$B$ meson spectroscopy

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Abstract

We study the $B$ meson spectroscopy allowing the mixture of conventional $P$ wave quark-antiquark states and four-quark components. A similar picture was used to describe the new $D_J$ and $D_{sJ}$ open charm mesons. The four-quark components shift the masses of some positive parity $B_{sJ}$ states below their corresponding isospin preserving two-meson threshold and therefore they are expected to be narrow. Electromagnetic decay widths are analyzed.

Keywords: Bottom mesons, Bottom-strange mesons, quark models.
Heavy-light mesons play in QCD a similar role as the hydrogen atom in QED. This analogy provides a simple way to make predictions for their excited states. In the limit $M_Q \to \infty$ heavy-light mesons can be characterized by the spin of the heavy quark, $S_Q$, the total angular momentum of the light quark, $\vec{j}_q = \vec{S}_q + \vec{L}$, and the total angular momentum, $\vec{J} = \vec{S}_Q + \vec{j}_q$. For $P$ wave excited states there appear two degenerate doublets: one corresponding to $j_q = 1/2$, and the other to $j_q = 3/2$, with quantum numbers $J^P = 0^+, 1^+$ and $J^P = 1^+, 2^+$, respectively. Those states with $j_q = 1/2$ can only decay via an $S$ wave transition, whereas the $j_q = 3/2$ states undergo a $D$ wave transition. Therefore the decay widths are expected to be much broader for $j_q = 1/2$ than for $j_q = 3/2$ states. We denote the $J^P = (0^+, 1^+)$ states as $(A_0^+, A_1^+)$ and the $J^P = (1^+, 2^+)$ states as $(A_1, A_2^*)$, with $A = B$ (for $b\pi$ states, where $n$ stands for a light $u$ or $d$ quark) or $B_s$ (for $b\sigma$ states). Based on this description the expected decay properties of the $P$ wave $B_{sJ}$ mesons are rather simple, they are summarized in Table I.

A few predictions of masses and widths are available from theory \cite{1, 2, 3, 4, 5, 6} and lattice simulations \cite{7}, see Table I. Similar masses and widths of the $L = 1$ excited state are predicted, around 5.7 GeV for $B$ and 5.8 GeV for $B_s$, except for Refs. \cite{3} and \cite{4} reporting lower masses for the $B_{s0}^+$ and $B_{s1}^+$. These works use model parameters fitted on the $D_s$ meson sector and, therefore, they incorporate the anomalies we will comment below.

From the experimental point of view the spectroscopy of excited mesons containing bottom quarks is still not well known. Only the ground $0^-$ and the excited $1^-$ states are well established in the PDG \cite{8}. There are some experimental data by the L3 Collaboration \cite{9} reporting the first measurement of the $B_1^+$ and $B_2^+$ masses, 5670±10 ± 13 MeV and 5768±5 ± 6 MeV, respectively. Recently, D0 and CDF Collaborations have reported results on the spectroscopy of orbitally excited bottom mesons \cite{10}. CDF found two states, $B_1$ and $B_2^*$, with masses $M(B_1) = 5734 ± 3 ± 2$ MeV and $M(B_2^*) = 5738 ± 6 ± 1$ MeV. D0 also found the same states but with slightly different masses, $M(B_1) = 5720.8 ± 2.5 ± 5.3$ MeV and $M(B_2^*) - M(B_1) = 25.2 ± 3.0 ± 1.1$ MeV. In the strange sector CDF reported two narrow $B_{s1}$ and $B_{s2}^*$ states with masses $M(B_{s1}) = 5829.4$ MeV and $M(B_{s2}^*) = 5839$ MeV while D0 measured only the $B_{s2}$, with a mass of 5839.1 ± 1.4 ± 1.5 MeV.

