambda(q^2, m_{Bc}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}} \sqrt{q^2}} \frac{A_+(q^2)}{m_{Bc} + m_{c\bar{c}}} \right)
\]
(38)

---

7 Note this is in accordance with the definition of $x_l$ and the fact that we have taken $\vec{P}_{c\bar{c}}$ in the negative $z$ direction. Furthermore there can be no dependence on the $\varphi_l$ azimuthal angle so we can take $\vec{P}_l$, and then $\vec{P}_\nu$, in the OZX plane.
- Case $0^- \to 2^-, 2^+$.  

\[
\begin{align*}
  h_{(\lambda)1}(P_{Bc}, P_{\bar{c}e}) & = -i\delta\lambda_0 \sqrt{\frac{2}{3} \frac{\lambda(q^2, m_{Bc}^2, m_{\bar{c}e}^2)}{4m_{\bar{c}e}^2 \sqrt{q^2}}} \left( T_1(q^2) + (m_{Bc}^2 - m_{\bar{c}e}^2) T_2(q^2) + q^2 T_3(q^2) \right) \\
  h_{(\lambda)+1}(P_{Bc}, P_{\bar{c}e}) & = i\delta\lambda - \frac{1}{\sqrt{2}} \frac{\lambda^{1/2}(q^2, m_{Bc}^2, m_{\bar{c}e}^2)}{2m_{\bar{c}e}} \left( T_1(q^2) - \lambda^{1/2}(q^2, m_{Bc}^2, m_{\bar{c}e}^2) T_4(q^2) \right) \\
  h_{(\lambda)-1}(P_{Bc}, P_{\bar{c}e}) & = i\delta\lambda + \frac{1}{\sqrt{2}} \frac{\lambda^{1/2}(q^2, m_{Bc}^2, m_{\bar{c}e}^2)}{2m_{\bar{c}e}} \left( T_1(q^2) + \lambda^{1/2}(q^2, m_{Bc}^2, m_{\bar{c}e}^2) T_4(q^2) \right) \\
  h_{(\lambda)0}(P_{Bc}, P_{\bar{c}e}) & = -i\delta\lambda_0 \sqrt{\frac{2}{3} \frac{\lambda(q^2, m_{Bc}^2, m_{\bar{c}e}^2)}{4m_{\bar{c}e}^2 \sqrt{q^2}}} \left( (m_{Bc}^2 - q^2 - m_{\bar{c}e}^2) T_1(q^2) + \lambda(q^2, m_{Bc}^2, m_{\bar{c}e}^2) T_2(q^2) \right)
\end{align*}
\]  

(39)

We see that the helicity amplitudes, and thus the helicity components of the hadron tensor, only depend on $q^2$. The expressions for the latter are collected in appendix D.

We can now define the combinations

\begin{align*}
  H_U & = \mathcal{H}_{+1+1} + \mathcal{H}_{-1-1} \\
  H_L & = \mathcal{H}_{00} \\
  H_P & = \mathcal{H}_{+1+1} - \mathcal{H}_{-1-1} \\
  H_S & = 3\mathcal{H}_{tt}
\end{align*}


Finally the double differential decay width is written in terms of the above defined combinations as

\[
\frac{d^2\Gamma}{dq^2 dx_1} = \frac{G_F^2}{8\pi^3} |V_{bc}|^2 \left( \frac{q^2 - m_l^2}{12m_{B_c}^2 q^2} \right) \lambda^{1/2}(q^2, m_{B_c}^2, m_{\bar{c}e}^2) \left\{ \frac{3}{8} (1 + x_1^2) H_U + \frac{3}{4} (1 - x_1^2) H_L + \frac{3}{4} x_1 H_P \right. \\
+ \left. \frac{3}{4} (1 - x_1^2) \bar{H}_U + \frac{3}{2} x_1 \bar{H}_L + \frac{1}{2} \bar{H}_S + 3x_1 \bar{H}_{SL} \right\}
\]

(41)

Note that for antiparticle decay $H_P$ has the opposite sign to the case of particle decay while all other hadron tensor helicity components combinations defined in Eq. (40) do not change (See appendix C for details). The sign change of $H_P$ compensates the extra sign coming from the lepton tensor. This means that in fact the double differential decay width is the same for $B_c^-$ or $B_c^{+}$ decay.

Integrating over $x_1$ we obtain the differential decay width

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2}{8\pi^3} |V_{bc}|^2 \left( \frac{q^2 - m_l^2}{12m_{B_c}^2 q^2} \right) \lambda^{1/2}(q^2, m_{B_c}^2, m_{\bar{c}e}^2) \left\{ H_U + H_L + \bar{H}_U + \bar{H}_L + \bar{H}_S \right\}
\]

(42)

from where, integrating over $q^2$, we obtain the total decay width that we write, following Ref. $\mathbb{R}$, as

\[
\Gamma = \Gamma_U + \Gamma_L + \Gamma_U + \Gamma_L + \Gamma_S
\]

(43)

with $\Gamma_J$ and $\bar{\Gamma}_J$ partial helicity widths defined as

\[
\Gamma_J = \int dq^2 \frac{G_F^2}{8\pi^3} |V_{bc}|^2 \left( \frac{q^2 - m_l^2}{12m_{B_c}^2 q^2} \right) \lambda^{1/2}(q^2, m_{B_c}^2, m_{\bar{c}e}^2) H_J
\]

(44)

and similarly for $\bar{\Gamma}_J$ in terms of $\bar{H}_J$. 


Another quantity of interest is the forward-backward asymmetry of the charged lepton measured in the leptons CMF. This asymmetry is defined as

$$A_{FB} = \frac{\Gamma_{x_1 > 0} - \Gamma_{x_1 < 0}}{\Gamma_{x_1 > 0} + \Gamma_{x_1 < 0}}$$

and it is given in terms or partial helicity widths as

$$\frac{3}{4} \frac{\pm \Gamma_{\rho} + 4 \tilde{\Gamma}_{SL}}{\Gamma_U + \Gamma_L + \Gamma_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S}$$

being the same for a negative charged lepton $l^-$ ($B^-_c$ decay) than for a positive charged one $l^+$ ($B^+_c$ decay), as $\Gamma_{\rho}$ for antiparticle decay has the opposite sign as for particle decay.

Finally for the decay channel $B_c \to J/\Psi \ell\bar{\nu}_\ell$ with the $J/\Psi$ decaying into $\mu^-\mu^+$ we can evaluate the differential cross section

$$\frac{d\Gamma_{B_c \to \mu^-\mu^+(J/\Psi)\ell\bar{\nu}_\ell}}{dx_{\mu}} = \left(1 + \frac{\sqrt{1 - 4m^2_{\mu}/m^2_{J/\Psi}} - 2m^2_{J/\Psi}}{2 - 2\sqrt{1 - 4m^2_{\mu}/m^2_{J/\Psi}}} \right) \frac{\Gamma_{U} + \tilde{\Gamma}_{U}}{2} \frac{\Gamma_{L} + \tilde{\Gamma}_{L} + \tilde{\Gamma}_{S}}{\Gamma_{U} + \tilde{\Gamma}_{U} + \tilde{\Gamma}_{L} + \tilde{\Gamma}_{S}} \frac{\Gamma_{\rho}}{3\mu^2}$$

TABLE VI: Partial helicity widths in units of $10^{-15}$ GeV for $B_c^-$ decays. Central values have been evaluated with the AL1 potential.

<table>
<thead>
<tr>
<th>$B_c^- \to$</th>
<th>$\Gamma_U$</th>
<th>$\tilde{\Gamma}_U$</th>
<th>$\Gamma_L$</th>
<th>$\tilde{\Gamma}_L$</th>
<th>$\Gamma_{\rho}$</th>
<th>$\tilde{\Gamma}_S$</th>
<th>$\tilde{\Gamma}_{SL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c e^-\bar{\nu}_e$</td>
<td>0.00</td>
<td>0.13+0.01</td>
<td>10^{-5}</td>
<td>0.44+0.01</td>
<td>10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_c \mu^-\bar{\nu}_\mu$</td>
<td>0.00</td>
<td>0.28+0.02</td>
<td>10^{-1}</td>
<td>0.10+0.01</td>
<td>10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_c \tau^-\bar{\nu}_\tau$</td>
<td>0.00</td>
<td>0.17+0.01</td>
<td>10^{-1}</td>
<td>0.15+0.04</td>
<td>0.29+0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_{11} e^-\bar{\nu}_e$</td>
<td>0.00</td>
<td>0.03+0.02</td>
<td>10^{-1}</td>
<td>0.23+0.02</td>
<td>10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_{11} \mu^-\bar{\nu}_\mu$</td>
<td>0.00</td>
<td>0.71+0.02</td>
<td>10^{-1}</td>
<td>0.84+0.07</td>
<td>10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_{11} \tau^-\bar{\nu}_\tau$</td>
<td>0.00</td>
<td>0.50+0.02</td>
<td>10^{-1}</td>
<td>0.36+0.02</td>
<td>0.21+0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_{12} e^-\bar{\nu}_e$</td>
<td>0.00</td>
<td>0.03+0.02</td>
<td>10^{-1}</td>
<td>0.09+0.03</td>
<td>10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_{12} \mu^-\bar{\nu}_\mu$</td>
<td>0.00</td>
<td>0.03+0.02</td>
<td>10^{-1}</td>
<td>0.39+0.02</td>
<td>10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_{12} \tau^-\bar{\nu}_\tau$</td>
<td>0.00</td>
<td>0.03+0.02</td>
<td>10^{-1}</td>
<td>0.36+0.02</td>
<td>0.21+0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Psi(3836) e^-\bar{\nu}_e$</td>
<td>0.00</td>
<td>0.03+0.02</td>
<td>10^{-1}</td>
<td>0.09+0.03</td>
<td>10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Psi(3836) \mu^-\bar{\nu}_\mu$</td>
<td>0.00</td>
<td>0.03+0.02</td>
<td>10^{-1}</td>
<td>0.39+0.02</td>
<td>10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Psi(3836) \tau^-\bar{\nu}_\tau$</td>
<td>0.00</td>
<td>0.03+0.02</td>
<td>10^{-1}</td>
<td>0.36+0.02</td>
<td>0.21+0.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The forward direction is determined by the momentum of the final meson that we have chosen in the negative z-direction.
TABLE VII: Asymmetry parameters in semileptonic $B_c \to \ell\bar{\nu}$ decays. Our central values have been evaluated with the AL1 potential. We also show the results obtained by Ivanov et al. $^2$.

<table>
<thead>
<tr>
<th>Process</th>
<th>$A_{FB}(\ell)$</th>
<th>$A_{FB}(\mu)$</th>
<th>$A_{FB}(\tau)$</th>
<th>$\alpha^*(\ell)$</th>
<th>$\alpha^*(\mu)$</th>
<th>$\alpha^*(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c \to \eta_c$</td>
<td>This work</td>
<td>0.60$^{+0.01}_{-0.01}$ $10^{-6}$</td>
<td>0.13$^{+0.01}_{-0.01}$ $10^{-1}$</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^5$</td>
<td>0.953 $10^{-6}$</td>
<td>0.36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c \to \chi_{c0}$</td>
<td>This work</td>
<td>0.72$^{+0.02}_{-0.02}$ $10^{-6}$</td>
<td>0.15 $10^{-1}$</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^5$</td>
<td>1.31 $10^{-6}$</td>
<td>0.39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c \to J/\Psi$</td>
<td>This work</td>
<td>$-0.19$ $-0.18_{-0.01}$ $10^{-1}$</td>
<td>$-0.35^{+0.02}_{-0.02}$ $10^{-1}$</td>
<td>$-0.29_{-0.01}$ $-0.29$</td>
<td>$-0.19$ $-0.24$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^5$</td>
<td>$-0.21$ $-0.48$ $10^{-1}$</td>
<td>$-0.34$ $-0.29$</td>
<td>$-0.24$ $-0.19$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c \to \chi_{c1}$</td>
<td>This work</td>
<td>$-0.60_{-0.01}$ $-0.60_{-0.01}$ $10^{-1}$</td>
<td>$-0.46$ $10^{-2}$</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^5$</td>
<td>0.19 $10^{-2}$</td>
<td>0.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c \to h_c$</td>
<td>This work</td>
<td>$-0.83_{-0.05}$ $10^{-2}$</td>
<td>$0.97^{+0.01}_{-0.05}$ $10^{-2}$</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^5$</td>
<td>$-3.6$ $10^{-2}$</td>
<td>0.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c \to \chi_{c2}$</td>
<td>This work</td>
<td>$-0.14$ $-0.13$ $10^{-1}$</td>
<td>$0.55^{+0.02}_{-0.01}$ $10^{-1}$</td>
<td>$-0.42$ $10^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$^5$</td>
<td>$-0.16$ $10^{-1}$</td>
<td>0.44</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE VII: Asymmetry parameters in semileptonic $B_c \to \ell\bar{\nu}$ decays. Our central values have been evaluated with the AL1 potential. We also show the results obtained by Ivanov et al. $^2$.

\[
\times \frac{\Gamma_{J/\Psi - \mu^-\mu^+}}{\Gamma_{J/\Psi}} \frac{1}{1 + 2m_\mu^2/m_{J/\Psi}^2} \left[ \frac{3}{4} \left( \Gamma_L + \overline{\Gamma}_L + \overline{\Gamma}_S \right) + \frac{3}{8} \left( 2 - \sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2} \right) \left( \Gamma_U + \overline{\Gamma}_U \right) \right] \tag{47}
\]

where $x_\mu$ is the cosine of the polar angle for the final $\mu^-\mu^+$ pair, relative to the momentum of the decaying $J/\Psi$, measured in the $\mu^-\mu^+$ CMF, $\Gamma_{J/\Psi - \mu^-\mu^+}$ is the $J/\Psi$ decay width into the $\mu^-\mu^+$ channel, and $\Gamma_{J/\Psi}$ is the total $J/\Psi$ decay width. The asymmetry parameter

\[
\alpha^* = \frac{\sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2} \left( \Gamma_U + \overline{\Gamma}_U - 2(\Gamma_L + \overline{\Gamma}_L + \overline{\Gamma}_S) \right)}{(2 - \sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2}) \left( \Gamma_U + \overline{\Gamma}_U \right) + 2 \left( \Gamma_L + \overline{\Gamma}_L + \overline{\Gamma}_S \right)} \tag{48}
\]

governs the muons angular distribution in their CMF.

