DEEPLY BOUNDED PIONIC STATES WITH THE (Σ⁻, A) REACTION.

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ABSTRACT

We study the reaction Σ⁻ + A → Λ + (Λ n⁻) with the n⁻ bound in the nucleus, as a means of producing deeply bound pionic states in nuclei, so far unobserved. The reaction is similar to the (n, p) reaction but, because of the Σ⁻. A mass difference, it allows the reaction to occur with smaller momentum transfer, thus increasing the transition probability and reducing the effects of distortion. The ratios of signal to background are one to two orders of magnitude better than in the (n, p) reaction.
momentum transfer given to the pionic atom, of the order of 200 MeV/c at \( T_n = 400 \) MeV, which is difficult to accommodate in the atom. This large momentum transfer has also another unwanted side effect: the distortion of the \( n \) and \( p \) waves reduces the cross section appreciably with reduction factors of 30–100 as an average. It is easy to realize why the momentum transfer influences the distortion. The distortion eliminates much of the contribution from small impact parameters and concentrates on large values of \( b \). Hence the transition matrix elements, involving the factor \( \exp \left( i \vec{q} \cdot \vec{T} \right) \) with large \( \vec{q} \) and \( \vec{T} \), will have large cancellations.

This is why some of the new proposals go in the direction of making the reaction recoiless or near recoiless like those of refs.\(^{15}\),\(^{15}\). In ref.\(^{15}\) the momentum transfer is tuned such that \( qR \) (\( R \), nuclear radius) is equal to 1, the angular momentum of the pionic nuclear state formed. However, there is one price to pay for this gain: these reactions, ejecting one nucleon from the nucleus, involve directly only the valence nucleons, while in the \((n, p)\) reaction all the nucleons contribute coherently to the amplitude and one benefits from the \( A^2 \) factor in the cross section.

It would be thus ideal to find a reaction combining these two features: recoiless and coherent. The reason why this is not possible with the \((n, p)\) reaction is that the pion energy comes from the difference between the neutron and proton kinetic energies and as a consequence there must be a difference between the neutron and proton momenta. The transfer of energy and no momentum is possible if we have a reaction of the type \((a, b)\) with two particles where \( m_a - m_b = m_\pi \). There is no such possibility using stable particles and a reaction which proceeds through strong or electromagnetic interaction to ensure reasonable cross sections. However there is a close case in the \((2\Sigma^-, A)\) reaction, where 82 MeV of the pion energy can come from the \( \Sigma^-\). A mass difference, thus resulting in smaller momentum transfer to the pionic atom. This is indeed the case as one can see in fig. 1 where we plot the momentum transfer given to the pionic atom for the \((n, p)\) and \((2\Sigma^-, A)\) reactions as a function of the \( n \) or \( \Sigma^- \) kinetic energies. As we can see, for...
kinetic energies around 400–800 MeV the momentum transfer in the (\(\Sigma^-,\Lambda\))
reaction is appreciably smaller than in the (n, p) reaction.

From the technical point of view the (\(\Sigma^-,\Lambda\)) reaction is very similar
to the (n, p). Indeed the \(\Sigma^-\text{p} \rightarrow \Lambda\text{N}\), in the meson exchange language, is
mediated in both cases by \(\pi\) and \(\rho\) exchange. Hence only the coupling
constants and isospin factors are different from one case to the other,
although the peculiar hard core interaction of the Nijmegen model\(^{[7,10]}\),
which we use, and the lack of form factor in the hyperon–meson vertices,
produces slight differences. We use version D of the Nijmegen model\(^{[7]}\).
We follow exactly the same steps as in ref.\(^{10}\) and the same equations are valid
with minor changes. The couplings of the mesons to the baryons are given
in detail in ref.\(^{10}\) and with the help of these two references we can
immediately write

\[
\frac{d^2\sigma}{d\Omega_A dE_A} = \frac{1}{|\mathbf{P}_\Sigma|} \frac{M_\Sigma M_\Lambda}{2\omega_\pi} \frac{1}{2\pi^3} \frac{\Gamma}{[E_\Sigma + M_\Lambda - E_A - E(A\pi^-)]^2 + (\Gamma/2)^2} \times \sum_s |T_s|^2
\]

(1)

where \(\omega_\pi, \Gamma\) are the energy and width of the pionic state and \(T\) is the transition
t matrix from the model depicted diagramatically in fig. 2, which gets
contribution with \(\pi\) exchange followed by the \(\pi N \rightarrow \pi N\) s-wave interactions,
\(\pi \rightarrow \rho\) exchange with \(p\)-wave meson–N coupling exciting a \(\Delta\) (including the
indirect effect of correlations which generate the Landau Migdal force in the
formalism of ref.\(^{20}\)) and \(\pi\) exchange followed by the Coulomb interaction of
the pion with the nucleus. Thus

\[
T = T^{(s)} + T^{(p)} + T^{(e)}
\]