The poorly known experimental situation in the open-beauty sector is far from the one observed for open-charm mesons. Since 2003, when the $D_{sJ}(2317)$ and the $D_{sJ}(2460)$ were discovered by BABAR Collaboration \cite{11}, eight $c\bar{s}$ new states have been reported, more than needed to fill the $L = 0$ doublet and the four $L = 1$ states. Also the number of $c\bar{c}$ states has grown with the Belle observation \cite{12} of a broad scalar resonance, $D_0^*$, with a mass of 2308 ± 36 MeV and a width $\Gamma = 276 ± 66$ MeV. Some of these new states present unexpected masses, quite different from those predicted by quark potential models if a pure $c\bar{c}$ configuration is considered. If they would correspond to standard $P$ wave mesons, their masses would be similar to the already known $L = 1$ $c\bar{q}$ states, namely around 2.4 GeV for $c\pi$ and 2.5 GeV for $c\rho$. They would be therefore above the $D\pi$, $D^*\pi$ and $DK$, $D^*K$ thresholds, respectively, being broad resonances. However, the $D_{sJ}(2317)$ and $D_{sJ}(2460)$ states are below the $DK$ and $D^*K$ thresholds and therefore they can only decay through the isospin forbidden channels $D_s\pi^0$ and $D^*_s\pi^0$ with a very narrow width. In the case of the $D_0^*(2308)$ the large width observed would be theoretically expected although not its low mass.

These unexpected properties of the $D_{sJ}(2317)$ and the $D_{sJ}(2460)$ mesons have been explained in Ref. \cite{13} assuming a mixture of $q\bar{q} (L = 1, S = 1, 0)$ and $qqq\bar{q} (L = 0, S = 1, 0)$ components. The reason why four-quark configurations, which mimic quark loop contribu-
tions, are important in this case is that whereas the $q\bar{q}$ pair is in a $L = 1$ state, the four-quark state is an $S$ wave and thus its contribution may be relevant. It is well known that the existence of four-quark configurations is favored in the heavy-quark sector because, due to the coulombic character of the systems, the binding energy augments proportionally to the mass whereas the kinetic energy contribution gets reduced when the mass increases. Therefore, if sizeable contributions of four-quark structures appear in the $D$ meson sector, they should also be present for $B$ mesons. Therefore, one may wonder which are the consequences of the mixing between two- and four-quark components in the $B$ meson spectra.

In the present work we have extended the analysis done on Ref. [13] for open-charm mesons to excited $P$ wave open-beauty mesons. Let us first briefly resume the basic features of the constituent quark model used [14]. The model includes a dynamical quark mass appearing as a consequence of the spontaneous breaking of the original QCD $SU(3)_L \otimes SU(3)_R$ chiral symmetry at some momentum scale. Once the dynamical quark mass is generated, whatever the mechanism, such quarks have inevitably to interact through Goldstone modes such that the rotation of the quark fields can be compensated by the boson fields. A simple lagrangian invariant under the chiral transformation has been derived in Ref. [15],

$$L = \bar{\psi}(i\partial - M(q^2)U^{\gamma_5})\psi,$$

(1)

where $U^{\gamma_5} = \exp(i\pi^a\lambda^a\gamma_5/f_\pi)$, $\pi^a$ denotes nine pseudoscalar fields ($\eta_0, \vec{\pi}, K_i, \eta_8$) with $i = 1, ..., 4$ and $M(q^2)$ is the constituent mass. The constituent quark mass can be explicitly obtained from the theory and parametrized as $M(q^2) = m_qF(q^2)$ with

$$F(q^2) = \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^{\frac{1}{2}}.$$  

(2)

Once this has been done $U^{\gamma_5}$ can be expanded in terms of boson fields,

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi} \gamma^5 \lambda^a \pi^a - \frac{1}{2f_\pi^2} \pi^a \pi^a + ...$$

(3)

The first term of the expansion generates the constituent quark mass while the second gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which has been simulated by a scalar exchange potential.

For higher momenta we assume that quarks still interact through gluon exchanges. Following Ref. [16] the gluon exchange can be described as an effective interaction between constituent quarks given by

$$L_{gqq} = i\sqrt{4\pi\alpha_s}\bar{\psi}\gamma_\mu G_\mu^a\lambda_c\psi.$$  

(4)

Finally, lattice calculations in the quenched approximation derived, for heavy quarks, a confining interaction linearly dependent on the interquark distance. The consideration of sea quarks apart from valence quarks (unquenched approximation) suggests a screening effect on the potential when increasing the interquark distance [17]. In terms of a potential, the color screening produces a gradual decrease of the potential slope, i.e.,

$$V_{CON}(\vec{r}_{ij}) = \{-a_c (1 - e^{-\mu_c r_{ij}}) + \Delta\}(\vec{\lambda}_i \cdot \vec{\lambda}_j)$$

(5)

where $\Delta$ is a global constant to fit the origin of energies. At short distances this potential presents a linear behavior while it becomes constant at large distances.
Using this model we have solved the Schrödinger equation for the two- and four-body problems. The two-body case has been solved exactly, while to solve the four-body case we have used a variational method with a radial trial wave function taken as the most general combination of generalized gaussians \cite{18}. This wave function includes all possible flavor-spin-color channels contributing to a given configuration \cite{19}.