1. Results

In Table VII we give our results for the partial helicity widths corresponding to $B_c^- \to \ell\bar{\nu}$ decays. For $B_c^+ \to \ell\bar{\nu}$ decays, the “P” column changes sign while all others remain the same. The central values have been evaluated with the AL1 potential and the theoretical errors quoted reflect the dependence of the results on the inter-quark potential.

In Table VII we show the asymmetry parameters. Our values for $\alpha^*$ compare well with the results obtained in Ref. $^5$. The same is true for the forward–backward asymmetry with some exceptions: most notably we get opposite signs for $B_c \to \chi_{c1}$ and $B_c \to \Psi(3836)$.

In Fig. 5 we show the differential decay width $d\Gamma/dq^2$ for the decay channels $\eta_c l^-\bar{\nu}_l$ and $\chi_{c1} l^-\bar{\nu}_l$ for the case where the final lepton is a light one $l = e, \mu$ or a heavy one $l = \tau$. We show the results obtained with the AL1 and BHAD

\[9\] We show the distribution corresponding to a final electron. The distribution for a final muon differs from the former only for $q^2$ around $m_\mu^2$. 


Table VIII: Decay widths in units of $10^{-13}$ GeV for semileptonic $B_c^- \rightarrow c\bar{c}$ decays. Our central values have been evaluated with the AL1 potential. Here $l$ stands for $l = e, \mu$.

<table>
<thead>
<tr>
<th>Decay</th>
<th>This work</th>
<th>[5]</th>
<th>[2]</th>
<th>[9, 10]</th>
<th>[12]</th>
<th>[13]</th>
<th>[14]</th>
<th>[18]</th>
<th>[19]</th>
<th>[22]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^- \rightarrow \eta_c l^- \bar{\nu}_l$</td>
<td>$6.95^{+0.29}_{-0.20}$</td>
<td>$10.7$</td>
<td>$5.9$</td>
<td>$14.2$</td>
<td>$11.1$</td>
<td>$8.31$</td>
<td>$11 \pm 1$</td>
<td>$2.1 \ (6.9)$</td>
<td>$8.6$</td>
<td>$10$</td>
</tr>
<tr>
<td>$B_c^- \rightarrow \eta_c \tau^- \bar{\nu}_\tau$</td>
<td>$2.46^{+0.07}_{-0.03}$</td>
<td>$3.52$</td>
<td>$2.52$</td>
<td>$1.69$</td>
<td>$0.19^{+0.01}_{-0.08}$</td>
<td>$0.26$</td>
<td>$0.25$</td>
<td>$7 \pm 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \chi_{c0} l^- \bar{\nu}_l$</td>
<td>$1.55^{+0.14}_{-0.08}$</td>
<td>$2.52$</td>
<td>$1.69$</td>
<td>$0.19^{+0.01}_{-0.08}$</td>
<td>$0.26$</td>
<td>$0.25$</td>
<td>$7 \pm 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \chi_{c0} \tau^- \bar{\nu}_\tau$</td>
<td>$1.55^{+0.14}_{-0.08}$</td>
<td>$2.52$</td>
<td>$1.69$</td>
<td>$0.19^{+0.01}_{-0.08}$</td>
<td>$0.26$</td>
<td>$0.25$</td>
<td>$7 \pm 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow J/\Psi l^- \bar{\nu}_l$</td>
<td>$21.9^{+1.3}_{-0.8}$</td>
<td>$28.2$</td>
<td>$17.7$</td>
<td>$34.4$</td>
<td>$30.2$</td>
<td>$20.3$</td>
<td>$28 \pm 5$</td>
<td>$21.6 \ (48.3)$</td>
<td>$17.2$</td>
<td>$42$</td>
</tr>
<tr>
<td>$B_c^- \rightarrow J/\Psi \tau^- \bar{\nu}_\tau$</td>
<td>$5.86^{+0.23}_{-0.17}$</td>
<td>$7.82$</td>
<td>$7 \pm 2$</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \chi_{c1} l^- \bar{\nu}_l$</td>
<td>$0.94^{+0.05}_{-0.03}$</td>
<td>$1.40$</td>
<td>$2.21$</td>
<td>$0.10$</td>
<td>$0.17$</td>
<td>$0.35$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \chi_{c1} \tau^- \bar{\nu}_\tau$</td>
<td>$0.94^{+0.05}_{-0.03}$</td>
<td>$1.40$</td>
<td>$2.21$</td>
<td>$0.10$</td>
<td>$0.17$</td>
<td>$0.35$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow h_c l^- \bar{\nu}_l$</td>
<td>$2.40^{+0.23}_{-0.11}$</td>
<td>$4.42$</td>
<td>$2.51$</td>
<td>$0.21^{+0.01}_{-0.08}$</td>
<td>$0.38$</td>
<td>$0.36$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow h_c \tau^- \bar{\nu}_\tau$</td>
<td>$2.40^{+0.23}_{-0.11}$</td>
<td>$4.42$</td>
<td>$2.51$</td>
<td>$0.21^{+0.01}_{-0.08}$</td>
<td>$0.38$</td>
<td>$0.36$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \chi_{c2} l^- \bar{\nu}_l$</td>
<td>$1.89^{+0.11}_{-0.08}$</td>
<td>$2.92$</td>
<td>$2.73$</td>
<td>$0.13^{+0.01}_{-0.08}$</td>
<td>$0.20$</td>
<td>$0.42$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \chi_{c2} \tau^- \bar{\nu}_\tau$</td>
<td>$1.89^{+0.11}_{-0.08}$</td>
<td>$2.92$</td>
<td>$2.73$</td>
<td>$0.13^{+0.01}_{-0.08}$</td>
<td>$0.20$</td>
<td>$0.42$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \Psi(3836) l^- \bar{\nu}_l$</td>
<td>$0.062_{-0.008}^{+0.005}$</td>
<td>$0.13$</td>
<td>$0.10$</td>
<td>$0.17$</td>
<td>$0.35$</td>
<td></td>
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<tr>
<td>$B_c^- \rightarrow \Psi(3836) \tau^- \bar{\nu}_\tau$</td>
<td>$0.0012_{-0.0002}^{+0.0002}$</td>
<td>$0.0031$</td>
<td>$0.0031$</td>
<td>$0.0031$</td>
<td>$0.0031$</td>
<td>$0.0031$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Finally in Tables VIII, IX we give the total decay widths and corresponding branching ratios for the different transitions. The branching ratios evaluated by Ivanov et al. [4], where they have used the new $B_c$ mass determination by the CDF Collaboration [26], are in reasonable agreement with our results. Discrepancies are larger for the decay widths in Table VIII where they use the larger mass value $m_{B_c} = 6400$ MeV quoted by the PDG [25].
FIG. 6: Differential decay width for the $B_c^- \rightarrow J/\Psi l^{-} \bar{\nu}_l$, $B_c^- \rightarrow h_c l^{-} \bar{\nu}_l$, $B_c^- \rightarrow \chi_{c1} l^{-} \bar{\nu}_l$, $B_c^- \rightarrow \chi_{c2} l^{-} \bar{\nu}_l$ and $B_c^- \rightarrow \Psi(3836) l^{-} \bar{\nu}_l$ decay channels obtained with the AL1 potential (solid lines) and the Bhaduri (BHAD) potential (dotted lines). The distribution for a final muon is not explicitly shown. It differs appreciably from the corresponding to a final electron only for $q^2$ around $m_{\mu}^2$. 
TABLE IX: Branching ratios in % for semileptonic $B_c^- \to c\bar{c}$ decays. Our central values have been evaluated with the AL1 potential. Here $l$ stands for $l = e, \mu$.

C. Heavy quark spin symmetry

As mentioned in the Introduction one can not apply HQS to systems with two heavy quarks due to flavor symmetry breaking by the kinetic energy terms. The symmetry that survives for such systems is HQSS amounting to the decoupling of the two heavy quark spins. Using HQSS Jenkins et al. [2] obtained relations between different form factors for semileptonic $B_c$ decays into ground state vector and pseudoscalar mesons. Let us check the agreement of our calculations with their results. For that purpose let us re-write the general form factor decompositions in Eq. (9) introducing the four vectors $v$ and $k$ such that

$$P_{B_c} = m_{B_c} v ; \quad P_{c\bar{c}} = m_{c\bar{c}} v + k$$

(49)

$v$ is the four-velocity of the initial $B_c$ meson whereas $k$ is a residual momentum. In terms of those we have

$$P = P_{B_c} + P_{c\bar{c}} = (m_{B_c} + m_{c\bar{c}}) v + k ; \quad q = P_{B_c} - P_{c\bar{c}} = (m_{B_c} - m_{c\bar{c}}) v - k$$

(50)

and we can write for the $\eta_c (c\bar{c} (0^-))$ final state case

$$\left< \eta_c, \bar{P}_{\eta_c} | J^{b}_{\mu} (0) | B_c^-, \bar{P}_{B_c} \right> = \left< \eta_c, \bar{P}_{\eta_c} | J^{b}_{\mu} (0) | B_c^-, \bar{P}_{B_c} \right>$$

$$= \left< \eta_c, \bar{P}_{\eta_c} | J^{b}_{\mu} (0) | B_c^-, \bar{P}_{B_c} \right>$$

$$= \left( (m_{B_c} + m_{\eta_c}) F_+(q^2) + (m_{B_c} - m_{\eta_c}) F_-(q^2) \right) v_{\mu} + \left( F_+(q^2) - F_-(q^2) \right) k_{\mu}$$

$$= \sqrt{2m_{B_c}2m_{\eta_c}} \left( \Sigma_1^{(0^-)} (q^2) v_{\mu} + \Sigma_2^{(0^-)} (q^2) k_{\mu} \right)$$

(51)

where we have introduced the new form factors

$$\Sigma_1^{(0^-)} (q^2) = \frac{1}{\sqrt{2m_{B_c}2m_{\eta_c}}} \left( (m_{B_c} + m_{\eta_c}) F_+(q^2) + (m_{B_c} - m_{\eta_c}) F_-(q^2) \right)$$

$$\Sigma_2^{(0^-)} (q^2) = \frac{1}{\sqrt{2m_{B_c}2m_{\eta_c}}} \left( F_+(q^2) - F_-(q^2) \right)$$

(52)

Similarly for the $J/\Psi (c\bar{c} (1^-))$ final state case we have

$$\left< J/\Psi, \lambda \bar{P}_{J/\Psi} | J^{b}_{\mu} (0) | B_c^-, \bar{P}_{B_c} \right> = \left< J/\Psi, \lambda \bar{P}_{J/\Psi} | J^{b}_{\mu} (0) - J^{b}_{\mu} (0) | B_c^-, \bar{P}_{B_c} \right>$$

$$= \left< J/\Psi, \lambda \bar{P}_{J/\Psi} | J^{b}_{\mu} (0) - J^{b}_{\mu} (0) | B_c^-, \bar{P}_{B_c} \right>$$
\[
\begin{align*}
\Sigma^{(0)}(q^2) &= \frac{2m_{B_c}}{m_{B_c} + m_{J/\Psi}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\nu\beta}_{(\lambda)}(\vec{p}_{J/\Psi}) V^\alpha k^\beta V(q^2) \\
&\quad - i \left\{ \left( m_{B_c} - m_{J/\Psi} \right) \varepsilon^*_{(\lambda)_\mu}(\vec{p}_{J/\Psi}) A_0(q^2) \\
&\quad + \frac{m_{B_c}}{m_{J/\Psi}} k \cdot \varepsilon^*_{(\lambda)}(\vec{p}_{J/\Psi}) \left( A_+(q^2) \left( m_{B_c} + m_{J/\Psi} \right) v_\mu + k_\mu \right) \\
&\quad + \frac{A_-(q^2)}{m_{B_c} + m_{J/\Psi}} \left( (m_{B_c} - m_{J/\Psi}) v_\mu - k_\mu \right) \right\} \\
\Sigma^{(1)}_{2}(q^2) &= \frac{1}{\sqrt{2m_{B_c}m_{J/\Psi}}} \left( m_{B_c} - m_{J/\Psi} \right) A_0(q^2) \\
&\quad - \sqrt{2m_{B_c}m_{J/\Psi}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\nu\beta}_{(\lambda)}(\vec{p}_{J/\Psi}) V^\alpha V(q^2) \\
\Sigma^{(1)}_{3}(q^2) &= \frac{1}{\sqrt{2m_{B_c}m_{J/\Psi}}} \left( m_{B_c} + m_{J/\Psi} \right) \left( A_+(q^2) - A_-(q^2) \right)
\end{align*}
\]

with

\[
\begin{align*}
\Sigma^{(0)}_{1}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}} \left( m_{B_c} - m_{J/\Psi} \right) A_0(q^2) \\
\Sigma^{(1)}_{1}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}} \left( m_{B_c} - m_{J/\Psi} \right) \left( A_+(q^2) + \frac{m_{B_c} - m_{J/\Psi}}{m_{B_c} + m_{J/\Psi}} A_-(q^2) \right) \\
\Sigma^{(1)}_{2}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}} \left( m_{B_c} + m_{J/\Psi} \right) \left( A_+(q^2) - A_-(q^2) \right) \\
\Sigma^{(1)}_{3}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}} \left( m_{B_c} + m_{J/\Psi} \right) \left( A_+(q^2) - A_-(q^2) \right)
\end{align*}
\]

\(\Sigma^{(0)}_{1}\) and \(\Sigma^{(1)}_{1}\) are dimensionless, \(\Sigma^{(0)}_{2}\), \(\Sigma^{(1)}_{2}\), \(\Sigma^{(1)}_{3}\) have dimensions of \(E^{-1}\), and \(\Sigma^{(1)}_{3}\) has dimensions of \(E^{-2}\).

**FIG. 7**: \(\Sigma^{(0)}_{1}\) (solid line) and \(\Sigma^{(0)}_{2}\) (dashed line) of the \(B_c^- \rightarrow \eta_c\) and \(\Sigma^{(1)}_{1}\) (solid line), \(\Sigma^{(1)}_{2}\) (dashed line), \(\Sigma^{(1)}_{3}\) (dotted line) and \(\Sigma^{(1)}_{3}\) (dashed-dotted line) of the \(B_c^- \rightarrow J/\Psi\) semileptonic decays evaluated with the AL1 potential.