(2)

where

\[
-iT^{(s)} = -\frac{f_{\Sigma\Lambda\pi}}{\mu} \frac{1}{q^2 - q^{02} - q^2_{\mu} - \mu^2} (1) 4\pi \left\{ \frac{2\Lambda_s}{\mu} + \frac{2\Lambda_q}{\mu} \left( \frac{N-Z}{A} \right) \right\}
\]

\[(\text{Continued in the next page})\]
the trapped pion $\hat{p}(\Phi^*_1 m) = \hat{p}(\Phi^*_1 m)$, $s' = (q + q_N)^2 - (q_o + M_N)^2 - q_N^2$ (with $M_N$ the nucleon mass), $\rho(z)$ is the nuclear density and $\Phi^*_1 m(\tau)$ the wave function of the pion in the atomic orbit $n, l, m$.

The set of parameters taken are $15, 19$.

The distortion of the $\Sigma^-$ and $\Lambda$ waves is done in the eikonal approximation as in ref. $15$ with the replacement

$$e^{i \beta \tau} = e^{i \hat{p}_\Sigma \hat{\tau}^\Sigma} e^{-i \hat{p}_\Lambda \hat{\tau}^\Lambda} \rightarrow$$

$$e^{i \hat{p}_\Sigma \hat{\tau}^\Sigma} e^{-i/2} \int_{-\infty}^{\infty} d\Sigma N(p, \Sigma) \rho(0, 0') d\Sigma'$$

$$e^{-i/2} \int_{-\infty}^{\infty} d\Lambda N(p, \Lambda) \rho(0, 0') d\Lambda$$

with $\hat{\tau}^\Sigma = \hat{\tau}^\Sigma = \frac{p}{|p_N|}$.

The values for the cross sections of $\Sigma N$ and $\Lambda N$ are taken from experiment $20, 21$ and from refs. $17, 19$. For the $\Sigma N$ total cross sections entering the distortion through eq. (4) we take $\sigma_{\Sigma N} = 30$ mb and $\sigma_{\Lambda N} = 20$ mb at the two energies $T_N = 600$ and $800$ MeV which we have considered.

In figs. 3 and 4 we show the results for the excitation of the 1s and 2p pionic states of $^{208}$Pb at the peak of the Lorentzian distribution with and without distortion, as a function of the scattering angle for $T_N = 600$ MeV. As we can see the effect of distortion on the 1s state is more pronounced than in the 2p state, as we might expect since the s-state gets contribution from the inner part of the nucleus, which is eliminated by the distortion, while the p-state gets contribution more from the surface. The peak of the distributions corresponds to $\theta^o$.

It is interesting to compare the effect of the distortion with the case of the $(n, p)$ reaction $15$. The effect of distortion depends very much on the momentum transfer. The momentum transfer at $T_N$ or $T_N = 600$ MeV and zero degrees are of the order of 175 MeV and 107 MeV respectively. If we concentrate at zero degrees for the purpose of comparison, the reduction factor here is 45 for the 1s state while this factor is 650 in the $(n, p)$ reaction at the same kinetic energy ($600$ MeV). For the 2p states the reduction factors are 2.65 and 60 respectively. In both cases there is a gain of a factor 15 to 20 in the $(\Sigma^-, \Lambda)$ reaction with respect to the $(n, p)$, because of the more moderate role played by the distortion in the $(\Sigma^-, \Lambda)$ reaction. For the absolute values of the cross sections without distortion we obtain $0.4$ mb/sr MeV here, versus $0.08$ mb/sr MeV of the $(n, p)$ reaction for the 1s state at zero degrees. In the case of the 2p states these numbers are $0.23$ mb/sr MeV and $0.35$ mb/sr MeV. This is in spite of a factor 8 smaller from the coupling constants in the $(\Sigma^-, \Lambda)$ case versus the $(n, p)$ reaction, which means that we have also gained in the undistorted transition amplitude because of the smaller momentum transfer.

The results at $T_N = 800$ MeV are very similar with slightly larger cross sections. We have also evaluated the results for the $\Sigma^- + ^{40}$Ca $\rightarrow \Lambda + (^{40}$Ca $\pi^-)$ reaction. In Table I we show the results for the two nuclei at zero degrees and $T_N = 800$ MeV. For the $^{40}$Ca case we evaluate the cross section for the 1s state only since the 2p state is already observable with the X-ray technique.