To describe the mixing between $q\bar{q}$ and four-quark components we use the simplest version of the coupled channel model. Namely, it is assumed that a meson state is given by

$$|\psi\rangle = \sum_i \alpha_i |q\bar{q}\rangle_i + \sum_j \beta_j |qq\bar{q}\bar{q}\rangle_j$$

where $q$ stands for quark degrees of freedom and the coefficients $\alpha_i$ and $\beta_j$ take into account the mixing. The meson systems could then be described in terms of a hamiltonian $H = H_0 + H_1$, being

$$H_0 = \begin{pmatrix} H_{q\bar{q}} & 0 \\ 0 & H_{qq\bar{q}\bar{q}} \end{pmatrix} \quad \text{and} \quad H_1 = \begin{pmatrix} 0 & V_{qq\bar{q}\bar{q}} \\ V_{qq\bar{q}\bar{q}} & 0 \end{pmatrix},$$

where $H_0$ is the constituent quark model hamiltonian described above and $H_1$ takes into account the mixing between $q\bar{q}$ and $qq\bar{q}\bar{q}$ configurations. It includes the annihilation operator of a quark-antiquark pair into the vacuum, that can be obtained from the $^3P_0$ model. Since this model depends on a vertex parameter, we determine this parameter by looking to the quark pair that is annihilated (and not to the spectator quarks that will form the final state). Therefore we have taken $V_{q\bar{q}\rightarrow qq\bar{q}\bar{q}} = \gamma$. If this coupling is weak enough one can solve independently the eigenproblem for the hamiltonians $H_{q\bar{q}}$ and $H_{qq\bar{q}\bar{q}}$, treating $H_1$ perturbatively. To ensure that the perturbative treatment is justified, $\gamma$ cannot take all possible values, being restricted to $|\gamma/(E_{j^p}^n - E_{j^p+1}^n)|^2 \leq 1$. This restriction will limit the energy range of the mixed states once the unmixed energies are calculated. The parameter $\gamma$ has been fixed in Ref. \cite{13} to reproduce the mass of the $D_{sJ}^*(2317)$, being $\gamma = 240$ MeV.

Let us first compare the experimental results with the predictions of our model for the $b\bar{n}$ and $b\bar{s}$ quark pairs. In Table III it can be seen that the model nicely reproduces the known experimental data. The two states which have not been measured, $(A_1^*, A_1)$ lie above the $BK = 5774$ MeV and $B^*K = 5820$ MeV thresholds for the strange sector, and above the $B\pi = 5417$ MeV threshold for the non-strange one.

Once the mixing of the $q\bar{q}$ pairs with the $bq\bar{q}\bar{q}$ states is considered, the $J^P = 0^+$ and $1^+$ states acquire almost a 30% of four-quark component (see Table IV). Without being dominant, this component is the responsible for shifting the mass of the unmixed states below the $BK$ and $B^*K$ thresholds. As a consequence, the only allowed strong decay to $B^*_s\pi$ would not preserve isospin and the resonances are expected to be narrow, of the order of a few keV. A second $b\bar{s}$ $J = 1^+$ resonance appears at $M = 5857$ MeV, with almost 99% of $q\bar{q}$ component which may correspond with the new $B_{s1}$ state reported by CDF with a mass around 5829 MeV. The fourth state appears at 6174 MeV. A similar calculation in the non-strange sector is much more involved and we can only predict the existence of a $B_0^*$ state with $M = 5615$ MeV and 48% of four-quark component and 51% of $b\bar{n}$ pair. The lowest state, representing the $B_0^*$, is 200 MeV above the isospin preserving threshold $B\pi$, therefore it would be broad. The orthogonal state appears higher in energy, at 6086 MeV, also with an important four-quark component.