We can take the infinite heavy quark mass limit \(m_b \gg m_c \gg \Lambda_{QCD}\) with the result that near zero recoil

\(\Sigma^{(0)}_{1} = \Sigma^{(1)}_{1}\)
\[ \Sigma_2^{(0)} = \Sigma_2^{(1)} = \Sigma_2^{(1')} = 0 \]
\[ \Sigma_3^{(1)} = 0 \]  

(55)

This agrees perfectly with the result obtained in Ref. [2] using HQSS\(^{10}\).

In Fig. 7 we give our results for the above quantities for the semileptonic \( B_c^- \to \eta_c \) and \( B_c^- \to J/\Psi \) decays for the actual heavy quark masses. Even though we are not in the infinite heavy quark mass limit we find that \( \Sigma_1^{(0)} \) and \( \Sigma_1^{(1)} \) dominate over the whole \( q^2 \) interval. This dominant behavior would be more so near the zero–recoil point where \( k \approx 0 \) and thus the contributions from the terms in \( \Sigma_2^{(0)} \), \( \Sigma_2^{(1)} \), \( \Sigma_2^{(1')} \) and \( \Sigma_3^{(1)} \) are even more suppressed. Thus, even for the actual heavy quark masses we find that near zero recoil

\[
\left\langle \eta_c, \tilde{P}_{\eta_c} \left| J_{\mu}^{cb}(0) \right| B_c^-, \tilde{P}_{B_c} \right\rangle \approx \sqrt{2m_{B_c^*}m_{\eta_c}} \Sigma_1^{(0)}(q^2) v_\mu \\
\left\langle J/\Psi, \lambda \tilde{P}_{J/\Psi} \left| J_{\mu}^{cb}(0) \right| B_c^-, \tilde{P}_{B_c} \right\rangle \approx -i\sqrt{2m_{B_c^*}m_{J/\Psi}} \varepsilon_{\lambda\mu}^* (\tilde{P}_{J/\Psi}) \Sigma_1^{(1)}(q^2) 
\]

(56)

Besides, as seen in In Fig. 8 \( \Sigma_1^{(0)} \) of the \( B_c^- \to \eta_c \), and \( \Sigma_1^{(1)} \) of the \( B_c^- \to J/\Psi \) semileptonic decays are very close to each other over the whole \( q^2 \) interval. This implies that the result obtained in Ref. [2] near zero recoil using HQSS seems to be valid, to a very good approximation, outside the infinite heavy quark mass limit.

![Graph showing \( \Sigma_1^{(0)} \) and \( \Sigma_1^{(1)} \) for \( B_c^- \to \eta_c \) and \( B_c^- \to J/\Psi \) semileptonic decays](image)

FIG. 8: \( \Sigma_1^{(0)} \) (solid line) of the \( B_c^- \to \eta_c \), and \( \Sigma_1^{(1)} \) (dashed line) of the \( B_c^- \to J/\Psi \) semileptonic decays evaluated with the AL1 potential.

**IV. NONLEPTONIC \( B_c^- \to c\bar{c} M^- \) TWO–MESON DECAYS.**

In this section we will evaluate decay widths for nonleptonic \( B_c^- \to c\bar{c} M^- \) two–meson decays where \( M^- \) is a pseudoscalar or vector meson. These decay modes involve a \( b \to c \) transition at the quark level and they are governed, neglecting penguin operators, by the effective Hamiltonian \([4, 7, 18]\)

\[
H_{\text{eff.}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} \left[ c_1(\mu) Q_1^{cb} + c_2(\mu) Q_2^{cb} \right] + H.c. \right\} 
\]

(57)

where \( c_1, c_2 \) are scale–dependent Wilson coefficients, and \( Q_1^{cb}, Q_2^{cb} \) are local four–quark operators given by

\[
Q_1^{cb} = \Psi_{c}(0)\gamma_\mu(I-\gamma_5)\Psi_b(0) \\
Q_2^{cb} = V_{ud}^{*} \Psi_d(0)\gamma^\mu(I-\gamma_5)\Psi_u(0) + V_{us}^{*} \Psi_s(0)\gamma^\mu(I-\gamma_5)\Psi_u(0) 
\]

Note, however, the different global phases and notation used in Ref. [2].

\(^{10}\) Note, however, the different global phases and notation used in Ref. [2].
\[ Q_{2}^{ch} = \bar{\psi}_d(0)(I - \gamma_5)\gamma^\mu(I - \gamma_5)\psi_c(0) + V_{cs}^* \bar{\psi}_s(0)(I - \gamma_5)\gamma^\mu(I - \gamma_5)\psi_c(0) + V_{ud}^* \bar{\psi}_c(0)(I - \gamma_5)\gamma^\mu(I - \gamma_5)\psi_u(0) + V_{cs}^* \bar{\psi}_c(0)(I - \gamma_5)\gamma^\mu(I - \gamma_5)\psi_c(0) \]

(58)

where the different \( V_{jk} \) are CKM matrix elements.

We shall work in the factorization approximation which amounts to evaluate the hadron matrix elements of the effective Hamiltonian as a product of quark–current matrix elements: one of these is the matrix element for the \( B_c^- \) transition to one of the final mesons, while the other matrix element corresponds to the transition from the vacuum to the other final meson. The latter is given by the corresponding meson decay constant. This factorization approximation is schematically represented in Fig. 9.

![Diagrammatic representation of \( B_c^- \) two–meson decay in the factorization approximation.](image)

When writing the factorization amplitude one has to take into account the Fierz reordered contribution so that the relevant coefficients are not \( c_1 \) and \( c_2 \) but the combinations

\[
\begin{align*}
\alpha_1(\mu) &= c_1(\mu) + \frac{1}{N_C} c_2(\mu) ; \quad \alpha_2(\mu) = c_2(\mu) + \frac{1}{N_C} c_1(\mu) \\
\end{align*}
\]

(59)

with \( N_C = 3 \) the number of colors. The energy scale \( \mu \) appropriate in this case is \( \mu \simeq m_b \) and the values for \( \alpha_1 \) and \( \alpha_2 \) that we use are

\[
\begin{align*}
\alpha_1 &= 1.14 ; \quad \alpha_2 = -0.20 \\
\end{align*}
\]

(60)

1. \( M_F^- = \pi^-, \rho^-, K^-, K^{*-} \)

This is the simplest case. The decay width is given by

\[
\Gamma = \frac{G_F^2}{16\pi m_{M_F}^2} |V_{cb}|^2 |V_{F}|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{c\bar{c}}^2, m_{F}^2)}{2m_{B_c}} \mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \hat{\mathcal{H}}^{\alpha\beta}(P_F) 
\]

(61)

with \( m_F \) the mass of the \( M_F^- \) final meson, and \( V_F = V_{ud} \) or \( V_F = V_{us} \) depending on whether \( M_F^- = \pi^-, \rho^- \) or \( M_F^- = K^-, K^{*-} \). \( \mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \) is the hadron tensor for the \( B_c^- \to c\bar{c} \) transition and \( \hat{\mathcal{H}}^{\alpha\beta}(P_F) \) is the hadron tensor for the vacuum \( \to M_F^- \) transition. The latter is

\[
\begin{align*}
\hat{\mathcal{H}}^{\alpha\beta}(P_F) &= P_F^\alpha P_F^\beta f_F^2 \\
\hat{\mathcal{H}}^{\alpha\beta}(P_F) &= (P_F^\alpha P_F^\beta - m_F^2 g^{\alpha\beta}) f_F^2 \\
\end{align*}
\]

(62)
TABLE X: Decay widths in units of $10^{-15}$ GeV, and for general values of the Wilson coefficient $a_1$, for exclusive nonleptonic decays of the $B_c^-$ meson. Our central values have been obtained with the AL1 potential.

<table>
<thead>
<tr>
<th>$B_c^-$</th>
<th>$\Gamma$ [$10^{-15}$ GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^- \to \eta_c \pi^-$</td>
<td>$1.02^{+0.07}_{-0.01}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \eta_c \rho^-$</td>
<td>$2.60^{+0.16}_{-0.16}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \eta_c K^-$</td>
<td>$0.88^{+0.04}_{-0.01}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \eta_c K^{*-}$</td>
<td>$0.15^{+0.01}_{-0.01}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to J/\Psi \pi^-$</td>
<td>$0.83^{+0.07}_{-0.07}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to J/\Psi \rho^-$</td>
<td>$2.61^{+0.27}_{-0.27}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to J/\Psi K^-$</td>
<td>$0.66^{+0.07}_{-0.07}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to J/\Psi K^{*-}$</td>
<td>$0.16^{+0.01}_{-0.01}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c0} \pi^-$</td>
<td>$0.28^{+0.03}_{-0.03}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c0} \rho^-$</td>
<td>$0.73^{+0.07}_{-0.07}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c0} K^-$</td>
<td>$0.022^{+0.003}_{-0.003}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c0} K^{*-}$</td>
<td>$0.041^{+0.005}_{-0.005}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c1} \pi^-$</td>
<td>$0.0015^{+0.0002}_{-0.0002}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c1} \rho^-$</td>
<td>$0.11^{+0.01}_{-0.01}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c1} K^-$</td>
<td>$0.00012^{+0.00001}_{-0.00001}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c1} K^{*-}$</td>
<td>$0.000007^{+0.000007}_{-0.000007}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to h_c \pi^-$</td>
<td>$0.58^{+0.01}_{-0.01}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to h_c \rho^-$</td>
<td>$1.41^{+0.17}_{-0.17}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to h_c K^-$</td>
<td>$0.045^{+0.006}_{-0.006}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to h_c K^{*-}$</td>
<td>$0.078^{+0.009}_{-0.008}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c2} \pi^-$</td>
<td>$0.24^{+0.02}_{-0.02}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c2} \rho^-$</td>
<td>$0.71^{+0.07}_{-0.07}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c2} K^-$</td>
<td>$0.018^{+0.002}_{-0.002}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \chi_{c2} K^{*-}$</td>
<td>$0.041^{+0.004}_{-0.004}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \Psi(3836) \pi^-$</td>
<td>$0.00045^{+0.00005}_{-0.00005}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \Psi(3836) \rho^-$</td>
<td>$0.021^{+0.001}_{-0.001}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \Psi(3836) K^-$</td>
<td>$0.00034^{+0.00002}_{-0.00002}$ $a_1^2$</td>
</tr>
<tr>
<td>$B_c^- \to \Psi(3836) K^{*-}$</td>
<td>$0.0015^{+0.0002}_{-0.0002}$ $a_1^2$</td>
</tr>
</tbody>
</table>

with $F_F$ being the $M_{F^-}$ decay constant.

Similarly to the semileptonic case, the product $\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}})$ $\hat{F}^{\alpha\beta}(F_F)$ can now be easily written in terms of helicity amplitudes for the $B_c^- \to c\bar{c}$ transition so that the width is given as [41]

$$
\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cb}|^2 |V_{c\bar{c}}|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{c\bar{c}}^2, m_T^2)}{2m_{B_c} m_T^2} \frac{f_F}{f_F} \mathcal{H}_{\alpha\beta}(P_{B_c}) \hat{F}^{\alpha\beta}(F_F) \quad M_{F^-} \equiv 0^- \text{ case}
$$

$$
\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cb}|^2 |V_{c\bar{c}}|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{c\bar{c}}^2, m_T^2)}{2m_{B_c} m_T^2} \frac{f_F}{f_F} \times \left( \mathcal{H}_{\alpha\beta}(P_{B_c}) \hat{F}^{\alpha\beta}(F_F) \right) \quad M_{F^-} \equiv 1^- \text{ case}
$$

with the different $\mathcal{H}_{\alpha\beta}$ evaluated at $q^2 = m_T^2$. In Table XI we show the decay widths for a general value of the Wilson coefficient $a_1$, whereas in Table XII we give the corresponding branching ratios evaluated with $a_1 = 1.14$. Our results for a final $\eta_c$ or $J/\psi$ are in good agreement with the ones obtained by Ebert et al. [3], El-Hady et al. [12] and Anisimov et al. [19], but they are a factor 2 smaller than the results by Ivanov et al. [4] and Kiselev [16]. Large discrepancies with Ivanov’s results show up for the other transitions.

2. $M_{F^-} = D^-, D^{*-}, D^*_+, D^{*-}_+$

In this subsection we shall evaluate the nonleptonic two-meson $B_c^- \to \eta_c D^-, \eta_c D^{*-}, J/\Psi D^-, J/\Psi D^{*-}$ and $B_c^- \to \eta_c D^*_+, \eta_c D^{*-}_+, J/\Psi D^*_+, J/\Psi D^{*-}_+$ decay widths. In these cases there are two different contributions in the factorization
approximation. Following the same steps that lead to Eq. (63) we shall get

\[ \Gamma = \frac{G_F^2}{16\pi m_{B_s}^2} |V_{cb}|^2 |V_F|^2 \frac{\lambda_1^{1/2}(m_{B_s}^2, m_{B_c}^2, m_H^2)}{2m_{B_c}} \mathcal{H}\mathcal{H} \]  

(64)

where now \( V_F = V_{cd} \) for \( M_F = D^-_s, D^{*-}_s \) and \( V_F = V_{cs} \) for \( M_F = D^-_s, D^{*-}_s \). The quantity \( \mathcal{H}\mathcal{H} \) incorporates all information on the hadron matrix elements and depends on the transition as \[ \mathcal{H}\mathcal{H}^{B^-\to D'^-} = a_1 h_t^{B^-\to D'-} (m_{D^-}^2) m_{f_D^-} f_{D^-} + a_2 h_t^{B^-\to D'^-} (m_{D^-}^2) m_{f_D^-} f_{D^-} \]

and similarly for \( D_s^-, D_s^{*-} \). Note that the helicity amplitudes corresponding to \( B^- \to D^-_s, D^{*-}_s, D_s^-, D_s^{*-} \) have been evaluated from the matrix elements for the effective current operators \( \bar{\Psi}_{d,s}(0)\gamma^\mu(I - \gamma_5)\Psi_{d,s}(0) \) in Eq. (63). While in

\[ \text{Note the different phases used in Ref. [4].} \]
practice this is a $b \to d, s$ transition, the momentum transfer ($m_{q'}^2$ or $m_{J/P}$) is neither too high, so that one has to include a $B_c^*$ resonance, nor too low, so as to have too high three-momentum transfers\textsuperscript{12}. Besides the contribution is weighed by the much smaller $a_2$ Wilson coefficient. In Table XIII we give the decay widths for general values of the Wilson coefficients $a_1$ and $a_2$, and in Table XIII we show the branching ratios. We are in reasonable agreement with the results by Ivanov et al. \textsuperscript{4}, El-Hady et al. \textsuperscript{12} and Kiselev \textsuperscript{16}. For decays with a final $D_s^+, D^+$ the agreement is also reasonable with the results by Colangelo et al. \textsuperscript{18} and Anisimov et al. \textsuperscript{14}.