The background for the reaction comes from the inclusive $(\Sigma^-, \Lambda)$ reaction while in the $(n, p)$ reaction it comes from the inclusive $(n, p)$.
of the $\Sigma$, $\Lambda$ mass difference $2(P_{\Sigma} - P_{\Lambda} - M_{\Sigma} - M_{\Lambda})$, the invariant $q^2$ factor in 

\[ q^2 \leq 10 \text{ GeV}^2/\text{c}^2 \text{ for } \Sigma^{-}\Lambda \rightarrow \Lambda n \text{ and } \Sigma^+ n \rightarrow \Sigma^+ \Lambda \text{ at the higher energies needed here gives } q \leq 5 \text{ GeV} \] 

and we have $\sigma \approx 20 \text{ mb}$. A smooth extrapolation of the theoretical results of ref. 17 at the higher energies needed here gives $\sigma \leq 5 - 10 \text{ mb}$. The np \rightarrow np cross section is about $35 \text{ mb}$. The $\Sigma \pi \rightarrow \Sigma \pi$ and $\Sigma \pi \rightarrow \Sigma \pi$ cross sections are similar to the NN \rightarrow NN cross sections. Hence we should expect a background for the inclusive $(\Sigma^{-}, \Lambda)$ at least 5 times smaller than the one for the $(n, p)$ reaction which is of the order of $0.8 \text{ mb/sr MeV}$ for $200 \text{ Pb}$ from the experiment of ref. 63. Therefore we will assume a background of the order of $0.16 \text{ mb/sr MeV}$ for this nucleus and, assuming a simple $A$ scaling, $0.03 \text{ mb/sr MeV}$ for the $40 \text{ Ca}$ case. With these results the ratio of signal to background with distorted waves is given in brackets in table I. The ratios of signal to background are $10\%$ for the $1s$ state in $200 \text{ Pb}$, $20\%$ for the $1s$ state in $40 \text{ Ca}$ and $56\%$ for the $2p$ state in $200 \text{ Pb}$.

For the pionic wave functions we have used the potential of ref. 24. If we use the potential of ref. 24 the results change at the level of $25\%$ for the $2p$ state in $\text{ Pb}$ and $50\%$ in the $1s$ state in $\text{ Pb}$. We have also investigated the sensitivity of the results to the distortion parameters. If instead of $30 \text{ mb}$, $20 \text{ mb}$ for the $\Sigma \pi$ and $\Lambda \pi$ cross sections we use $35 \text{ mb}$, $25 \text{ mb}$ ($25 \text{ mb}$, $15 \text{ mb}$), we decrease (increase) the cross section by a factor two for the $1s$ states and reduce (increase) it by about $30\%$ for the $2p$ states. Even with uncertainties in the estimation of the ratio of signal to background of the order of a factor $2$, it is clear that the reaction offers much cleaner signals than the $(n, p)$ even at the optimal energy of $T_{n} \approx 1000 \text{ MeV}$ investigated in ref. 63. Indeed, with the results of Table I, we gain a factor $50\%$ increase in this ratio for the $1s$ state and about a factor $10$ for the $2p$ state in $200 \text{ Pb}$.

Note that the $1s$ state is more easily populated here than in the $(n, p)$ reaction. Apart from the more moderate role of the distortion already discussed, the excitation of the $1s$ state in the forward direction is only sensitive to the $V_{11}$ part of eq. (3) (apart from the $s$-wave and Coulomb pieces). This part is larger here than in the $(n, p)$ reaction as a consequence.

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Fig. 1.- Momentum transfer in the \((n, p)\) and \((\Sigma^-, \Lambda)\) reaction creating a \(\pi^-\) bound, as a function of the kinetic energy of the \(n\) or the \(\Sigma^-\) respectively.

Fig. 2.- Feynman diagrams considered in the \(\Sigma^- N \rightarrow \Lambda N \pi^-\) processes. \(a\) pion exchange with \(\pi N \rightarrow \pi N\) s-wave scattering; \(b\) \(\pi \pi \rho\) exchange including the indirect effect of nuclear correlations with p-wave \(\pi N \rightarrow \pi N\) scattering mediated by \(\Delta\) excitation; \(c\) \(\pi\) exchange with \(\pi\) Coulomb interaction with the protons.

Fig. 3.- \(d^2\sigma / d\Omega dE\) at the peak of the Lorentzian distribution for the \(\Sigma^- + ^{208}Pb \rightarrow \Lambda + (^{208}Pb \pi^-)\) at \(T_\Sigma = 600\) MeV creating a pion in the 1s state as a function of the scattering angle. Dashed line: results without distortion. Continuous line: results with distortion.

Fig. 4.- Same as fig. 3 for the 2p state.

**Table 1**

\(d^2\sigma / d\Omega dE\) [mb/sr MeV] at the peak for \(T_\Sigma = 800\) MeV and \(\theta = 0^\circ\).

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<tr>
<th></th>
<th>40Ca</th>
<th>208Pb</th>
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<tr>
<td></td>
<td>without/with distortion</td>
<td>without/with distortion</td>
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<tr>
<td></td>
<td>(\frac{\text{Signal}}{\text{Background}})</td>
<td>(\frac{\text{Signal}}{\text{Background}})</td>
</tr>
<tr>
<td>1s</td>
<td>0.066/0.0060 (20%)</td>
<td>0.49/0.015 (10%)</td>
</tr>
<tr>
<td>2p</td>
<td></td>
<td>0.29/0.090 (56%)</td>
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