Our results are in agreement with Refs. \cite{3} and \cite{4} obtained with different approaches. Ref. \cite{3} interprets the two $D_{sJ}$ resonances as the chiral partners of the $(0^-, 1^-)$ ground state.
spin multiplet. Fitting the chiral mass gap between the \((0^-,1^-)\) and \((0^+,1^+)\) multiplets to the \(D_{sJ}^*(2317)\) experimental mass, they reproduce the \(D_{sJ}^*(2460)\) mass. Assuming that in the \(M_Q \to \infty\) limit the chiral mass gap for the charm and bottom sectors is the same, they predict the masses for the \(B\) meson chiral multiplet \((0^+,1^+)\). In Ref. [4] the authors impose invariance under heavy quark spin-flavor and chiral transformations to build an effective QCD lagrangian. The effective parameters are determined using as inputs the experimental \(D, D_s, B_s\) masses together with the assumption that the mass splitting between positive and negative parity doublets is the same in the charm and bottom sectors. Their results are summarized in Table II. These calculations strengthen the idea that the anomalies observed in the charm sector must appear in the bottom one.

Our approach is similar to Ref. [20] considering one loop chiral corrections to calculate the \(D_{sJ}^*(2317)\) mass. Such corrections lower the mass of the \(0^+\) state and therefore can account for the unusually small mass of the \(D_{sJ}^*(2317)\) and the small mass difference between \(D_{sJ}^*(2317)\) and \(D_s^0(2308)\). However, our approach has the additional property that considering four-quark configurations, besides playing a similar role to the one loop corrections, it augments the number of states. This provides a plausible interpretation of the recently measured \(D_{s}^*(2860)\) [21] as the orthogonal partner of the mixture of conventional \(P\) wave quark-antiquark states and four-quark components describing the \(D_{sJ}^*(2317)\) [22].

In the L3 Collaboration results [9] the masses of the \(B_0^*\) and \(B_1\) mesons, although not explicitly given, are constrained by heavy quark effective theory relations [23], \(M(B_2^*) - M(B_1^0) \approx M(B_1^0) - M(B_0^*) \approx 12\) MeV. This allows to estimate the masses of the \(B_0^*\) and \(B_1\) mesons from the \(B_1^0\) and \(B_2^*\) experimental masses, obtaining \(M(B_0^*) \approx 5658\) MeV and \(M(B_1^0) \approx 5756\) MeV. The \(B_1\) mass value agrees with the recent measurement of CDF and D0 Collaborations [10] and with our prediction (see Table III) whereas the \(B_0^*\) mass is very close to our \(I = 1/2, J^P = 0^+\) state with a 48% of four-quark component (see Table IV). Furthermore the L3 Collaboration observed an excess of events above the expected background in the 5.9-6.0 GeV region of the \(B\pi\) mass spectrum, what might correspond to our second \(I = 1/2, J^P = 0^+\) mixed state at 6086 MeV. The L3 data seem to indicate that the \(L = 1\) excited \(B\) mesons show the same behavior as the corresponding excited states in the open charm sector.

The isovector four-quark states do not couple to the \(b\bar{s}\) system and they are therefore much higher in energy, being both close to 6 GeV and above the strong decay threshold. Although they should be very broad, and therefore very difficult to observe, any indication of its existence would be a definite signal of the presence of four-quark states in the heavy-meson sector.

The structure of the excited \(B\) mesons can also be analyzed studying their electromagnetic decay widths. The formalism needed to evaluate the \(\Gamma[b\bar{q} \to b\bar{q} + \gamma], \Gamma[b\bar{n}\bar{q} \to b\bar{n}\bar{q} + \gamma],\) and \(\Gamma[b\bar{n}\bar{q} \to b\bar{q} + \gamma]\) widths has been described in Ref. [13]. We compare in Table V our results for the radiative transitions of the \(0^+\) and \(1^+\) states for the cases of a pure \(b\bar{q}\) structure, QM(b\(\bar{q}\)), a mixed one, QM(b\(\bar{q}\) + b\(\bar{q}\)), and those of Ref. [3]. The difference between the QM(b\(\bar{q}\)) results and those of Ref. [3] are due to phase space. The difference of the previous two cases with the QM(b\(\bar{q}\) + b\(\bar{q}\)) results can be traced back to the more involved wave function of this mixed case. The mixing among the \(3P(b\bar{q}), 1P(b\bar{q})\) and the four-quark components generates a very small \(3P(b\bar{q})\) probability for some particular states, as for example the \(J^P = 1^+\), of the order of 1%. The electromagnetic decay of the \(1P(b\bar{q})\) component to the \(J^P = 1^+\) state is forbidden. The transition from the four-quark component to the meson-photon state does only occur for a very particular component of the tetraquark
wavefunction: the one where the light quark-antiquark pair is in a color singlet, spin one, isospin zero state, with a very small probability for the state under consideration. The most significant consequence is the suppression predicted for the $1^+ \rightarrow 1^- + \gamma$ decay as compared to the $1^+ \rightarrow 0^- + \gamma$. A ratio $\frac{1^+ - 0^- + \gamma}{1^+ - 1^- + \gamma} \approx 1$ has been obtained in Ref. [3] and in our results with a pure $b\bar{q}$ structure. For the mixed case we find a much larger value for this ratio, $\frac{1^+ - 0^- + \gamma}{1^+ - 1^- + \gamma} \approx 100$ due to the small $1^3P_1 c\bar{s}$ probability of the $1^+$ state. In view of these predictions, once experimentally measured, the electromagnetic decay widths would be an important diagnostic tool to understand the nature of these states.