| Branching ratios in % for exclusive nonleptonic decays of the $B_c^-$ meson. | This work | \textsuperscript{[4]} | \textsuperscript{[9]} | \textsuperscript{[12]} | \textsuperscript{[16]} | \textsuperscript{[18]} | \textsuperscript{[19]} |
|---|---|---|---|---|---|---|
| $B_c^- \to \eta_c D^*$ | 0.014 $^{+0.005}_{-0.001}$ | 0.019 | 0.0012 | 0.014 | 0.015 | 0.005 | 0.010 |
| $B_c^- \to \eta_c D^{*+}$ | 0.012 $^{+0.001}_{-0.001}$ | 0.019 | 0.0010 | 0.013 | 0.010 | 0.002 | 0.0055 |
| $B_c^- \to J/\Psi D^-$ | 0.0083 $^{+0.0005}_{-0.0005}$ | 0.015 | 0.0009 | 0.009 | 0.009 | 0.013 | 0.0044 |
| $B_c^- \to J/\Psi D^{*-}$ | 0.031 $^{+0.001}_{-0.001}$ | 0.045 | 0.028 | 0.028 | 0.019 | 0.010 |
| $B_c^- \to \eta_c D_s^*$ | 0.44 $^{+0.02}_{-0.02}$ | 0.44 | 0.054 | 0.26 | 0.28 | 0.50 | 0.35 |
| $B_c^- \to \eta_c D_s^{*-}$ | 0.24 $^{+0.02}_{-0.02}$ | 0.37 | 0.044 | 0.24 | 0.27 | 0.038 | 0.36 |
| $B_c^- \to J/\Psi D_s^*$ | 0.24 $^{+0.02}_{-0.02}$ | 0.34 | 0.041 | 0.15 | 0.17 | 0.34 | 0.12 |
| $B_c^- \to J/\Psi D_s^{*-}$ | 0.68 $^{+0.03}_{-0.03}$ | 0.97 | 0.55 | 0.67 | 0.59 | 0.62 |

TABLE XII: Decay widths in units of $10^{-15}$ GeV, and for general values of the Wilson coefficients $a_1$ and $a_2$, for exclusive nonleptonic decays of the $B_c^-$ meson. Our central values have been obtained with the AL1 potential. For vector–vector final state we show the three different contributions corresponding to $r = +1, −1, 0$ (see Eq. (66)).

V. SEMILEPTONIC $B_c^- \to \bar{B}^0, \bar{B}_s^0, \bar{B}_s^0, \bar{B}_d^0$ DECAYS

In this section we shall study the semileptonic $B_c^- \to \bar{B}^0, \bar{B}_s^0, \bar{B}_s^0, \bar{B}_d^0$ decays. With obvious changes the calculations are done as before, with the only novel thing that now it is the antiquark that suffers the transition (we have $c \to d, s$ ), and thus we have to take into account the changes in the form factors according to the results in appendix C.

\textsuperscript{12} Our experience with the $B \to \pi$ decay where we have a similar $b \to u$ quark transition, shows that the naive nonrelativistic quark model gives reliable results for $q^2 \approx 9$ GeV$^2$. 

\[ \]
FIG. 10: $F_+$ and $F_-$ form factors (solid lines) for $B_c^- \to B^0$ and $B_c^- \to B^0_s$ semileptonic decay, and $V$ (solid line), $A_+$ (dashed line), $A_-$ (dotted line) and $A_0$ (dashed–dotted line) form factors for $B_c^- \to B^0$ and $B_c^- \to B^0_s$ semileptonic decay evaluated with the AL1 potential. For the first two cases, and for comparison, we also show with dotted lines the results obtained with the Bhaduri (BHAD) potential.

A. Form factors

In Fig. 10 we show the form factors for the above transitions evaluated with the AL1 potential. For the $B^0$ and $B^0_s$ cases we also show the results obtained with the BHAD potential. Although they are less visible in the figures, the larger differences, of up to 25%, occur for the $F_-$ form factor.

In Table XIV we show $F_+$, $F_-$ and $F_0$ (defined as in Eq. (15) changing the mass of the final meson) of the $B_c^- \to \overline{B}^0$, $B_c^- \to \overline{B}^0_s$ transitions evaluated at $q^2_{\text{min}}$ and $q^2_{\text{max}}$ and compare them with the results by Ivanov et al. and Ebert et al. Notice that, to favor comparison, we have changed the signs of the form factors by Ebert et al. (they evaluate $B_c^+ \to \tau^-$ decay) in accordance with the results in appendix C. The agreement with the results by Ebert et al. is good. We also agree with Ivanov et al. for $F_+$, but get very different results for $F_-$. As fermion masses are very small the disagreement in the $F_-$ form factor will have a negligible effect on the decay width.

In Table XV we show $V$, $A_+$, $A_-$, $A_0$ and $\tilde{A}_0$ (defined as in Eq. (19) changing the mass of the final meson) of the $B_c^- \to \overline{B}^0$, $B_c^- \to \overline{B}^0_s$ evaluated at $q^2_{\text{min}}$ and $q^2_{\text{max}}$ and compare them with the results by Ivanov et al. and Ebert et al. With some exceptions the agreement with Ebert’s results is bad in this case. We are also in clear disagreement with Ivanov’s results.
B. Decay width

In Tables XV, XVI, XVII we give respectively our results for the partial helicity widths and forward-backward asymmetries.

In Fig. 11 we show the differential decay width for the $B_c^- \to \overline{B}^0 l^- \bar{v}_l$, $B_c^- \to \overline{B}_s^0 l^- \bar{v}_l$, $B_c^- \to B^0 l^- \bar{v}_l$ and $B_c^- \to \overline{B}_s^{*0} l^- \bar{v}_l$ transitions ($l = e, \mu$). In Tables XVIII, XIX we give the total decay widths and branching ratios and compare them with determinations by other groups. Our results are in better agreement with the ones obtained by Ebert et al. [5], Colangelo et al. [18], Anisimov et al. [17] and Lu et al. [14].
TABLE XVI: Partial helicity widths in units of $10^{-15}$ GeV for $B_c^-$ decay. Central values have been evaluated with the AL1 potential.

<table>
<thead>
<tr>
<th>$B_c^- \to$</th>
<th>$\Gamma_U$</th>
<th>$\bar{\Gamma}_U$</th>
<th>$\Gamma_L$</th>
<th>$\bar{\Gamma}_L$</th>
<th>$\Gamma_P$</th>
<th>$\bar{\Gamma}_S$</th>
<th>$\bar{\Gamma}_{SL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{B}^0 e^- \bar{\nu}_e$</td>
<td>0</td>
<td>0</td>
<td>$0.63^{+0.07}_{-0.09}$</td>
<td>$0.14^{+0.02}_{-0.02}$</td>
<td>$10^{-5}$</td>
<td>0</td>
<td>$0.47^{+0.05}_{-0.07}$</td>
</tr>
<tr>
<td>$\bar{B}^0 \mu^- \bar{\nu}_\mu$</td>
<td>0</td>
<td>0</td>
<td>$0.55^{+0.06}_{-0.07}$</td>
<td>$0.15^{+0.01}_{-0.02}$</td>
<td>$10^{-1}$</td>
<td>0</td>
<td>$0.64^{+0.16}_{-0.09}$</td>
</tr>
<tr>
<td>$\bar{B}_s^0 e^- \bar{\nu}_e$</td>
<td>0</td>
<td>0</td>
<td>$15.4^{+0.7}_{-0.3}$</td>
<td>$0.43^{+0.02}_{-0.02}$</td>
<td>$10^{-4}$</td>
<td>0</td>
<td>$0.14^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>$\bar{B}<em>s^0 \mu^- \bar{\nu}</em>\mu$</td>
<td>0</td>
<td>0</td>
<td>$12.4^{+0.5}_{-0.3}$</td>
<td>$0.40^{+0.02}_{-0.01}$</td>
<td>0</td>
<td>$1.69^{+0.08}_{-0.03}$</td>
<td>$0.47^{+0.02}_{-0.01}$</td>
</tr>
<tr>
<td>$\bar{B}^0 e^- \bar{\nu}_e$</td>
<td>$0.83^{+0.08}_{-0.11}$</td>
<td>$0.26^{+0.03}_{-0.04}$</td>
<td>$10^{-6}$</td>
<td>$0.76^{+0.09}_{-0.11}$</td>
<td>$0.10^{+0.02}_{-0.01}$</td>
<td>$10^{-5}$</td>
<td>$0.36^{+0.03}_{-0.05}$</td>
</tr>
<tr>
<td>$\bar{B}^0 \mu^- \bar{\nu}_\mu$</td>
<td>$0.79^{+0.10}_{-0.11}$</td>
<td>$0.97^{+0.11}_{-0.11}$</td>
<td>$10^{-2}$</td>
<td>$0.68^{+0.08}_{-0.10}$</td>
<td>$0.14^{+0.01}_{-0.03}$</td>
<td>$10^{-1}$</td>
<td>$0.34^{+0.02}_{-0.05}$</td>
</tr>
<tr>
<td>$\bar{B}_s^0 e^- \bar{\nu}_e$</td>
<td>$16.7^{+0.8}_{-0.7}$</td>
<td>$0.66^{+0.04}_{-0.03}$</td>
<td>$10^{-5}$</td>
<td>$16.8^{+1.1}_{-0.8}$</td>
<td>$0.33^{+0.02}_{-0.02}$</td>
<td>$10^{-4}$</td>
<td>$6.11^{+0.21}_{-0.20}$</td>
</tr>
<tr>
<td>$\bar{B}<em>s^0 \mu^- \bar{\nu}</em>\mu$</td>
<td>$15.6^{+0.8}_{-0.6}$</td>
<td>$0.24^{+0.02}_{-0.02}$</td>
<td>$14.5^{+1.0}_{-0.6}$</td>
<td>$0.37^{+0.02}_{-0.02}$</td>
<td>$5.66^{+0.18}_{-0.19}$</td>
<td>$0.77^{+0.06}_{-0.04}$</td>
<td>$0.30^{+0.02}_{-0.01}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$A_{FB}(e)$</th>
<th>$A_{FB}(\mu)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^- \to \bar{B}^0$</td>
<td>$0.67^{+0.02}_{-0.01}$</td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}_s^0$</td>
<td>$0.89^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}_s^0$</td>
<td>$0.17^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}_s^0$</td>
<td>$0.14^{+0.01}_{-0.01}$</td>
</tr>
</tbody>
</table>

TABLE XVII: Forward-backward asymmetry. Our central values have been evaluated with the AL1 potential. We would obtain the same results for $B_c^+ \to \bar{B}_s^0$ decays.

<p>| $\Gamma$ [10^{-15} GeV] |</p>
<table>
<thead>
<tr>
<th>This work</th>
<th>[6]</th>
<th>[8]</th>
<th>[9]</th>
<th>[12]</th>
<th>[13]</th>
<th>[16]</th>
<th>[18]</th>
<th>[19]</th>
<th>[20]</th>
<th>[24]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^- \to \bar{B}^0 e^- \bar{\nu}_e$</td>
<td>$0.65^{+0.07}_{-0.09}$</td>
<td>2.1</td>
<td>0.6</td>
<td>2.30</td>
<td>1.14</td>
<td>1.90</td>
<td>4.9</td>
<td>0.9(1.0)</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}^0 \mu^- \bar{\nu}_\mu$</td>
<td>$0.63^{+0.07}_{-0.09}$</td>
<td>29</td>
<td>12</td>
<td>26.6</td>
<td>14.3</td>
<td>26.8</td>
<td>59</td>
<td>11.1(12.9)</td>
<td>15</td>
<td>12.3</td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}_s^0 e^- \bar{\nu}_e$</td>
<td>$15.1^{+0.7}_{-0.3}$</td>
<td>37</td>
<td>25</td>
<td>44.0</td>
<td>50.4</td>
<td>34.6</td>
<td>65</td>
<td>33.5(37.0)</td>
<td>34</td>
<td>19.0</td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}<em>s^0 \mu^- \bar{\nu}</em>\mu$</td>
<td>$14.5^{+0.6}_{-0.3}$</td>
<td>33.5^{+1.9}_{-1.5}</td>
<td>37</td>
<td>25</td>
<td>44.0</td>
<td>50.4</td>
<td>34.6</td>
<td>65</td>
<td>33.5(37.0)</td>
<td>34</td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}_s^0 e^- \bar{\nu}_e$</td>
<td>$15.0^{+0.17}_{-0.22}$</td>
<td>2.3</td>
<td>1.7</td>
<td>3.32</td>
<td>2.34</td>
<td>3.53</td>
<td>8.5</td>
<td>2.8(3.2)</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>$B_c^- \to \bar{B}<em>s^0 \mu^- \bar{\nu}</em>\mu$</td>
<td>$15.2^{+0.17}_{-0.21}$</td>
<td>33.5^{+1.8}_{-1.2}</td>
<td>37</td>
<td>25</td>
<td>44.0</td>
<td>50.4</td>
<td>34.6</td>
<td>65</td>
<td>33.5(37.0)</td>
<td>34</td>
</tr>
</tbody>
</table>

TABLE XVIII: Decay widths in units of $10^{-15}$ GeV. Our central values have been evaluated with the AL1 potential.

C. Heavy quark spin symmetry

In Fig. 12 we give $\Sigma_1^{(0^-)}$ and $\Sigma_2^{(0^-)}$ of the $B_c^- \to \bar{B}^0$ and $B_c^- \to \bar{B}_s^0$ transitions, and $\Sigma_1^{(1^-)}$, $\Sigma_2^{(1^-)}$, $\Sigma_2^{(1^-)}$ and $\Sigma_3^{(1^-)}$ of the $B_c^- \to \bar{B}_s^0$ and $B_c^- \to \bar{B}_s^0$ transitions.

We can take the infinite heavy quark mass limit on our analytic expressions with the result that near zero recoil

$$\Sigma_1^{(1^-)} = \Sigma_1^{(0^-)}$$
$$\Sigma_2^{(1^-)} = \Sigma_2^{(1^-)} = -\Sigma_2^{(0^-)}$$
$$\Sigma_3^{(1^-)} = 0$$ (66)
**TABLE XIX:** Branching ratios in %. Our central values have been evaluated with the AL1 potential.

<table>
<thead>
<tr>
<th>Branching Ratio</th>
<th>This work</th>
<th>[4]</th>
<th>[8]</th>
<th>[9]</th>
<th>[12]</th>
<th>[16]</th>
<th>[18]</th>
<th>[19]</th>
<th>[20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^- \rightarrow \overline{B}^0_\ell \ell \bar{\nu}_\ell$</td>
<td>0.046$^{+0.003}_{-0.007}$</td>
<td>0.071</td>
<td>0.042</td>
<td>0.16</td>
<td>0.078</td>
<td>0.34</td>
<td>0.06</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>$B_c^- \rightarrow \overline{B}^0_\mu \mu \bar{\nu}_\mu$</td>
<td>0.044$^{+0.005}_{-0.006}$</td>
<td>1.06</td>
<td>0.84</td>
<td>1.82</td>
<td>0.98</td>
<td>4.03</td>
<td>0.8</td>
<td>0.99</td>
<td>0.92</td>
</tr>
<tr>
<td>$B_c^- \rightarrow \overline{B}^0_\ell \ell \bar{\nu}_\ell$ (for $\ell = e, \mu$ transitions)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FIG. 11:** Differential decay width for the $B_c^- \rightarrow \overline{B}^0_\ell \ell \bar{\nu}_\ell$ and $B_c^- \rightarrow \overline{B}^0_\mu \mu \bar{\nu}_\mu$ transitions. Solid line: results for a final $e$ evaluated with the AL1 potential; dotted line: results for a final $e$ evaluated with the Bhaduri (BHAD) potential; dashed line: results for a final $\mu$ evaluated with the AL1 potential; dashed–dotted line: results for a final $\mu$ evaluated with the Bhaduri (BHAD) potential.
When compared to the results of HQSS by Jenkins et al. \[2\] we see differences. In Ref. \[2\] we find $\Sigma_2^{(0)} = \Sigma_2^{(0)}$ instead. This is wrong as there is a misprint in Ref. \[2\] that has not been noted before: the sign of the term in $v^\mu$ in the last expression of Eqs. (2.9) and (2.10) in Ref. \[2\] should be a minus \[42\]. Also from Ref. \[2\] one would expect $\Sigma_2^{(1)} = \Sigma_2^{(0)}$ contradicting our result in Eq. (66) were we find $\Sigma_2^{(1)} = -\Sigma_2^{(0)}$. Our result is a clear prediction of the quark model and comes from the extra signs that appear due to the fact that it is the antiquark that decays (See appendix \[C\]). This difference between quark and antiquark decay was not properly reflected in their published work \[42\].

How far are we from the infinite heavy quark mass limit? In Fig. 12 we show $\Sigma_1^{(0)}$ of the semileptonic $B_c^- \to B^0$ and $B_c^- \to B^0_s$ transitions, and $\Sigma_1^{(1)}$ of the semileptonic $B_c^- \to B^*_0$ and $B_c^- \to B^*_0$ transitions. The differences

---

13 Note the different notation and global phases used.

14 One would have to look at Eq. (2.10) in Ref. \[2\], even though it refers to $B_c^+$ decay into $D^0, D^{*0}$, because that is the reaction where you have antiquark decay in their case.
between the corresponding $\Sigma_1^{(0^{-})}$ and $\Sigma_1^{(1^{-})}$ are at the level of 10%. The differences are much more significant for $\Sigma_2^{(0^{-})}$, $\Sigma_2^{(1^{-})}$ and $\Sigma_2^{(1^{-})}$ that we show in Fig. 14. In each case the three curves shown would be the same in the infinite heavy quark mass limit. Clearly in this case corrections on the inverse of the heavy quark masses seem to be important.

VI. NONLEPTONIC $B_c^-$ \( \rightarrow B M_F \) TWO-MESON DECAYS

In this section we will evaluate decay widths for nonleptonic $B_c^- \rightarrow B M_F$ two-meson decays where $M_F$ is a pseudoscalar or vector meson with no $b$ quark content, and, at this point, $B$ represents a meson with a $b$ quark. These decay modes involve a $\bar{c} \rightarrow \bar{d}$ or $\bar{c} \rightarrow \bar{s}$ transition at the quark level and they are governed, neglecting penguin
operators, by the effective Hamiltonian

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{cd} \left[ c_1(\mu) Q_1^{cd} + c_2(\mu) Q_2^{cd} \right] + V_{cs} \left[ c_1(\mu) Q_1^{cs} + c_2(\mu) Q_2^{cs} \right] + H.c. \right\}$$

(67)

where $c_1, c_2$ are scale–dependent Wilson coefficients, and $Q_1^{cd}, Q_2^{cd}, Q_1^{cs}, Q_2^{cs}$ are local four–quark operators given by

$$Q_1^{cd} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_d(0) \left[ V_{ud} \bar{\Psi}_d(0) \gamma^\mu (I - \gamma_5) \Psi_u(0) + V_{us} \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_u(0) \right]$$

$$Q_2^{cd} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_u(0) \left[ V_{ud} \bar{\Psi}_d(0) \gamma^\mu (I - \gamma_5) \Psi_d(0) + V_{us} \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_d(0) \right]$$

$$Q_1^{cs} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_s(0) \left[ V_{us} \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_u(0) + V_{us} \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_u(0) \right]$$

$$Q_2^{cs} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_u(0) \left[ V_{us} \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_s(0) + V_{us} \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_s(0) \right]$$

(68)

We shall work again in the factorization approximation taking into account the Fierz reordered contribution so that the relevant coefficients are not $c_1$ and $c_2$ but the combinations

$$a_1(\mu) = c_1(\mu) + \frac{1}{N_C} c_2(\mu) \quad ; \quad a_2(\mu) = c_2(\mu) + \frac{1}{N_C} c_1(\mu)$$

(69)

The energy scale $\mu$ appropriate in this case is $\mu \simeq m_c$ and the values for $a_1$ and $a_2$ that we use are

$$a_1 = 1.20 \quad ; \quad a_2 = -0.317$$

(70)

## 1. $M_F = \pi^-, \rho^-, K^-, K^{*-}$

### \[ \Gamma \left[ 10^{-15} \text{ GeV} \right] \]

<table>
<thead>
<tr>
<th>[ B^- \rightarrow \bar{B}^\prime \pi^- ]</th>
<th>[ B^- \rightarrow \bar{B}^\prime \rho^- ]</th>
<th>[ B^- \rightarrow \bar{B}^\prime K^- ]</th>
<th>[ B^- \rightarrow \bar{B}^\prime K^{*-} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>This work</td>
<td>This work</td>
<td>This work</td>
</tr>
<tr>
<td>-------------------------------------------------</td>
<td>-------------------------------------------------</td>
<td>-------------------------------------------------</td>
<td>-------------------------------------------------</td>
</tr>
<tr>
<td>$1.00^{+0.14}_{-0.16} , m_c^2$</td>
<td>$1.10^{+0.17}_{-0.15} , m_c^2$</td>
<td>$0.44^{+0.07}_{-0.06} , m_c^2$</td>
<td>$0.12^{+0.04}_{-0.03} , m_c^2$</td>
</tr>
<tr>
<td>$2.36^{+0.12}_{-0.11} , m_c^2$</td>
<td>$2.54^{+0.13}_{-0.12} , m_c^2$</td>
<td>$0.14^{+0.06}_{-0.06} , m_c^2$</td>
<td>$0.03^{+0.04}_{-0.04} , m_c^2$</td>
</tr>
<tr>
<td>$34^{+2.9}_{-2.9} , m_c^2$</td>
<td>$31^{+3.0}_{-3.0} , m_c^2$</td>
<td>$2.87^{+0.13}_{-0.12} , m_c^2$</td>
<td>$0.13^{+0.13}_{-0.13} , m_c^2$</td>
</tr>
<tr>
<td>$22.8^{+2.2}_{-2.2} , m_c^2$</td>
<td>$132^{+9.5}_{-9.5} , m_c^2$</td>
<td>$1.29^{+0.10}_{-0.10} , m_c^2$</td>
<td>$0.13^{+0.01}_{-0.01} , m_c^2$</td>
</tr>
</tbody>
</table>

### \[ B^- \rightarrow \bar{B}^0 \pi^- \] \[ B^- \rightarrow \bar{B}^0 \pi^- \] \[ B^- \rightarrow \bar{B}^0 K^- \] \[ B^- \rightarrow \bar{B}^0 K^- \] \[ B^- \rightarrow \bar{B}^0 K^- \] \[ B^- \rightarrow \bar{B}^0 K^- \] \[ B^- \rightarrow \bar{B}^0 K^- \] \[ B^- \rightarrow \bar{B}^0 K^- \]

\[ \Gamma_{B^- \rightarrow \bar{B}^0} \]

<table>
<thead>
<tr>
<th>[ \Gamma_{B^- \rightarrow \bar{B}^0} \left[ 10^{-15} \text{ GeV} \right] ]</th>
<th>[ \text{B.R. in } % ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>This work</td>
</tr>
<tr>
<td>-------------------------------------------------</td>
<td>-------------------------------------------------</td>
</tr>
<tr>
<td>$0.11_{-0.01} , m_c^2$</td>
<td>$0.12_{-0.02} , m_c^2$</td>
</tr>
<tr>
<td>$0.20_{-0.02} , m_c^2$</td>
<td>$0.20_{-0.03} , m_c^2$</td>
</tr>
<tr>
<td>$0.85_{-0.08} , m_c^2$</td>
<td>$0.85_{-0.09} , m_c^2$</td>
</tr>
<tr>
<td>$0.04_{-0.06} , m_c^2$</td>
<td>$0.04_{-0.07} , m_c^2$</td>
</tr>
</tbody>
</table>

### TABLE XX: Decay widths in units of $10^{-15} \text{ GeV}$, and for general values of the Wilson coefficient $a_1$, and branching ratios in $\%$ for exclusive nonleptonic decays of the $B^-\rightarrow B^-$ meson. Our central values have been obtained with the AL1 potential.

In this case $B$ denotes one of the $\bar{B}^0, \bar{B}^{*-0}, \bar{B}^{0*}, \bar{B}^{*0}$. The decay widths are

$$\Gamma = \frac{G_F^2}{16\pi m_{B^-}} \left| V_{cd} \right|^2 \left| V_F \right|^2 a_1^{1/2} \left( m_{B^+}^2, m_{B^{*-0}}^2, m_{B^{0*}}^2, m_{B^{*0}}^2 \right) \frac{m_{B^-}^2}{m_{B^+}^2} \frac{m_{B^+}^2}{m_{B^{*-0}}^2} \frac{m_{B^{*-0}}^2}{m_{B^{0*}}^2} \frac{m_{B^{0*}}^2}{m_{B^{*0}}^2} H_{\text{fit}}(m_F^2) \quad M_F \equiv 0^- \text{ case}$$

(68)
\[
\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cd}|^2 |V_F|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{B^-}^2, m_{B^0}^2, m_{B^+}^2)}{2m_{B_c}} \frac{m_F^2 f_F}{f_F} \times \left( \mathcal{H}_{1+1+}^{B^- - B^-} (m_F^2) + \mathcal{H}_{1-1-}^{B^- - B^-} (m_F^2) + \mathcal{H}_{00}^{B^- - B^-} (m_F^2) \right) \quad M_F \equiv 1^- \text{ case (71)}
\]

and similarly for $\mathcal{B}_s^+, \mathcal{B}_s^{0}$ with $|V_{cd}| \rightarrow |V_{cs}|$, $\mathcal{B}_s^+, \mathcal{B}_s^{0} \rightarrow \mathcal{B}_s^+, \mathcal{B}_s^{0}$. $V_F = V_{ud}$ or $V_F = V_{us}$ depending on whether $M_F = \pi^-, \rho^-$ or $M_F = K^-, K^{*0}$. $f_F$ is the decay constant of the $M_F$ meson, and the different $\mathcal{H}_{t\tau}$ have been evaluated at $q^2 = m_{F0}^2$. In Table XXI we show the decay widths for a general value of the Wilson coefficient $a_1$, and the corresponding branching ratios evaluated with $a_1 = 1.20$. The transition $B_c^- \rightarrow \mathcal{B}_s^- K^{*-}$ is not allowed with the new $B_c$ mass value from Ref. [26]. Our branching ratios for a final $\mathcal{B}_s^+$ or $\mathcal{B}_s^{0}$ are in very good agreement with the results by Ivanov et al. [3], while for a final $\mathcal{B}_s^+$ or $\mathcal{B}_s^{0}$ we are in very good agreement with the results by Ebert et al. [8] (with the exception of the $B_c \rightarrow B^* \pi^-$ decay).

2. $M_F = \pi^0, \rho^0, K^0, K^{*0}$

Here the generic name $B$ stands for a $B^-$ or a $B^{*-}$ meson. The different decay widths are given by

\[
\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{ud}|^2 |V_F|^2 a_2^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{B^-}^2, m_{B^0}^2, m_{B^+}^2)}{2m_{B_c}} \frac{m_F^2 f_F}{f_F} \mathcal{H}_{t\tau}^{B^- - B^-} (m_F^2) \quad M_F \equiv 0^- \text{ case}
\]

\[
\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{ud}|^2 |V_F|^2 a_2^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{B^-}^2, m_{B^0}^2, m_{B^+}^2)}{2m_{B_c}} \frac{m_F^2 f_F}{f_F} \times \left( \mathcal{H}_{1+1+}^{B^- - B^-} (m_F^2) + \mathcal{H}_{1-1-}^{B^- - B^-} (m_F^2) + \mathcal{H}_{00}^{B^- - B^-} (m_F^2) \right) \quad M_F \equiv 1^- \text{ case (72)}
\]

where $V_F = V_{ud}$ or $V_F = V_{cs}$ depending on whether $M_F = \pi^0, \rho^0$ or $M_F = K^0, K^{*0}$, $f_F = f_F$ for $M_F = K^0, K^{*0}$ whereas $f_F = f_F/\sqrt{2}$ for $M_F = \pi^0, \rho^0$, with $f_F$ the $M_F$ meson decay constant, and the different $\mathcal{H}_{t\tau}$ evaluated at $q^2 = m_{F0}^2$. The latter have been obtained from the matrix elements for the effective current operator $\Psi_c(0)\gamma^\mu(I - \gamma_5)\Psi_u(0)$.

The decay widths, for a general value of the Wilson coefficient $a_2$, and the corresponding branching ratios are shown in Table XXI. With the exception of the $B_c^- \rightarrow B^{*-}\pi^0$ case, our results are in a global good agreement with the ones by Ebert et al. [8].