In brief, we have performed an exploratory study of the $L = 1$ excited $B$ mesons in terms of two- and four-quark components based in our experience on the open-charm mesons. Our results agree with the recently measured $B$ meson states by CDF and D0 Collaborations. In addition we predict the existence of two resonances, $B_{s0}^*$ and $B_{s1}^*$, with almost 30% of four-quark component, which lie below the $BK$ and $B^*K$ thresholds, respectively. Thus, the only allowed strong decays would violate isospin and the resonances would be narrow. In the non-strange sector we did not find such narrow resonances but our results give support to the L3 Collaboration findings. Therefore the mixing between two and four-quark components, which explains the unexpected low masses of $D_{sJ}^{*}(2317)$ and $D_{sJ}^{*}(2460)$ open-charm states, would also play a relevant role in the open-beauty sector. We also found that the ratio $\frac{1^+ - 0^- + \gamma}{1^+ - 1^- + \gamma}$ would provide an experimental signature of the proposed structure.

We encourage experimentalists on the measurement of the spectroscopic and electromagnetic properties of the positive parity $B_{J}$ and $B_{sJ}$ states, that would help to clarify the exciting situation of the open-bottom and open-charm mesons.

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TABLE I: Expected decay properties of the $P$ wave $B_s$ mesons. $J$ is the total angular momentum, $P$ its parity, and $j_q$ the total angular momentum of the light quark.

<table>
<thead>
<tr>
<th>$j_q$</th>
<th>$J^P$</th>
<th>State</th>
<th>Decay mode</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>0+</td>
<td>$B_{s0}^*$</td>
<td>$BK$</td>
<td>broad</td>
</tr>
<tr>
<td>1/2</td>
<td>1+</td>
<td>$B_{s1}'$</td>
<td>$B^*K$</td>
<td>broad</td>
</tr>
<tr>
<td>3/2</td>
<td>1+</td>
<td>$B_{s1}$</td>
<td>$B^*K$</td>
<td>narrow</td>
</tr>
<tr>
<td>3/2</td>
<td>2+</td>
<td>$B_{s2}^*$</td>
<td>$BK, B^*K$</td>
<td>narrow</td>
</tr>
</tbody>
</table>

TABLE II: $b\pi$ and $b\eta$ spectra, in MeV, using different approaches.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{s0}^*$</td>
<td>5738</td>
<td>5760</td>
<td>5627</td>
<td>5700</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$B_0^*$</td>
<td>5719</td>
<td>5780</td>
<td>5674</td>
<td>5750</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$B_1$</td>
<td>5757</td>
<td>5780</td>
<td>–</td>
<td>5774</td>
<td>5780</td>
<td>5755</td>
<td>–</td>
</tr>
<tr>
<td>$B_2^*$</td>
<td>5733</td>
<td>5800</td>
<td>–</td>
<td>5790</td>
<td>5846</td>
<td>5767</td>
<td>–</td>
</tr>
</tbody>
</table>

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<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$B_{s0}^*$</td>
<td>5841</td>
<td>5830</td>
<td>5718</td>
<td>5710</td>
<td>–</td>
<td>–</td>
<td>5756±31</td>
</tr>
<tr>
<td>$B_0^*$</td>
<td>5831</td>
<td>5786</td>
<td>5765</td>
<td>5770</td>
<td>–</td>
<td>–</td>
<td>5804±31</td>
</tr>
<tr>
<td>$B_1$</td>
<td>5859</td>
<td>5786</td>
<td>–</td>
<td>5877</td>
<td>5886</td>
<td>5834</td>
<td>5892±52</td>
</tr>
<tr>
<td>$B_{s2}^*$</td>
<td>5844</td>
<td>5808</td>
<td>–</td>
<td>5893</td>
<td>5899</td>
<td>5846</td>
<td>5904±52</td>
</tr>
</tbody>
</table>

TABLE III: $b\bar{s}$ and $b\bar{n}$ quark model (QM) masses, in MeV. Experimental data (Exp.) are from Ref. [8], except for the states denoted by a dagger from Ref. [10] and by a double dagger from Ref. [9].