<table>
<thead>
<tr>
<th>$B_c \rightarrow B^- \pi^0$</th>
<th>$B_c \rightarrow B^- \pi^0$</th>
<th>$B_c \rightarrow B^- \pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.54 (\pm 0.05) (\pm 0.02)</td>
<td>0.54 (\pm 0.05) (\pm 0.02)</td>
<td>0.54 (\pm 0.05) (\pm 0.02)</td>
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<tr>
<td>$B_c \rightarrow B^- \rho^0$</td>
<td>$B_c \rightarrow B^- \rho^0$</td>
<td>$B_c \rightarrow B^- \rho^0$</td>
</tr>
<tr>
<td>0.71 (\pm 0.09) (\pm 0.02)</td>
<td>0.71 (\pm 0.09) (\pm 0.02)</td>
<td>0.71 (\pm 0.09) (\pm 0.02)</td>
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<tr>
<td>$B_c \rightarrow B^- K^0$</td>
<td>$B_c \rightarrow B^- K^0$</td>
<td>$B_c \rightarrow B^- K^0$</td>
</tr>
<tr>
<td>35.3 (\pm 4.0) (\pm 2.1)</td>
<td>35.3 (\pm 4.0) (\pm 2.1)</td>
<td>35.3 (\pm 4.0) (\pm 2.1)</td>
</tr>
<tr>
<td>$B_c \rightarrow B^- K^{*0}$</td>
<td>$B_c \rightarrow B^- K^{*0}$</td>
<td>$B_c \rightarrow B^- K^{*0}$</td>
</tr>
<tr>
<td>13.1 (\pm 0.9) (\pm 0.6)</td>
<td>13.1 (\pm 0.9) (\pm 0.6)</td>
<td>13.1 (\pm 0.9) (\pm 0.6)</td>
</tr>
<tr>
<td>$B_c \rightarrow B^{*-} \pi^0$</td>
<td>$B_c \rightarrow B^{*-} \pi^0$</td>
<td>$B_c \rightarrow B^{*-} \pi^0$</td>
</tr>
<tr>
<td>0.35 (\pm 0.05) (\pm 0.02)</td>
<td>0.35 (\pm 0.05) (\pm 0.02)</td>
<td>0.35 (\pm 0.05) (\pm 0.02)</td>
</tr>
<tr>
<td>$B_c \rightarrow B^{*-} \rho^0$</td>
<td>$B_c \rightarrow B^{*-} \rho^0$</td>
<td>$B_c \rightarrow B^{*-} \rho^0$</td>
</tr>
<tr>
<td>2.84 (\pm 0.27) (\pm 0.02)</td>
<td>2.84 (\pm 0.27) (\pm 0.02)</td>
<td>2.84 (\pm 0.27) (\pm 0.02)</td>
</tr>
<tr>
<td>$B_c \rightarrow B^{*-} K^0$</td>
<td>$B_c \rightarrow B^{*-} K^0$</td>
<td>$B_c \rightarrow B^{*-} K^0$</td>
</tr>
<tr>
<td>16.9 (\pm 2.4) (\pm 0.7)</td>
<td>16.9 (\pm 2.4) (\pm 0.7)</td>
<td>16.9 (\pm 2.4) (\pm 0.7)</td>
</tr>
<tr>
<td>$B_c \rightarrow B^{*-} K^{*0}$</td>
<td>$B_c \rightarrow B^{*-} K^{*0}$</td>
<td>$B_c \rightarrow B^{*-} K^{*0}$</td>
</tr>
<tr>
<td>103 (\pm 8) (\pm 0.8)</td>
<td>103 (\pm 8) (\pm 0.8)</td>
<td>103 (\pm 8) (\pm 0.8)</td>
</tr>
</tbody>
</table>

TABLE XXI: Decay widths in units of \(10^{-15}\) GeV, and for general values of the Wilson coefficient $a_2$, and branching ratios in % for exclusive nonleptonic decays of the $B_c$ meson. Our central values have been obtained with the AL1 potential.

VII. SUMMARY

We have made a comprehensive and exhaustive study of exclusive semileptonic and nonleptonic two–meson decays of the $B_c$ meson within a nonrelativistic quark model. We have left out semileptonic processes involving a $b \rightarrow u$ transition at the quark level to avoid known deficiencies both at high and low $q^2$ transfers [3]. For similar reasons we have only considered two–meson nonleptonic decay channels that include a $c\bar{c}$ or $B$ meson. Our model respects HQSS constraints in the infinite heavy quark mass limit but hints at sizeable corrections away from that limit for some form
factors. Unfortunately such corrections have not been worked out in perturbative QCD as they have for heavy–light mesons [10].

To check the sensitivity of our results to the inter–quark interaction we have used five different quark–quark potentials. Most observables change only at the level of a few per cent when changing the interaction. There is another source of theoretical uncertainty in the use of nonrelativistic kinematics in the evaluation of the orbital wave functions and the construction of our states in Eq.(11). While this is a very good approximation for the $B_c$ itself it is not necessarily so for mesons with a light quark. We nevertheless think that, to a certain extent, the ignored relativistic effects are contained in an effective way in the free parameters of the inter–quark interaction, which are fitted to experimental data.

Our results for the observables analyzed are in a general good agreement (whenever comparison is possible) with the results obtained within the quasi–potential approach to the relativistic quark model of Ebert et al. [7, 8].

The branching ratios for the leptonic $B^-_c \rightarrow c\bar{c}$ and $B^-_c \rightarrow B$ decays are also in reasonable agreement with the relativistic constituent quark model results of Ivanov et al. [4, 5].

For the nonleptonic $B^-_c \rightarrow \eta \, M_F^-$ and $B^-_c \rightarrow J/\Psi \, M_F^-$ two–meson decay channels with $M_F^- = \pi^-, \rho^-, K^-, K^{*-}$, we find also reasonable agreement (better for the $J/\Psi$ channel) with the Bethe–Salpeter calculation by El-Hady et al. [12] and the light front calculation by Anisimov et al. [19], while our results are a factor of two smaller than the ones by Ivanov et al. [4, 5] and Chang et al. [8, 10, 11], the latter obtained within the nonrelativistic approach to the Bethe–Salpeter equation. For the two–meson decay channels with $\chi_{c0}, \chi_{c1}, h_c, \chi_{c2}$ or $\Psi(3836)$ as the final $c\bar{c}$ meson and $M_F = \pi^-, \rho^-, K^-, K^{*-}$ our results are generally a factor of two smaller than the ones of Ivanov et al. [4, 5], whereas for some channels ($\chi_{c0}\pi^-, \chi_{c0}\rho^-, h_c, \pi^-, h_c\rho^-, \chi_{c1}\pi^-, \chi_{c2}\pi^-, \chi_{c2}\rho^-$) we find very good agreement with the results by Chang et al. [8, 10, 11]. The disagreement with Ivanov et al. extend to the two–meson decay channels with a final $c\bar{c}$ and $D$ mesons. There we find good agreement with the results by El-Hady et al. [12] and the ones obtained within the sum rules of QCD and nonrelativistic QCD by Kiselev [10].

As for the two–meson nonleptonic decay channels $B^-_c \rightarrow M_F^-$ with $M_F = \pi^-, \rho^-, K^-, K^{*-}$ we are in a reasonable good agreement with the results by Anisimov et al. [19]. For the case of a final $\overline{B}_s^0$ or $B_s^0$ the agreement with the results by Ivanov et al. [4, 5] is very good. For the $B^-_c \rightarrow B^- M_F^-$ case with $M_F = \pi^0, \rho^0, K^0, K^{*0}$, and apart from the results by Ebert et al. [8], we find no global agreement with other calculations, neither do they agree with each other.

From the above comparison one sees that there are different models producing sometimes very different results for the same observables. Accurate experimental data will shed light into this issue.

Acknowledgments

We thank J.G. Körner for a critical reading of our work. We also thank A.V. Manohar for discussions on the form factor decomposition in the the infinite heavy quark mass limit. This research was supported by DGI and FEDER funds, under contracts FIS2005-00810, BFM2003-00856 and FPAP-2004-05616, by Junta de Andalucía and Junta de Castilla y León under contracts FQM0225 and SA104/04, and it is part of the EU integrated infrastructure initiative Hadron Physics Project under contract number RII3-CT-2004-506078. J. M. V.-V. acknowledges an E.P.I.F. contract with the University of Salamanca.

APPENDIX A: $\varepsilon^\mu_{(\lambda)}(\vec{P})$ POLARIZATION VECTORS

Different sets of polarization vectors used in this paper:

\[
\vec{P} = \vec{0}
\]

\[
\begin{align*}
\varepsilon^\mu_{(+1)}(\vec{P}) &= (0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon^\mu_{(-1)}(\vec{P}) &= (0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon^\mu_{(0)}(\vec{P}) &= (0, 0, 0, 1)
\end{align*}
\]
\[ \mathbf{\bar{P}} = |\mathbf{\bar{P}}| \mathbf{k} \]

Spin or helicity bases

\[
\begin{align*}
\varepsilon_{(+1)}^\mu (\mathbf{\bar{P}}) &= (0, -\frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(+1)}^\mu (\mathbf{\bar{P}}) &= (0, \frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(+0)}^\mu (\mathbf{\bar{P}}) &= (\frac{|\mathbf{\bar{P}}|}{m}, 0, 0, E(\mathbf{\bar{P}})_m) \\
\varepsilon_{(+1)}^\mu (\mathbf{\bar{P}}) &= (0, -\frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(-1)}^\mu (\mathbf{\bar{P}}) &= (0, -\frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(-0)}^\mu (\mathbf{\bar{P}}) &= (\frac{|\mathbf{\bar{P}}|}{m}, 0, 0, -E(\mathbf{\bar{P}})_m)
\end{align*}
\]

\[ \mathbf{\bar{P}} = -|\mathbf{\bar{P}}| \mathbf{k} \]

Spin base

\[
\begin{align*}
\varepsilon_{(+1)}^\mu (\mathbf{\bar{P}}) &= (0, -\frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(-1)}^\mu (\mathbf{\bar{P}}) &= (0, \frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(+0)}^\mu (\mathbf{\bar{P}}) &= (-\frac{|\mathbf{\bar{P}}|}{m}, 0, 0, E(\mathbf{\bar{P}})_m)
\end{align*}
\]

Helicity base

\[
\begin{align*}
\varepsilon_{(+1)}^\mu (\mathbf{\bar{P}}) &= (0, -\frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(-1)}^\mu (\mathbf{\bar{P}}) &= (0, -\frac{i}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0) \\
\varepsilon_{(+0)}^\mu (\mathbf{\bar{P}}) &= (\frac{|\mathbf{\bar{P}}|}{m}, 0, 0, -E(\mathbf{\bar{P}})_m)
\end{align*}
\]

APPENDIX B: EXPRESSION FOR THE $V^\mu(|\mathbf{\bar{q}}|)$, $V^\mu_{(\lambda)}(|\mathbf{\bar{q}}|)$, $V^\mu_{T(\lambda)}(|\mathbf{\bar{q}}|)$ AND $A^\mu(|\mathbf{\bar{q}}|)$, $A^\mu_{(\lambda)}(|\mathbf{\bar{q}}|)$, $A^\mu_{T(\lambda)}(|\mathbf{\bar{q}}|)$ MATRIX ELEMENTS

Here we give general expressions valid for transitions between a pseudoscalar meson $M_T$ at rest with quark content $q_{f_1} \bar{q}_{f_2}$ and a final $M_F$ meson with total angular momentum and parity $J^\pi = 0^-, 0^+, 1^-, 1^+, 2^+, 2^-, 3^-, 3^+$, three-momentum $-|\mathbf{q}| \mathbf{k}$ and quark content $q_{f_1} \bar{q}_{f_2}$. In the transition it is the quark that changes flavor. The phases of the wave functions are the ones chosen in Eqs. (5, 7, 8). We generally have

\[
V^\mu(|\mathbf{\bar{q}}|) - A^\mu(|\mathbf{\bar{q}}|) = \sqrt{2m_T^2E_F(-q^2)} \sum_{NR} \left< M_F(J^\pi), \lambda = -|\mathbf{\bar{q}}| \mathbf{k} \right| J^{f_1, f_2}_{\mu}(0) \left| M_T(0), \tilde{0} \right>^R
\]

\[
= \sqrt{2m_T^2E_F(-q^2)} \int d^3p \sum_{s_{1}, s_{2}} (\hat{\phi}^{(M_F(J^\pi), \lambda)}_{s_1, f_1}(\mathbf{p}))^* \hat{\phi}^{(M_T(0^-))}_{s_2, f_2}(\mathbf{p}) (\mathbf{p} - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k})
\]

\[
\frac{1}{\sqrt{2E_{f_1}^2-2E_{f_2}}} u_{s_1, f_1} (-\frac{m_{f_1}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k} - \mathbf{p}) \gamma^\mu (I - \gamma_5) u_{s_2, f_2} (\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| - \mathbf{p})
\]

where $V^\mu(A^\mu)$ represent any of the $V^\mu(A^\mu)$, $V^\mu_{(\lambda)}(A^\mu_{(\lambda)})$ or $V^\mu_{T(\lambda)}(A^\mu_{T(\lambda)})$, and where $E_{f_1}$ and $E_{f_2}$ are shorthand notations for $E_{f_1}(-\frac{m_{f_1}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k} - \mathbf{p})$ and $E_{f_1}(-\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k} - \mathbf{p})$ respectively. Defining also $\hat{E}_{f_1} = E_{f_1} + m_{f_1}$ and $\hat{E}_{f_1} = E_{f_1} + m_{f_1}$ we arrive at the following final expressions:

- Case $J^\pi = 0^-$

\[
V^0(|\mathbf{\bar{q}}|) = \sqrt{2m_T^2E_F(-q^2)} \int d^3p \frac{1}{4\pi} (\hat{\phi}^{(M_T(0^-))}_{f_1, f_2}(|\mathbf{p}|))^* \hat{\phi}^{(M_F(J^\pi))}_{f_1, f_2}(\mathbf{p} - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k})
\]

\[
\frac{\hat{E}_{f_1} \hat{E}_{f_2}}{4E_{f_1}E_{f_2}} \left( 1 + \frac{(-\frac{m_{f_1}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k} - \mathbf{p}) \cdot (\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k} - \mathbf{p})}{\hat{E}_{f_1} \hat{E}_{f_2}} \right)
\]

\[
V^3(|\mathbf{\bar{q}}|) = \sqrt{2m_T^2E_F(-q^2)} \int d^3p \frac{1}{4\pi} (\hat{\phi}^{(M_T(0^-))}_{f_1, f_2}(|\mathbf{p}|))^* \hat{\phi}^{(M_F(J^\pi))}_{f_1, f_2}(\mathbf{p} - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| \mathbf{k})
\]

\[
\frac{\hat{E}_{f_1} \hat{E}_{f_2}}{4E_{f_1}E_{f_2}} \left( \frac{\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| - p_z}{\hat{E}_{f_1}} + \frac{\frac{m_{f_1}}{m_{f_1} + m_{f_2}} |\mathbf{\bar{q}}| - p_z}{\hat{E}_{f_2}} \right)
\]
• Case $J^\pi = 0^+$

$$A^0(|q^\prime|) = \sqrt{2m_fE_F(-\bar{q})} \int d^3p \frac{1}{4\pi|p|} \left( \hat{\phi}^{(M_f(0^+))}(\bar{p}|) \right)^{\ast} \hat{\phi}^{(M_i(0^-))}(\bar{p} - \frac{m_f}{m_f + m_z}|\bar{q}|\bar{k})$$

$$= \sqrt{\frac{m_{f_1}E_{f_1}}{E_{f_1}E_{f_1}}} \left( \bar{p} \cdot (-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}) + \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p} \right)$$

$$A^3(|q^\prime|) = \sqrt{2m_fE_F(-\bar{q})} \int d^3p \frac{1}{4\pi|p|} \left( \hat{\phi}^{(M_f(0^+))}(\bar{p}|) \right)^{\ast} \hat{\phi}^{(M_i(0^-))}(\bar{p} - \frac{m_f}{m_f + m_z}|\bar{q}|\bar{k})$$

$$= \sqrt{\frac{m_{f_1}E_{f_1}}{E_{f_1}E_{f_1}}} \left( p_z \left( 1 - \frac{(-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}) \cdot \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}}{E_{f_1}E_{f_1}} \right) \right)$$

$$+ \frac{1}{E_{f_1}E_{f_1}} \left( \frac{(-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}| - p_z) \cdot \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}}{E_{f_1}E_{f_1}} \right)$$

$$+ \frac{1}{E_{f_1}E_{f_1}} \left( \frac{(-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}| - p_z) \cdot \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}}{E_{f_1}E_{f_1}} \right) \right) \right) \right) \right)$$

(B3)

• Case $J^\pi = 1^-$

$$V_{\lambda=-1}^{(1^-)}(|q^\prime|) = -\frac{i}{\sqrt{2}} \sqrt{2m_fE_F(-\bar{q})} \int d^3p \frac{1}{4\pi|p|} \left( \hat{\phi}^{(M_f(1^-))}(\bar{p}|) \right)^{\ast} \hat{\phi}^{(M_i(0^-))}(\bar{p} - \frac{m_f}{m_f + m_z}|\bar{q}|\bar{k})$$

$$= \sqrt{\frac{m_{f_1}E_{f_1}}{E_{f_1}E_{f_1}}} \left( \bar{p} \cdot (-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}) + \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p} \right)$$

and similarly

$$A_{\lambda=0}^{(1^-)}(|q^\prime|) = i \sqrt{2m_fE_F(-\bar{q})} \int d^3p \frac{1}{4\pi|p|} \left( \hat{\phi}^{(M_f(1^-))}(\bar{p}|) \right)^{\ast} \hat{\phi}^{(M_i(0^-))}(\bar{p} - \frac{m_f}{m_f + m_z}|\bar{q}|\bar{k})$$

$$= \sqrt{\frac{m_{f_1}E_{f_1}}{E_{f_1}E_{f_1}}} \left( \bar{p} \cdot (-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}) + \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p} \right)$$

$$A_{\lambda=1}^{(1^-)}(|q^\prime|) = \frac{1}{\sqrt{2}} \sqrt{2m_fE_F(-\bar{q})} \int d^3p \frac{1}{4\pi|p|} \left( \hat{\phi}^{(M_f(1^-))}(\bar{p}|) \right)^{\ast} \hat{\phi}^{(M_i(0^-))}(\bar{p} - \frac{m_f}{m_f + m_z}|\bar{q}|\bar{k})$$

$$= \sqrt{\frac{m_{f_1}E_{f_1}}{E_{f_1}E_{f_1}}} \left( \bar{p} \cdot (-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}) + \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p} \right)$$

$$A_{\lambda=0}^{(1^-)}(|q^\prime|) = i \sqrt{2m_fE_F(-\bar{q})} \int d^3p \frac{1}{4\pi|p|} \left( \hat{\phi}^{(M_f(1^-))}(\bar{p}|) \right)^{\ast} \hat{\phi}^{(M_i(0^-))}(\bar{p} - \frac{m_f}{m_f + m_z}|\bar{q}|\bar{k})$$

$$= \sqrt{\frac{m_{f_1}E_{f_1}}{E_{f_1}E_{f_1}}} \left( \bar{p} \cdot (-\frac{m_{f_1}}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p}) + \frac{m_f}{m_{f_1} + m_f} |\bar{q}|\bar{k} - \bar{p} \right)$$

(B4)
• Case $J^z = 1^+$

\[
 V^{(1^+, S_q=0)}_{\lambda=0} (|q|) = i\sqrt{3} \sqrt{2 m_f^2 E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi^{(M_F(1^+, S_q=0))}_{\vec{p}_f', f_2} (|\vec{p}|) \right)^* \left( \phi^{(M_f(0^-))}_{\vec{p}_f', f_2} \right) \left( \vec{p} - \frac{m_f}{m_{f_1} + m_f} |\vec{q} - \vec{p}| \right)
\]

\[
 \left( \frac{\vec{E}_{f_1}' \vec{E}_{f_1}}{4E_{f_1}' E_{f_1}} \right) \left( \frac{\vec{q} (|\vec{p}_z^2 - \vec{p}^2|)}{4E_{f_1}' E_{f_1}} \right) \left( \frac{1}{\vec{E}_{f_1}'} + \frac{1}{\vec{E}_{f_1}} \right)
\]

\[
 V^{(1^+, S_q=1)}_{\lambda=0} (|q|) = -i \sqrt{3} \sqrt{2 m_f^2 E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi^{(M_F(1^+, S_q=1))}_{\vec{p}_f', f_2} (|\vec{p}|) \right)^* \left( \phi^{(M_f(0^-))}_{\vec{p}_f', f_2} \right) \left( \vec{p} - \frac{m_f}{m_{f_1} + m_f} |\vec{q} - \vec{p}| \right)
\]

\[
 \left( \frac{\vec{E}_{f_1}' \vec{E}_{f_1}}{4E_{f_1}' E_{f_1}} \right) \left( \frac{\vec{q} (|\vec{p}_z^2 - \vec{p}^2|)}{4E_{f_1}' E_{f_1}} \right) \left( \frac{1}{\vec{E}_{f_1}'} + \frac{1}{\vec{E}_{f_1}} \right)
\]

\[
 V^{(1^+, S_q=0)}_{\lambda=0} (|q|) = i\sqrt{3} \sqrt{2 m_f^2 E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi^{(M_F(1^+, S_q=0))}_{\vec{p}_f', f_2} (|\vec{p}|) \right)^* \left( \phi^{(M_f(0^-))}_{\vec{p}_f', f_2} \right) \left( \vec{p} - \frac{m_f}{m_{f_1} + m_f} |\vec{q} - \vec{p}| \right)
\]

\[
 \left( \frac{\vec{E}_{f_1}' \vec{E}_{f_1}}{4E_{f_1}' E_{f_1}} \right) \left( \frac{\vec{q} (|\vec{p}_z^2 - \vec{p}^2|)}{4E_{f_1}' E_{f_1}} \right) \left( \frac{1}{\vec{E}_{f_1}'} + \frac{1}{\vec{E}_{f_1}} \right)
\]

\[
 V^{(1^+, S_q=1)}_{\lambda=0} (|q|) = -i \sqrt{3} \sqrt{2 m_f^2 E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi^{(M_F(1^+, S_q=1))}_{\vec{p}_f', f_2} (|\vec{p}|) \right)^* \left( \phi^{(M_f(0^-))}_{\vec{p}_f', f_2} \right) \left( \vec{p} - \frac{m_f}{m_{f_1} + m_f} |\vec{q} - \vec{p}| \right)
\]

\[
 \left( \frac{\vec{E}_{f_1}' \vec{E}_{f_1}}{4E_{f_1}' E_{f_1}} \right) \left( \frac{\vec{q} (|\vec{p}_z^2 - \vec{p}^2|)}{4E_{f_1}' E_{f_1}} \right) \left( \frac{1}{\vec{E}_{f_1}'} + \frac{1}{\vec{E}_{f_1}} \right)
\]

\[
 A^{(1^+, S_q=0)}_{\lambda=-1} (|q|) = -i \sqrt{3} \sqrt{2 m_f^2 E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi^{(M_F(1^+, S_q=0))}_{\vec{p}_f', f_2} (|\vec{p}|) \right)^* \left( \phi^{(M_f(0^-))}_{\vec{p}_f', f_2} \right) \left( \vec{p} - \frac{m_f}{m_{f_1} + m_f} |\vec{q} - \vec{p}| \right)
\]

\[
 \left( \frac{\vec{E}_{f_1}' \vec{E}_{f_1}}{4E_{f_1}' E_{f_1}} \right) \left( \frac{\vec{q} (|\vec{p}_z^2 - \vec{p}^2|)}{4E_{f_1}' E_{f_1}} \right) \left( \frac{1}{\vec{E}_{f_1}'} + \frac{1}{\vec{E}_{f_1}} \right)
\]

\[
 \left( \frac{\vec{E}_{f_1}' \vec{E}_{f_1}}{4E_{f_1}' E_{f_1}} \right) \left( \frac{\vec{p}_z^2 |\vec{q}|}{4E_{f_1}' E_{f_1}} \right) \left( \frac{1}{\vec{E}_{f_1}'} + \frac{1}{\vec{E}_{f_1}} \right)
\]

\[
 (B5)
\]

\[
 (B6)
\]
\[ A_{\lambda=-1}^{(1+S_q=1)}(|q|) = i \frac{\sqrt{3}}{2} \sqrt{2 m_f E_F(-q)} \int d^3p \frac{1}{4\pi|\vec{p}|^2} \left( \hat{\phi}_{(M_F(1^+))}^{(M_F(1^+))}(|\vec{p}|) \right)^* \hat{\phi}_{j_f,j_f}^{(M_F(0^-))} \left( \vec{p} - \frac{m_{f_1} + m_{f_2}}{m_{f_1} + m_{f_2}} |q| \vec{k} \right) \]
\[
\sqrt{\frac{E_{f_1} E_{f_1}}{E_{f_1} E_{f_1}}} \left\{ p_z \left( 1 - \frac{(-m_{f_1} + m_{f_2}) |q| \vec{k} - \vec{p} \cdot (m_{f_1} + m_{f_2}) |q| \vec{k} - \vec{p} \right) \right. \\
+ \frac{m_{f_2} - m_{f_1} - p_z^2 |q|}{m_{f_1} + m_{f_2} E_{f_1} E_{f_1}} \} \\
\]

\( \bullet \) Case \( J^\pi = 2^- \)

\[ V_{T_\lambda=0}^{(2^-)}(|q|) = i \frac{\sqrt{15}}{2} \sqrt{2 m_f E_F(-q)} \int d^3p \frac{1}{4\pi|\vec{p}|^2} \left( \hat{\phi}_{j_f,j_f}^{(M_F(2^-))}(|\vec{p}|) \right)^* \hat{\phi}_{j_f,j_f}^{(M_F(0^-))} \left( \vec{p} - \frac{m_{f_1} + m_{f_2}}{m_{f_1} + m_{f_2}} |q| \vec{k} \right) \]
\[
\sqrt{\frac{E_{f_1} E_{f_1}}{E_{f_1} E_{f_1}}} \left\{ p_z \left( -p_z - \frac{m_{f_1} - m_{f_2}}{E_{f_1}} |q| \vec{k} \right) \\
- p_z p_y^2 \left( \frac{1}{E_{f_1}} - \frac{1}{E_{f_1}} \right) \} \\
\]
\[ V_{T_\lambda=1}^{(2^-)}(|q|) = i \frac{\sqrt{15}}{2} \sqrt{2 m_f E_F(-q)} \int d^3p \frac{1}{4\pi|\vec{p}|^2} \left( \hat{\phi}_{j_f,j_f}^{(M_F(2^-))}(|\vec{p}|) \right)^* \hat{\phi}_{j_f,j_f}^{(M_F(0^-))} \left( \vec{p} - \frac{m_{f_1} + m_{f_2}}{m_{f_1} + m_{f_2}} |q| \vec{k} \right) \]
\[
\sqrt{\frac{E_{f_1} E_{f_1}}{E_{f_1} E_{f_1}}} \left\{ p_z \left( 1 - \frac{(-m_{f_1} - m_{f_2}) |q| \vec{k} - \vec{p} \cdot (m_{f_1} + m_{f_2}) |q| \vec{k} - \vec{p} \right) \right. \\
- p_z p_y^2 |q| \frac{m_{f_2} - m_{f_1} - p_z^2 |q|}{m_{f_1} + m_{f_2} E_{f_1} E_{f_1}} \{ \frac{1}{E_{f_1}} \} \\
\]

and similarly

\[ A_{T_\lambda=1}^{(2^-)}(|q|) = i \frac{\sqrt{5}}{2} \sqrt{2 m_f E_F(-q)} \int d^3p \frac{1}{4\pi|\vec{p}|^2} \left( \hat{\phi}_{j_f,j_f}^{(M_F(2^-))}(|\vec{p}|) \right)^* \hat{\phi}_{j_f,j_f}^{(M_F(0^-))} \left( \vec{p} - \frac{m_{f_1} + m_{f_2}}{m_{f_1} + m_{f_2}} |q| \vec{k} \right) \]
\[
\sqrt{\frac{E_{f_1} E_{f_1}}{E_{f_1} E_{f_1}}} \left\{ p_z \left( 1 - \frac{(-m_{f_1} + m_{f_2}) |q| \vec{k} - \vec{p} \cdot (m_{f_1} + m_{f_2}) |q| \vec{k} - \vec{p} \right) \right. \\
- p_z p_y^2 |q| \frac{m_{f_2} - m_{f_1} - p_z^2 |q|}{m_{f_1} + m_{f_2} E_{f_1} E_{f_1}} \{ \frac{1}{E_{f_1}} \} \\
\]