<table>
<thead>
<tr>
<th>$nL \ J^P$</th>
<th>QM $(b\bar{s})$</th>
<th>Exp.</th>
<th>QM $(b\bar{n})$</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1S \ 0^-$</td>
<td>5355</td>
<td>5369.6±2.4</td>
<td>5281</td>
<td>5279.2±0.5</td>
</tr>
<tr>
<td>$1S \ 1^-$</td>
<td>5400</td>
<td>5416.6±3.5</td>
<td>5321</td>
<td>5325.0±0.6</td>
</tr>
<tr>
<td>$1P \ 0^+$</td>
<td>5838</td>
<td>−</td>
<td>5848</td>
<td>−</td>
</tr>
<tr>
<td>$1P \ 1^+$</td>
<td>5837</td>
<td>5829.4±0.2 ± 0.6$^\dagger$</td>
<td>5768</td>
<td>5734 ± 3 ± 2$^\dagger$</td>
</tr>
<tr>
<td></td>
<td>5869</td>
<td>5839.6±0.4 ± 0.5$^\dagger$</td>
<td>5876</td>
<td>5720.8 ± 2.5 ± 5.3$^\dagger$</td>
</tr>
<tr>
<td></td>
<td>5853</td>
<td>5839.1±1.4 ± 1.5$^\dagger$</td>
<td>5768</td>
<td>5768 ± 5 ± 6$^\dagger$</td>
</tr>
</tbody>
</table>

TABLE IV: Masses (QM), in MeV, and probability of the different wave function components for $B_s$ mesons once the mixing between $b\bar{q}$ and $bq\bar{q}\bar{q}$ configurations is considered.

<table>
<thead>
<tr>
<th>$I = 0$</th>
<th>$I = 1/2$</th>
</tr>
</thead>
</table>

$$
\begin{array}{c|cc|c|cc|c}
\hline
\hline
 & \text{QM} & 5679 & 6174 & \text{QM} & 5713 & 5857 & \text{QM} & 5615 & 6086 \\
\hline
P(bn\bar{s}\bar{n}) & 0.30 & 0.51 & P(bn\bar{s}\bar{n}) & 0.24 & \sim 0.01 & P(bn\bar{n}\bar{n}) & 0.48 & 0.46 \\
P(b\bar{s}_13P) & 0.69 & 0.26 & P(b\bar{s}_11P) & 0.74 & \sim 0.01 & P(bn\bar{n}_1P) & 0.51 & 0.47 \\
P(b\bar{s}_22P) & \sim 0.01 & 0.23 & P(b\bar{s}_13P) & \sim 0.01 & 0.99 & P(bn\bar{n}_2P) & \sim 0.01 & 0.07 \\
\hline
\end{array}
$$

TABLE V: Comparison of radiative decay widths (keV) assuming only a pure $b\bar{q}$ structure, QM($b\bar{q}$), a combination of two- and four-quark components, QM($b\bar{q} + bq\bar{q}\bar{q}$), and those of Ref. [3].

<table>
<thead>
<tr>
<th>Transition</th>
<th>QM($b\bar{q}$)</th>
<th>QM($b\bar{q} + bq\bar{q}\bar{q}$)</th>
<th>Ref. [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+ \rightarrow 1^- + \gamma$</td>
<td>171.4</td>
<td>31.9</td>
<td>58.3</td>
</tr>
<tr>
<td>$1^+ \rightarrow 1^- + \gamma$</td>
<td>75.6</td>
<td>0.6</td>
<td>56.9</td>
</tr>
<tr>
<td>$1^+ \rightarrow 0^- + \gamma$</td>
<td>106.5</td>
<td>60.7</td>
<td>39.1</td>
</tr>
<tr>
<td>$1^+ \rightarrow 0^- + \gamma$</td>
<td>1.41</td>
<td>101.17</td>
<td>0.69</td>
</tr>
</tbody>
</table>

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