\( \bullet \) Case \( J^\pi = 2^+ \)

\[ V_{T_\lambda=1}^{(2^+)}(|q|) = i \frac{\sqrt{3}}{2} \sqrt{2 m_f E_F(-q)} \int d^3p \frac{1}{4\pi|\vec{p}|^2} \left( \hat{\phi}_{j_f,j_f}^{(M_F(2^+))}(|\vec{p}|) \right)^* \hat{\phi}_{j_f,j_f}^{(M_F(0^+))} \left( \vec{p} - \frac{m_{f_1} + m_{f_2}}{m_{f_1} + m_{f_2}} |q| \vec{k} \right) \]
\[
A_{T=0}^{(2^+)}(|q'|) = \frac{-i}{\sqrt{2}} \sqrt{2m_1 2E_F(-q')} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi_{\vec{f}_1', \vec{f}_2}^{(M_F^{(2^+)})}(|\vec{p}|) \right)^* \phi_{\vec{f}_1, \vec{f}_2}^{(M_f^0)} \left( \vec{p} - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} \right) \\
\frac{\hat{E}_{\vec{f}_1} \hat{E}_{\vec{f}_2}}{4E_{\vec{f}_1} E_{\vec{f}_2}} \left( \frac{p_1^2 - p_2^2 - p_z |q'| m_{f_1}'}{m_{f_1}'} - \frac{p_2^2 - p_z |q'| m_{f_2}'}{m_{f_2}'} \right)
\]

and similarly

\[
A_{T=1}^{(2^+)}(|q'|) = \frac{i \sqrt{3}}{2} \sqrt{2m_1 2E_F(-q')} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi_{\vec{f}_1', \vec{f}_2}^{(M_F^{(2^+)})}(|\vec{p}|) \right)^* \phi_{\vec{f}_1, \vec{f}_2}^{(M_f^0)} \left( \vec{p} - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} \right) \\
\frac{\hat{E}_{\vec{f}_1} \hat{E}_{\vec{f}_2}}{4E_{\vec{f}_1} E_{\vec{f}_2}} \left\{ p_z \left( 1 - \frac{\left(-\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} - \vec{p} \right) \cdot \left(\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} - \vec{p} \right)}{E_{\vec{f}_1} E_{\vec{f}_2}} \right) \right. \\
+ \frac{4p_z p_2^2 - p_2^2 |q'| m_{f_2}'}{m_{f_1} + m_{f_2}} \right\}
\]

\[
A_{T=0}^{(2^+)}(|q'|) = -i \sqrt{2} \sqrt{2m_1 2E_F(-q')} \int d^3 p \frac{1}{4\pi |\vec{p}|} \left( \phi_{\vec{f}_1', \vec{f}_2}^{(M_F^{(2^+)})}(|\vec{p}|) \right)^* \phi_{\vec{f}_1, \vec{f}_2}^{(M_f^0)} \left( \vec{p} - \frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} \right) \\
\frac{\hat{E}_{\vec{f}_1} \hat{E}_{\vec{f}_2}}{4E_{\vec{f}_1} E_{\vec{f}_2}} \left\{ p_z \left( 1 - \frac{\left(-\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} - \vec{p} \right) \cdot \left(\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} - \vec{p} \right)}{E_{\vec{f}_1} E_{\vec{f}_2}} \right) \right. \\
+ \frac{1}{E_{\vec{f}_1} E_{\vec{f}_2}} \left[ 2p_z \left(-\frac{m_{f_1}'}{m_{f_1} + m_{f_2}} |q'| - p_z \right) \cdot \left(\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| - p_z \right) \right. \\
+ \left( p_z^2 + p_2^2 \right) \left(-p_z + \frac{m_{f_2} - m_{f_1}'}{2(m_{f_1} + m_{f_2})} |q'| \right) \left\}
\]

**APPENDIX C: TRANSITIONS INVOLVING ANTIQUARKS**

When it is the antiquark that suffers the decay the expressions are modified as described below for a general transition between a pseudoscalar meson $M_f$ at rest with quark content $q_f, \bar{q}_{f_2}$ and a final $M_F$ meson with total angular momentum and parity $J^P = 0^-, 0^+, 1^-, 1^+, 2^-, 2^+$, three-momentum $-|\vec{q}'| \vec{k}'$ and quark content $q_{f_1}, \bar{q}_{f_2}$. We have

\[
\mathcal{V}^\mu(|\vec{q}'|) - A^\mu(|\vec{q}'|) = \sqrt{2m_1 2E_F(-q')} \left< M_F(J^P), \lambda - |q'| \vec{k}' \left| Jf_2 f_2' \bar{\mu} (0) \right| M_f(0^+), 0 \right>_{NR} \\
= -\sqrt{2m_1 2E_F(-q')} \int d^3 p \sum_{s_1, s_2} \left( \phi_{(s_1 f_1, s_2 f_2)}^{(M_F(J^P), \lambda)}(\vec{p}) \right)^* \phi_{(s_1 f_1, s_2 f_2)}^{(M_f^0)}(\vec{p} + \frac{m_{f_1}}{m_{f_1} + m_{f_2}} |q'| \vec{k}') \\
\frac{-1}{{\sqrt{2E_{f_2} E_{f_1} E_{f_2}}} \bar{v}_{s_1, f_2} \left( \frac{m_{f_1}}{m_{f_1} + m_{f_2}} |q'| \vec{k} - \vec{p} \right) \gamma^\mu (I - \gamma_5) v_{s_1, f_2} \left(-\frac{m_{f_2}}{m_{f_1} + m_{f_2}} |q'| \vec{k} - \vec{p} \right)}
\]

\[(C1)\]
where $\Psi$ represent any of the $V^\mu(A^\mu), V^{\mu}_{(\lambda)}(A^\mu_{(\lambda)}),$ or $V^\mu_{(\lambda)}(A^\mu_{(\lambda)}),$ and $E_{f_2}^2, E_{f_2}$ are shorthand notation for $E_{f_2}(-\frac{m_{f_2}}{m_f + m_{f_2}}|\vec{q}|\vec{k} + \vec{p}), E_{f_2}(-\frac{m_{f_2}}{m_f + m_{f_2}}|\vec{q}|\vec{k} + \vec{p})$. We can use now that

$$v_{s,f}(\vec{p}) = (-1)^{(1/2) - s}C\bar{u}_{s,f}(\vec{p}) \quad ; \quad \bar{v}_{s,f}(\vec{p}) = -(1)^{(1/2) - s}u^T_{s,f}(\vec{p})C$$

(C2)

where $C$ is a matrix given in the Fermi–Dirac representation that we use by

$$C = i\gamma^2\gamma^0$$

(C3)

and that satisfies

$$C = -C^{-1} = -C^\dagger = -C^T \quad ; \quad C\gamma^\mu C^\dagger = -\gamma_\mu$$

(C4)

Using the above information and making the change of variable $\vec{p} \rightarrow -\vec{p}$ we can rewrite

$$\Psi^\dagger(|\vec{p}|) - A^\mu(|\vec{p}|) = \sqrt{2m_f}2E_F(\vec{q}) \int d^4p \sum s_2 \sum s_1, s_2 \left(\gamma(\hat{s}_f(\lambda), \hat{\lambda}) \right)^* \phi_{s_1, f_1}(s_2, f_2) \left(-\left(\vec{p} - \frac{m_{f_2}}{m_f + m_{f_2}}|\vec{q}|\vec{k}\right)\right)$$

$$\frac{1}{\sqrt{2E_{f_2}^22E_{f_2}}} \bar{u}_{s_2, f_2}(-\frac{m_{f_2}}{m_f + m_{f_2}}|\vec{q}|\vec{k} - \vec{p}) \gamma^\mu(-I - \gamma_5)u_{s_2, f_2}(\frac{m_f}{m_f + m_{f_2}}|\vec{q}|\vec{k} - \vec{p})$$

(C5)

By comparison with the corresponding expressions involving quarks we find that, apart from the changes in the masses involved, there is an extra minus sign for the vector part, and, due to Clebsch–Gordan re-arrangements and the fact that $Y_{lm}(-\vec{p}) = (-1)^l Y_{lm}(\vec{p})$, a global sign given by $(-1)^{l_+ + s_1 - l_F - s_F}$ where $l_1, s_1, s_F$ are the orbital and spin angular momenta of the initial (final) meson.

In any case this implies a change of sign in the relative phase between vector and axial contributions, which in its term produces a sign change in the tensor helicity components combination $H_\mu$ due to the fact that $\mathcal{H}_{+1+1}$ goes into $\mathcal{H}_{-1-1}$ and vice versa. All other tensor helicity components combinations defined in Eq. (39) keep their signs.

A simple way of anticipating the above result is the following: the current for $\bar{q}_{f_2}$ decay into $\bar{q}_{f_2}$ is

$$\Psi(0)\gamma^\mu(I - \gamma_5)\Psi(0)$$

(C6)

But for antiquarks the fields that play the similar role as the $\Psi$ fields play for quarks are the charge conjugate ones $\Psi^C$. In terms of the latter the above current is written as

$$\Psi(0)\gamma^\mu(I - \gamma_5)\Psi(0) = \Psi^C(0)\gamma^\mu(I - \gamma_5)\Psi^C(0)$$

(C7)

Now this is similar to the current for quark decay but with an extra minus sign in the vector part. Whatever other changes might come from reorderings in the wave functions we will have an extra relative sign between the vector and axial part.

**APPENDIX D: EXPRESSIONS FOR THE HELICITY COMPONENTS OF THE HADRON TENSOR**

In this appendix we give the expressions for the non–zero helicity components $\mathcal{H}_{+1}$ of the hadron tensor, as defined in Eq. (39), corresponding to a $B^-_c \rightarrow c\bar{c}$ transition. The different cases correspond to the ones discussed in the main text.

- **Case 0**
  $$\mathcal{H}_{+0}(P_{B_c}, P_{c\bar{c}}) = \frac{(m_{B_c}^2 - m_{c\bar{c}}^2)}{\sqrt{q^2}} F_+(q^2) + \sqrt{q^2} F_-(q^2)$$
  $$\mathcal{H}_{+0}(P_{B_c}, P_{c\bar{c}}) = \mathcal{H}_{+0}(P_{B_c}, P_{c\bar{c}}) = \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2) \left(\frac{m_{B_c}^2 - m_{c\bar{c}}^2}{q^2} F_+(q^2) + F_+(q^2) F_-(q^2)\right)$$
  $$\mathcal{H}_{00}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{q^2} F_+(q^2)$$

(D1)
Case $0^{-} \to 1^{-}, 1^{+}$.

\[
\mathcal{H}_{tt}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{4m_{c\bar{c}}^2 q^2} \left( (m_{B_c} - m_{c\bar{c}}) (A_0(q^2) - A_+(q^2)) - \frac{q^2}{m_{B_c} + m_{c\bar{c}}} A_-(q^2) \right)^2
\]

\[
\mathcal{H}_{t0}(P_{B_c}, P_{c\bar{c}}) = \mathcal{H}_{0t}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}} \sqrt{q^2}} \left( (m_{B_c} - m_{c\bar{c}}) (A_0(q^2) - A_+(q^2)) - \frac{q^2}{m_{B_c} + m_{c\bar{c}}} A_-(q^2) \right)
\times \left[ (m_{B_c} - m_{c\bar{c}}) \frac{m_{B_c}^2 - q^2 - m_{c\bar{c}}^2}{2m_{c\bar{c}} \sqrt{q^2}} A_0(q^2) - \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}} \sqrt{q^2}} A_+(q^2) \right]
\]

\[
\mathcal{H}_{t+1}(P_{B_c}, P_{c\bar{c}}) = \left( \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{m_{B_c} + m_{c\bar{c}}} V(q^2) + (m_{B_c} - m_{c\bar{c}}) A_0(q^2) \right)^2
\]

\[
\mathcal{H}_{-1-t}(P_{B_c}, P_{c\bar{c}}) = \left( -\frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{m_{B_c} + m_{c\bar{c}}} V(q^2) + (m_{B_c} - m_{c\bar{c}}) A_0(q^2) \right)^2
\]

\[
\mathcal{H}_{00}(P_{B_c}, P_{c\bar{c}}) = \left( (m_{B_c} - m_{c\bar{c}}) \frac{m_{B_c}^2 - q^2 - m_{c\bar{c}}^2}{2m_{c\bar{c}} \sqrt{q^2}} A_0(q^2) - \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}} \sqrt{q^2}} A_+(q^2) \right)^2
\]

(D2)

For a $B_c^+ \to B$ transition (with $B$ representing any of the $B = T^0, B^0, B^-, B^{*-}$, where it is the $\bar{c}$ antiquark that decays, we have to change the mass of the final meson in the expressions above and take into account the changes in the form factors that derive from the discussions in appendix C.

Case $0^{-} \to 2^{-}, 2^{+}$.

\[
\mathcal{H}_{tt}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda^2(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{24m_{c\bar{c}}^2 q^2} \left( T_1(q^2) + (m_{B_c}^2 - m_{c\bar{c}}^2) T_2(q^2) + q^2 T_3(q^2) \right)^2
\]

\[
\mathcal{H}_{t0}(P_{B_c}, P_{c\bar{c}}) = \mathcal{H}_{0t}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda^{3/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{24m_{c\bar{c}}^2 q^2} \left( T_1(q^2) + (m_{B_c}^2 - m_{c\bar{c}}^2) T_2(q^2) + q^2 T_3(q^2) \right)
\times \left( (m_{B_c}^2 - q^2 - m_{c\bar{c}}^2) T_1(q^2) + \lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2) T_2(q^2) \right)
\]

\[
\mathcal{H}_{+1-t}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{8m_{c\bar{c}}^2} \left( T_1(q^2) - \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2) T_4(q^2) \right)^2
\]

\[
\mathcal{H}_{-1-t}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{8m_{c\bar{c}}^2} \left( T_1(q^2) + \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2) T_4(q^2) \right)^2
\]

\[
\mathcal{H}_{00}(P_{B_c}, P_{c\bar{c}}) = \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{24m_{c\bar{c}}^2 q^2} \left( (m_{B_c}^2 - q^2 - m_{c\bar{c}}^2) T_1(q^2) + \lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2) T_2(q^2) \right)^2
\]

(D3)
